A1 Amperometric sensitivity

\[
\frac{dI_{DS}}{dpH} = \frac{d}{dpH} \left[ \frac{\mu C_{ox}}{L} \left( V_{ref} - V_{dh(sfer)} \right) V_{DS} - \frac{V_{DS}^2}{2} \right]
\]

\[
= \mu C_{ox} \left( \frac{W}{L} \right) \left( \frac{dV_{dh(sfer)}}{dpH} \right) V_{DS}
\]

\[
= \mu C_{ox} \left( \frac{W}{L} \right) V_{DS} \left( \frac{d\phi_{eq}}{dpH} \right)
\]

A2 The cylindrical coaxial capacitance

The cylindrical coaxial capacitance is given as

\[
C_{cyl-coax} = \frac{2 \pi \varepsilon L}{\ln \left( \frac{b}{a} \right)}
\]

Where \( L \) = Length of the coaxial cable

\( b \) = Outer radius

\( a \) = Inner radius

\( C_{cyl-coax} \) = coaxial capacitance
Fig. A1: Cylindrical capacitance cross sectional view
FigA.2: Cross sectional view of a cylindrical ISFET surrounded by electrolyte.
Therefore the coaxial capacitance per unit area is

\[
C_{\text{cyl-coax}} = \frac{C_{\text{cyl-coax}}}{\text{Area}} = \frac{C_{\text{cyl-coax}}}{2\pi a.L}
\]

\[
= \frac{\varepsilon}{a \left( \ln \left( \frac{b}{a} \right) \right)}
\]

If we put \( a = \frac{t_u}{2}, \quad b = \frac{t_u + 2t_{ox}}{2} \)

Then cylindrical oxide capacitance per unit area will be

\[
C_{\text{ox-cox}} = \frac{2\varepsilon_s}{t_u \left( \ln \left( 1 + \frac{2t_{ox}}{t_u} \right) \right)}
\]

From the figure A2 we can see that for IHP capacitance per unit area the inner radius ‘a’ and outer radius ‘b’ is given by

\[
a = \frac{t_u + 2t_{ox}}{2}, \quad b = \frac{t_u + 2t_{ox} + 2t_{IHP}}{2}
\]

And the IHP capacitance per unit area becomes

\[
C_{\text{IHP-cox}} = \frac{2\varepsilon_{IHP}}{(t_u + 2t_{ox}) \left( \ln \left( 1 + \frac{2t_{IHP}}{t_u + 2t_{ox}} \right) \right)}
\]
Similarly for OHP capacitance per unit area the inner radius ‘a’ and outer radius ‘b’ is given by

\[ a = \frac{t_n + 2t_{ox} + 2t_{OH}}{2}, \quad b = \frac{t_n + 2t_{ox} + 2t_{OH} + 2t_{OH}}{2} \]

And the OHP capacitance per unit area becomes

\[ C_{OH,P,OL} = \frac{2e_{OH}}{(t_n + 2t_{ox} + 2t_{OH}) \left( \ln \left( \frac{2t_{OH}}{t_n + 2t_{ox} + 2t_{OH}} \right) \right)} \]

A3 The effective channel width of a cylindrical MOSFET

For a planar device neglecting the effect of drain bias, the shape of the channel can be approximated as shown below.

Here

- \( L = \) channel length
- \( W = \) channel width
- \( t_{inv} = \) thickness of the inversion layer
- \( A = \) area of the current path

The width ‘W’ of the channel for a planar can be expressed as

\[ W = \frac{A}{t_{inv}} \]
For a cylindrical device neglecting the effect of drain bias, the shape of the channel can be approximated as shown in Fig: A4.

The figure A3 shows the body of a cylindrical MOSFET.

\[ L = \text{channel length} \]
W = channel width (not shown in the figure)

t_{inv} = thickness of the inversion layer

The area of the current path 'A' can be expressed as (This is shown in the figure with the dotted texture)

Fig: A4: Cylindrical silicon pillar
A = area of the circle of diameter \( t_{si} \) – area of the circle of diameter \((t_{si} - 2t_{inv})\)

\[
A = \frac{\pi t_{si}^2}{4} - \frac{\pi (t_{si} - 2t_{inv})^2}{4}
\]

Now the effective channel width ‘W’ can be found by dividing the current path area by thickness of the current path (as it was done in the case of a planar device)

\[
W = A \frac{t_{si}^2 - (t_{si} - 2t_{inv})^2}{4t_{inv}}
\]

A4 Calculation of reference electrode potential(\(E_{ref}\)) relative to vacuum:

The reference electrode potential is given by the following expression [2],[3]. The filling solution is assumed to be of 3.5M KCl, saturated with AgCl.

\[
E_{ref} (T) = E_{bas} (H^+|H_2) + E_{rel} (Ag|AgCl) + \left( \frac{dE_{ref}}{dT} \right) (T - 298.16)
\]

\[
= 4.7 + .205 + 1.410^{-4}(300 - 298.16)
\]

\[
= 5.37 \text{volts}
\]
\[ E_{obs}(H^+|H_2) = \text{Absolute potential of hydrogen electrode} \]

\[ E_{rel}(Ag|AgCl) = \text{Relative potential of Ag/AgCl electrode with reference} \]
\[ \text{to hydrogen electrode} \]
\[ \frac{dE_{ref}}{dT} = \text{Temperature coefficient} \]

**A5. References**

