Chapter 3:

Transformation Relationship of Directional Coupler with Two Mode Interference (TMI) Coupler and Multimode Interference (MMI) Coupler by using Simple Effective Index Method (SEIM)

Introduction
Directional Coupler
Two Mode Interference (TMI) Coupler
Multimode Interference (MMI) Coupler
Comparative Study of Directional Coupler with TMI Coupler and MMI Coupler
Conclusion
3.1. Introduction

As reviewed in chapter -1 and -2, the Photonic Integrated Devices (PID) have become essential devices in application of all optical networks due to its reliability, immunity to vibration and electromagnetic interference, low loss transmission, small size, light weight and low power consumption. Further, in comparison to fiber devices, PID’s are more compact and of low cost due to its capability of mass production. The basic components of these PIDs are Directional Coupler (DC) [1]-[4], Two Mode Interference (TMI) coupler [5]-[8], Multimode Interference (MMI) coupler [9]-[12], X-branches [13], Y-branches [13] and Mach-Zhender (MZ) structure [13]-[14] etc. Further, directional coupler [15]-[16], MMI coupler [17]-[19] and TMI coupler [20]-[21] based devices have been widely used in the applications of optical networks due to its attractive properties such as compactness, tolerance to a range of fabrication parameters, an inherent balance and low optical loss.

In this chapter, a mathematical model using Simple Effective Index Method (SEIM) [3],[4],[22]-[25] based on sinusoidal modes have been developed for theoretical analysis of coupling characteristics of directional coupler, TMI coupler and MMI coupler with embedded rectangular core waveguide. In section-3.2, coupling behavior of DC and its coupling characteristics has been discussed using SEIM. Further, theoretical analysis of coupling characteristics of TMI coupler using the SEIM based numerical model is mentioned and the results of TMI coupler reported by different authors are also reviewed. Section-3.4 describes the mathematical model based on SEIM, coupling characteristics of MMI coupler and results demonstrated by previous authors. The coupling characteristics of DC, TMI couplers and MMI couplers are also compared. It is found that the TMI coupler provides the lower coupling length than the other two couplers. The normalized coupling power at the cross state and bar state of TE polarized light for these devices are also discussed and analyzed. Finally, the SEIM based results are compared with the other numerical techniques such as Marcuse theory [26] and Beam Propagation Method (BPM) [27]-[28] results obtained by using commercially available software.
based on finite difference time domain method (FDTD).

3.2. Directional Coupler (DC)

Directional Coupler (DC) consists of two dielectric waveguides placed in close proximity to each other for coupling of guided power based on phase difference of two guided modes— even mode and odd mode. As discussed in the chapter-2, the basic principle of a directional coupler is based on the coupled mode theory which describes the coupling of evanescent lightwave that occurs between two adjacent parallel waveguides through the overlapping of the evanescent waves of the propagating modes. In order to study the above, a mathematical model has been derived using Simple Effective Index Method (SEIM) based on sinusoidal modes for accurate estimation of coupling power. The Effective Index Method (EIM) is used to find approximate solutions for the propagation constants of three dimensional waveguides as details are discussed in following sections.

3.2.1 Mathematical Model Based on SEIM for DC

Fig-3.1 shows the schematic three dimensional (3D) view of a 2x2 conventional directional coupler with coupling gap 'h' consisting of a coupling region of length L, two single mode input access waveguides and two single mode output access waveguides. The coupling region consists of two parallel identical waveguides of core width 'a' and thickness 'b' placed close to each other showing small coupling separation gap 'h'. The separation between center of cores is 2d (where 2d=a+h) and n₁, n₃ are the refractive indices of the core and coupling gap’s cladding region respectively whereas n₂ is refractive index of cladding region other than coupling gap cladding region. The input single mode field of propagation constant β, incident through the access waveguide-2 excites even and odd modes in coupling region where these modes propagate with propagation constants βₑ and βₒ (where βₑ=β+C, βₒ=β−C, C=coupling coefficient). P₁ is the input incident power in waveguide-2 and output power for output access waveguide-3 and waveguide-4 are
P_3 and P_4 respectively.

Fig-3.1: Schematic 3D view of 2x2 conventional directional coupler (a) device layout and (b) waveguide layer

If the coupling takes place in the region 0<z< L, in which the even mode and odd mode are propagate with propagation constants $\beta_e$ and $\beta_o$ respectively. When the phase shift between the even and odd modes become 180° (or $\pi$), the propagation distance $L_\pi$ (also known as the beat length) is defined by [14],
\[ L_n = \frac{\pi}{\beta_c - \beta_o} \]  

(Fig-3.2: Schematic directional coupler with coupling gap, \( h \) and coupling length \( L \))

(a) 2D top view  (b) Cross sectional view along line AA'

Fig-3.2(a) shows the two dimensional (2D) schematic top view of guiding (core) layer as shown in Fig-3.1 of a conventional directional coupler with coupling gap -\( h \), whereas Fig-3.2(b) shows the cross sectional view along line AA' with core and cladding refractive indices respectively.

As reviewed in section-2.2.1 of previous chapter-2, the basic idea of the simple effective index method (SEIM) is to approximate a 2D waveguide by a one dimensional one with an effective index profile of the original structure. Considering
effective index method, 3D waveguides of DC is divided in to two 2D waveguides: firstly with light confinement along x axis and then 2D waveguides with light confinement along y-axis as shown in Fig-3.3(a).

**Fig-3.3 (a):** Effective index method solving a single slab for effective refractive index $n_x$ and resulting in an array of two slabs with refractive index, $n_x$

**Fig-3.3 (b):** Cross sectional view of SEIM applies to the directional coupler consisting of two parallel rectangular waveguide cores
In order to approximate a non-separable mode field of the original structure of the DC into separable mode field along different axis, there must be existence of a separate refractive index profile. Such separable refractive index profile can be defined by using the simple effective index method (SEIM-X) along x-direction to the original structure as shown in Fig-3.3(a). Fig-3.3(b) shows a cross sectional view of SEIM-X applies to the directional coupler along with the refractive index profile. The refractive index in the cladding region at both sides of the waveguide cores is increased by an amount of \((n_2^r - n_2^c)\) whereas at the corner region of both waveguides, it is decreased by an amount of \((n_2^2 - n_2^c)\). Considering asymptotic approximation [3], the mode field (based on sinusoidal) of a single rectangular core waveguide can be defined in terms of the function \(\psi_0(x, y)\) for the core and different cladding region as

\[
\psi_0(x, y) = \frac{\pi b}{4 a V_1} \exp \left[-V_1 \left(\frac{|x| - \frac{a}{2}}{\frac{b}{2}}\right)\right] \left[-V_1 \left(\frac{|y| - \frac{b}{2}}{\frac{b}{2}}\right)\right] \quad \text{if} \quad \frac{a}{2} \leq x \leq \frac{b}{2}, \quad \frac{b}{2} \leq y \leq \frac{b}{2}.
\]

\[
\psi_0(x, y) = \frac{\pi b}{2 a V_1} \exp \left[-V_1 \left(\frac{|x| - \frac{a}{2}}{\frac{b}{2}}\right)\right] \left[-V_1 \left(\frac{|y| - \frac{b}{2}}{\frac{b}{2}}\right)\right] \quad \text{if} \quad -\infty \leq x \leq -\frac{a}{2}, \quad -\frac{b}{2} \leq y \leq \frac{b}{2}.
\]

\[
\psi_0(x, y) = \frac{\pi b}{2 a V_1} \exp \left[-V_1 \left(\frac{|x| - \frac{a}{2}}{\frac{b}{2}}\right)\right] \left[-V_1 \left(\frac{|y| - \frac{b}{2}}{\frac{b}{2}}\right)\right] \quad \text{if} \quad \frac{a}{2} \leq x \leq \frac{b}{2}, \quad -\infty \leq y \leq -\frac{b}{2}.
\]
Core Region:

$$\psi_0(x, y) = \sin \left[ \pi \left( \frac{x + \frac{a}{2}}{a} \right) \right] \sin \left[ \pi \left( \frac{y + \frac{b}{2}}{b} \right) \right] ; -\frac{a}{2} \leq x \leq \frac{a}{2} \text{ and } -\frac{b}{2} \leq y \leq \frac{b}{2}$$

(3.3)

where $$V_1 = \frac{b}{2} k(n_1^2 - n_2^2)^{1/2}$$, $$V_2 = \frac{b}{2} k(n_1^2 - n_3^2)^{1/2}$$

3.2.2 Coupling Coefficient of Directional Coupler

According to the coupled mode theory, the normalized coupling coefficient can be defined as [26][29]-[30],

$$V \int \int_{-\infty}^{\infty} \psi_0(x + d, y)\psi_0(x - d, y) \, dx \, dy$$

$$C = \frac{\frac{b}{2} d \frac{d}{2} a}{2 \int \int_{-\infty}^{\infty} \psi_0^2(x, y) \, dx \, dy}$$

(3.4)

where $$V = \frac{b}{2} k(n_{core}^2 - n_{clad}^2)^{1/2}$$

Applying coupled mode theory for directional coupler (as shown in Fig-3.3(b)); the Eq. (3.4) can be express as,

$$V_1 \int \int_{-\infty}^{\infty} \psi_0(x + d, y)\psi_0(x - d, y) \, dx \, dy$$

$$V_2 \int \int_{-\infty}^{\infty} \psi_0(x + d, y)\psi_0(x - d, y) \, dx \, dy$$

$$C = \frac{\frac{b}{2} d \frac{d}{2} a}{2 \int \int_{-\infty}^{\infty} \psi_0^2(x, y) \, dx \, dy} + \frac{\frac{b}{2} d \frac{d}{2} a}{2 \int \int_{-\infty}^{\infty} \psi_0^2(x, y) \, dx \, dy}$$

(3.5)

where $$V_1 = \frac{b}{2} k(n_1^2 - n_2^2)^{1/2}$$ and $$V_2 = \frac{b}{2} k(n_1^2 - n_3^2)^{1/2}$$

(3.6)

Applying the boundary conditions from Eq. (3.2) and Eq. (3.3) in the Eq. (3.5) and taking integration with respect to different boundary limits, finally we obtain,

©Tezpur University 3.7
\[ C \frac{V_1}{C_0} = \frac{V_1}{2ab} \times \int_{\frac{a}{2}d+\frac{a}{2}}^{\frac{b}{2}d+\frac{a}{2}} \int_{\frac{a}{2}d+\frac{a}{2}}^{\frac{b}{2}d+\frac{a}{2}} \left[ \pi \left( \frac{y+b}{2} \right) \right] \sin \left( \frac{\pi y}{2aV_1} \right) \exp \left[ -V_1 \left( \frac{x+d-a}{b/2} \right) \right] \exp \left[ -V_1 \left( \frac{x-d-a}{b/2} \right) \right] \] 

\[ + \frac{V_2}{2ab} \times \int_{\frac{a}{2}d+\frac{a}{2}}^{\frac{b}{2}d+\frac{a}{2}} \int_{\frac{a}{2}d+\frac{a}{2}}^{\frac{b}{2}d+\frac{a}{2}} \left[ \pi \left( \frac{y+b}{2} \right) \right] \sin \left( \frac{\pi y}{2aV_2} \right) \exp \left[ -V_2 \left( \frac{x+d-a}{b/2} \right) \right] \exp \left[ -V_2 \left( \frac{x-d-a}{b/2} \right) \right] \] 

where the resultant integrand values of the denominator is \( \int_{\frac{a}{2}d+\frac{a}{2}}^{\frac{b}{2}d+\frac{a}{2}} \int_{\frac{a}{2}d+\frac{a}{2}}^{\frac{b}{2}d+\frac{a}{2}} \psi_0^2(x, y) dx dy = ab \).

\[ C \frac{V_1}{C_0} = \frac{V_1}{2ab} \times \left( \frac{\pi b}{2aV_1} \right)^2 \times \int_{\frac{a}{2}d+\frac{a}{2}}^{\frac{b}{2}d+\frac{a}{2}} \sin \left( \frac{\pi y}{2aV_1} \right) \int_{\frac{a}{2}d+\frac{a}{2}}^{\frac{b}{2}d+\frac{a}{2}} \left[ \pi \left( \frac{y+b}{2} \right) \right] \sin \left( \frac{\pi y}{2aV_1} \right) \exp \left[ -V_1 \left( \frac{x+d-a}{b/2} \right) \right] \exp \left[ -V_1 \left( \frac{x-d-a}{b/2} \right) \right] \] 

\[ + \frac{V_2}{2ab} \times \left( \frac{\pi b}{2aV_1} \right)^2 \times \int_{\frac{a}{2}d+\frac{a}{2}}^{\frac{b}{2}d+\frac{a}{2}} \sin \left( \frac{\pi y}{2aV_2} \right) \int_{\frac{a}{2}d+\frac{a}{2}}^{\frac{b}{2}d+\frac{a}{2}} \left[ \pi \left( \frac{y+b}{2} \right) \right] \sin \left( \frac{\pi y}{2aV_2} \right) \exp \left[ -V_2 \left( \frac{x+d-a}{b/2} \right) \right] \exp \left[ -V_2 \left( \frac{x-d-a}{b/2} \right) \right] \] 

\[ C \frac{V_1}{C_0} = \frac{V_1}{2ab} \times \left( \frac{\pi b}{2aV_1} \right)^2 \times \frac{b}{2} \times \frac{-b}{4V_1} \left[ \exp \left( \frac{-4V_1d}{b} \right) - \exp \left( \frac{-4V_1(d-a)}{b} \right) \right] 

\[ + \frac{V_2}{2ab} \times \left( \frac{\pi b}{2aV_2} \right)^2 \times \frac{b}{2} \times \frac{-b}{4V_2} \left[ \exp \left( \frac{-4V_2d}{b} \right) - \exp \left( \frac{-4V_2(d-a)}{b} \right) \right] \] 

\[ C \frac{V_1}{C_0} = \left( \frac{-\pi^2b^3}{64a^3V_1^2} \right) \times \left[ \exp \left( \frac{-4V_1d}{b} \right) - \exp \left( \frac{-4V_1(d-a)}{b} \right) \right] 

\[ + \left( \frac{-\pi^2b^3}{64a^3V_2^2} \right) \times \left[ \exp \left( \frac{-4V_2d}{b} \right) - \exp \left( \frac{-4V_2(d-a)}{b} \right) \right] \]
\[
\frac{C}{C_0} = \left( \frac{\pi^2 b^3}{64a^3 V_1^2} \right) \times \left[ \exp \left( -\frac{4V_1 (d-a)}{b} \right) - \exp \left( -\frac{4V_1 d}{b} \right) \right] \\
+ \left( \frac{\pi^2 b^3}{64a^3 V_2^2} \right) \times \left[ -\exp \left( -\frac{4V_2 (d-a)}{b} \right) - \exp \left( -\frac{4V_2 d}{b} \right) \right]
\]

\[
\frac{C}{C_0} = \left( \frac{\pi^2 b^3}{64a^3 V_1^2} \right) \times \left[ \exp \left( -\frac{2V_1 (2d-2a)}{b} \right) - \exp \left( -\frac{2V_1 (h+a)}{b} \right) \right] \\
+ \left( \frac{\pi^2 b^3}{64a^3 V_2^2} \right) \times \left[ -\exp \left( -\frac{2V_2 (2d-2a)}{b} \right) - \exp \left( -\frac{2V_2 (h+a)}{b} \right) \right]
\]

From the Fig.-3.3(b), substituting 2d=a + h ⇒ h=2d-a, we have

\[
\frac{C}{C_0} = \left( \frac{\pi^2 b^3}{64a^3 V_1^2} \right) \times \left[ \exp \left( -\frac{2V_1 (h-a)}{b} \right) - \exp \left( -\frac{2V_1 (h+a)}{b} \right) \right] \\
+ \left( \frac{\pi^2 b^3}{64a^3 V_2^2} \right) \times \left[ -\exp \left( -\frac{2V_2 (h-a)}{b} \right) - \exp \left( -\frac{2V_2 (h+a)}{b} \right) \right]
\]

\[
\frac{C}{C_0} = \left( \frac{\pi^2 b^3}{64a^3 V_1^2} \right) \times \exp \left( -\frac{2V_1 h}{b} \right) \times \left[ \exp \left( \frac{2V_1 a}{b} \right) - \exp \left( \frac{-2V_1 a}{b} \right) \right] \\
+ \left( \frac{\pi^2 b^3}{64a^3 V_2^2} \right) \times \exp \left( -\frac{2V_2 h}{b} \right) \times \left[ -\exp \left( \frac{2V_2 a}{b} \right) - \exp \left( \frac{-2V_2 a}{b} \right) \right]
\]

Thus, using the asymptotic analysis of SEIM model of DC [shown in Fig-3.3(a)] and equation (3.4), the normalized coupling coefficient is approximated as

\[
\frac{C}{C_0} = \left( \frac{\pi^2 b^3}{64a^3 V_1^2} \right) \times \exp \left( -\frac{2V_1 h}{b} \right) \times \left[ \exp \left( \frac{2V_1 a}{b} \right) - \exp \left( \frac{-2V_1 a}{b} \right) \right] \\
+ \left( \frac{\pi^2 b^3}{64a^3 V_2^2} \right) \times \exp \left( -\frac{2V_2 h}{b} \right) \times \left[ -\exp \left( \frac{2V_2 a}{b} \right) - \exp \left( \frac{-2V_2 a}{b} \right) \right]
\]

Considering \( a=b \) for square embedded channel waveguide and substituting the values of \( V_1, V_2 \) from Eqn. (3.6) in Eqn. (3.7) we obtain,
\[
\frac{C}{C_0} = \frac{\pi^2 b^3}{64a^3} \times \left[ \frac{2h_x \times b \times k(n_{\text{eff}}^2 - n_2^2)^{3/2}}{b^2} \right] \times \left[ \exp\left( \frac{2a \times b \times k(n_2^2 - n_2^2)^{3/2}}{2} \right) - \exp\left( \frac{-2a \times b \times k(n_2^2 - n_2^2)^{3/2}}{2} \right) \right] \\
+ \frac{\pi^2 b^3}{64a^3} \times \left[ \frac{2h_x \times b \times k(n_{\text{eff}}^2 - n_2^2)^{3/2}}{b^2} \right] \times \left[ \exp\left( \frac{2a \times b \times k(n_2^2 - n_2^2)^{3/2}}{2} \right) - \exp\left( \frac{-2a \times b \times k(n_2^2 - n_2^2)^{3/2}}{2} \right) \right] \\
\frac{C}{C_0} = \frac{\pi^2}{16b^2 k^2 (n_1^2 - n_2^2)} \exp\left( -\frac{hk(n_{\text{eff}}^2 - n_2^2)^{3/2}}{2} \right) \left[ \exp\left( \frac{hk(n_2^2 - n_2^2)^{3/2}}{2} \right) - \exp\left( \frac{-bk(n_2^2 - n_2^2)^{3/2}}{2} \right) \right] \\
+ \frac{\pi^2}{16b^2 k^2 (n_1^2 - n_2^2)} \exp\left( -\frac{hk(n_1^2 - n_2^2)^{3/2}}{2} \right) \left[ \exp\left( \frac{bk(n_1^2 - n_2^2)^{3/2}}{2} \right) - \exp\left( \frac{-bk(n_1^2 - n_2^2)^{3/2}}{2} \right) \right]
\]  

(3.8)

where \( C_0 = \frac{0.4}{(1 + 0.2h) \times \left( n_{\text{eff}}^2 - n_2^2 \right)} \times \left( n_{\text{eff}}^2 - n_2^2 \right) \left[ \frac{1}{n_{\text{eff}}^2 (n_1^2 - n_2^2)} W + \frac{2}{k_0 n_{\text{eff}}^2 (n_1^2 - n_2^2)} \right] \); for TE mode  

(3.9)

\[
C_0 = \frac{0.4}{(1 + 0.2h) \times \left( n_{\text{eff}}^2 - n_2^2 \right)} \times \left( n_{\text{eff}}^2 - n_2^2 \right) \left[ \frac{1}{n_{\text{eff}}^2 (n_1^2 - n_2^2)} W + \frac{2}{k_0 n_{\text{eff}}^2 (n_1^2 - n_2^2)} \right] \); for TM mode  

(3.10)

\[
n_{\text{eff}}(\text{TE}) = \beta_{\text{TE}} \left( \frac{\lambda}{2\pi} \right) \\
n_{\text{eff}}(\text{TM}) = \beta_{\text{TM}} \left( \frac{\lambda}{2\pi} \right)
\]  

(3.11)

where \( \beta_{\text{TE}}, \beta_{\text{TM}} \) are the propagation constants of TE and TM mode respectively that are determined from dispersive relations as discussed in section 2.3.2 of chapter-2.

In this directions, the propagation constants \( \beta_{\text{TE}}, \beta_{\text{TM}} \) for TE or TM modes as discussed above, are estimated using dispersion relations [14] of a 3D waveguide.
(shown in Fig-3.4) as follows:

![Fig-3.4: Basic Optical Waveguide Structure with three layers: Cladding layer, Waveguide Core layer (thickness T) and Substrate layer of refractive indices $n_2$, $n_1$, and $n_s$ respectively.]

The dispersion equation of TE modes for a two dimensional asymmetric step index planar waveguide along x-axis as shown in Fig-3.4, can be written as

$$V_i \sqrt{1-b_i} = (m+1)\pi - \tan^{-1} \left( \frac{1-b_i}{b_i} \right) - \tan^{-1} \left( \frac{1-b_i}{b_i+a_i} \right)$$  \hspace{1cm} (3.12)

where $m$ is an integer, $a_i$ asymmetric factor for the waveguide structure with normalized guide index, $b_i = (N^2-n_i^2)/(n_i^2-n_s^2)$ and normalized frequency, $V_i = k_0 T \sqrt{(n_i^2-n_s^2)}$ with $k_0 = 2\pi/\lambda$.

The above dispersion equation (3.12) for a two dimensional symmetric step index planar waveguide reduces to,

$$V_i \sqrt{1-b_i} = (m+1)\pi - 2 \tan^{-1} \left( \frac{1-b_i}{b_i} \right)$$  \hspace{1cm} (3.13)

where $m$ is an integer and $a_i = 0$ for symmetric waveguide and normalized frequency,

$$V_i = k_0 T \sqrt{(n_i^2-n_s^2)} \, , \, k_0 = 2\pi/\lambda$$.
Now, substituting the values of $b_1$ that satisfying the equation (3.13) in the following equation, $N_i$ is calculated as follows

$$N_i = \sqrt{n_i^2 + b_1 (n_i^2 - n_i^2)}$$

(3.14)

The dispersion equation for the 2D waveguide as shown in Fig.3.4 along y-axis, can be written as,

$$V_0 \sqrt{1 - b_{II}} = (n + 1)\pi - 2 \tan^{-1} \sqrt{\frac{1 - b_{II}}{b_{II}}}$$

(3.15)

where $n$ is an integer, normalized guide index, $b_{II} = (n_{eff}^2 - n_i^2) i (N_i^2 - n_i^2)$ and normalized frequency, $V_0 = k_o b \sqrt{(N_i^2 - n_i^2)}$ ; $k_o = \frac{2\pi}{\lambda}$ respectively.

Further, substituting the values of $b_{II}$ that satisfying the equation (3.15) in the following equation, $n_{eff}$ is calculated.

$$n_{eff} = \sqrt{n_i^2 + b_{II} (N_i^2 - n_i^2)}$$

(3.16)

Thus by calculating effective refractive index ($n_{eff}$) using simple effective index method for a three dimensional waveguide, the propagation constant is estimated as,

$$\beta = k_o n_{eff \ (TE)} = \left( \frac{2\pi}{\lambda} \right) n_{eff \ (TE)}$$

(3.17)

Similarly for TM modes, $n_{eff \ (TM)}$ can be estimated from dispersion equation of TM mode [14] and propagation constant can be estimated as,

$$\beta = k_o n_{eff \ (TM)} = \left( \frac{2\pi}{\lambda} \right) n_{eff \ (TM)}$$

(3.18)

Again, the coupling takes place in the 0<z<L region of directional coupler in which the even and odd modes are propagated with propagation constants $\beta_e$ and $\beta_o$. The phase shift between the even and odd modes becomes $\pi$ when the propagation distance $L_\pi$ is given by

$$L_\pi = \frac{\pi}{\beta_e - \beta_o} = \frac{\pi}{2C}$$

(3.19)
The coupling coefficients \( C \) for the SEIM model of directional coupler are estimated using equation (3.8)-(3.18) for both TE/TM polarizations.

3.2.3 Coupling Characteristics of DC

![Coupling Gap vs. Coupling Coefficient Graph](image)

**Fig-3.5:** Coupling characteristics of DC using SEIM with \( n_3=1.45, 1.47, 1.49, 1.4945 \) with \( a=b=1.5 \ \mu m, \ n_1=1.5, \ n_2=1.45 \) and \( \Delta n=5\% \)

Fig-3.5 shows the coupling characteristics versus coupling gap \( -h \) of directional coupler (DC) obtained by using simple effective index method (SEIM) based on sinusoidal mode for different coupling gap refractive indices \( n_3=1.45, 1.47, 1.49, 1.4945 \) with \( a=b=1.5 \ \mu m, \ n_1=1.5, \ n_2=1.45 \) and \( \Delta n=5\% \) respectively. It is found that the coupling coefficient of DC decreases as the coupling gap increases and the rate of
decrease of coupling coefficient with respect to $h$ increases as coupling gap refractive index $n_3$ increases. The cross point in the figure represents the experimental point demonstrated by previous authors [31]-[32] with SiON/SiO$_2$ matching well with the theoretical curves.

**Fig-3.6:** Normalized coupled power versus beat length for DC with coupling gap $h$=0.5 μm, a=b=1.5 μm, $n_2$=1.45, $\Delta n$=5 % and $\lambda$=1.55 μm.

The coupled power to the output access waveguide of directional coupler can be estimated using coupled mode theory [29] as discussed in Section-2.3.2 of Chapter-2 which can be defined as,

\[
\frac{P_3}{P_i} = \cos^2(CZ) \tag{3.20}
\]

\[
\frac{P_4}{P_i} = \sin^2(CZ) \tag{3.21}
\]

where $P_3$ and $P_4$ are the output power in the bar state and cross state respectively whereas $P_1$ is the incident power. The coupling coefficient (C) is determined by using
the equation (3.8) and (3.11) where Z is the length along the direction of propagation.

Fig-3.6 shows the normalized coupled power for the bar coupling (P₂/P₁) state and the cross coupling (P₂/P₁) state versus beat length (Lₐ) obtained by using the equations (3.17)-(3.18), (3.20) and (3.21) for directional coupler with coupling gap, h~0.5 μm, a=b=1.5 μm, n₂=1.45 and Δn =5 % respectively. It is seen from the figure that the peak cross-coupling power (P₂/P₁) is obtained at beat length ~91 μm for the conventional directional coupler with n₂=1.45, λ=1.55 μm and Δn = 5 % respectively.

3.2.4 Beat Length of DC

![Graph showing the relationship between Beat Length (μm) and Coupling Gap (μm)](image)

**Fig-3.7:** Beat length (Lₐ) vs coupling gap (h) for directional coupler with n₃=1.45, 1.47 and 1.49 with n₂=1.45, a=b=1.5 μm, Δn =5 % and λ=1.55 μm.

Fig-3.7 shows the beat length (Lₐ) vs coupling gap (h) of directional coupler for different coupling gap refractive index n₃=1.45, 1.47 and 1.49 with a=b=1.5 μm, n₂=1.45, Δn=5% and λ=1.55 μm respectively. From the graph, it is seen that beat
length increases as \( h \) increases. The cross signs in the figure indicates the previously reported experimental results [31] that are matching well with the theoretical results as discussed in section-3.2.2.

**Fig-3.8:** Beat length \((L_a)\) versus index contrast \((\Delta n)\) of conventional DC with \( h=1 \) \( \mu \)m, 0.5 \( \mu \)m, 0.2 \( \mu \)m with \( a=b=1.5 \mu m, \Delta n =5 \% \), \( n_2=1.45 \) and \( \lambda=1.55 \mu m \).

The Beat length \((L_a)\) versus index contrast \((\Delta n)\) of directional coupler for different waveguide separation gap \( h=1 \mu m, 0.5 \mu m, 0.2 \mu m \) with \( a=b=1.5 \mu m, \Delta n=5\% \), \( n_2=1.45 \) and \( \lambda=1.55 \mu m \) is shown in the Fig-3.8. The cross points in the figures indicate the experimental results of previous authors [31][33], which are matching well with the curves that obtain by using simple effective index method (SEIM) based on sinusoidal mode. It is seen from the figure that the beat length of directional coupler for index contrast \((\Delta n)\)~5 \% with \( a=b=1.5 \mu m, n_2=1.45, \lambda=1.55 \)
μm are obtained as 45 μm, 91 μm, and 140 μm for different coupling gaps (h) of 0.2 μm, 0.5 μm and 1 μm respectively.

3.2.5 Comparison of Coupling Characteristics Obtained by SEIM and Marcuse Theory

![Graph](image)

**Fig-3.9**: Coupling characteristics of DC using SEIM and Marcuse theory for \(n_3=1.45, 1.47, 1.49, 1.4945\) with \(a=b=1.5 \mu m, n_1=1.5, n_2=1.45\) and \(\Delta n =5 \%\).

Fig-3.9 shows the coupling coefficient versus coupling gap \(h\) for TE polarization of DC obtained by using the equation (3.8)-(3.15) of SEIM and Marcuse theory [as details are discussed in section 2.2.2 of chapter 2] for coupling gap refractive index, \(n_3=1.45, 1.47, 1.49\) and 1.4945 with \(a=b=1.5 \mu m, n_1=1.5, n_2=1.45\) and \(\Delta n =5 \%\) (where \(\Delta n =n_1-n_2\)) are compared. It is evident from the figure that the coupling coefficient of DC decreases as the coupling gap increases and the rate of decrease of
coupling coefficient with respect to \( h \) increases as coupling gap refractive index \( n_3 \) increases. The figure also shows that the curve obtained by SEIM is close to the same obtained by using Marcuse relations as details are mentioned in chapter-2.

The cross point in the figure represents the experimental point demonstrated by previous authors [31] with SiON/SiO\(_2\), matching well with the theoretical curves. The coupling coefficients of directional coupler obtain by using SEIM are compared with Marcuse theory for different coupling gaps (h) ranges from 0 \( \mu \)m to 3 \( \mu \)m and \( n_3=1.45, 1.47, 1.49, 1.4945 \) with \( a=b=1.5 \) \( \mu \)m, \( n_2=1.45 \) and \( \Delta n =5 \% \) which is shown in the Table-3.1.

### Table-3.1: Comparison of Coupling Coefficients obtained by using SEIM and Marcuse theory

<table>
<thead>
<tr>
<th>Coupling gap, ( h ) (( \mu )m)</th>
<th>Calculated Coupling Coefficient (( \mu )m(^{-1} )) for ( \Delta n=5 % )</th>
<th>Experimental results reported by previous authors [31]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SEIM</td>
<td>Marcuse theory</td>
</tr>
<tr>
<td>0</td>
<td>0.30152</td>
<td>0.291</td>
</tr>
<tr>
<td>0.5</td>
<td>0.2258</td>
<td>0.2106</td>
</tr>
<tr>
<td>1.0</td>
<td>0.14232</td>
<td>0.1331</td>
</tr>
<tr>
<td>1.5</td>
<td>0.08645</td>
<td>0.079</td>
</tr>
<tr>
<td>2.0</td>
<td>0.05387</td>
<td>0.048</td>
</tr>
<tr>
<td>2.5</td>
<td>0.034408</td>
<td>0.0314</td>
</tr>
<tr>
<td>3.0</td>
<td>0.0212</td>
<td>0.0209</td>
</tr>
</tbody>
</table>

#### 3.2.6 Beam Propagation Method (BPM) Simulation Results of Directional Coupler

From the above studies and as mentioned in the previous chapters, it is necessary to study the lightwave beam propagation with the designed device parameters before fabrication. In this direction, the design waveguide device components are studied for beam propagation with the help of commercially
available optiBPM software (version 9.0) which is based on beam propagation method (BPM) and these results are compared with results obtained by SEIM.

**Fig-3.10:** Normalized coupled power versus beat length for DC with coupling gap $h \sim 0.5 \, \mu\m$, $n_2=1.45$, $\Delta n=5 \%$, $\lambda=1.55 \, \mu\m$ respectively and the BPM output results at beatlength $\sim 91 \, \mu\m$ and 3 dB coupling length $\sim 45 \, \mu\m$ respectively.

Fig-3.10 shows normalized coupled power distribution for the bar coupling ($P_b/P_1$) state and the cross coupling ($P_c/P_1$) state versus beat length ($L_b$) for directional coupler with coupling gap, $h \sim 0.5 \, \mu\m$, $a=b=1.5 \, \mu\m$, $n_2=1.45$ and $\Delta n=5 \%$ respectively as details are discussed in section-3.2.3. The figure also shows the lightwave propagation at half coupling point (3 dB) and cross coupling point of directional coupler obtained by optiBPM software matching well with the results obtained by SEIM model. It is seen from the figure that the peak cross-coupling power ($P_c/P_1$) is obtained at beat length $\sim 91 \, \mu\m$ whereas 3dB coupling power is obtained at beat length $\sim 45.1 \, \mu\m$ for the directional coupler with $a=b=1.5 \, \mu\m$, $h=0.5 \, \mu\m$, $n_2=1.45$ $\Delta n=5 \%$ respectively.
Fig-3.11: BPM results of Conventional Directional Coupler for (a) Layout structure (b) Cross state (c) 3-dB coupler and (c) bar state

The Beam Propagation Method (BPM) results of conventional directional coupler (DC) obtained by using optiBPM software at the cross state, bar state and 3-dB directional coupler are shown in Fig-3.11. From the BPM output results, it is found that the beat length of conventional directional coupler at cross point~ 91.1 μm, 3 dB state ~45.3 μm and bar point~182 μm with a=b=1.5 μm, n₀=1.45, h=0.5 μm and Δn=5% respectively which are analogous with the theoretical results obtained by SEIM.

3.2.7 Fabrication Tolerances and Polarization Dependence of Directional Coupler

Since it may be difficult for precise fabrication of device structure with exact designed parameters, it is necessary to study its performance degradation with small
unwanted variation of waveguide parameters. Therefore, the effect of fabrication tolerances ($\delta w$) of DC width on power imbalance of directional coupler has been studied. Fig-3.12 shows plot for power imbalance [$=10 \log_{10} (P_3/P_4)$] versus fabrication tolerances ($\pm \delta w$) of DC width with cladding index~1.45, $n_1=1.45$, index contrast ~5%, $a=1.5 \mu m$ and wavelength ~1.55 $\mu m$. It is seen from the figure that power imbalance increases almost symmetrically in both sides of minimum power imbalance obtained at $\delta w=0 \mu m$ as 3 dB directional coupler is designed for this. The rate of increase of power imbalance (dB) with respect to width tolerance for the conventional directional couplers is approximately obtained as $\frac{\partial}{\partial \delta w} \text{[Power Imbalance (dB)]} \approx 0.15 \text{dB}/\mu m$ respectively.

Fig-3.12 Power imbalance characteristics versus width tolerance ($\pm \delta w$) of conventional directional coupler with cladding index~1.45, index contrast ~5%, $a=b=1.5 \mu m$ and wavelength~1.55 $\mu m$.

Fig-3.13 shows the dependence of power imbalance on wavelength for 3 dB conventional directional coupler with $a=1.5 \mu m$, $b=1.5 \mu m$, index contrast ~5% and
cladding index~1.45. It is seen from the graph that power imbalance increases almost symmetrically in both sides of minimum power imbalance obtained at \( \lambda \sim 1.55 \, \mu \text{m} \) as 3dB directional coupler is designed for this wavelength.

![Power Imbalance Characteristics](image)

**Fig-3.13:** Power Imbalance characteristics versus wavelength variation for conventional directional coupler with \( a=1.5 \, \mu \text{m}, b=1.5 \, \mu \text{m} \), index contrast \( \sim 5 \, \% \) and cladding index~1.45.

Further the polarization dependence of coupling characteristics are also studied for conventional directional coupler. Fig-3.14 shows the normalized coupling power distribution versus beat length of conventional directional coupler for both TE-mode and TM-mode with \( h=0.5 \, \mu \text{m}, a=1.5 \, \mu \text{m}, b=1.5 \, \mu \text{m} \), cladding index~1.45, \( \Delta n=5 \, \% \) and \( \lambda \sim 1.55 \, \mu \text{m} \) respectively. It is found that for TM-polarization the value of beat length \( (L_b) \) is \( \sim 0.22 \, \% \) more than that of the TE-polarization.
Fig-3.14: Normalized coupling power distribution of conventional directional coupler for both TE-mode (solid line) and TM-mode (dashed line) with \( h=0.5 \) \( \mu \text{m} \), \( a=1.5 \) \( \mu \text{m} \), \( b=1.5 \) \( \mu \text{m} \), cladding index \( \sim 1.45 \), \( \Delta n = 5 \% \) and \( \lambda \sim 1.55 \) \( \mu \text{m} \) respectively.

3.3. Two Mode Interference (TMI) Coupler

The Two Mode Interference (TMI) coupler is based on two mode interference phenomena, where the input field excites two modes–fundamental and first order modes and interfered with each other along the direction of propagation [31]. Two mode interference (TMI) coupler are consist of two single mode waveguides placed with zero separation gaps where due to the coupling and depending upon the phase difference between two excited modes, light propagates along the direction of propagation. Depending upon the phase difference at the end of coupling region, light signal can be obtained at the cross state or bar state output access waveguides respectively.
3.3.1 Mathematical Model of TMI Coupler Using SEIM

Fig. 3.15: 3D Schematic view of TMI coupler (h=0 μm) based general interference

Fig. 3.15 shows the three dimensional (3D) schematic view of the basic geometry of a two-mode interference (TMI) coupler with coupling gap (h) zero. It consists of two-mode coupling region of width 2a and coupling length L with four single mode access waveguides of core width a and core thickness b. $n_1$ and $n_2$ are the refractive index of the core and cladding region respectively, whereas the single mode access waveguides— (Waveguide-1 & Waveguide-2) are attached to the input portion of the TMI region and the other two access waveguides (Waveguide-3 &
Waveguide-4) are attached to the output portion.

\[ L_x = \frac{\pi}{\beta_0 - \beta_1} = \frac{\pi}{2C} \]  \hspace{1cm} (3.22)

where \( \beta_0 \), \( \beta_1 \) are the propagation constants of fundamental and first order modes, whereas coupling coefficient (C) for two-mode interference coupler (h=0 \( \mu \)m) can be estimated using equation (3.8)-(3.11).

The normalized coupling coefficient for two-mode interference (TMI) coupler can be derived using the following equation (3.23) of asymptotic analysis of SEIM model of DC as already discussed in the previous section-3.2.1, with a consideration
as coupling gap (h) tends to zero, we have

\[
\frac{C}{C_0} = \frac{\pi^2 b^3}{64a^3} \left[ \frac{1}{V_1^2} \exp\left(-\frac{2V_1}{b}h\right) \exp\left(\frac{2V_1}{b}\right) - \exp\left(-\frac{2V_1}{b}\right) \right]
\]

\[
+ \frac{1}{V_2^2} \exp\left(-\frac{2V_2}{b}h\right) \exp\left(\frac{2V_2}{b}\right) - \exp\left(-\frac{2V_2}{b}\right) \right]
\]

(3.23)

where \( V_1 = \frac{b}{2} k(n_1^2 - n_2^2)^{1/2} \) and \( V_2 = \frac{b}{2} k(n_1^2 - n_3^2)^{1/2} \)

3.3.2 Coupling Coefficient of TMI Coupler

For TMI coupler (h→0), as h tends to zero the value of the exponential term contains ‘h’ in equation (3.22) will be 1. Since there is no separation gap between the two cores (n_3 does not exist and n_1 ≈ n_3), V_2 will be vanishes. Finally the equation (3.22) for TMI coupler can be approximate as,

\[
\frac{C}{C_0} = \frac{\pi^2 b^3}{64a^3} \left[ \frac{1}{V_1^2} \left( \exp\left(\frac{2V_1}{b}\right) - \exp\left(-\frac{2V_1}{b}\right) \right) \right]
\]

(3.24)

Similar to the directional coupler as mentioned in section-3.2.2, considering square embedded channel waveguide (a=b, h→0) and substituting the values of \( V_1 \), \( V_2 \) from equation (3.2) and (3.3) in the above equation (3.24) we have,

\[
\frac{C}{C_0} = \frac{\pi^2 b^3}{16b^2 k^2 (n_1^2 - n_2^2)^2} \times \left[ \exp\left(\frac{2a}{b} x \frac{b}{2} k(n_1^2 - n_2^2)^{1/2}\right) - \exp\left(-\frac{2a}{b} x \frac{b}{2} k(n_1^2 - n_2^2)^{1/2}\right) \right]
\]

(3.25)

where \( C_0 = \frac{0.4}{(1+0.2h)^2} \times \frac{(n_1^2 - n_{\text{eff}(TE)}^2)(n_{\text{eff}(TE)}^2 - n_2^2)}{n_{\text{eff}(TE)}(n_1^2 - n_3^2)} \frac{W + \frac{2}{k_0\sqrt{n_{\text{eff}(TE)}^2 - n_2^2}}}{W + \frac{2}{k_0\sqrt{n_{\text{eff}(TE)}^2 - n_2^2}}} \) ; for TE mode (3.26)
\[ C_0 = \frac{0.4}{(1 + 0.2h)} \times \frac{\left( n_1^2 - n_{\text{eff(TM)}}^2 \right)}{n_{\text{eff(TM)}}(n_1^2 - n_2^2)} \sqrt{n_{\text{eff(TM)}}^2 - n_2^2} \] for TM mode \hspace{1cm} (3.27)

\[ n_{\text{eff(TE)}} = \beta_{\text{TE}} \left( \frac{\lambda}{2\pi} \right) \]

\[ n_{\text{eff(TM)}} = \beta_{\text{TM}} \left( \frac{\lambda}{2\pi} \right) \]

\( \beta_{\text{TE}}, \beta_{\text{TM}} \) are the propagation constants of TE and TM mode respectively which can be estimated for two mode interference coupler from dispersive relation as discussed in section-3.2.2 of the current chapter.

3.3.3 Coupling Characteristics of TMI Coupler

For the calculation of power transfer to the output access waveguides of TMI coupler, the same coupled mode relations to that of the directional coupler as discussed in section-3.2.3 are used. The coupled power into the single mode output access waveguides of TMI coupler can be approximate as,

\[ \frac{P_2}{P_1} = \cos^2 (\Delta \phi / 2) \] \hspace{1cm} (3.28)

\[ \frac{P_4}{P_1} = \sin^2 (\Delta \phi / 2) \] \hspace{1cm} (3.29)

where, \( \Delta \phi = \Delta \beta \cdot L \) and \( \Delta \beta = \beta_{00} - \beta_{01} \); and \( \beta_{00}, \beta_{01} \) are the propagation constant of fundamental and first order modes respectively.

The coupling length for maximum power transfer \( (\Delta \phi = \pi) \) can be defined as,

\[ L = \frac{\Delta \phi}{\Delta \beta} = \frac{\pi}{\Delta \beta} \] \hspace{1cm} (3.30)
Fig-3.17: Normalized coupled power versus beat length with $\Delta n=5\%$ for two mode interference (TMI) coupler with coupling gap $h=0\ \mu m$, $2a=3\ \mu m$.

Fig-3.17 shows normalized coupled power ($P_3/P_1$ and $P_4/P_1$) obtained by using the equations (3.28) and (3.29) for two mode interference (TMI) coupler with coupling gap, $h=0\ \mu m$, $2a=3\ \mu m$, $n_2=1.45$ and $\Delta n=5\%$ respectively. It is seen that the normalized coupled power of two mode interference (TMI) coupler is transferred to the cross state at the beat length which can be determined using equation (3.30). It is found that the beat length of the TMI coupler with $\Delta n=5\%$, $h=0\ \mu m$, $n_2=1.45$, $a=b=1.5\ \mu m$ are obtained as $\sim 45\ \mu m$.

3.3.4 Beat Length of TMI Coupler

The Beat length ($L_b$) with respect to the index contrast ($\Delta n$) for TMI coupler with a separation gap of $h=0\ \mu m$ and $a=b=1.5\ \mu m$, $\Delta n=5\%$, $n_2=1.45$, $\lambda=1.55\ \mu m$ has
been shown in the Fig-3.18. It is evident from the figure that the beat length of TMI coupler decreased with the increase of index contrast (\(\Delta n\)). It is found that the beat length of TMI coupler for index contrast (\(\Delta n\))\% with \(a=b=1.5\, \mu\text{m}\), \(n_2=1.45\), \(\lambda=1.55\, \mu\text{m}\) are obtained as \(\sim45\, \mu\text{m}\). The results obtained by using simple effective index method (SEIM) based on sinusoidal mode are also compared with beam propagation method that is discussed later in this chapter.

![Graph](image)

**Fig-3.18:** Beat length versus index contrast (\(\Delta n\)) of two mode interference (TMI) coupler with \(a=b=1.5\, \mu\text{m}\), \(n_2=1.45\) and \(\lambda\sim1.55\, \mu\text{m}\).

### 3.3.5 Beam Propagation Method (BPM) Simulation Results of TMI Coupler

The normalized coupled power (\(P_2/P_1\) and \(P_d/P_1\)) obtained by using the equations (3.28) and (3.29) for SEIM based two mode interference (TMI) coupler with coupling gap, \(h\sim0\, \mu\text{m}\), \(2a=3\, \mu\text{m}\), \(n_2=1.45\) and \(\Delta n=5\%\) has been shown in Fig-3.19. The figure also indicates the light wave propagation at half coupling point (\(\sim22.3\, \mu\text{m}\)) and cross coupling point (\(\sim45\, \mu\text{m}\)) of TMI coupler obtained (by optiBPM
version 9.0), matching well with the results obtained by SEIM model.

**Fig-3.19:** Normalized coupled power versus beat length with $\Delta n=5\%$ for two mode interference (TMI) coupler with coupling gap $h=0.5 \mu m$, $2a=3 \mu m$.

The beam propagation method (BPM) results of conventional Two-Mode Interference (TMI) coupler obtained by using optiBPM software for cross state, bar state and 3-dB TMI coupler have been shown in the Fig.-3.20. The results obtained by SEIM based model of TMI coupler is matching well with the BPM results. From the BPM simulation results, it is found that the beat lengths of conventional TMI coupler at the cross point, 3 dB coupling point and bar point are obtained as $\sim 45 \mu m$, $22.1 \mu m$ and $90.2 \mu m$ respectively.
Fig-3.20: BPM results of Conventional Two-Mode Interference (TMI) Coupler for
(a) Layout structure (b) Cross state (c) 3-dB coupler and (d) bar state

3.3.6 Fabrication Tolerances and Polarization Dependence of TMI Coupler

As already mentioned the necessity to study the performance degradation of designed devices with small unwanted variation of waveguide parameters due to difficulties in realization of precise fabrication of device structure with exact designed parameters; the effect of fabrication tolerances ($\delta w$) of TMI width on power imbalance of designed TMI coupler also has been studied. Fig-3.21 shows the plot for power imbalance [=$10 \log_{10} (P_3/P_4)$] versus fabrication tolerances ($\pm \delta w$) of TMI width with cladding index~1.45, $n_2$=1.45, index contrast ~5 %, a=b=1.5 $\mu$m and wavelength ~ 1.55 $\mu$m respectively.

The graph shows that the power imbalance is increases almost symmetrically in both sides of minimum power imbalance obtained at $\delta w$~0 $\mu$m as 3 dB TMI coupler
is designed for this. The rate of increase of power imbalance (dB) with respect to width tolerance for conventional TMI coupler is approximately obtained as \( \frac{\partial}{\partial \delta w} \) [Power Imbalance (dB)] \( \sim \) 0.18 dB/\( \mu \)m respectively.

**Fig-3.21:** Power imbalance characteristics versus width tolerance (\( \pm \delta w \)) of conventional 3-dB TMI coupler with cladding index \( \sim \) 1.45, index contrast \( \sim \) 5\%, \( a=b=1.5 \) \( \mu \)m and wavelength \( \sim \) 1.55 \( \mu \)m.

The dependence of power imbalance on wavelength for 3 dB conventional TMI coupler with \( a=1.5 \) \( \mu \)m, \( b=1.5 \) \( \mu \)m, index contrast \( \sim \) 5\% and cladding index \( \sim \) 1.45 also have been studied which is shown in Fig-3.22. It is found that power imbalance increases almost symmetrically in both sides of minimum power imbalance obtained at \( \lambda \sim 1.55 \) \( \mu \)m as 3 dB TMI coupler is designed for this wavelength.

Fig-3.23 shows the normalized coupling power distribution versus beat length of conventional TMI coupler for both TE-mode and TM-mode with \( h=0 \) \( \mu \)m, \( a=1.5 \) \( \mu \)m, \( b=1.5 \) \( \mu \)m, cladding index \( \sim \) 1.45, \( \Delta n=5 \)\% and \( \lambda \sim 1.55 \) \( \mu \)m respectively. From this polarization dependences plot, it is obtained that for TM-polarization the value of beat length (\( L_B \)) is \( \sim 0.25 \)\% more than that of the TE-polarization.
Fig-3.22: Power Imbalance characteristics versus wavelength variation for conventional TMI coupler with $a=b=1.5 \, \mu m$, index contrast $\sim 5 \%$ and cladding index $\sim 1.45$.

Fig-3.23: Normalized coupling power distribution of conventional TMI coupler for both TE-mode (solid line) and TM-mode (dashed line) with $h=0 \, \mu m$, $a=1.5 \, \mu m$, $b=1.5 \, \mu m$, cladding index $\sim 1.45$, $\Delta n=5 \%$ and $\lambda \sim 1.55 \, \mu m$ respectively.
3.4. Multimode Interference (MMI) Coupler

The MMI coupler is based on self-imaging phenomena where the input excited field profile is reproduced in single or multiple images of the exciting field at a periodic interval along the direction of wave propagation. The MMI coupling length depends on consideration of structure based on either restricted interference (where, there is a restriction of excitation of some selected modes) or general interference (where, self imaging mechanism is independent of modal excitation). The conventional MMI structures based on general interference and restricted interference has been studied by previous authors as details are reviewed in chapter-2. In case of MMI coupler, the input field excites higher order modes in addition to the fundamental mode and first order mode and interfered with each other along the direction of propagation. Based on self imaging principles, there will multiple images.

3.4.1 Mathematical Model of MMI Coupler Using SEIM

Multi mode interference (MMI) couplers are basically consisting of two single mode waveguides placed with a gap and gap is filled with same core material. When light is launched to one of the input waveguide, more than two modes are excited. Due to the coupling and depending upon the phase difference between excited modes propagated along the direction of propagation, at the end of coupling region light signal can be obtain at the cross state or bar state output access waveguides

Fig-3.24 shows the three dimensional (3D) schematic view of the basic geometry of a multimode interference (MMI) coupler that is consisting of multimode coupling region of width \((2a+h)\) and coupling length \(L\) with four single mode access waveguides of core width \(a\) and core thickness \(b\) whereas \(h\) represents the coupling waveguide separation gap. \(n_1\) and \(n_2\) are the refractive index of the core and cladding region respectively. A pair of single mode access waveguides is attached to the input portion of the MMI region and the other pair of single mode access waveguides is attached to the output portion.
Fig-3.24: 3D schematic view of 2x2 MMI coupler based general interference

Fig-3.25 shows the two dimensional (2D) schematic view of multimode interference coupler as shown in Fig-3.24. When the input signal light is launch into one of the input access waveguides, higher order modes are excited in addition to the fundamental and first order modes of propagation constant $\beta_{00}$ and $\beta_{01}$ respectively are excited in the coupling region. At the end of the coupling region, depending upon relative phase differences between these excited modes, light power is either coupled into two output waveguides or vanishes.
The beat length (length required for $\pi$ phase difference) of the MMI coupler is written as

$$L_{\pi} = \frac{\pi}{\beta_{oo} - \beta_{oo}}$$  \hspace{1cm} (3.31)

where $\beta_{oo}, \beta_{01}$ = propagation constant of fundamental and first order mode that can be obtained by using dispersion equations[14].

The normalized coupling coefficient for multimode interference (MMI) coupler can be derived using the following relation (3.32) of asymptotic analysis of SEIM model of DC with the assumption ($a=b$, $n_3\rightarrow n_1$) as discussed in previous section-3.2.2,

$$\frac{C}{C_0} = \frac{\pi^2 b^4}{64 a^4} \left[ \frac{1}{V_1} \exp \left( \frac{-2V_1 h}{b} \right) \left( \exp \left( \frac{2V_1 a}{b} \right) - \exp \left( \frac{-2V_1 a}{b} \right) \right) \right]$$

$$+ \frac{1}{V_2} \exp \left( \frac{-2V_2 h}{b} \right) \left( \exp \left( \frac{2V_2 a}{b} \right) - \exp \left( \frac{-2V_2 a}{b} \right) \right)$$  \hspace{1cm} (3.32)

where $V_1 = \frac{b}{2} k(n_1^2 - n_2^2)^{1/2}$ and $V_2 = \frac{b}{2} k(n_3^2 - n_1^2)^{1/2}$.

### 3.4.2 Coupling Coefficient of MMI Coupler

The MMI coupler consists of two waveguides having a separation gap ($h$) of refractive index ($n_3$) similar to that of the core refractive index ($n_1$). As discussed in the previous sections for a $2\times2$ directional coupler, the coupling gap between the
access waveguides should be filled with same refractive index to that of the core refractive index. As the coupling gap refractive index, \( n_3 \) tends to the waveguide's core refractive index, \( n_1 \) of a directional coupler as discussed in previous section; the equation (3.32) can be approximated as follows.

\[
\frac{C}{C_0} = \frac{\pi b^2}{64a^3} \left[ \frac{1}{V_i} \exp \left( \frac{-2V_1}{b} \right) \left( \exp \left( \frac{2V_1}{b} \right) - \exp \left( \frac{-2V_1}{b} \right) \right) \right]^{\frac{2}{b^2}}
\]

(3.33)

Similar to the directional coupler as mentioned in section-3.2.2, considering square embedded channel waveguide for MMI coupler \((a=b, n_1=n_3)\) and substituting the values of \( V_1, V_2 \) from equation (3.2) and (3.3) in the above equation (3.33) we have,

\[
\frac{C}{C_0} = \frac{\pi b^2}{64a^3} \frac{1}{\left( \frac{b}{2} \right)^2} \exp \left\{ \frac{\lambda}{b} (n_1^2 - n_2^2) \right\} \left[ \exp \left( \frac{2a \times b}{b} \right) \left( \exp \left( \frac{2a \times b}{b} \right) - \exp \left( \frac{-2a \times b}{b} \right) \right) \right]^{\frac{2}{b^2}}
\]

\[
\frac{C}{C_0} = \frac{\pi^2}{16b^2k^2(n_1^2 - n_2^2)} \exp \left\{ \frac{\lambda}{b} (n_1^2 - n_2^2) \right\} \left[ \exp \left( \frac{2a \times b}{b} \right) \left( \exp \left( \frac{2a \times b}{b} \right) - \exp \left( \frac{-2a \times b}{b} \right) \right) \right]^{\frac{2}{b^2}}
\]

(3.34)

where

\[
C_0 = \frac{0.4}{(1 + 0.2h)} \frac{\left( n_1^2 - n_{\text{eff}}^{(TE)} \right)}{n_{\text{eff}}^{(TE)}} \left[ \left( n_1^2 - n_2^2 \right) W + \frac{2}{k_0 \sqrt{n_{\text{eff}}^{(TE)} - n_2^2}} \right]
\]

for TE mode (3.35)

\[
C_0 = \frac{0.4}{(1 + 0.2h)} \frac{\left( n_1^2 - n_{\text{eff}}^{(TM)} \right)}{n_{\text{eff}}^{(TM)}} \left[ \left( n_1^2 - n_2^2 \right) W + \frac{2}{k_0 \sqrt{n_{\text{eff}}^{(TM)} - n_2^2}} \right]
\]

for TM mode (3.36)

\[
n_{\text{eff}}^{(TE)} = \beta_{TE} \frac{\lambda}{2\pi}
\]

\[
n_{\text{eff}}^{(TM)} = \beta_{TM} \frac{\lambda}{2\pi}
\]

\( \beta_{TE}, \beta_{TM} \) = Propagation constants of TE and TM mode respectively which are determined from dispersive relations [14].
3.4.3 Coupling Characteristics of MMI Coupler

![Diagram showing normalized coupled power versus beat length with Δn=5% for MMI coupler with h=4 μm.](image)

**Fig-3.26:** Normalized coupled power versus beat length with Δn=5% for MMI coupler with h=4 μm.

The normalized coupled power (P₂/P₁ and P₄/P₁) obtained by using the equations (3.25) and (3.26) applying for conventional multimode interference (MMI) coupler with coupling gap, h=4 μm, w_{mm}=7 μm, n₂=1.45 and Δn=5% is shown in the Fig-3.26. It is observed that the normalized coupled power of MMI coupler is transferred to the cross state at the beat length which can be determined using equation (3.31). It is found that the beat length of the MMI coupler for TE-mode with Δn=5% with h=4 μm, n₂=1.45, a=b=1.5 μm are obtained as ∼ 80 μm.

3.4.4 Beat Length of MMI Coupler

The beat length (Lₐ) for TE polarization obtained by using the equation (3.31) for coupling gap (h) varying for 2 μm to 3 μm of MMI coupler is shown in Fig-3.27. For TM-polarization, the value of Lₐ is estimated 0.6% more than that of TE-
polarization. From the figure, it is seen that beat length increases with increase of coupling separation gap due to the increase of excited modes. For \( h > 3 \ \mu m \) (not shown in the figure), it is seen that the beat length increases sharply with \( h \).

**Fig-3.27:** Beat length versus coupling gap of MMI coupler with \( a=b=1.5 \ \mu m \) and \( \Delta n=5 \% \) respectively.

Fig 3.28 shows the comparison of \( L_{\pi} \) versus coupling gap for DC and MMI coupler with \( \Delta n=5 \% \), \( a=b=1.5 \ \mu m \). It is seen that beat length of DC with \( n_3=1.45 \) is much more than that of MMI coupler and the rate of increase of beat length is more for \( n_3=1.45 \). As \( n_3 \) increases, the beat length of DC decreases and at \( n_3=1.4945 \), the beat of DC is almost close to that of MMI coupler. So DC with \( n_3=1.4945 \) behaves as MMI coupler with \( \Delta n=5 \% \).

Fig-3.29 shows beat length \( (L_{\pi}) \) versus index contrast \( (\Delta n) \) for conventional MMI coupler with \( a=b=1.5 \ \mu m, h=4 \ \mu m, n_2=1.45 \) and \( \Delta n=5 \% \) respectively. It is evident from the figure that the beat length decreases with increase of \( \Delta n \) and it decreases slowly with \( \Delta n \) for \( \Delta n \geq 5 \% \). So \( \Delta n=5 \% \) is chosen for details study of MMI coupler.
Fig-3.28: Beat length versus coupling gap of directional coupler (dashed lines) with $n_3=1.45$, 1.49, 1.4945 and MMI coupler (solid line) with $a=b=1.5 \, \mu m$ and $\Delta n = 5\%$.

Fig-3.29: Beat length ($L_c$) versus index contrast ($\Delta n$) of conventional MMI coupler with $a=b=1.5 \, \mu m$, $h=4 \, \mu m$, $n_2=1.45$ and $\Delta n = 5\%$ respectively.
3.4.5 Beam Propagation Method (BPM) Simulation Results of MMI Coupler

![Graph showing normalized coupling power versus beat length with Δn=5% for multimode interference (MMI) coupler with coupling gap h~4 μm.](image)

**Fig-3.30:** Normalized coupled power versus beat length with Δn=5 % for multimode interference (MMI) coupler with coupling gap h~4 μm.

Fig.3.30 shows normalized coupled power (\( P_3/P_1 \) and \( P_4/P_1 \)) obtained by using the equations (3.25) and (3.26) for multimode interference (MMI) coupler with coupling gap, h~4 μm, \( n_2=1.45 \) and Δn=5 %. The figure also shows light wave propagation at half coupling point (39.9 μm) and cross coupling point (80 μm) of multimode interference (MMI) coupler obtained (by optiBPM version 9.0), matching well with the results obtained by SEIM model.

Fig.-3.31 shows the beam propagation method (BPM) results of multimode interference (MMI) coupler obtained by using optiBPM software for cross state, bar state and 3-dB MMI coupler respectively. It is found that the beat length of multimode interference (MMI) coupler at cross state, 3-dB MMI coupler and bar state with h=4 μm, \( n_2=1.45 \), Δn=5 % are obtained as ~80.1 μm, 40.2 μm and 160 μm respectively.
Fig-3.31 : BPM results of Conventional Multimode Interference (MMI) Coupler for (a) Layout structure (b) Cross state (c) 3-dB coupler and (d) bar state

3.4.6 Fabrication Tolerances and Polarization Dependence of MMI Coupler

In order to study the performance degradation of designed devices with a small unwanted variation of waveguide parameters during fabrication process step; the effect of fabrication tolerances ($\delta w$) of MMI width on power imbalance of conventional MMI coupler has been estimated. Fig-3.32 shows plot for power imbalance [$=10 \log_{10}(P_3/P_4)$] versus fabrication tolerances ($\pm \delta w$) of MMI width with $h\sim 4.0$ µm, $a=1.5$ µm, $b=1.5$ µm, index contrast~5 %, cladding index~1.45, and $\lambda\sim 1.55$ µm respectively. It is found that the power imbalance increases symmetrically for both side of with $\pm \delta w=0$ µm. The rate of increase of power imbalance (dB) with
respect to width tolerance for conventional MMI coupler is approximately obtained as \( \frac{\partial}{\partial (\delta w)} \) [Power Imbalance (dB)] \(-0.13\) dB/\(\mu m\) respectively.

![Diagram](image)

**Fig-3.32:** Power Imbalance characteristics versus width tolerances (\(\delta w\)) for conventional MMI coupler with index contrast \(-5\%\), cladding index \(-1.45\),

\[ h \approx 4.0 \ \mu m, \ a = 1.5 \ \mu m, \ b = 1.5 \ \mu m \ and \ \lambda \approx 1.55 \ \mu m. \]

Since, it is also essential to study the dependence of power imbalance on wavelength; the power imbalance versus wavelength characteristics for conventional MMI coupler with \(a=b=1.5 \ \mu m, \ h=4.0 \ \mu m\), index contrast \(-5\%\) and cladding index \(-1.45\) is shown in Fig-3.33. In the figure, the solid line indicates the curve for 3 dB conventional MMI coupler of coupling length \(-40.1 \ \mu m\) and the minimum power imbalance is obtained at \(\lambda \approx 1.55 \ \mu m\). It is found that power imbalance is almost symmetrically increased in both sides of \(\lambda \approx 1.55 \ \mu m\).
Fig-3.33: Power Imbalance characteristics versus wavelength variation for conventional MMI coupler with $a=b=1.5 \, \mu m$, $h=4.0 \, \mu m$, $\Delta n \sim 5$ % and $n_2 \sim 1.45$.

Fig-3.34: Normalized coupling power distribution of conventional MMI coupler for both TE-mode (solid line) and TM-mode (dashed line) with $h=4.0 \, \mu m$, $a=1.5 \, \mu m$, $b=1.5 \, \mu m$, cladding index $\sim 1.45$, $\Delta n=5$ % and $\lambda \sim 1.55 \, \mu m$ respectively.
The polarization dependence characteristic for conventional MMI coupler is shown in the Fig-3.34. The figures shows the normalized coupling power distribution versus beat length of conventional MMI coupler for both TE-mode and TM-mode with h=4 µm, a=1.5 µm, b=1.5 µm, cladding index=1.45, Δn=5 % and λ=1.55 µm respectively. It is found that for TM-polarization the value of beat length (Lₜ) is ~0.24 % more than that of the TE-polarization.

3.5. Transformation Relationship of DC, TMI Coupler and MMI Coupler

From the above studies, transformation relationships between directional coupler, two-mode interference coupler and multimode interference coupler have been observed and details are discussed in the following two sub-sections 3.5.1 and 3.5.2 respectively.

3.5.1 Transformation from DC to TMI Coupler

![Diagram of TMI coupler](image)

**Fig-3.35:** Schematic of TMI coupler (h=0) based general interference

Fig-3.35 shows a 2x2 conventional TMI coupler that is consisting of a two-mode coupling region of length L and width 2a (where h=0); and a pair of single mode input access waveguides of width a and thickness b and another pair of single mode output access waveguides of same size. Fig-3.36 shows beat length (Lₜ) versus
index contrast, $\Delta n$ (%) of DC obtained by using equation (3.17)-(3.19) for different values of coupling gap $h=0.02\ \mu m$, $0.2\ \mu m$, $0.5\ \mu m$, $1\ \mu m$ and $n_2=n_1=1.45$, $a=b=1.5\ \mu m$ respectively. The cross points in the figure are the experimental results demonstrated by previous authors [31], [33] matching well with theoretical curves. It is found that as index contrast ($\Delta n$) increases, the beat length decreases and the rate of decrease of $L_\pi$ with $\Delta n$ becomes slower in lower values of $h$. As $h$ becomes closer to zero coupling gap ($h=0$), the curves become closer and closer. For $\Delta n \geq 2\%$, the decrease of beat length is smaller. It is obtained that the beat lengths of DC with $h \leq 0.02\ \mu m$ are almost same with those of TMI coupler.

![Graph](image)

**Fig.3.36:** Beat length ($L_\pi$) versus index contrast ($\Delta n$) of DC (dashed line) with $h=1\ \mu m$, $0.5\ \mu m$, $0.2\ \mu m$, $0.02\ \mu m$ and TMI coupler ($h=0\ \mu m$, solid line) respectively.

We have also estimated the beat length ($L_\pi$) of conventional TMI coupler for index contrast, $\Delta n$ (%) varying from 1 to 10 obtained by using the equation (3.27) which is shown by the solid line in Fig.3.36. It is evident from the graph that the curve for the
TMI coupler is almost overlapped with that of DC with $h=0.02 \, \mu m$, showing the equivalence of TMI coupler with DC having coupling gap $h \leq 0.02 \, \mu m$. For $\Delta n > 10\%$ (not shown in figure), it is seen that $L_a$ decreases slowly with $\Delta n$. As $h$ tends to zero, the equation (3.7) is approximately written for TMI coupler as

$$
\frac{C}{C_0} = \frac{\pi^2 b^3}{64 a^3} \left[ \frac{1}{V^2} \left( \exp \left( \frac{2V}{a} b \right) - \exp \left( -\frac{2V}{a} b \right) \right) \right]
$$

(3.37)

Thus, as $h$ tends to zero ($\leq 0.02 \, \mu m$), DC is equivalence of TMI coupler. So the coupling coefficient in the equation (3.37) estimated from the equation (3.7) is approximately equal to coupling coefficient of TMI coupler.

### 3.5.2 Transformation from DC to MMI Coupler

![Graph showing beat length versus coupling gap](#)

**Fig-3.37:** Beat length versus coupling gap of DC (dashed line) with $n_3=1.45$, 1.49, 1.4945 and MMI coupler (solid line) with $a=b=1.5 \, \mu m$ and $\Delta n=5 \%$ respectively.

Fig-3.37 shows the beat length ($L_a$) versus coupling gap $h$ of DC obtained by using the equation (3.27) for TE polarization with different values of $n_3=1.45$, 1.47,
1.49, 1.4945 with Δn=5 %, a=b=1.5 μm respectively. It is found that as coupling gap increases, the beat length increases and the rate of increase of $L_α$ with h becomes slower in higher values of $n_3$. As $n_3$ becomes closer and closer to $n_1$, the curves also become closer and closer. We have also obtained (not shown in fig) that for $1.5 > n_3 > 1.45945$, the curves are almost superposed. It is important to note that the values of beat length for DC with $n_3 \geq 1.4945$ are almost same with those of MMI coupler based on general inference as shown in Fig-3.37.

![Fig-3.38: Schematic of 2x2 MMI coupler based general interference](image)

Fig-3.38 shows a 2x2 conventional MMI coupler consisting of a multimode coupling region of length $L$ and width $(2a+h)$, thickness b; a pair of single mode input access waveguides of width $a$ and thickness b and another pair of single mode output access waveguides of same dimensions. The beat length of the MMI coupler is written as

$$L_α = \frac{\pi}{β_{eo} - β_{01}}$$

(3.38)

where $β_{eo}$, $β_{01}$ =propagation constant of fundamental and first order mode obtained by using dispersion equations. We have determined $L_α$ for TE polarization by using the equation (3.35) for h varying for 2 μm to 3 μm, as shown by the solid line in Fig-3.37. For TM-polarization, the value of $L_α$ is estimated 0.6 % more than that of TE-polarization. The cross point in the figure is the experimental result demonstrated by previous authors [32], matching well with theoretical curves. The figure shows the plot of the MMI coupler is almost overlapped with that of DC with $n_3 = 1.4945$. 

©Tezpur University
showing the equivalence of MMI coupler with DC having \( n_3 \) close to core refractive index. For \( h>3 \) \( \mu \text{m} \) (not shown in the figure), it is seen that the beat length increases with \( h \). As \( n_3 \) tends to \( n_1 \), the equation (3) is approximated as follows.

\[
\frac{C}{C_0} = \frac{\pi^2 b^3}{64a} \left[ \frac{1}{V_1} \exp\left(-\frac{2V_1 h}{b}\right) \left\{ \exp\left(\frac{2V_1 a}{b}\right) - \exp\left(-\frac{2V_1 a}{b}\right) \right\} \right]
\]

(3.39)

From Fig-3.37, it is also shown that as \( n_3 \) tends to \( n_1 \) (\( n_1-n_3\leq 0.0055 \), which is \( \sim 0.43\% \)), DC is equivalent to MMI coupler. So the coupling coefficient for DC with \( n_3 \) close to \( n_1 \), satisfies also the coupling coefficient formula for MMI coupler. The beat length of MMI coupler with \( \Delta n=5 \% \) and width \( \sim 7 \mu \text{m} \) is obtained as 80 \( \mu \text{m} \).

Thus form the above studies, the following two observations have notice:

(i) When the waveguide separation gap in DC decrease (<0.02 \( \mu \text{m} \)), DC shows the coupling characteristics equivalent to TMI couplers.

(ii) When the refractive index of the waveguide separation gap region of DC is increases and almost equivalent [(\( n_1-n_3 \) \( \sim 0.005 \)) to the refractive index of the core region; the DC shows the coupling characteristics equivalent to MMI couplers.

3.6. Comparison of Coupling Characteristics for DC, TMI Coupler and MMI Coupler

A comparative normalized coupling characteristic of DC, TMI coupler and MMI coupler is shown in the Fig-3.39. The figure shows the normalized coupled power obtained by using the equations (3.20) and (3.21) for DC with coupling gap-0.5 \( \mu \text{m} \), \( \Delta n=5 \% \), TMI coupler with same \( \Delta n \) and 2a=3 \( \mu \text{m} \); and MMI coupler with \( h=4 \) \( \mu \text{m} \) and \( \Delta n=5\% \) respectively. The figure also shows the light wave propagation at half coupling points and cross coupling points of DC, TMI and MMI coupler obtained (by optiBPM version 9.0), are matching well with the results obtained by SEIM model. It is also found that the peak power of MMI coupler is slightly more than that of DC and TMI coupler due to presence of bending loss of

©Tezpur University 3.49
DC and TMI coupler at bent access waveguides.

![Diagram showing normalized coupling power versus beat length with Δn=5% for DC with coupling gap h~0.5 μm (solid line), TMI coupler (dotted line) with h~0 μm and MMI coupler (dashed line) with h~4 μm.]

**Fig-3.39:** Normalized coupled power versus beat length with Δn=5% for DC with coupling gap h~0.5 μm (solid line), TMI coupler (dotted line) with h~0 μm and MMI coupler (dashed line) with h~4 μm.

Further, it is found that the beat length of TMI coupler (h=0 μm) and MMI coupler with h=4 μm, Δn=5% are obtained as ~45 μm and ~80 μm respectively and that for DC with h=0.5 μm and same Δn is ~91 μm respectively. The dot and star signs in the normalized coupling characteristics graph indicate the respective cross and 3 dB coupling point of DC, TMI coupler and MMI coupler respectively. The design parameters for DC, TMI coupler and MMI coupler that are considered in the above studies are summarized in the Table-3.2 as given below. The designed DC, TMI coupler and MMI coupler with these waveguide parameters are then fabricated and experimentally tested using SiON as the waveguide core material with SiO₂ cladding layer. The detail fabrication process steps and experimental results are
discussed later on in the chapter-6 of this thesis.

<table>
<thead>
<tr>
<th>Design Parameters</th>
<th>Directional Coupler</th>
<th>MMI Coupler</th>
<th>TMI Coupler</th>
</tr>
</thead>
<tbody>
<tr>
<td>Core waveguide width (a), µm</td>
<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>Core waveguide Thickness (b), µm</td>
<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>Index Contrast (∆n)</td>
<td>5%</td>
<td>5%</td>
<td>5%</td>
</tr>
<tr>
<td>Core RI (n₁), ∆n=5%</td>
<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>Cladding RI (n₂)</td>
<td>1.45</td>
<td>1.45</td>
<td>1.45</td>
</tr>
<tr>
<td>Coupling Gap</td>
<td>1.45</td>
<td>1.4945</td>
<td>----</td>
</tr>
<tr>
<td>Cladding RI (n₁)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coupling gap (h), µm</td>
<td>0.5</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>Wavelength (λ), µm</td>
<td>1.55</td>
<td>1.55</td>
<td>1.55</td>
</tr>
<tr>
<td>Beat Length (L₀), µm</td>
<td>91</td>
<td>80</td>
<td>45</td>
</tr>
</tbody>
</table>

3.7. Conclusion

In this chapter the coupling characteristics of DC, TMI and MMI are shown. Polarization dependence property and fabrication tolerance of DC, TMI and MMI couplers are discussed. For accurate estimation of these characteristics, a mathematical model using simple effective index method (SEIM) based on sinusoidal modes have been developed. Finally, a transformation relationship has been established for DC with TMI coupler and DC with MMI coupler. From the transformation relationship, it is observed that the beat length of TMI coupler is half of that of DC and 0.65 of that of MMI coupler with h=4 µm. Further, this transformation relationship of DC, TMI and MMI coupler have been used to estimate the coupling characteristics for the proposed structures of directional coupler, two
mode interference (TMI) and multi mode interference (MMI) couplers as discussed in chapter 4, 5 and 7 respectively.

Reference:


37. Miya, T. Silica-based planar lightwave circuits: passive and thermally active


********