Chapter 4

On Solving Chance Constrained Programming Problems involving Uniform Distribution with Fuzzy Parameters

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4.1 Introduction

Most of the real world optimization problems contain multiple, non commensurable and conflicting objectives known as multiobjective optimization problem. CCP in the framework of conventional GP with dependent and independent chance constraints has been studied by many researchers [156; 312] in the past. FP in the form of GP was first introduced by Narasimhan [265] in 1980 for solving multiobjective decision making (MODM) problems. Thereafter FGP had been deeply studied extensively by Pal et al. [281], Iskander [164], Biswas and Bose [34], and others. The methodological aspects of CCP problems using FGP were studied by Mohammed [258] in 2000. Luhandjula [240] did a survey work on the developments of fuzzy SP in 2006.

For handling simultaneous occurrence of randomness and fuzziness in an MODM context, Sinha et al. [345] proposed FP technique by implicitly assuming that the fuzzy decision or the min-operator is proper representation of DM’s ambiguous preferences. Realizing the drawbacks of the model proposed by Sinha et al. [345], Jana and Biswal [166] proposed a genetic algorithm based GP approach as a more general method for solving MOCCP problems. A procedure for solving MOCCP model involving discrete random variables in chance constraints has been presented by Jana and Sharma [168] in 2010.

Due to the ambiguity presence in formulating models of MOCCP, it is very often found that the mean and standard deviation of random variables associated with the parameters of chance constraints cannot be articulate precisely. So the concept of FRVs [50] was introduced in the decision making context. An efficient FGP technique for solving CCP with normally distributed FRVs associated with the system constraints have been presented by Biswas and Modak [35] in the recent past.

In this chapter, the FGP approach to MODM problems with chance constraints having uniformly distributed FRV in right side of the system constraints has been presented. In the model formulation process the FMOCCP problem is converted into an FP model by applying chance constrained methodology developed for fuzzy uniform distribution with the help of $\alpha$-cuts of the fuzzy numbers. After that considering the fuzzy nature of the parameters associated with the system constraints, the problem is decomposed on the basis of the tolerance ranges of the parameters.
Then the membership function of the each of the individual objectives is defined in isolation to measure the degree of achievements of the goal levels of the objectives. Then FGP model is developed to achieve the highest degree of each of the defined membership functions to the extent possible. To explore the potentiality of the proposed approach, an illustrative example is solved and the solution is compared with other existing technique.

### 4.2 Prerequisites

In deriving FMOCCP model the following concepts are derived to make most satisfactory solution by the DMs in the decision making context. In this section, the concepts of uniform distribution have been extended in a fuzzy environment.

#### 4.2.1 Fuzzy Uniform

Let the uniform density on an interval \((\delta, \gamma]\) is written as \(U(\delta, \gamma)\). Again let \(X\) be a uniformly distributed random variable. Then the density function of \(U(\delta, \gamma)\), of the random variable \(X\) is given by

\[
f(x; \delta, \gamma) = \begin{cases} \frac{1}{\gamma - \delta} & \text{for } \delta < x \leq \gamma \\ 0 & \text{otherwise} \end{cases}
\]

where the mean and variance of the distribution is defined as \(\mu = \frac{\gamma + \delta}{2}\) and \(\sigma^2 = \frac{(\gamma - \delta)^2}{12}\).

Now if \(\tilde{\delta}\) and \(\tilde{\gamma}\) are considered as fuzzy numbers, then the uniform density for the fuzzy numbers \(\tilde{\delta}\) and \(\tilde{\gamma}\) is written as \(U(\tilde{\delta}, \tilde{\gamma})\). It is to be noted here that there is some ambiguity at the end points of the uniform density function. A uniform density function for fuzzily described random variable \(X\) defined on \(\tilde{\delta} < x \leq \tilde{\gamma}\) is given as

\[
f(x; \tilde{\delta}, \tilde{\gamma}) = \begin{cases} \frac{1}{t - s} & \text{for } s < x \leq t \\
0 & \text{otherwise} \end{cases} \quad |s \in \tilde{\delta}[\alpha], \quad t \in \tilde{\gamma}[\alpha]\
\]

where \(\tilde{\delta}[\alpha]\) and \(\tilde{\gamma}[\alpha]\) represent the \(\alpha\)-cuts of the corresponding fuzzy numbers \(\tilde{\delta}\) and \(\tilde{\gamma}\). Now the mean of the uniformly distributed FRV is represented by the \(\alpha\)-cuts of fuzzy numbers as \([50]\)

\[
\bar{\mu}[\alpha] = \left\{ \int_{s \in \tilde{\delta}[\alpha]}^{t} \frac{x}{t - s} dx \mid s \in \tilde{\delta}[\alpha], \quad t \in \tilde{\gamma}[\alpha], s < t \right\}
\]

For different values of \(\alpha\), the value of the integral is equals to \(\frac{\gamma + \delta}{2}\). Hence the mean of the distribution \(U(\tilde{\delta}, \tilde{\gamma})\) is written as \(\bar{\mu} = \frac{\gamma + \delta}{2}\).
In this way the mean of the distribution is fuzzified from the crisp mean value $\mu = \frac{y + \delta}{2}$.

Similarly the variance of the distribution is defined as

$$\sigma^2[\alpha] = \left\{ \int_s^t \left[ \frac{(x - \mu)^2}{t - s} \right] dx \mid s \in \delta[\alpha], \ t \in \gamma[\alpha], \ s < t \right\}$$

where $\mu = \frac{s + t}{2}$.

For different values of $\alpha$ the value of the integral is equal to $\frac{(t - s)^2}{12}$ which can be expressed as $\sigma^2 = \frac{(\gamma - \delta)^2}{12}$, which is the fuzzification of the crisp variance $\sigma^2 = \frac{(\gamma - \delta)^2}{12}$.

Now with the help of the above results the following lemma can be stated as

**4.2.1.1 Lemma.**

If $X$ be a uniformly distributed FRV over $[\delta, \gamma]$ then the mean $\bar{\mu}$ and variance $\bar{\sigma}^2$ can be defined as $\bar{\mu} = \frac{y + \delta}{2}$ and $\bar{\sigma}^2 = \frac{(\gamma - \delta)^2}{12}$.

With the above considerations the FMOCCP model has been developed in the subsequent Section.

**4.3 FMOCCP Model Formulation**

Jana and Biswal [166] discussed multiobjective linear programming problems with crisp random variables. But in many decision making situations, the mean and variance of the random variables cannot be described precisely. An FMOCCP model involving uniformly distributed FRVs with $K$ number of objectives is considered as

Find

$$X(x_1, x_2, \ldots, x_n)$$

so as to

Maximize $Z_k = \sum_{j=1}^{n} c_{kj} x_j$; \quad $k = 1, 2, \ldots, K$

Subject to

$$P\left[ \sum_{j=1}^{n} \tilde{a}_{ij} x_j \leq \tilde{b}_i \right] \geq 1 - p_i$; \quad $i = 1, 2, \ldots, m$

$$x_j \geq 0; \quad j = 1, 2, \ldots, n$$

(4.1)

where $\tilde{b}_i$ represent continuously distributed FRVs; $\tilde{a}_{ij}$ are triangular fuzzy numbers; $p_i$ ($0 \leq p_i \leq 1$) denote any real number for $i = 1, 2, \ldots, m$.

Since $\tilde{b}_i$ are uniformly distributed FRVs, the probability density function of $\tilde{b}_i$ using the concept of fuzzy uniform described in Section 4.2 is presented as

$$f(x; \tilde{\delta}, \tilde{\gamma}) = \begin{cases} \frac{1}{t - s} & \text{for} \quad s < x \leq t \mid s \in \delta[\alpha], \ t \in \gamma[\alpha] \\ 0 & \text{otherwise} \end{cases}$$
where the mean and variance of the distribution is given by \( E(\tilde{\beta}_i) = \frac{\tilde{\delta}_i + \tilde{\gamma}_i}{2} \) and \( Var(\tilde{\beta}_i) = \frac{(\tilde{\gamma}_i - \tilde{\delta}_i)^2}{12} \), respectively.

### 4.3.1 Construction of FP Model

Using CCP technique for uniform distribution in fuzzy environment, the constraints in the FMOCCP model (4.1) is transformed into the following form as

\[
\left\{ \int_{t}^{s} \frac{1}{t-s} db_i \big| s \in \tilde{\delta}[\alpha], t \in \tilde{\gamma}[\alpha], u \in \tilde{a}_{ij}[\alpha], s < t \right\} \geq 1 - p_i
\]

i.e.,

\[
\left\{ \int_{t}^{s} \frac{1}{t-s} db_i \big| s \in \tilde{\delta}[\alpha], t \in \tilde{\gamma}[\alpha], u \in \tilde{a}_{ij}[\alpha], s < t \right\} \geq 1 - p_i
\]

i.e.,

\[
u x_j \leq s + p_i(t - s)
\]

with \( s < t, s \in \tilde{\delta}[\alpha], t \in \tilde{\gamma}[\alpha] \) and \( u \in \tilde{a}_{ij}[\alpha] \).

Since this inequality is true for all \( \alpha \in (0, 1) \), the expression can be written in terms of \( \alpha \)-cut as

\[
\sum_{j=1}^{n} \tilde{a}_{ij}[\alpha] x_j \leq \tilde{\delta}[\alpha] - p_i(\tilde{\gamma}[\alpha] - \tilde{\delta}[\alpha])
\]

i.e.,

\[
\sum_{j=1}^{n} \tilde{a}_{ij}[\alpha] x_j \leq E(\tilde{\beta}_i)[\alpha] - \sqrt{3Var(\tilde{\beta}_i)[\alpha]} + 2p_i\sqrt{3Var(\tilde{\beta}_i)[\alpha]}. \quad (4.2)
\]

Now applying first decomposition theorem as stated in Subsection 1.3.2.1, chapter-1, the equation (4.2) is reduced to the following form as

\[
\sum_{j=1}^{n} \tilde{a}_{ij} x_j \leq E(\tilde{\beta}_i) - \sqrt{3Var(\tilde{\beta}_i)} + 2p_i\sqrt{3Var(\tilde{\beta}_i)}. \quad (4.3)
\]

Again, considering the probabilistic behavior of FMOCCP model (4.1), the problem is converted into the equivalent FP problem by using the derived methodology as

\[
\text{Find } X(x_1, x_2, \ldots, x_n) \text{ so as to Maximize } Z_k = \sum_{j=1}^{n} c_{kj} x_j ; \quad k = 1, 2, \ldots, K
\]

subject to

\[
\sum_{j=1}^{n} \tilde{a}_{ij} x_j \leq E(\tilde{\beta}_i) - \sqrt{3Var(\tilde{\beta}_i)} + 2p_i\sqrt{3Var(\tilde{\beta}_i)}
\]

\[
x_j \geq 0; \quad j = 1, 2, \ldots, n. \quad (4.4)
\]

In fuzzy decision making situation it is to be assumed that the mean and variance associated with the continuously distributed FRVs, \( E(\tilde{\beta}_i) \) and \( Var(\tilde{\beta}_i) \) are right sided fuzzy numbers having tolerance limit \( \xi_i \) and \( \zeta_i \), respectively. So the respective membership functions of these fuzzy numbers are represented using Subsection 1.3.2.1 of chapter -1, as
\[ \mu_{E(b_i)}(x) = \begin{cases} 
1 & \text{if } x \leq E(b_i) \\
\frac{(E(b_i) + \xi_i) - x}{\xi_i} & \text{if } E(b_i) \leq x \leq E(b_i) + \xi_i \\
0 & \text{if } x \geq E(b_i) + \xi_i 
\end{cases} \]

and

\[ \mu_{var(b_i)}(x) = \begin{cases} 
1 & \text{if } x \leq \text{Var}(b_i) \\
\frac{(\text{Var}(b_i) + \eta_i) - x}{\delta} & \text{if } \text{Var}(b_i) \leq x \leq \text{Var}(b_i) + \eta_i \\
0 & \text{if } x \geq \text{Var}(b_i) + \eta_i 
\end{cases} \]

Further the coefficients \( \bar{a}_{ij} \) related to the system constraints in (4.4) are considered as triangular fuzzy numbers with the form \( \bar{a}_{ij} = (a_{ij}^l, a_{ij}, a_{ij}^r) \) with the membership function defined in terms of Subsection 1.3.2.1 of chapter-1.

On the basis of above types of fuzzy numbers associated with different parameters, model (4.4) is decomposed into the following form as

\[
\begin{align*}
\text{Find} & \quad X(x_1, x_2, \ldots, x_n) \\
\text{so as to} & \quad \text{Maximize } Z_k = \sum_{j=1}^{n} c_{kj} x_j ; \quad k = 1, 2, \ldots, K \\
\text{subject to} & \quad \sum_{j=1}^{n} (a_{ij}^l + (a_{ij} - a_{ij}^l) \alpha)x_j \leq (E(b_i))^r - \alpha \xi_i - \sqrt{3(\text{Var}(b_i))^r - \alpha \xi_i} - 2\alpha \sqrt{3(\text{Var}(b_i))^r - \alpha \xi_i} \\
& \quad \sum_{j=1}^{n} (a_{ij}^r - (a_{ij} - a_{ij}) \alpha)x_j \leq (E(b_i))^r - \alpha \xi_i - \sqrt{3(\text{Var}(b_i))^r - \alpha \xi_i} + 2\alpha \sqrt{3(\text{Var}(b_i))^r - \alpha \xi_i} \\
& \quad 0 \leq \alpha \leq 1; \quad x_j \geq 0; \quad i = 1, 2, \ldots, m; \quad j = 1, 2, \ldots, n. \quad (4.5)
\end{align*}
\]

4.3.1.1 Characterization of Membership Functions

Let \( Z_k^b \) and \( Z_k^w \) (\( k = 1, 2, \ldots, K \)) be the respective best and worst values obtained by solving each objective in isolation under the system constraints defined in (4.5). Hence the fuzzy objective goal for each of the objectives is expressed as

\[ Z_k \geq Z_k^b \text{ for } k = 1, 2, \ldots, K. \]

Thus the membership function for each of the objectives are expressed as

\[ \mu_{Z_k}(x) = \begin{cases} 
1 & \text{if } Z_k \geq Z_k^b \\
\frac{Z_k - Z_k^w}{Z_k^b - Z_k^w} & \text{if } Z_k^w \leq Z_k \leq Z_k^b \\
0 & \text{if } Z_k \leq Z_k^w 
\end{cases} \quad \text{for } k = 1, 2, \ldots, K. \quad (4.6) \]

Now the aim of DMs is to achieve the highest membership value (unity) of each of the defined membership functions to the extent possible. Considering this aspect, an FGP model is formulated in the next section based on the developed membership functions.
4.4 FGP Model Development

In a fuzzy decision making environment, achievement of a fuzzy goal to its aspired level means achievement of the corresponding membership function to its highest degree. Since highest value of each membership function is unity, the DMs’ problem is to achieve the highest aspiration level (i.e., unity) of each of the membership function to the extent possible by minimizing the under deviational variables of the flexible membership goals obtained by the introduction of under- and over-deviational variables to each of the membership functions and assigning unity as the aspiration level to each of them. Under this situation the following model is developed to find the most satisfactory solution in the decision making environment as

\[
\text{Find} \quad X(x_1, x_2, \ldots, x_n)
\]

so as to

\[
\text{Min } D = \sum_{k=1}^{K} W_k d_k^-
\]

and satisfy

\[
\frac{z_k - z_k^w}{z_k^p - z_k^w} + d_k^- - d_k^+ = 1
\]

subject to system constraints in (4.5)

\[
\text{(4.6)}
\]

where \( D \) represents the fuzzy goal achievement function consisting of the weighted under deviational variables; \( d_k^- \), \( d_k^+ \) represent over and under deviational variable respectively and \( W_k, k = 1, 2, \ldots, K, \) are the fuzzy weights as defined earlier.

Now the resultant model (4.6) is solved for finding the most satisfactory solution in the fuzzily defined chance constrained decision making environment.

4.5 An Illustrative Example

To demonstrate potential use and efficiency of the proposed approach, a modified version of the MOCCP problem studied by Sinha et al. [345] is considered in fuzzy environment and is solved. The FMOCCP problem is presented as

\[
\text{Find} \quad X(x_1, x_2)
\]

so as to

\[
\text{Maximize } Z_1 = x_1 + 3x_2 \\
\text{Maximize } Z_2 = x_1 + 5x_2
\]

subject to

\[
\begin{align*}
\text{P}[2x_1 + \bar{X} x_2 &\leq \bar{b}_1] \geq 1 - 0.9 \\
\text{P}[5x_1 + \bar{Y} x_2 &\leq \bar{b}_2] \geq 1 - 0.95 \\
\text{P}[\bar{X} x_1 + \bar{Y} x_2 &\leq \bar{b}_3] \geq 1 - 0.8 \\
x_1, x_2 &\geq 0.
\end{align*}
\]

\[
\text{(4.8)}
\]
Here $\tilde{b}_1, \tilde{b}_2, \tilde{b}_3$ are independent FRVs following uniform distribution with respective mean and variance represented by the following right sided fuzzy numbers

$$
\mu_{E(\tilde{b}_1)}(x) = \begin{cases} 
\frac{1}{0.4} & \text{if } E(b_1) \leq 6 \\
0 & \text{if } 6 \leq E(b_1) \leq 6.4 \\
0 & \text{if } E(b_1) \geq 6.4 
\end{cases}
$$

$$
\mu_{Var(\tilde{b}_1)}(x) = \begin{cases} 
\frac{1}{0.5} & \text{if } Var(b_1) \leq 4 \\
0 & \text{if } 4 \leq Var(b_1) \leq 4.5 \\
0 & \text{if } Var(b_1) \geq 4.5 
\end{cases}
$$

$$
\mu_{E(\tilde{b}_2)}(x) = \begin{cases} 
\frac{1}{0.5} & \text{if } E(b_2) \leq 6 \\
0 & \text{if } 6 \leq E(b_1) \leq 6.5 \\
0 & \text{if } E(b_1) \geq 6.5 
\end{cases}
$$

$$
\mu_{Var(\tilde{b}_2)}(x) = \begin{cases} 
\frac{1}{0.6} & \text{if } Var(b_1) \leq 4 \\
0 & \text{if } 4 \leq Var(b_1) \leq 4.6 \\
0 & \text{if } Var(b_1) \geq 4.6 
\end{cases}
$$

$$
\mu_{E(\tilde{b}_3)}(x) = \begin{cases} 
\frac{1}{0.7} & \text{if } E(b_2) \leq 6 \\
0 & \text{if } 6 \leq E(b_1) \leq 6.7 \\
0 & \text{if } E(b_1) \geq 6.7 
\end{cases}
$$

$$
\mu_{Var(\tilde{b}_3)}(x) = \begin{cases} 
\frac{1}{0.4} & \text{if } Var(b_1) \leq 4 \\
0 & \text{if } 4 \leq Var(b_1) \leq 4.4 \\
0 & \text{if } Var(b_1) \geq 4.4 
\end{cases}
$$

Also the fuzzy parameters $\tilde{1}, \tilde{2}, \tilde{3}, \tilde{4}, \tilde{5}$ are taken as triangular fuzzy numbers with the form, $\tilde{1} = (0.95, 1, 1.05), \tilde{2} = (3.95, 4, 4.05), \tilde{3} = (2.95, 3, 3.05), \tilde{2} = (1.95, 2, 2.05), \text{ and } \tilde{5} = (4.95, 5, 5.05)$.

Now applying CCP technique developed for uniform distribution, the following model (4.9) is derived from (4.8) as

Maximize $Z_1 = x_1 + 3x_2$

Maximize $Z_2 = x_1 + 5x_2$

subject to $\tilde{2}x_1 + \tilde{1}x_2 \leq E(\tilde{b}_1) - \sqrt{3Var(\tilde{b}_1)} + 1.8\sqrt{3Var(\tilde{b}_1)}$

$\tilde{5}x_1 + \tilde{3}x_2 \leq E(\tilde{b}_2) - \sqrt{3Var(\tilde{b}_2)} + 1.9\sqrt{3Var(\tilde{b}_2)}$

$\tilde{1}x_1 + \tilde{4}x_2 \leq E(\tilde{b}_3) - \sqrt{3Var(\tilde{b}_3)} + 1.6\sqrt{3Var(\tilde{b}_3)}$

$x_1, x_2 \geq 0$  

(4.9)
Further considering the fuzzy numbers associated with the system constraints of model (4.9), the model can be decomposed into the following FP problem as

Maximize $Z_1 = x_1 + 3x_2$

Maximize $Z_2 = x_1 + 5x_2$

Subject to

$$(1.95 + 0.05\alpha)x_1 + (0.95 + 0.05\alpha)x_2 \
\leq (6.4 - 0.4\alpha) - \sqrt{3(4.5 - 0.5\alpha)} + 1.8\sqrt{3(4.5 - 0.5\alpha)}$$

$$(2.05 - 0.05\alpha)x_1 + (1.05 - 0.05\alpha)x_2 \
\leq (6.4 - 0.4\alpha) - \sqrt{3(4.5 - 0.5\alpha)} + 1.8\sqrt{3(4.5 - 0.5\alpha)}$$

$$(4.95 + 0.05\alpha)x_1 + (2.95 + 0.05\alpha)x_2 \
\leq (6.5 - 0.5\alpha) - \sqrt{3(4.6 - 0.6\alpha)} + 1.9\sqrt{3(4.6 - 0.6\alpha)}$$

$$(5.05 - 0.05\alpha)x_1 + (3.05 - 0.05\alpha)x_2 \
\leq (6.5 - 0.5\alpha) - \sqrt{3(4.6 - 0.6\alpha)} + 1.9\sqrt{3(4.6 - 0.6\alpha)}$$

$$(0.95 + 0.05\alpha)x_1 + (3.95 + 0.05\alpha)x_2 \
\leq (6.7 - 0.7\alpha) - \sqrt{3(4.4 - 0.4\alpha)} + 1.6\sqrt{3(4.4 - 0.4\alpha)}$$

$$(1.05 - 0.05\alpha)x_1 + (4.05 - 0.05\alpha)x_2 \
\leq (6.7 - 0.7\alpha) - \sqrt{3(4.4 - 0.4\alpha)} + 1.6\sqrt{3(4.4 - 0.4\alpha)}$$

$x_1, x_2 \geq 0, \ 0 \leq \alpha \leq 1.$ (4.10)

Now to find the fuzzy goals of each of objectives of the above model, each objective is considered independently and is solved with respect to the same set of system constraints defined in (4.10). The best and worst values of the objectives are obtained as

$Z_1^b = 6.742, \ Z_1^w = 0; \ Z_2^b = 10.962; \ Z_2^w = 0.$

So the fuzzy goals of the objectives are found as $Z_1 \geq 6.742, Z_2 \geq 10.962.$

Hence the membership functions are constructed by using the procedure described in (4.6) as

$$\mu_{Z_1}(x_1, x_2) = \frac{x_1 + 3x_2}{6.742}, \ \mu_{Z_2}(x_1, x_2) = \frac{x_1 + 5x_2}{10.962}.$$

Assigning unity as the aspiration level of the defined membership functions and introducing under and over deviational variables to each of them following FGP model is presented as
Find \( X(x_1, x_2) \)

so as to

\[
\begin{align*}
\text{Min } D &= 0.148d_1^- + 0.091d_2^- \\
\text{and satisfy } & 0.148(x_1 + 3x_2) + d_1^- - d_1^+ = 1 \\
& 0.091(x_1 + 5x_2) + d_2^- - d_2^+ = 1
\end{align*}
\]

subject to the system constraints defined in (4.10)

\[
\begin{align*}
x_1, x_2 & \geq 0, \ 0 \leq \alpha \leq 1 \\
d_1^-, d_1^+, d_2^-, d_2^+ & \geq 0 \text{ with } d_1^- \cdot d_1^+ = d_2^- \cdot d_2^+ = 0
\end{align*}
\]  \( (4.11) \)

The software LINGO (ver. 11.0) is used to solve the problem.

The optimal solution of the problem is obtained as \( x_1 = 0.739 \) and \( x_2 = 2.001 \) with the objective values \( Z_1 = 6.742, Z_2 = 10.744 \). The achieved membership values of the objective goals are obtained as \( \mu_{Z_1} = 1, \ \mu_{Z_2} = 0.98 \).

The solution achieved here is most satisfactory in terms of achieving aspired goal levels of the objectives in the decision making environment.

Again, the solution obtained by Sinha et al. [345] is found as \( x_1 = 0.282, x_2 = 1.729 \) with the corresponding objective values \( Z_1 = 5.47 \) and \( Z_2 = 8.929 \). The result shows that both the value of objectives obtained by Sinha et al. [345] is inferior in compare to the objective values obtained by using the developed methodology. The comparison of achieved goal values of the objectives is clearly presented by the following diagrammatic representation.

![Comparison of Membership Values](image)

**Figure 4.1: Comparison of Membership Values**
4.6 Conclusions and Scope for Future Studies

The developed methodology in this chapter is very much efficient to solve FMOCCP problems with uniform distribution from a view point to solve the problems with joint occurrence of fuzziness and randomness under simultaneous considerations. The proposed model can also be applied to the FMOCCP models in which the objective functions also follow uniform or some other type of probability distributions. This technique can also be extended to solve FMOCCP problems in an imprecisely defined probabilistic hierarchical decision making context for making most satisfactory decision at each hierarchical levels. However it is hoped that the derived methodology may open up new vistas into the way of making decisions from a current complex ambiguous and probabilistic decision making environment.