Chapter 5:

Tooth Shaped Grating Assisted Geometry
for Compact Multimode Interference Coupler

Introduction

Mathematical Model of Grating Assisted Multimode Interference (GA-MMI) Couplers

Results and Discussion

Conclusion
5.1. Introduction

Since total length of photonic integrated device (PID) component is contributed by beat length and access waveguide length; the compactness of device requires reduction of access waveguide length having S-bend. As discussed in chapter-4, the reduction of beat length has been considered in the components as the length of large scale integrated optic devices such as wavelength division multiplexer/demultiplexer, optical matrix switches etc. for all optical networks. For the reduction of total device length it is very much essential to study the longitudinal access waveguide length (also known as transition length) for Two Mode Interference (TMI) coupler as well as Multimode Interference (MMI) coupler with other potential structures such as grating assisted geometry and also in the conventional structures. In this direction, Multimode Interference (MMI) coupler [1]-[5] based devices have become attractive due to having lower access waveguide bending losses than that of TMI coupler. As per our knowledge no study is made on surface relief grating assisted structure of multimode interference coupler. So, tooth shaped grating assisted geometry has been proposed for the reduction of device length of MMI coupler and studied the same structure for MMI coupler in this chapter. The coupling behavior of grating assisted MMI (GA-MMI) coupler have been analyzed theoretically using the mathematical model based on sinusoidal mode Simple Effective Index Method (SEIM) [6]-[10] as mentioned in chapter-3 and chapter-4. The coupling characteristics, beat length and fabrication tolerances for GA-MMI coupler have been compared with GA-TMI coupler. It is observed that, although beat length of GA-TMI coupler with grating width ($\Delta W$) =0.25 $\mu$m is $\sim$1.6 times less than that of GA-MMI coupler with grating width ($\Delta W$)=0.25 $\mu$m, but the total device length of GA-MMI coupler by inclusion of access waveguide length with permissible bending loss of 0.01 dB is $\sim$1.5 % less than GA-TMI coupler. The dependence of access waveguide length on h with fixed value of S-bending loss for grating assisted MMI (GA-MMI) structure and tooth shaped grating assisted two-mode interference (GA-TMI) structure are discussed. The effect of fabrication tolerance on power imbalance of GA-MMI coupler is also
studied whereas these SEIM results are compared with the results obtained by commercially available beam propagation method (BPM) [11]-[12] based optiBPM software (V 9.0).

5.2. Grating Assisted MMI (GA-MMI) Coupler

Like conventional MMI coupler the tooth shaped grating assisted multimode interference (GA-MMI) coupler is based on the principle of self imaging principles[9]. When light is launched through access waveguide of MMI coupler, higher order modes are excited with fundamental and first order modes. These excited modes are interfered with each other along the direction of propagation where multiple reflections of evanescent lightwave occur within the grating geometry.

![Diagram of 2x2 tooth shaped grating assisted multimode interference (GA-MMI) coupler](image)

Fig-5.1: Schematic 3D diagram of 2x2 tooth shaped grating assisted multimode interference (GA-MMI) coupler

Fig-5.1 shows the schematic three dimensional (3D) diagram of a 2x2 compact tooth shaped grating assisted multimode interference (GA-MMI) coupler with a
channel waveguide consisting of two single mode input S-bent access waveguides of core width a and thickness b (Waveguide-1 & Waveguide-2), two single mode output S-bent access waveguides of same core size (Waveguide-3 & Waveguide-4) and a coupling region of length L with guiding width, \( W_m = 2a + h \) (where \( h \) is the gap between two input access waveguides) and grating width, \( W_g = W_m + 2\Delta W \) (where \( \Delta W \) is tooth width of grating) placed alternately. As mentioned in the previous chapter-4, tooth shaped rectangular grating geometry has been considered for the case study due to better compactness and simpler for applications. The coupling region consists of N total numbers of grating period (\( \Lambda = l_m + l_g \), where \( l_m \) and \( l_g \) are the length of guiding width (\( K=m \)) and grating width (\( K=g \)) in each grating period respectively. The \( n_1 \) is the refractive index of core whereas \( n_2 \) is the refractive index of the cladding region. When the input power \( P_1 \) is launched into lower most input S-bent access waveguide (Waveguide-2), the output powers \( P_3 \) and \( P_4 \) are obtained as bar state and cross state respectively.

When the input signal mode field of propagation constant \( \beta_i (\lambda) \) is incident through input single mode S-bent access waveguide (Waveguide-2), multiple modes are excited in the grating assisted MMI coupling region. At the end of GA-MMI region based on relative phase assisted MMI coupling region. At the end of GA-MMI region based on relative phase difference between these modes in the region, the light power is coupled into two single mode S-bent output access waveguides (Waveguide-3 and Waveguide-4). Since fundamental and first order modes carry most of signal power, the beat length (defined as the length for \( \pi \) phase shift) for the MMI couplers assisted with total N numbers of grating period is obtained as,

\[
L_x = \frac{\pi}{[\beta_{00}^m - \beta_{01}^m] + (\beta_{00}^g - \beta_{01}^g)}
\]

(5.1)

where \( \beta_{00}^m, \beta_{01}^m \) and \( \beta_{00}^g, \beta_{01}^g \) are the propagation constants of fundamental and first order modes in the guiding region and grating region respectively.

Fig-5.2(b) shows a two dimensional (2D) cross sectional schematic view of 2x2 tooth shaped grating assisted multimode interference (GA-MMI) coupler of Fig-5.1
whereas Fig-5.2(a) shows the 3D view of the guided layer. As GA-MMI coupling region in transverse dimension (along Y-axis) is smaller (minimum two times as mentioned later in this chapter) than the lateral dimension (along X-axis) and have the same transverse behavior in everywhere of GA-MMI coupling region (in the XZ-plane), it is justified to be assumed that the waveguide structure is to be single mode in transverse dimension. So the mode fields in grating assisted MMI couplers can be represented in two dimensionally.

**Fig-5.2:** Schematic diagram of tooth shaped grating assisted multimode interference (GA-MMI) coupler (a) 3D view (b) 2D cross sectional view with x and z axis.

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The input field profile $H(x,0)$ launched into tooth shaped grating assisted multimode interference (MMI) coupler ($z=0$) is composed of mode field distribution of all modes excited in GA-MMI region and in 2D approximation expressed as

$$H(x,0) = \sum_{i=0}^{c-1} b_i H_i(x)$$  \hspace{1cm} (5.2)

where $b_i$ is field contribution coefficient of tooth shaped grating assisted MMI coupler for $i^{th}$ order mode and $H_i(x)$ is mode field distribution of $i^{th}$ order mode at $z=0$.

The composite field profile at a distance $z$ inside GA-MMI region can be represented in 2D approximation as a summation of all the guided modes.

$$H(x,z) = \sum_{i=0}^{c-1} H_i(x,z) = \sum_{i=0}^{c-1} b_i H_i(x) \exp \left[ j \left( \beta_i^K - \beta_0^K \right) z \right]$$  \hspace{1cm} (5.3)

where $i = 0, 1, 2, \ldots, (r-1)$ denotes the order of guided modes, $\beta_0^K$ is the propagation constant of zeroth order (fundamental mode) and $\beta_i^K$ represents the propagation constant for $i^{th}$ order mode respectively. The $K=m$ represents the width of guided region whereas $K=g$ is the width of grating region.

Since the width of the access waveguide ($a=1.5 \, \mu m$) is required to be small for single mode operation of the access waveguide by keeping the normalization frequency $V \sim 2.3$, the lateral penetration of the mode field outside the waveguide is negligible for the lateral high index contrast ($\Delta n$). Thus input mode field profile $H_i(x)$ for the $i^{th}$ mode can be approximated for tooth shaped grating assisted multimode interference (GA-MMI) region as,

$$H_i(x) = \sin \left[ (i+1) \frac{\pi x}{W_g} \right]$$  \hspace{1cm} (5.4)

At the end of tooth shaped grating assisted MMI coupling section, optical power is either transferred to the output S-bent access waveguide or lost out at the end of tooth shaped grating assisted MMI waveguide. The mode field of output access
waveguides is contributed by all guided modes propagated in grating assisted MMI region. The mode fields at M-th output S-bent access waveguide can be written as

\[
H_M^K(x, L) = \sum_{i=0}^{N-1} H_{M,i}^K(x, L) = \sum_{i=0}^{N-1} c_{M,i}^K \exp[j(\beta_0^K - \beta_i^K) L]
\]

(5.5)

where \( L = (N+1)l_m N l_g \) and \( c_{M,i} = \sqrt{C_{M,i}^K} \) is the \( i \)th order mode's contribution coefficient to the M-th output access waveguide (M=3 for the \( 3^{rd} \) access waveguide and M=4 for the \( 4^{th} \) access waveguide), which can be calculated by using mathematical model based on sinusoidal mode simple effective index method (SMSEIM) as discussed in section-4.2.1 of previous chapter-4 with consideration \( n_3 \rightarrow n_1 \) (\( h \neq 0 \)), we have

\[
\frac{C_{M,i}^K}{C_0} = \frac{\pi^2}{16 \beta^2 k^4 (n_2^2 - n_3^2)} \exp\left[-h(k_0^2 n_2^2 - n_3^2)\right] \times \exp\left[k_0^2 (n_2^2 - n_3^2)\right] \exp\left[k_0^2 (n_2^2 - n_3^2)\right]
\]

(5.6)

where for TE mode,

\[
C_s = \frac{0.4}{F_c} \times \frac{(n_2^2 - n_{\text{eff}(TE),K}^2) \sqrt{n_{\text{eff}(TE),K}^2 - n_2^2}}{n_{\text{eff}(TE),K} \left(n_i^2 - n_3^2\right) \left[W_K + \frac{2}{k_0 \sqrt{n_{\text{eff}(TE),K}^2 - n_2^2}}\right]}
\]

(5.7)

\[
F_c = \frac{3(1 + 0.2 h)}{[13.5 + 185 (\beta_0^K - \beta_i^K)] h}
\]

(5.8)

\[
n_{\text{eff}(TE),K} = \beta_0^K \left(\frac{\lambda}{2 \pi}\right) ; K = m, g
\]

(5.9)

Similarly, for TM mode,

\[
C_v = \frac{0.4}{F_c} \times \frac{(n_2^2 - n_{\text{eff}(TM),K}^2) \sqrt{n_{\text{eff}(TM),K}^2 - n_2^2}}{n_{\text{eff}(TM),K} \left(n_i^2 - n_3^2\right) \left[W_K + \frac{2}{k_0 \sqrt{n_{\text{eff}(TM),K}^2 - n_2^2}}\right]}
\]

(5.10)

\[
F_c = \frac{3(1 + 0.2 h)}{[13.5 + 185 (\beta_0^K - \beta_i^K)] h}
\]

(5.11)
\[ n_{\text{eff}}^{(\text{TM} \ \text{+} \ K)} = \beta_{\text{TM} \ \text{+} \ K} \left( \frac{\lambda}{2\pi} \right) \ ; \ K = m, g \] (5.12)

The contributed power to the M-th output S-bent access waveguide by \( i^{th} \) mode is
given by [13]-[14],

\[ P_{\text{M}}^{i} = \left| H_{\text{M},j}^{K} (x,L) \right|^2 \] (5.13)

Normalized power coupled to the \( M^{th} \) output access waveguide for tooth shaped GA-
MMI coupler can be approximated as,

\[
\frac{P_{\text{M},j} (x,L)}{P_{\text{M},j} (x,0)} = \left[ \sum_{i=0}^{r-1} \left| H_{\text{M},i}^{K} (x,L) \right|^2 \right]^{-1} \left[ \sum_{i=0}^{r-1} \left| H_{1,i}^{K} (x,0) \right|^2 \right]
\]

\[
= \sum_{i=0}^{r-1} \sum_{j=0}^{N} \left[ C_{M,i}^{K} C_{M,j}^{K} H_{i}^{K} (x) \right] \left[ \sum_{j=0}^{q} q_{j} \cos \left( \sum_{i=0}^{r-1} \left( N+q_{j} \right) \left| \epsilon_{i}^{K} - \beta_{L}^{K} \right| \right) \right]\] (5.14)

where \( i, j = 0, 1, 2, \ldots, (r-1) \) are the order of modes provided \( j \geq i, q_{K} = 0, 1 \) for
grating region \( (K=m) \) and guided region \( (K=g) \) respectively, \( N=\)Number of grating period
and \( C_{M,i}^{K}, C_{M,j}^{K} \) =contribution coefficients \( (\text{measure of field contribution of } i^{th} \text{ and } j^{th}
\text{ order modes to lower output access waveguides}) \) that are obtained using equations
(5.6)-(5.12), \( \beta_{L}, \beta_{L}=\)propagation constant for \( i^{th} \) and \( j^{th} \) modes which are determined
from dispersive equations \( (\text{as discussed in section-3.2.2 of chapter-3}) \). The length of
the guiding width \( (l_{m}) \) and grating width \( (l_{g}) \) is determined by using the following
relation (5.15) [15][16],

\[ l_{K} = \frac{\lambda}{a_{\text{eff}}(x,K)} \ ; \ K = m, g \] (5.15)

Thus, the contributed power to the 3\textsuperscript{rd} S-bent access waveguide-3 \( (\text{bar state}) \) by \( i^{th}
\text{ order mode is given by,}

\[ P_{j}^{i} = \left| H_{3,i}^{K} (x,L) \right|^2 \] (5.16)
Similarly, normalized power coupled to the 3rd S-bent access waveguide by $i^{th}$ order mode for tooth shaped GA-DC can be approximated as,

$$\frac{P_{3i}(x,L)}{P_{1i}(x,0)} = \left| \sum_{i=0}^{r-1} H_{3i}^{K}(x,L) \right|^2 \left( \sum_{j=0}^{r-1} H_{1j}^{K}(x,0) \right)^2$$

$$\approx \sum_{m=0}^{r-1} C_{3i}^{K} H_{3i}^{K}(x) + \sum_{m=0}^{r-1} \sum_{j|m+1}^{r-1} 2 \sum_{K=m+1}^{r-1} C_{3i}^{K} C_{3j}^{K} H_{i}^{K}(x) H_{j}^{K}(x) \cos \left( \sum_{K=m+1}^{r-1} \left( N + q_{K} \right) (\beta_{K}^{i} - \beta_{K}^{j}) \lambda_{K} \right) \right)$$

where $C_{3i}^{K} = (c_{3i}^{K})^2$ and $c_{3i}^{K}$ is the contribution coefficient of $i^{th}$ mode (which can be calculated by using a mathematical model based on SM-SEIM) for the 3rd output access Waveguide-3.

Normalized power coupled to the output access waveguide-4 (cross state) by $i^{th}$ order mode for tooth shaped GA-MMI can be approximated as,

$$\frac{P_{4i}(x,L)}{P_{1i}(x,0)} = \left| \sum_{i=0}^{r-1} H_{4i}^{K}(x,L) \right|^2 \left( \sum_{j=0}^{r-1} H_{1j}^{K}(x,0) \right)^2$$

$$\approx \sum_{m=0}^{r-1} C_{4i}^{K} H_{4i}^{K}(x) + \sum_{m=0}^{r-1} \sum_{j|m+1}^{r-1} 2 \sum_{K=m+1}^{r-1} C_{4i}^{K} C_{4j}^{K} H_{i}^{K}(x) H_{j}^{K}(x) \cos \left( \sum_{K=m+1}^{r-1} \left( N + q_{K} \right) (\beta_{K}^{i} - \beta_{K}^{j}) \lambda_{K} \right) \right)$$

where $C_{4i}^{K} = (c_{4i}^{K})^2$ and $c_{4i}^{K}$ is the contribution coefficient of $i^{th}$ mode (which can be calculated by using a mathematical model based on SM-SEIM) for the 4th output access Waveguide-4.

The transition length ($L_T$) of the S-bent access waveguide (along the $z$
direction) from the Fig.-5.2(b) can be obtained as follows,

\[ L_T = \sqrt{\left( H - \frac{h}{2} \right)^2 + \frac{4R + h}{2} - H} \]  \hspace{1cm} (5.19)

where R, H and h are bending radius, height and coupling gap between two access waveguides respectively. The bending loss (T_s) in dB for S-bent access waveguides can be approximated as [17]-[18],

\[ T_s = 4.343 \alpha S \]  \hspace{1cm} (5.20)

where \( \alpha \) = loss coefficient that mainly depends on bending radius R and

\[ S = 2R \cos^{-1}\left[1 - \frac{\left(H - \frac{h}{2}\right)}{2R}\right] \]  \hspace{1cm} (5.21)

5.2.1 Multiple Reflection in Grating Assisted MMI Coupler

![Diagram](image)

**Fig-5.3**: Schematic ray diagram showing the multiple reflections occur in the tooth shaped grating structures
Fig. 5.3 shows the schematic ray diagram of the multiple reflections that occur in the tooth shaped grating geometry. The ray with red colour is reflected at the guiding width (K=m) whereas the yellow colored rays are getting multiple reflections in the grating width (K=g). It is observed that the path travelled by light in grating assisted structure is more than that of conventional structures, showing the path difference in grating assisted geometry is more that in conventional structures.

5.2.2 Coupling Characteristics of GA-MMI Coupler

![Graph showing coupling characteristics](image)

**Fig-5.4:** Normalized coupling power distribution of tooth shaped grating assisted geometry of two mode interference (GA-TMI) coupler with coupling gap, h=0.0 μm (solid line) and multimode interference (GA-MMI) couplers (dashed lines) for h=3.0 μm, 4.0 μm, 5.0 μm, 6.0 μm with ΔW=0.25 μm, a=1.5 μm, b=1.5 μm, cladding index~1.45, Δn=5 % and λ~1.55 μm respectively.

Fig-5.4 shows the normalized coupling power distribution versus number of grating (N) obtained by using the equations (5.6) and (5.18) for different waveguide
separation gaps, h=0.0 μm, 3.0 μm, 4.0 μm, 5.0 μm and 6.0 μm for the tooth shaped grating assisted multimode interference (GA-MMI) coupler with \( \Delta W=0.25 \) μm, \( a=1.5 \) μm, \( b=1.5 \) μm, \( l_m=0.26 \) μm, \( l_p=0.26 \) μm, \( \Delta n=5 \) %, cladding index\textasciitilde1.45 and wavelength (\( \lambda \))\textasciitilde1.55 μm respectively. In the figure h=0.0 μm corresponds to grating assisted TMI coupler. It is seen from the figure that peak cross coupling power (\( P_d/P_1 \)) is obtained at beat lengths where N values are 41, 70, 85, 105 and 134 for h=0.0 μm, 3.0 μm, 4.0 μm, 5.0 μm, and 6.0 μm respectively. So the beat lengths obtained using equation (5.1) are \( \approx 22.2 \) μm, \( 36.0 \) μm, \( 40.0 \) μm, \( 57.8 \) μm and \( 70.5 \) μm for h=0.0 μm, 3.0 μm, 4.0 μm, 5.0 μm and 6.0 μm respectively. The increase of beat length with increase of h is mainly due to excitation of higher order modes (apart from lower order modes) having less coupling efficiency as these modes are partly transferred. It is seen (not mentioned in the figure) that the number of modes excited in GA-TMI and GA-MMI coupler for h=0.0 μm, 3.0 μm, 4.0 μm, 5.0 μm, and 6.0 μm is two, four, five and six respectively. It is evident from the figure that as h decreases, the peak normalized cross coupling power decreases. This is due to increase of the radiation losses at the bending portion of the input/output access waveguides with decrease of h, which is evident from the equation (5.20) and (5.21) respectively. It is also seen (discussed latter on in this chapter) that the polarization dependence of GA-MMI coupler is almost equivalent to the GA-TMI coupler (h=0.0 μm) but is slightly more than conventional MMI/TMI couplers (as details are discussed in chapter-3, section-3.3.6 and section-3.4.6) because the number of waveguide parameters in the grating assisted geometry is more than that of conventional structures. Although \( L_p \) of GA-MMI with h=3 μm is lower than that of GA-MMI with h=4 μm, we have chosen h=4 μm because of lower bending loss which is discussed later in this chapter.

5.2.3 Beat Length of GA-MMI Coupler

Fig-5.5 shows the beat length (\( L_\alpha \)) versus index contrast (\( \Delta n \)) of tooth shaped
Grating assisted multimode interference (GA-MMI) coupler for $\Delta W=0.05 \, \mu m$, 0.1 $\mu m$, 0.25 $\mu m$ and conventional MMI coupler ($\Delta W=0 \, \mu m$) with $a=1.5 \, \mu m$, $b=1.5 \, \mu m$, $h=4.0 \, \mu m$, $W_m (\sim 2a+h)=7.0 \, \mu m$, cladding index $\sim 1.45$ and wavelength $\sim 1.55 \, \mu m$.

**Fig-5.5:** Beat length ($L_\pi$) versus index contrast ($\Delta n$) of tooth shaped grating assisted multimode interference (MMI) couplers (dashed lines) with $\Delta W=0.05 \, \mu m$, 0.1 $\mu m$, 0.25 $\mu m$, $h=4 \, \mu m$ and conventional MMI coupler ($\Delta W=0 \, \mu m$, $h=4 \, \mu m$) (solid line).

It is observed from the plot that as the index contrast ($\Delta n$) increases, the beat length decreases and it slowly decreases for $\Delta n>5 \%$. The variation of the beat length with $\Delta n$ for $\Delta W=0.05 \, \mu m$ are almost close to that for $\Delta W=0.1 \, \mu m$ and 0.25 $\mu m$ but the beat length for conventional MMI coupler is $\sim 2$ times higher than that for tooth shaped grating assisted MMI coupler ($\Delta W\neq0 \, \mu m$). For fabrication advantage, we have chosen $\Delta W=0.25 \, \mu m$ and we have also chosen $\Delta n=5 \%$ for further study. For $\Delta n=5 \%$ and $\Delta W=0.25 \, \mu m$, it is found that the beat length of tooth shaped grating assisted multimode interference (MMI) couplers is $\sim 50 \%$ lower than that of conventional
MMI couplers. The less beat length in GA-MMI coupler than that of conventional MMI coupler is due to multiple reflection occurred in the tooth shaped grating region as shown in section-5.2.1.

![Graph showing beat length vs index contrast for conventional and GA-MMI couplers](image)

**Fig-5.6:** Beat length ($L_\alpha$) versus index contrast ($\Delta n$) of tooth shaped grating assisted multimode interference (MMI) couplers (dashed line) for waveguide separation gaps, $h \sim 4 \mu m$ with $\Delta W = 0.25 \mu m$ and conventional MMI coupler ($\Delta W = 0 \mu m$), $h = 4 \mu m$ (solid line) respectively.

Fig-5.6 shows the beat length ($L_\alpha$) versus index contrast ($\Delta n$) of tooth shaped grating assisted structures of MMI (GA-MMI) coupler with $h = 4.0 \mu m$ for $\Delta W = 0.25 \mu m$ and conventional MMI couplers ($\Delta W = 0 \mu m$) with $a = 1.5 \mu m$, $b = 1.5 \mu m$, $h = 4.0 \mu m$, $W_m \sim 2a + h$, $W_g \sim W_m + 2\Delta W$, cladding index $\sim 1.45$ and $\lambda \sim 1.55 \mu m$ respectively. It is seen that beat length decreases as the index contrast ($\Delta n$) increases and for $\Delta n > 5\%$, $L_\alpha$ decreases slowly. It is also seen that the beat lengths with $\Delta n = 5\%$ for GA-MMI coupler ($\Delta W = 0.25 \mu m$) and conventional MMI coupler ($\Delta W = 0 \mu m$) are obtained as $\sim 40 \mu m$ and $81 \mu m$ respectively. So the beat length of tooth shaped grating assisted
multimode interference (GA-MMI) couplers is ~50% lower than that of conventional MMI coupler. The lesser beat length in GA-MMI coupler than that of conventional MMI coupler is due to multiple reflections occurred in the tooth shaped grating region as seen in section 5.2.1.

5.2.4 Beam Propagation Method (BPM) Simulation Results for GA-MMI Coupler

Since as mentioned in the previous chapters: -3 and -4, before fabrication it is required to study the beam propagation performance with the designed parameters of the structures.

![Figure 5.7](image_url)

**Fig-5.7:** Normalized coupling power distribution of tooth shaped GA-MMI coupler for $h=4.0 \, \mu m$ with $\Delta W=0.25 \, \mu m$, $a=b=1.5 \, \mu m$, $\Delta n=5 \%$ and $\lambda=1.55 \, \mu m$ respectively.

Fig-5.7 shows the beam propagation results with the bar coupling ($P_3/P_1$) state and the cross coupling ($P_4/P_1$) state for tooth shaped grating assisted MMI (GA-MMI) coupler with $\Delta W=0.25 \, \mu m$, $h=4 \, \mu m$, $W_n=7.0 \, \mu m$, $a=1.5 \, \mu m$, $b=1.5 \, \mu m$, $\Delta n=5 \%$, $\lambda=1.55 \, \mu m$ obtained by using optiBPM software. The figure also shows the
lightwave propagation at half coupling point of 3 dB GA-MMI coupler and cross coupling point obtained by optiBPM software that is based on Finite Difference Time Domain (FDTD) method [6],[12]. It is seen that cross coupling point is obtained at coupling length of 40.1 μm which is almost close to that obtained by SEIM based on sinusoidal modes.

Fig-5.8: BPM results of grating assisted MMI (GA-MMI) coupler for (a) Layout with tooth shaped grating geometry, (b) Cross coupling state of beatlength~40.1 μm and (c) 3-dB coupler of beatlength~20.2 μm

From the BPM results as shown in Fig.-5.8, it is found that the beat lengths of GA-MMI and conventional MMI coupler are obtained as ~40.1 μm and 80.3 μm respectively which are almost close to that obtained with SM-SEIM method. It is also evident from the figures that the propagation loss in GA-MMI region is slightly
more than that in conventional TMI/MMI coupler due to multiple reflections in grating region. It is found that the beat length of tooth shaped grating assisted MMI (GA-MMI) coupler at cross point≈40.1 μm, 3 dB coupler≈20.2 μm and bar point≈80.3 μm with a=b=1.5 μm, h=4 μm, n₂=1.45 and Δn=5 % respectively, that are matching well with the theoretical results obtained by SEIM.

5.2.5 Fabrication Tolerances and Polarization Dependence of GA-MMI Coupler

![Diagram](image)

**Fig-5.9:** Power Imbalance characteristics versus width tolerances (δw) for tooth shaped grating assisted MMI coupler (solid line), tooth shaped grating assisted TMI coupler (dashed line) and conventional MMI coupler (dotted line) with index contrast ≈5 %, cladding index≈1.45, h≈4.0 μm, a=b=1.5 μm and λ≈1.55 μm respectively.

Since it may not be possible for accurate fabrication of device structure with exact designed parameters, it is required to study its performance degradation with a small
unwanted variation of waveguide parameters. So, the effect of fabrication tolerances \((\delta w)\) of MMI width on power imbalance of tooth shaped grating assisted MMI coupler and conventional MMI coupler \((\Delta W=0 \mu m)\) has been studied.

Fig-5.9 shows plot for power imbalance \([=10 \log_{10} (P_3/P_4)]\) versus fabrication tolerances \((\pm \delta w)\) of tooth shaped grating assisted MMI width with \(h\sim4.0 \mu m, a=1.5 \mu m, b=1.5 \mu m,\) index contrast\(-5 \%\), cladding index\(-1.45,\) and \(\lambda\sim1.55 \mu m.\) It is seen that the power imbalance increases with \(\pm \delta w\) symmetrically for both the structures and the increase of power imbalance for tooth shaped grating assisted MMI coupler is slightly more than that of conventional MMI coupler due to having more number of device parameters in tooth shaped grating assisted MMI coupler. The figure also shows the variation of power imbalance with \(\pm \delta w\) for GA-TMI coupler and the curve for the same is almost close to that of GA-MMI coupler. The rate of increase of power imbalance \((dB)\) with respect to width tolerance for GA-MMI, GA-TMI and conventional MMI couplers are approximately obtained as \(\frac{\partial}{\partial (\delta w)} [\text{Power Imbalance (dB)}] \sim0.17 \text{dB/\mu m}, 0.16 \text{dB/\mu m and 0.13 dB/\mu m respectively.}\) It is also required to study the dependence of power imbalance on wavelength for conventional MMI coupler and tooth shaped grating assisted MMI coupler.

Fig-5.10 shows power imbalance versus wavelength for \(a\sim1.5 \mu m, b\sim1.5 \mu m, h\sim4.0 \mu m,\) index contrast \(-5\%\) and cladding index\(-1.45.\) In the figure, the solid line indicates the curve for \(3 \text{ dB} \) tooth shaped grating assisted MMI coupler of coupling length \(-20.2 \mu m\) and the dotted line shows for \(3 \text{ dB} \) conventional MMI coupler of coupling length \(-40.1 \mu m.\) It is seen from the plot that and in both cases minimum power imbalance is obtained at \(\lambda\sim1.55 \mu m\) and it is almost symmetrically increased in both sides of \(\lambda\sim1.55 \mu m.\) The increase of power imbalance for tooth shaped grating assisted MMI coupler is sharp in comparison conventional MMI coupler. The dashed line in the figure represents the variation of power imbalance versus wavelength for GA-TMI coupler and the curve for the same is almost superposed to that of GA-MMI coupler. So the dependence of power imbalance on fabrication
tolerance and wavelength for GA-MMI coupler is almost same as that for GA-TMI coupler.

![Power Imbalance](image)

**Fig-5.10:** Power Imbalance characteristics versus wavelength variation for tooth shaped grating assisted MMI coupler (solid line), tooth shaped grating assisted TMI coupler (dashed line) and conventional MMI coupler (dotted line) with $a=1.5 \, \mu m$, $b=1.5 \, \mu m$, $h=4.0 \, \mu m$, index contrast $\sim 5\%$ and cladding index $\sim 1.45$.

The polarization dependence characteristic of tooth shaped grating assisted MMI (GA-MMI) coupler is shown Fig-5.11. The figures shows the normalized coupling power distribution versus grating number ($N$) for both TE-mode and TM-mode with $h=4.0 \, \mu m$, $\Delta W=0.25 \, \mu m$, $a=1.5 \, \mu m$, $b=1.5 \, \mu m$, cladding index $\sim 1.45$, $\Delta n=5\%$ and $\lambda \sim 1.55 \, \mu m$ respectively. It is found that for TM-polarization the value of beatlength ($L_\lambda$) is $\sim 0.22 \%$ more than that of the TE-polarization. It is also seen that the polarization dependence of GA-MMI coupler is almost equivalent to the GA-TMI coupler ($h=0.0 \, \mu m$) but is slightly more than conventional MMI/TMI.
couplers because the number of waveguide parameters in the grating assisted geometry is more than that of conventional structures.

![Graph showing normalized coupling power distribution of tooth shaped GA-MMI coupler for both TE-mode (solid line) and TM-mode (dashed line) with h=4.0 µm, ΔW=0.25 µm, a=1.5 µm, b=1.5 µm, cladding index~1.45, Δn=5% and λ~1.55 µm respectively.]

**Fig-5.11:** Normalized coupling power distribution of tooth shaped GA-MMI coupler for both TE-mode (solid line) and TM-mode (dashed line) with h=4.0 µm, ΔW=0.25 µm, a=1.5 µm, b=1.5 µm, cladding index~1.45, Δn=5% and λ~1.55 µm respectively.

### 5.3. Comparative Study of Access Waveguide Length of GA-MMI Coupler with GA-TMI Coupler

The dependence of transition length $L_T$ and beat length $L_\pi$ on $h$ of MMI structure with tooth shaped grating is studied by considering fixed S-bending loss $T_S$ of 0.2 dB with the equations (5.1) and (5.19)-(5.21) as shown in Fig-5.12. In the figure, $h=0$ µm corresponds to TMI coupler with tooth shaped grating where $L_T$ and $L_\pi$ are obtained as 132 µm and 22.2 µm respectively for same S-bending loss. The total device length $L_{\text{tot}}$ is obtained as $2L_T + L_\pi = 286.2$ µm. It is observed from the figure that for tooth shaped grating assisted MMI coupler, beat length increases with...
increase of $h$ whereas the transition length $L_T$ decreases with $h$ for same $T_S$. The optimum value of $h$ is obtained at crossing point of the curves ($L_T$ versus $h$ and $L_\pi$ versus $h$) as $\sim 4 \ \mu m$ at which the value of $L_T$ and beat length $L_\pi$ are $\sim 114.5 \ \mu m$ and $40 \ \mu m$ respectively (same value of $h$ is already chosen in section 5.2.2). The total device length $L_{tot}$ of MMI coupler with tooth shaped grating is obtained as $2L_T + L_\pi = 269 \ \mu m$ which is $17 \ \mu m$ less than that of tooth shaped grating based TMI coupler. The figure also shows dependence of transition length ($L_T$) and beat length ($L_\pi$) on $h$ of MMI region of the proposed structures by considering fixed $S$-bending loss ($T_S$) of $0.2 \ dB$. It is seen that the beat length of conventional MMI coupler is two times larger than that of tooth shaped grating assisted MMI coupler.

Fig.5.12: Transition length ($L_T$) and Beat length ($L_\pi$) versus waveguide separation gap ($h$) variation of tooth shaped grating assisted MMI coupler (solid line) and conventional MMI coupler (dotted line) with $a=1.5 \ \mu m$, $b=1.5 \ \mu m$, index contrast $\sim 5\%$ and cladding index $\sim 1.45$. ©Tezpur University
In NxN photonic matrix switching applications, it is required to keep maximum access waveguide bending loss of 0.1 dB due to large scale integration [19]-[20]. So we have studied the reduction of bending loss, $T_s$, (dotted line) with increase of $h$ for tooth shaped grating assisted MMI coupler with $a=1.5 \ \mu m$, $b=1.5 \ \mu m$, index contrast $\sim 5 \%$ and cladding index $\sim 1.45$, as shown in Fig-5.13. The figure also shows the variation of beat length ($L_\pi$) with coupling gap ($h$) as a solid line. It is found that as $h$ increases beat length increases whereas bending loss decreases with increase of $h$ and the optimum value of $h$ is obtained at crossing point of the curves (bending loss versus $h$ and $L_\pi$ versus $h$) as $\sim 4 \ \mu m$ at which the value of the bending loss and beat length $L_\pi$ are $\sim 0.1 \ dB$ and $40 \ \mu m$ respectively.

**Fig-5.13:** Bending loss ($T_s$) and Beat length ($L_\pi$) versus waveguide separation gap ($h$) variation for tooth shaped grating assisted MMI coupler with $a=1.5 \ \mu m$, $b=1.5 \ \mu m$, index contrast $\sim 5 \%$ and cladding index $\sim 1.45$ respectively.
5.4. Design Device Parameters

<table>
<thead>
<tr>
<th>Design Parameters</th>
<th>GA-TMI Coupler</th>
<th>GA-MMI Coupler</th>
<th>Conventional TMI Coupler</th>
<th>Conventional MMI Coupler</th>
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<tr>
<td>Core waveguide width (a), μm</td>
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<td>Grating region length (lₘ₂), μm</td>
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<td>Access Waveguide length (Lₘ₁), μm</td>
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<td>327</td>
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</table>

Table-5.1 shows the design parameters that is considered for the designed of tooth shaped grating assisted TMI (GA-TMI) coupler and grating assisted multimode interference (GA-MMI) coupler as discussed in the current chapter. The device
lengths \(2L_r+L_a\) of GA-TMI coupler and GA-MMI coupler are obtained as \(\sim 286.2\) \(\mu m\) and \(269\) \(\mu m\) respectively which shows that device length of GA-MMI coupler is \(\sim 17\) \(\mu m\) less than GA-TMI coupler. For comparison, the device parameters of conventional TMI coupler and MMI couplers are also mentioned. It is found that the beat lengths of tooth shaped grating assisted structures are \(\sim 50\%\) lower than that of conventional structures.

5.5. Conclusion

In this chapter, a compact 2x2 tooth shaped grating assisted multimode interference (MMI) coupler has been studied by using a mathematical model based on sinusoidal modes simple effective index method (SM-SEIM). It is found that the beat length of tooth shaped grating assisted MMI coupler (GA-MMI) with access waveguide separation \(h=4.0\) \(\mu m\) is \(40\) \(\mu m\) which is \(\sim 50\%\) less than that of conventional MMI coupler with same value of \(h\). We have also studied dependence of access waveguide length on \(h\) with fixed value of S-bending loss for GA-MMI coupler and compared with that of tooth shaped grating assisted two-mode interference (GA-TMI) coupler. It is obtained that the device length including access waveguide length of GA-MMI coupler is less than that of GA-TMI coupler for a fixed value of access waveguide bending loss. Although the effect of fabrication tolerance on power imbalance of GA-MMI coupler is more than that of conventional MMI coupler, it is almost same as that for GA-TMI coupler. In the designed structures, each tooth shaped grating periods \((\lambda)\) consist of a guiding region \((K=m)\) of length \(l_m=0.26\) \(\mu m\) and a grating region \((K=g)\) of length \(l_g=0.26\) \(\mu m\) respectively. The fabrication of such compact photonic integrated devices with dimensions<1 \(\mu m\) require processes such as electron beam, focused ion beam (FIB) method etc. The process is expensive and due to our limited access/availability of these process/techniques, the fabrication of tooth shaped grating assisted structures which essentially requires electron beam technique could not be done. And as such the designs without grating (specifically conventional structures and structures with
double S-bend) with dimensions≥1 µm have been fabricated with standard photolithography process which is discussed in the preceding chapter-6 and chapter-7 respectively.

References
10. Chiang, K. S. Analysis of the effective-index method for the vector modes of


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