SYNOPSIS

SOME UNSTEADY FLOWS OF IMMISCIBLE NEWTONIAN AND NON-NEWTONIAN FLUIDS IN CHANNELS

Fluid mechanics forms a foundation for understanding many aspects of applied sciences and engineering and concerns with the investigation of motion and equilibrium of fluids. It is a subject of prime importance in engineering, astrophysics, geo-physics, meteorology and medicine. The recent developments in aeronautical engineering, mechanical engineering and space technology are based on the principles of fluid mechanics. The frontiers of research in fluid dynamics have been extended to other areas like MHD and flow through porous media. In this connection it has become necessary to combine the knowledge of thermodynamics, mass transfer, heat transfer, electromagnetic theory and fluid mechanics in order to understand the physical phenomena involved.

It is possible to experience the applications of fluid mechanics principles in daily life. The flight of birds, aero plane in air and motion of fish in water are governed by principles of fluid mechanics. The information about the natural calamities (hurricanes, tsunamis, earthquakes, etc.) is now-a-days acquired mainly with the knowledge of laws of fluid mechanics.

In order to have logical understanding of fluid mechanics, it is necessary to differentiate solids and fluids, which are the parts of matter. Infact, the fluid represents liquid and gaseous states of matter. Very strong intermolecular attractive forces exist in solids which give them the property of rigidity. These forces are observed to be weaker in liquids and very small in gases. Further, continuum model is assumed for the description of the motion of a fluid.
The various types of flows in fluid mechanics are differentiated based on fluid properties which characterize the physical situation. The viscosity is one of the important properties of a fluid. This decides the fluid behavior and fluid motion near the solid boundary. For simple fluids such as gases, gaseous mixtures, low-molecular-weight liquids and their mixtures, it has been experimentally confirmed that in a simple shearing motion \( u = u_x(y) \), the flux of x-momentum in the positive y-direction is given by Newton’s law of viscosity

\[
\tau_{xx} = -\mu \frac{du}{dy}
\]  

(1)

where \( \mu \) is the viscosity of the fluid and \( 'u' \) is the velocity component in x-direction.

The appropriate generalization of (1.1) for time-dependent flows is

\[
\Pi = p\bar{\delta} + \bar{\tau} = p\bar{\delta} - \mu \left[ \nabla \bar{\nu} + (\nabla \bar{\nu})^T \right] + \left( \frac{2}{3} \mu - K \right) (\nabla \bar{\nu}) \bar{\delta}
\]  

(2)

where \( (\nabla \bar{\nu})^T \) is the transpose of the dyadic \( \nabla \bar{\nu} \), \( \bar{\tau} \) is the viscous stress tensor which is the part of the momentum flux tensor or stress tensor that is associated with the viscosity of the fluid, \( \Pi \) is the molecular stress tensor (or total momentum flux tensor or total stress tensor), \( K \) is the dilational viscosity and \( \bar{\delta} \) is the unit tensor.

Eq. (2) represents the constitutive equation for the Newtonian fluid.

For incompressible fluids, Eqn. (2) reduces to

\[
\Pi = p\bar{\delta} + \bar{\tau} = p\bar{\delta} - \mu \dot{\gamma}
\]  

(3)

where \( \dot{\gamma} = \nabla \bar{\nu} + (\nabla \bar{\nu})^T \) is the rate of strain tensor.
For incompressible Newtonian fluids the expressions for the stress tensor is given by

\[ \bar{\tau} = - \mu \dot{\gamma} \]  \hspace{1cm} (4)

The generalized Newtonian fluid model is obtained by replacing constant viscosity \( \mu \) by the non-Newtonian viscosity \( \eta \), a function of shear-rate. It is given by

\[ \bar{\tau} = - \eta \dot{\gamma}, \text{ Where } \eta = \eta(\dot{\gamma}) \]  \hspace{1cm} (5)

The empiricism for \( \eta(\dot{\gamma}) \) leads various non-Newtonian models which are used to describe polymeric liquid flows in industries and biofluid flows in physiological systems of living organisms. Many models are proposed for the analysis of such flows. The following three non-Newtonian models are used in the thesis

i) Power - Law Model

In Power-Law model, \( \eta(\dot{\gamma}) \) is defined as

\[ \eta = m \dot{\gamma}^{n-1} \]  \hspace{1cm} (6)

where 'm' and 'n' are constants of characterizing fluid. We note that hydroxyethylcellulose, banana pure and apple sauce are characterized as Power-Law fluids
ii) Jeffrey Model

The governing equation for this model is obtained by including a time derivative of the velocity gradients. It is given by

$$\ddot{\tau} + \dot{\lambda}_1 \frac{\partial \tau}{\partial t} = -\eta_0 \left( \dot{\gamma} + \dot{\lambda}_2 \frac{\partial \dot{\gamma}}{\partial t} \right)$$

(7)

Where

- $\lambda_1$ = a time constant (the relaxation time)
- $\lambda_2$ = the retardation time
- $\eta_0$ = the zero shear rate viscosity

iii) Couple Stress Model

The couple stress fluid model has wide applications in the steady of biofluids, colloidal fluids, liquid crystals, and synthetic lubricants. It is observed that slurries in general and animal blood in particular show, under certain circumstances, strongly deviated from a Newtonian fluid behavior. Some of these deviations may be explained by assuming that blood obeys couple stress model.

Stokes theory of couple stress fluid is the simplest generalization of the classical theory of fluids which allows the polar effects such as the presence of a non symmetric stress tensor, couple stress and body couples. The constitutive equations for couple stress fluid proposed by Stokes (1966) are

$$T_{(ij)} = (-p + \lambda D_{ik}) \delta_{ij} + 2\mu_j D_j$$

(8)

$$T_{[ij]} = -2\eta \omega_{ij, kk} - \frac{P}{2} \epsilon_{ij} G$$

(9)

$$M_{ij} = 4\eta \omega j_{i, j} + 4\eta' \omega j_{i, j}$$

(10)
Where \( D_{ij} = \frac{1}{2} (q_{i,j} + q_{j,i}) \), \( W_{ij} = -\frac{1}{2} (q_{i,j} - q_{j,i}) \), \( \omega_e = \frac{1}{2} \epsilon_{i,j} q_{k,j} \).

\( T_{ij} \) is the symmetric part and \( T_{[ij]} \) is the anti-symmetric part of the stress tensor. \( T_{ij}, M_{ij} \) the couple stress tensor, \( D \), the deformation rate tensor, \( W_i \), the velocity vector, \( G_i \), the body couple, \( \delta_i \), the kronecker delta, \( \rho \) the density, \( p \) the pressure \( \epsilon_{i,j} \), the alternating unit tensor, \( \lambda \) and \( \mu_i \) are the material constants have the dimension of viscosity, \( \eta \) and \( \eta' \) are the material constants have the dimension of momentum. The ratio of \( \left( \frac{\eta}{\sqrt{\mu_i}} \right) \) has the dimension of length squared and it characterizes the size of micro structure.

Viscous flow through and past porous media has important applications in engineering and medicine. In recent years the investigation of flow of fluids through porous media has become an important topic due to the recovery of crude oil from the pores of reservoir rocks. Also the flow through porous media is of interest in chemical engineering, petroleum engineering, hydrology, geo-physics and bio-physics. The flow in a porous medium can be described using either Darcy law or some non-Darcy law. In 1856, Darcy made experiments on sand filters. These experiments gave rise to the following Darcy law.

\[
\bar{V} = -\frac{k}{\mu} (\nabla P - \bar{F})
\]

(11)

where \( \bar{V} = \) Darcy velocity

\( \bar{F} = \) body force per unit volume

\( P = \) pressure

\( k = \) permeability

\( \mu = \) coefficient of viscosity
Darcy law is verified experimentally through a number of investigations. Hubbert (1956) and Hall (1956) derived Darcy law on integrating the Navier-Stokes equation. Yin (1956), Collins (1961), Scheidegger (1957), Longwell (1966), De weist (1969), Muskat (1982) and many others investigated various problems in flow through porous media using Darcy law.

In obtaining Darcy law it is assumed that the velocity of the fluid is small and the inertial forces are negligible. When the flow rates are large, Darcy law becomes invalid. At low pressure, Darcy law is not adequate to describe the gaseous flow through porous media. In view of the limitations of the Darcy law, it has become necessary to derive non-Darcy law. Brinkman (1947) proposed a non-Darcy law for the flow through highly permeable bed of spherical particles. He suggested the following boundary layer type of equation

$$\nabla p = \rho \bar{g} - \frac{\mu}{k} \bar{q} + \mu \nabla^2 \bar{q}$$ (12)

where $\bar{g} =$ gravitational force

$\rho =$ density

$\bar{\mu} =$ effective viscosity

$\mu =$ fluid viscosity

$\bar{q} =$ velocity vector

$k =$ permeability

The study of MHD of viscous conducting fluids is playing a significant role, owing to its practical interest and abundant applications in astro-physical and geo-physical phenomena. The main impetus to the engineering approach to the electromagnetic fluid interaction studies has come from the concept of the hydrodynamics. Ion propulsion study of flow problems of electrically conducting fluid,
particularly of ionized gases is currently receiving considerable interest. Such studies have been done for many years in connection with astro-physical and geo-physical problems such as sun spot theory, motion of the interstellar gas etc. Hartmann (1937) called the study of ionized gas (plasma) as mercury dynamics, as he worked with mercury. Astro-physists called it as magnetohydrodynamics. This is commonly referred to as plasma dynamics also.

Peristalsis is the mechanism of the fluid transport that occurs generally from a region of lower pressure to higher pressure when a progressive wave of area contraction and expansion travels along the flexible wall of the tube. Peristaltic flow occurs widely in the functioning of ureter, food mixing and chyme movement in the intestine, movement of egg in the fallopian tube, the transport of the spermatozoa in the cervical canal, transport of bile in bile duct, transport of cilia, circulation of blood in small blood vessels, etc. There are many important applications of this principle in designing the roller pumps, which are useful in pumping the fluids without being contaminated due to the contact with the pumping machinery.

Latham (1966) made an experiment to study the fluid mechanics of peristaltic transport. A detailed study on the peristaltic pumping through a tube and a channel under the assumptions of low Reynolds number and long wavelength is given by Shapiro et al. (1969). The complete review of peristaltic transport is given by Jaffrin and Shapiro (1971). Srivastava and Srivastava (1989) solved the problem of two-dimensional peristaltic transport of a mixture of a Newtonian fluid and small spherical solid particles using perturbation method. Mishra and Ramachandra Rao (2003) studied the peristaltic flow of a Newtonian fluid in a two dimensional asymmetric channel. Vajravelu et al. (2005a,b) studied peristaltic flow of Herschel-Bulkley fluid in an inclined channel and a tube. They have deduced and discussed the results for the Bingham and power-law fluids. Srinivas and Kothandapani (2008) have analyzed the magneto hydrodynamic (MHD) peristaltic flow of a viscous fluid in an asymmetric channel with heat transfer. Recently, Vajravelu et al. (2011) studied the influence of heat transfer on peristaltic transport of a Jeffrey fluid in a vertical porous stratum.
Shukla et al. (1980) first made an attempt to study the peristaltic flow by considering the peristaltic motion of two immiscible Newtonian fluids in a channel and axisymmetrical geometries. They specified the interface shape, independent of the fluid viscosities, and used a trivial solution of the law of conservation of mass over one wavelength. Brasseur et al. (1987) have proved the invalidity of the analysis mentioned above in the limit of infinite peripheral-layer viscosity, since the principle of mass conservation is not satisfied independently in the core and the peripheral layer across any cross-section of the tube. Following the analysis of Brasseur et al. (1987), peristaltic flow of two immiscible fluid layers in a circular tube has been studied by Ramachandra Rao and Usha (1995) under the assumptions of long wavelength and low Reynolds number. Peristaltic transport of two-layered Power-law fluids in a tube is studied by Usha and Ramachandra Rao (1997). Vajravelu et al. (2006) studied the Peristaltic transport of a Herschel-Bulkley fluid in contact with a Newtonian fluid. Vajravelu et al. (2009) studied the peristaltic flow of a Casson fluid in contact with a Newtonian fluid in a circular pipe with permeable wall. They have obtained the equation of the interface and discussed its variation with the yield stress.

Most of the problems relating to petroleum industry, geophysics, plasma physics, magneto-fluid dynamics, etc., involve multi-fluid flow situations. In the petroleum industry, as well as in other engineering fields, a stratified two-phase flow often occurs. Bird et al. (1960) obtained an exact solution for laminar flow of two immiscible fluids between two parallel plates. Bhattacharya (1968) investigated the flow of two immiscible fluids between two rigid parallel plates with a time dependent pressure gradient. Chaturani et al. (1981) studied a three layered Couette flow with applications to blood flow. Loharsbi and Sahai (1987) dealt with two-phase MHD flow and heat transfer in a parallel plate channel. Both phases were incompressible and the flow was assumed to be steady, one-dimensional and fully developed. Chamkha (2000) reported analytical solutions for the flow of two immiscible fluids in porous and non-porous channels. An analytical characterization of the heat transfer in an oscillating flow through a porous medium was presented by Byun et al. (2006). Umavathi et al. (2008) studied the unsteady flow and heat transfer of three immiscible fluids in a horizontal channel.
They transformed the partial differential equations to ordinary differential equations using the exponential function of periodic frequency parameter and obtained the exact solution.

In view of several physiological and industrial applications, some Newtonian and non-Newtonian time dependent flows are investigated in the thesis. This thesis is divided into five chapters. The nomenclature of each chapter is independent of other chapter.

In the first chapter, unsteady flow of two immiscible fluids of Couple stress fluid in contact with a Newtonian fluid between two permeable beds is analyzed. The flow region between the permeable beds is divided into two regions. The flow region between the nominal surface of the lower permeable bed and the interface \( y=0 \) is named as Region-I and the flow region between the interface \( y=0 \) and the upper permeable bed is designated as Region-II. The flow in Region-I is described by couple stress model and the flow in Region-II is governed by Navier-Stokes equations. The flow is assumed to be driven by an exponentially time dependent pressure gradient. Expressions for the velocity distributions in the two regions, interface velocity and the mass flow rate are obtained. The effects of physical parameters such as couple stress parameter and viscosity ratio on the flow are found and are shown graphically.

The second chapter deals with peristaltic flow of Jeffrey fluid in contact with a Power-Law fluid in a channel bounded by permeable walls under long wave length and low Reynolds number assumptions. The variation of time-mean flow \( \bar{Q} \) with pressure rise over one wave length for different values of flow behavior index \( k \) and permeability parameter \( \alpha \) are studied.

In the third chapter, unsteady flow of a conducting fluid in a horizontal composite porous medium is analyzed. A uniform transverse magnetic field of strength \( B_0 \) is applied perpendicular to the composite channel. The flow in the channel is divided into two regions, namely porous and non-porous regions. The flow in the porous region is modeled using Darcy-Brinkman equation. The viscous and Darcian dissipation terms are
also included in the energy equations governing the flow. The nonlinear governing equations are solved analytically using two-term harmonic and non-harmonic functions. The effects of the porous medium parameter, ratio of viscosities, oscillation amplitude, conductivity ratio, Prandtl number and Eckert number on the velocity and the temperature fields are shown graphically.

The fourth chapter deals with unsteady flow and heat transfer in a horizontal channel consisting of Jeffrey fluid sandwiched between two Newtonian fluids occupying equal length which are bounded by two rigid walls. The flow between the rigid walls is divided into three zones. The fluids in all the zones are assumed to be incompressible, and the transport properties of the fluids in all zones are assumed to be constant. The governing equations describing the flow in all the channels are partial differential equations and are solved assuming that the pressure and velocity vary exponentially with time. Effects of physical parameters such as viscosity ratio, Jeffrey parameter, and conductivity ratio on the velocity and the temperature are shown graphically. It is found that the velocity increases with increases Jeffrey parameter.

The fifth chapter deals with unsteady flow and heat transfer in a horizontal channel consisting of three immiscible Jeffrey fluids occupying equal length bounded by two rigid walls. The flow between the rigid walls is divided into three zones. The flow in each zone is governed by Jeffrey model. The bounding walls are maintained at different constant temperatures. Further the fluid in all the three zones is driven by a constant pressure gradient and the existence of heat transfer does not affect the pressure gradient. The velocity field and the temperature distributions in all three zones are obtained. The influence of physical parameters such as viscosity ratio, Jeffrey parameter, conductivity ratio frequency, and periodic frequency parameter on the velocity and temperature fields are computed numerically and presented graphically. It is observed that both velocity and temperature increase with the increment in the Jeffrey parameters.