Using Fuzzy Programming Technique to Solve Multiobjective Fuzzy Programming Problems in a probabilistic environment with exponentially distributed parameters

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5.1 Introduction

A methodology has been developed in the previous chapter for solving FMOCCP problems where the right side parameters associated with the system constraints were considered as uniformly distributed FRVs. In this chapter the right side parameters associated with system constraints are taken as exponentially distributed FRVs from the viewpoint that the exponential distribution has only one parameter and it is more robustly applicable to small sample sizes and uncertainties in parameter fitting than other distributions with two or more parameters.

In the proposed methodology, FGP technique is used to solve FMOCCP problems consisting of exponentially distributed FRVs. Also the coefficients of the system constraints considered as fuzzy numbers. The general CCP technique is used to remove the probabilistic nature of the problem and to convert it into an equivalent FP model in the model formulation process. Now considering the fuzzy aspects of the parameters associated with the system constraints of the problem, the problem is decomposed on the basis of tolerance ranges of the parameters in the decision making situation. Then the individual optimal solution of each objective are found to construct the membership function of them to measure the degree of satisfaction of each objectives. Finally a FGP model is used for achievement of the highest membership degree of each of the defined fuzzy goals to the extent possible. To explore the potentiality of the proposed approach an illustrative example is solved and compared with the existing methodology [345].

5.2 FMOCCP Model Formulation

The following FCCP model with K number of objectives is considered as

\[
\text{Find } x(x_1, x_2, \ldots, x_n) \\
\text{Maximize } Z_k = \sum_{j=1}^{n} c_{kj} x_j ; \quad k = 1, 2, \ldots, K \\
\text{Subject to } P\left[\sum_{j=1}^{n} \tilde{a}_{ij} x_j \leq \tilde{b}_i\right] \geq 1 - \alpha_i; \quad i = 1, 2, \ldots, m \\
x_j \geq 0; \quad j = 1, 2, \ldots, n
\] (5.1)

where \( \tilde{b}_i \) represent exponentially distributed FRVs; \( \tilde{a}_{ij} \) are triangular fuzzy numbers; \( \alpha_i \) (0 ≤ \( \alpha_i \) ≤ 1) denote any real number for \( i = 1, 2, \ldots, m; \ j = 1, 2, \ldots, n \).
Since $\tilde{b}_i$ is an exponentially distributed FRV. The probability density function of $\tilde{b}_i$ can be defined as
\[
f(\tilde{b}_i) = \begin{cases} \tilde{\lambda}_i \exp(-\tilde{\lambda}_i \tilde{b}_i) & \text{if } \tilde{b}_i \in \mathbb{R}^+ \\ 0 & \text{elsewhere} \end{cases}
\]
where $\mathbb{R}^+$ represents the set of positive real numbers. Here $\tilde{\lambda}_i \in \mathbb{R}^+$, and the corresponding mean and variance is given by
\[
E(\tilde{b}_i) = \frac{1}{\tilde{\lambda}_i} \text{ and } Var(\tilde{b}_i) = \frac{1}{\tilde{\lambda}_i^2}, \text{ respectively.}
\]

5.3 Formulation of equivalent FP Model

Now considering the probabilistic behavior of the above model (5.1), the problem is converted into the equivalent FP problem by using general CCP methodology defined in fuzzy environment. Then the resultant model is appeared as

Maximize $Z_k = \sum_{j=1}^{n} c_{kj} x_j$ ; $k = 1, 2, \ldots, K$

Subject to $\sum_{j=1}^{n} \tilde{a}_{ij} x_j \leq -\frac{\ln(1-a_{ij})}{\tilde{\lambda}_i}$, $i = 1, 2, \ldots, m$

$x_j \geq 0$; $j = 1, 2, \ldots, n$ \hspace{1cm} (5.2)

5.3.1 Characterization of Membership Functions

In fuzzy decision making situation it is to be assumed that the mean $E(\tilde{b}_i)$, associated with the exponentially distributed FRV $\tilde{b}_i$ are right sided fuzzy numbers having tolerance limit $\delta_i$ as described in the Chapter 1, Sub-Section 1.3.2.1

Further the coefficients related to the system constraints are considered as triangular fuzzy numbers as defined in (1.14), Sub-Section 1.3.2.1, Chapter 1.

Now associated with the system constraints in (5.2), $\tilde{a}_{ij}$ are considered as the triangular fuzzy numbers with the form $\tilde{a}_{ij} = (a_{ij}^l, a_{ij}, a_{ij}^R)$.

It is to be noted that the parameters $\tilde{a}_{ij}$, involved with the system constraints may be considered as trapezoidal fuzzy numbers also.

5.4 Decomposed FP Model Formulation

On the basis of above fuzzy numbers, model (5.2) can be decomposed as

Maximize $Z_k = \sum_{j=1}^{n} c_{kj} x_j$ ; $k = 1, 2, \ldots, K$

Subject to
\[
\sum_{j=1}^{n} (a_{ij}^l + (a_{ij} - a_{ij}^l)\beta) x_j \leq -\left(\frac{1}{\tilde{\lambda}_i} - \beta \delta_i\right) \ln(1 - \alpha_i) \\
\sum_{j=1}^{n} (a_{ij}^R - (a_{ij}^R - a_{ij})\beta) x_j \leq -\left(\frac{1}{\tilde{\lambda}_i} - \beta \delta_i\right) \ln(1 - \alpha_i)
\]
where \( \frac{1}{\lambda_i} = E(b_i) + \delta_i \) represents the right tolerance value of the fuzzy number \( E(\tilde{b}_i) \).

Let \( Z_k^b \) and \( Z_k^w \) \( (k = 1, 2, \ldots, K) \) be the respective best and worst values obtained by solving each objective independently. Hence the fuzzy objective goal for each of the objectives can be expressed as

\[
Z_k \geq Z_k^b \quad \text{for} \quad k = 1, 2, \ldots, K.
\]

Thus the membership function for each of the objectives can be written as

\[
\mu_{Z_k}(x) = \begin{cases} 
0 & \text{if } Z_k \leq Z_k^w \\
\frac{Z_k^b - Z_k^w}{Z_k^b - Z_k^w} & \text{if } Z_k^w \leq Z_k \leq Z_k^b, \\
1 & \text{if } Z_k \geq Z_k^b
\end{cases}
\]

Now on the basis of the developed membership functions the FGP model is formulated in the next section.

5.5 Fuzzy Goal Programming Model

The membership functions are converted into membership goals by introducing under and over deviational variables and the following model is developed to find the most satisfactory solution in the decision making environment.

Find \( X(x_1, x_2, \ldots, x_n) \)

so as to Minimize

\[
D = \sum_{k=1}^{K} W_k d_k^-
\]

and satisfy

\[
\sum_{k=1}^{K} \left( \frac{Z_k^b - Z_k^w}{Z_k^b - Z_k^w} - d_k^- + d_k^+ \right) = 1
\]

Subject to

\[
\begin{align*}
\sum_{j=1}^{n} (a_{ij}^b + (a_{ij} - a_{ij}^b)\beta) x_j & \leq -\left( \frac{1}{\lambda_i} - \beta \delta_i \right) \ln (1 - \alpha_i) \\
\sum_{j=1}^{n} (a_{ij}^w - (a_{ij}^w - a_{ij})\beta) x_j & \leq -\left( \frac{1}{\lambda_i} - \beta \delta_i \right) \ln (1 - \alpha_i)
\end{align*}
\]

\( 0 \leq \beta \leq 1; x_j \geq 0; j = 1, 2, \ldots, n; i = 1, 2, \ldots, m; k = 1, 2, \ldots, K \)

where \( D \) represent the fuzzy goal achievement function consisting of the weighted under deviational variables, and where \( W_k, k = 1, 2, \ldots, K \) are the fuzzy weights representing the relative importance of achieving the aspired levels of the goals defined by usual process.

The resultant model is solved for finding the most satisfactory decision in the fuzzily defined chance constrained decision making context.

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5.6 Illustrative Example

To demonstrate the efficiency of the proposed approach, the following FMOCCP is considered and is solved to explain the methodology in details. The FMOCCP is presented as

\[
\text{Find } X(x_1, x_2)
\]

so as to

\[
\text{Maximize } Z_1 = 10x_1 + 5x_2
\]

Maximize \( Z_2 = 3x_1 + 7x_2 \)

Subject to

\[
P\left[\bar{\tilde{1}}x_1 + \tilde{1}x_2 \leq \tilde{b}_1\right] \geq 0.94
\]

\[
P\left[\tilde{4}x_1 + \tilde{3}x_2 \leq \tilde{b}_2\right] \geq 0.93
\]

\[
P\left[\tilde{2}x_1 + \tilde{5}x_2 \leq \tilde{b}_3\right] \geq 0.91
\]

\[
x_1, x_2 \geq 0
\] (5.6)

Here \( \tilde{b}_1, \tilde{b}_2, \tilde{b}_3 \) are independent FRVs following exponential distribution with respective mean represented by the following right sided fuzzy numbers

\[
\mu_{E(b_1)}(x) = \begin{cases} 
1 & \text{if } E(b_1) \leq 7 \\
7.5 - E(b_1) & \text{if } 7 \leq E(b_1) \leq 7.5 \\
0 & \text{if } E(b_1) \geq 7.5
\end{cases}
\]

\[
\mu_{E(b_2)}(x) = \begin{cases} 
1 & \text{if } E(b_2) \leq 9 \\
9.5 - E(b_2) & \text{if } 9 \leq E(b_2) \leq 9.5 \\
0 & \text{if } E(b_2) \geq 9.5
\end{cases}
\]

\[
\mu_{E(b_3)}(x) = \begin{cases} 
1 & \text{if } E(b_3) \leq 8 \\
8.5 - E(b_3) & \text{if } 8 \leq E(b_3) \leq 8.5 \\
0 & \text{if } E(b_3) \geq 8.5
\end{cases}
\]

Also \( \tilde{1}, \tilde{4}, \tilde{3}, \tilde{2}, \tilde{5} \) are taken as triangular fuzzy numbers with the form,

\( \tilde{1} = (0.95, 1, 1.05), \tilde{4} = (3.95, 4, 4.05), \tilde{3} = (2.95, 3, 3.05), \tilde{2} = (1.95, 2, 2.05), \) and \( \tilde{5} = (4.95, 5, 5.05). \)

Now applying general CCP technique the following model (5.7) can be derived from (5.6) as

\[
\text{Find } X(x_1, x_2)
\]

\[
\text{Maximize } Z_1 = 10x_1 + 5x_2
\]

Maximize \( Z_2 = 3x_1 + 7x_2 \)

Subject to

\[
\tilde{1}x_1 + \tilde{1}x_2 \leq -E(\tilde{b}_1)\ln(1 - 0.06)
\]
Further considering the fuzzy numbers associated with the system constraints of model \((5.7)\) the model can be converted into the following FP problem as

\[
\begin{align*}
\text{Maximize } & Z_1 = 10x_1 + 5x_2 \\
\text{Maximize } & Z_2 = 3x_1 + 7x_2 \\
\text{subject to } & (0.95 + 0.05\alpha)x_1 + (0.95 + 0.05\alpha)x_2 \leq -(7.5 - 0.5\alpha)\ln(1 - 0.06) \\
& (1.05 - 0.05\alpha)x_1 + (1.05 - 0.05\alpha)x_2 \leq -(7.5 - 0.5\alpha)\ln(1 - 0.06) \\
& (3.95 + 0.05\alpha)x_1 + (2.95 + 0.05\alpha)x_2 \leq -(9.5 - 0.5\alpha)\ln(1 - 0.07) \\
& (4.05 - 0.05\alpha)x_1 + (3.05 - 0.05\alpha)x_2 \leq -(9.5 - 0.5\alpha)\ln(1 - 0.07) \\
& (1.95 + 0.05\alpha)x_1 + (4.95 + 0.05\alpha)x_2 \leq -(8.5 - 0.5\alpha)\ln(1 - 0.09) \\
& (2.05 - 0.05\alpha)x_1 + (5.05 - 0.05\alpha)x_2 \leq -(8.5 - 0.5\alpha)\ln(1 - 0.09) \\
& x_1, x_2 \geq 0, 0 < \alpha < 1
\end{align*}
\] (5.8)

Now to find the fuzzy goals of each of objectives of the above model, each objective is considered independently and is solved with respect to the same set of system constraints. The best and worst values of the objectives obtained as

\[
Z_1^b = 1.702259, Z_1^w = 0; \quad Z_2^b = 1.122741; \quad Z_2^w = 0.
\]

Then the fuzzy goals of the objectives are found as

\[
Z_1 \succeq 1.702259, Z_2 \succeq 1.122741.
\]

Hence the membership functions are constructed by using the procedure described in (5.4).

\[
\mu_{Z_1}(x_1, x_2) = \frac{10x_1 + 5x_2}{1.702259}, \quad \mu_{Z_2}(x_1, x_2) = \frac{3x_1 + 7x_2}{1.122741}.
\]

Considering unity as the aspiration level of the defined membership functions the following FGP model is derived as

\[
\text{Minimize } D = \frac{d_1^-}{1.702259} + \frac{d_2^-}{1.122741}
\]

so as to

\[
\begin{align*}
\text{subject to } & 10x_1 + 5x_2 + d_1^- - d_1^+ = 1 \\
& 3x_1 + 7x_2 + d_2^- - d_2^+ = 1
\end{align*}
\]

the system constraints in (5.8)

\[
d_1^-, d_1^+, d_2^-, d_2^+ \geq 0 \text{ with } d_1^- \cdot d_1^+ = d_2^- \cdot d_2^+ = 0
\] (5.9)
5.7 Results and Discussions

The problem (5.9) is solved by using the software LINGO (6.0). The optimal solution of the problem is obtained as

\[ x_1 = 0.07299711 \quad \text{and} \quad x_2 = 0.1291071 \]

with the achieved objective values

\[ Z_1 = 1.375507, Z_2 = 1.122741 \]

The solution obtained by using the methodology developed by Sinha et al. [345] is

\[ x_1 = 0.071536, x_2 = 0.122286 \]

and the corresponding objective values are

\[ Z_1 = 1.32679 \quad \text{and} \quad Z_2 = 1.07061. \]

The solution achieved here is most satisfactory in terms of the aspired goal levels of the objectives of the decision maker.

Both the achieved values of the objectives obtained by Sinha et al. [345] is inferior to the objective function values obtained by using the proposed methodology and the comparison is presented by using the following diagram.

![Figure 5.1: Comparison of achieved goal values](image)

5.8 Conclusion and Scope for Future Studies

In this chapter the developed methodology can also be treated to solve the problem involving probabilistic as well as fuzzy conventions under one roof. The superiority
of the proposed technique has also been reflected by comparing with other existing
techniques.

The proposed methodology can also be extended to solve FMOCCP problems
which follow other types of probability distributions and also the objectives involving
some fuzzily defined parameters. Also the proposed methodology can be used to
solve hierarchical decision making problems in a FMOCCP arena. However it is
concluded that the described methodology may add a new dimension into the way of
solving MOCCP in a fuzzily defined probabilistic decision making environment.