CHAPTER-III

Two Unit Standby Oil Delivering System with a Provision of Switching over to Another System
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Introduction

Development of the stochastic/reliability models coming under Mathematical Sciences, in general and Statistics, in particular is in progress day by day. Studies before 2004 done by various researchers including Nakagawa and Osaki (1975), Arora (1977), Mine and Kaiwal (1979), Murari and Maruthachalam (1984), Guo Tong De (1989), Tuteja and Taneja (1991, 92, 93), Saini and Kumar (1994), Sehgal (2000) and Taneja and Nanda (2003) discussed the stochastic models for one or two or more units standby systems assuming numerical values for various rates, probabilities, costs, etc. These researchers, while making the analysis through graphs, took the assumed values of failure, repair and other rates i.e. the real data on these rates were not taken into consideration.

Taneja (2004) first time collected the real data on failure and repair rates of 232 programmable logic controllers (PLC) and studied a single unit PLC considering the four types of failure. Taneja (2005) discussed reliability and profit analysis of a system which consists of one main unit (used for manufacturing) and two PLCs (used for controlling). Parasher and Taneja (2007) studied the reliability and profit evaluation of a standby system based on a master slave concept and two types of repair facilities. Goyal, Taneja and Singh (2009) evaluated the reliability and profit analysis of a two unit standby system working in a sugar mill with operating and rest periods.

Though the work on the basis of collecting real data is going on but to a very small extent. There is a lot more work on the basis of collecting real data is yet to be done. Putting a step in this direction, I.O.C.L, Panipat was visited by the author to gather information/data on failures/repairs of oil delivering system. One more feature observed therein was the concept of another line facility, not considered in the field of reliability/stochastic modelling so far.
Keeping the above in this view, a system is analysed with the concept of another line facility. Initially one unit is operative and the other is standby. It has been observed that the unit in the system may fail due to

1. Some Repairable fault

2. Some irreparable fault in the component which has to be replaced.

On the failure of the operative unit, it is repaired or its component is replaced with a new one according as it is repairable or irreparable. The standby unit becomes operative at this stage. In the situation when one unit is under repair and the other unit is waiting for repair switching over to the other similar system is done by suspending the repair of the first unit. A valve is used for this switching over.

Thus considering the above situations we studied the model and the assumptions of the model are

All the random variables are independent.

1. Failure times are assumed to have exponential distribution whereas repair/replacement/inspection times have general distribution

2. The refinery has single repairman facility.

3. The repairman comes immediately as soon as unit fails.

4. After each repair, the system works as good as new one.

The system is analysed by making use of semi-Markov processes and regenerative point technique and the following measures of the system effectiveness are obtained:

- Mean time to system failure
- Steady-state availability
- Expected busy period of the repairman per unit time for repair the failed unit.
- Expected busy period of the repairman per unit time for replacing the failed unit.
- Expected number of visits of the repairman
- Expected number of replacements.
- Expected time during which the operation is performed by some other system on the failure of both the units

The profit incurred to the system is evaluated and the graphical study is also made.
Notations and States of the System

O          operative unit
s          cold stand by
Fr         unit is under repair
Fwr        failed unit is waiting for repair
FR         repair is continuing from previous state
Frp        unit is under replacement
Fwrp       failed unit is waiting for replacement
FRp        replacement is continuing from previous state
Frs        repair of failed units is kept under suspension
Frpst      replacement of failed unit is kept under suspension
C          system gets connection
CV         valve change for being connected
λ          failure rate of main pump
α₁         repair rate of completely failed unit
α₂         replacement rate of completely failed unit
β          rate of change of valve
p          probability that unit is under repair
q          probability that unit is under replacement
p₁         probability of switching over to another line
q₁         probability of failure of switching over to another line
G₁(t),g₁(t) c.d.f. and p.d.f. of the repair time of unit.
G₂(t),g₂(t) c.d.f. and p.d.f. of the replacement time of unit

Diagram of Model and Results

Transition Probabilities and Mean Sojourn Times

A transition diagram showing the various states of the system is shown in Fig 3.1.

The epochs of entry into states 0, 1, 2, 4, 5, 8, 10,11,12,13 and 14 are regenerative points.
The transition probabilities are given below:
The non-zero elements $p_{ij}$ are given below:

\begin{align*}
p_{01} &= p \
p_{02} &= q \
p_{10} &= g_1(\lambda) \
p_{11}^{(3)} &= q_1 p \{1-g_1^*(\lambda)\} \
p_{12}^{(6)} &= q_1 q \{1-g_1^*(\lambda)\} \
p_{13} &= q_1 p \{1-g_1^*(\lambda)\} \
p_{14} &= pp_1 \{1-g_1^*(\lambda)\} \
p_{20} &= g_2(t) e^{-\lambda t} dt \
p_{21}^{(7)} &= q_1 p \{1-g_2^*(\lambda)\} \
p_{22}^{(9)} &= q_1 q \{1-g_2^*(\lambda)\} \
p_{27} &= q_1 p \{1-g_2^*(\lambda)\} \
p_{28} &= q_1 q \{1-g_2^*(\lambda)\} \
p_{29} &= q_1 q \{1-g_2^*(\lambda)\} \
p_{30} &= g_1(t) dt \
p_{31} &= g_1(t) dt \
p_{32} &= g_2(t) dt \
p_{33} &= g_2(t) dt \
p_{40} &= g_1(t) dt \
p_{41} &= g_1(t) dt \
p_{42} &= g_2(t) dt \
p_{43} &= g_2(t) dt \
p_{44} &= g_2(t) dt
\end{align*}
Good State  Failed State  Regeneration State

State Transition Diagram

Fig 3.1
\[ p_{15} = p_1 q \{ 1 - g_1^* (\lambda) \} \]
\[ p_{16} = q_1 q \{ 1 - g_1^* (\lambda) \} \]
\[ p_{20} = g_2^* (\lambda) \]
\[ p_{21}^{(7)} = q_1 p \{ 1 - g_2^* (\lambda) \} \]
\[ p_{2,2}^{(9)} = q_1 q \{ 1 - g_2^* (\lambda) \} \]
\[ p_{27} = q_1 p \{ 1 - g_2^* (\lambda) \} \]
\[ p_{28} = p_1 p \{ 1 - g_2^* (\lambda) \} \]
\[ p_{29} = q_1 q \{ 1 - g_2^* (\lambda) \} \]
\[ p_{2,10} = p_1 q \{ 1 - g_2^* (\lambda) \} \]

By these transition probabilities, it can be verified that

\[ p_{01} + p_{02} = 1 \]
\[ p_{10,14} = 1 \]
\[ p_{10} + p_{13} + p_{14} + p_{15} + p_{16} = 1 \]
\[ p_{10} + p_{11}^{(3)} + p_{12}^{(6)} + p_{1,4} + p_{1,5} = 1 \]
\[ p_{20} + p_{27} + p_{28} + p_{29} + p_{2,10} = 1 \]
\[ p_{20} + p_{2,1}^{(7)} + p_{2,2}^{(9)} + p_{2,8} + p_{2,10} = 1 \]
\[ p_{11,1} = p_{12,2} = p_{13,1} = p_{14,2} = 1 \]
\[ p_{4,11} = p_{14,2} = p_{8,13} = p_{5,12} = 1 \]

Also \( \mu_i \), the mean sojourn times in regenerative states \( i \) is defined as the time of stay in that state before transition to any other state. Thus

The mean sojourn time \( (\mu_i) \) in the regenerative state ‘\( i \)’ is given by

\[ \mu_0 = \frac{1}{\lambda} \]
\[ \mu_1 = \frac{1 - g_1^* (\lambda)}{\lambda} \]
\[ \mu_2 = \frac{1 - g_2^* (\lambda)}{\lambda} \]
\[ \mu_4 = \frac{1}{\beta} = \mu_5 \]
\[ \mu_{11} = g_1^* (0) = \mu_{12} \]
\[ \mu_{13} = g_2^* (0) = \mu_{14} \]
The unconditional mean time taken by the system to transit for any regenerative state ‘j’ when it (time) is counted from the epoch of entrance in to state ‘i’ is mathematically stated as
\[
m_{ij} = \int_0^\infty t \, dQ_{ij}(t) = -q_{ij}^*(0)
\]
Thus
\[
m_{01} + m_{02} = \mu_0
\]
\[
m_{10} + m_{13} + m_{14} + m_{15} + m_{16} = \mu_1
\]
\[
m_{10} + m_{11}^{(3)} + m_{12}^{(6)} + m_{14} + m_{15} = K_1
\]
\[
m_{20} + m_{27} + m_{28} + m_{29} + m_{2,10} = \mu_2
\]
\[
m_{20} + m_{21}^{(7)} + m_{22}^{(9)} + m_{28} + m_{2,10} = K_2
\]
\[
m_{5,12} = m_{8,13} = m_{10,14} = \mu_4 = \mu_8 = \mu_{10}
\]

**Mean Time to System Failure**

Let \(\phi_i(t)\) be the c.d.f. of the first passage time from regenerative state i to a failed state. To determine the mean time to system failure (MTSF) of the system, considering the failed state as absorbing states. By probabilistic arguments, we obtain the following recursive relation for \(\phi_i(t)\):
\[
\begin{align*}
\phi_0(t) &= Q_{01}(t) \phi_1(t) + Q_{02}(t) \phi_2(t) \\
\phi_1(t) &= Q_{10}(t) \phi_0(t) + Q_{13}(t) + Q_{14}(t) + Q_{15}(t) + Q_{16}(t) \\
\phi_2(t) &= Q_{20}(t) \phi_0(t) + Q_{27}(t) + Q_{28}(t) + Q_{29}(t) + Q_{2,10}(t) \\
\phi_{11}(t) &= Q_{11,1}(t) \phi_1(t) \\
\phi_{12}(t) &= Q_{12,2}(t) \phi_2(t) \\
\phi_{13}(t) &= Q_{13,1}(t) \phi_1(t) \\
\phi_{14}(t) &= Q_{14,2}(t) \phi_2(t)
\end{align*}
\]

Taking Laplace-Stieltjes Transforms (L.S.T.) of these relations and solving for \(\phi_0**(s)\), we obtain
\[
\phi_0**(s) = \frac{N(s)}{D(s)}
\]
where
\[ N(s) = Q_{01}(s)(Q_{13}(s) + Q_{14}(s) + Q_{15}(s) + Q_{16}(s)) + Q_{02}(s)(Q_{27}(s) + Q_{28}(s) + Q_{29}(s) + Q_{210}(s)) \]
\[ D(s) = 1 - Q_{10}(s)Q_{01}(s) - Q_{02}(s)Q_{20}(s) \]

Now, the mean time to system failure (MTSF) when the system starts from the state 0 is
\[ T_0 = \lim_{s \to 0} \frac{1 - \phi_0''(s)}{s} \]
Using L’ Hospital’s Rule and putting the value of \( \phi_0''(s) \) from equation we have
\[ T_0 = N/D \]

where
\[ N = \mu_0 + p_{01}\mu_1 + p_{02}\mu_2 \]
\[ D = 1 - p_{10}p_{01} - p_{02}p_{20} \]

**Availability Analysis**

Let \( A_i(t) \) be the probability that the system is in up state at instant \( t \) given that the system entered regenerative state \( i \) at \( t = 0 \). The availability \( A_i(t) \) is seen to satisfy the following recursive relations:
\[ A_0(t) = M_0(t) + q_{01}(t)A_1(t) + q_{02}(t)A_2(t) \]
\[ A_1(t) = M_1(t) + q_{10}(t)A_0(t) + q_{11}(t)A_1(t) + q_{11}(t)A_2(t) + q_{14}(t)A_4(t) \]
\[ A_2(t) = M_2(t) + q_{20}(t)A_0(t) + q_{21}(t)A_1(t) + q_{22}(t)A_2(t) + q_{28}(t)A_8(t) + q_{210}(t)A_{10}(t) \]
\[ A_4(t) = q_{4,11}(t)A_{11}(t) \]
\[ A_5(t) = q_{5,12}(t)A_{12}(t) \]
\[ A_8(t) = q_{8,13}(t)A_{12}(t) \]
\[ A_{10}(t) = q_{10,14}(t)A_{14}(t) \]
\[ A_{11}(t) = M_{11}(t) + q_{11,1}(t)A_1(t) \]
\[
A_{12}(t) = M_{12}(t) + q_{122}(t) A_2(t)
\]
\[
A_{13}(t) = M_{13}(t) + q_{131}(t) A_1(t)
\]
\[
A_{14}(t) = M_{14}(t) + q_{142}(t) A_2(t)
\]

where
\[
M_0(t) = e^{-\lambda_1 t}
\]
\[
M_1(t) = e^{-\lambda_1 t} \bar{N}_1(t)
\]
\[
M_2(t) = e^{-\lambda_1 t} \bar{N}_2(t)
\]
\[
M_{13}(t) = M_{12}(t) = \bar{N}_1(t)
\]
\[
M_{13}(t) = M_{14}(t) = \bar{N}_2(t)
\]

Taking Laplace transforms of the above equations and solving them for

\[A_0^*(s), \text{ we get}\]
\[
A_0^*(s) = \frac{N_1(s)}{D_1(s)}
\]

where
\[
N_1(s) = M_0^* (s) \left( (1-q_{11}^{(3)*}(s) - q_{14}^*(s)) (1- q_{22}^{(9)*}(s) - q_{210}^*(s)) + (q_{12}^{(6)*}(s) q_{15}^*(s)) (- q_{21}^{(7)*}(s) - q_{28}^*(s)) + q_{01}(M_0^* (s) (1- q_{22}^{(9)*}(s) - q_{210}^*(s)) + q_{02}^*(s) (M_2^* + M_{13}^*(s) q_{28}^*(s) + M_{14}^*(s) q_{210}^*(s)) - q_{02}^*(s)(M_0^* (s) (1- q_{11}^{(3)*}(s) - q_{14}^*(s)) + q_{01}^*(s)(M_1^* (s) + M_{11}^*(s) q_{14}^*(s) + M_{12}^*(s) q_{13}^*(s)))
\]
\[
D_1(s) = (1- q_{22}^{(9)*}(s)) (1- q_{11}^{(3)*}(s) - q_{14}^*(s) - q_{01}^*(s) q_{10}^*(s)) - (q_{12}^{(6)*}(s) + q_{15}^*(s)) (q_{27}^*(s) + q_{28}^*(s) + q_{20}^*(s) q_{01}^*(s)) + q_{02}^*(s)(-q_{10}^*(s) q_{27}^*(s) - q_{10}^*(s) q_{28}^*(s) - q_{20}^*(s) + q_{20}^*(s) q_{11}^{(3)*}(s) + q_{20}^*(s) q_{14}^*(s))
\]

In steady-state, availability of the system is given by

where
\[
N_1 = \mu_0 ((1- p_{11}^{(3)} - p_{14}) (1- p_{22}^{(9)} - p_{210}) + (p_{12}^{(6)} + p_{15}) (- p_{21}^{(7)} - p_{28}) + p_0 (1- p_{22}^{(9)} - p_{210} + p_0 (K_2 + \mu_{13} p_{28} + \mu_{14} p_{210})) - p_0 (\mu_0 (1- p_{11}^{(3)} p_{14}) + p_0 (K_1 + \mu_{11} p_{14} + \mu_{12} p_{13}))
\]
\[ D_1 = \mu_0(p_{10} + p_{27} + p_{10}p_{28} + p_{10}p_{20} + p_{20}p_{16} + p_{15}) + K_1(p_{27} + p_{28} + p_{01}p_{20}) + K_2(p_{16} + p_{15} + p_{20}p_{10}) + (\mu_4 + \mu_{11})(p_{27} + p_{28} + p_{01}p_{20} + p_{16} + p_{15}) + (\mu_4 + \mu_{13})(p_{20} + p_{16} + p_{01}p_{02})(p_{28} + p_{20}) \]

**Busy Period Analysis for Repair Time Only**

Let \( B_i(t) \) be the probability that the repairman is busy at instant \( t \), given that the system entered regenerative state \( i \) at \( t = 0 \). By probabilistic arguments we have the following recursive relations:

\[
B_0(t) = q_{01}(t)B_1(t) + q_{02}(t)B_2(t) \\
B_1(t) = W_1(t) + q_{10}(t)B_0(t) + q_{11}(t)B_1(t) + q_{12}(t)B_2(t) + q_{14}(t)B_4(t) + q_{15}(t)B_5(t) \\
B_2(t) = q_{20}(t)B_0(t) + q_{21}(t)B_1(t) + q_{22}(t)B_2(t) + q_{28}(t)B_8(t) + q_{2,10}(t)B_{10}(t) \\
B_4(t) = q_{4,11}(t)B_{11}(t) \\
B_5(t) = q_{5,12}(t)B_{12}(t) \\
B_8(t) = q_{8,13}(t)B_{13}(t) \\
B_{10}(t) = q_{10,14}(t)B_{14}(t) \\
B_{11}(t) = W_{11}(t) + q_{11,1}(t)B_1(t) \\
B_{12}(t) = W_{12}(t) + q_{12,2}(t)B_2(t) \\
B_{13}(t) = q_{13,1}(t)B_1(t) \\
B_{14}(t) = q_{14,2}(t)B_2(t)
\]

where

\[
W_1(t) = e^{-\lambda t} \overline{G_1(t)} + (q_1 \lambda e^{-\lambda t}) \overline{G_1(t)} \\
W_{11}(t) = \overline{G_1(t)} \\
W_{12}(t) = \overline{G_1(t)} \\
B_0^*(s) = \frac{N_2(s)}{D_1(s)}
\]
where

\[ N_2(s) = (W_1(s) + W_{11}(s) q_{14}(s) + W_{12}(s) q_{15}(s)) (q_{01}(s) q_{20}(s) + q_{27}(s) + q_{28}(s)) \]

\[ D_1(s) \text{ is already specified} \]

In steady-state, the total fraction of time for which system is under repair by the repairman, is given by

\[ B_0 = \lim_{s \to 0} B_0^*(s) = \frac{N_2}{D_1} \]

where

\[ N_2 = (W_1 + W_{11} p_{14} + W_{12} p_{15}) (p_{01} P_{20} + p_{27} + p_{28}) \]

and \( D_1 \) is already specified

**Busy Period Analysis for Replacement Time Only**

Let \( BR_i(t) \) be the probability that the repairman is busy for replacement at instant \( t \), given that the system entered regenerative state \( i \) at \( t = 0 \). By probabilistic arguments we have the following recursive relations:

\[ BR_0(t) = q_{01}(t) \odot BR_1(t) + q_{02}(t) \odot BR_2(t) \]

\[ BR_1(t) = q_{10}(t) \odot BR_0(t) + q_{11}(t) \odot BR_1(t) + q_{12}(t) \odot BR_2(t) + q_{14}(t) \odot BR_4(t) \]

\[ q_{15}(t) \odot BR_5(t) \]

\[ BR_2(t) = W_2(t) + q_{20}(t) \odot BR_0(t) + q_{21}(t) \odot BR_1(t) + q_{22}(t) \odot BR_2(t) + q_{28}(t) \odot BR_8(t) + q_{2,10}(t) \odot BR_{10}(t) \]

\[ BR_4(t) = q_{4,11}(t) \odot BR_{11}(t) \]

\[ BR_5(t) = q_{5,12}(t) \odot BR_{12}(t) \]

\[ BR_9(t) = q_{8,13}(t) \odot BR_{13}(t) \]

\[ BR_{10}(t) = q_{10,14}(t) \odot BR_{14}(t) \]

\[ BR_{11}(t) = q_{11,1}(t) \odot BR_1(t) \]

\[ BR_{12}(t) = q_{12,2}(t) \odot BR_2(t) \]

\[ BR_{13}(t) = W_{13}(t) + q_{13,1}(t) \odot BR_1(t) \]

\[ BR_{14}(t) = W_{14}(t) + q_{14,2}(t) \odot BR_2(t) \]
\[ \text{BR}_0 = \frac{N_3}{D_1} \]

where

\[ W_2(t) = e^{-\lambda t} \overline{G_2(t)} + (q_{11} e^{-\lambda t} \otimes 1) \overline{G_2(t)} \]
\[ W_{13}(t) = \overline{G_2(t)} \]
\[ W_{14}(t) = \overline{G_2(t)} \]
\[ \text{BR}_0^*(s) = \frac{N_3(s)}{D_1(s)} \]

where

\[ N_3(s) = (W_2^*(s) + W_{13}^*(s) q_{28}^*(s) + W_{14}^*(s) q_{2,10}^*(s)) (q_{02}^*(s) q_{10}^*(s) + q_{15}^*(s) + q_{16}^*(s)) \]
\[ D_1(s) \text{ is already specified} \]
\[ \text{BR}_0 = \lim_{s \to 0} \text{BR}_0^*(s) = \frac{N_3}{D_1} \]

where

\[ N_3 = (W_2 + W_{13} p_{28} + W_{14} p_{2,10}) (p_{02} p_{10} + p_{15} + p_{16}) \]
\[ \text{and } D_1 \text{ is already specified} \]

**Expected Number of Visits by Repairman**

By probabilistic arguments we have the following recursive relations for \( V_i(t) \)

\[ V_0(t) = Q_{00}(t) \overset{<}{\overset{>}{\overset{\sim}{\overset{\approx}{=}}} \ [1+V_1(t)]+ Q_{02}(t) \overset{<}{\overset{>}{\overset{\sim}{\overset{\approx}{=}}} \ [1+V_2(t)] \]
\[ V_1(t) = Q_{10}(t) \overset{<}{\overset{>}{\overset{\sim}{\overset{\approx}{=}}} \ V_0(t) + Q_{11}(t) \overset{<}{\overset{>}{\overset{\sim}{\overset{\approx}{=}}} \ V_1(t) + Q_{14}(t) \overset{<}{\overset{>}{\overset{\sim}{\overset{\approx}{=}}} \ V_4(t)+ Q_{15}(t) \overset{<}{\overset{>}{\overset{\sim}{\overset{\approx}{=}}} \ V_5(t)+ Q_{12}(t) \overset{<}{\overset{>}{\overset{\sim}{\overset{\approx}{=}}} \ V_2(t) \]
\[ V_2(t) = Q_{20}(t) \overset{<}{\overset{>}{\overset{\sim}{\overset{\approx}{=}}} \ V_0(t)+ Q_{21}(t) \overset{<}{\overset{>}{\overset{\sim}{\overset{\approx}{=}}} \ V_1(t)+ Q_{28}(t) \overset{<}{\overset{>}{\overset{\sim}{\overset{\approx}{=}}} \ V_8(t)+ Q_{22}(t) \overset{<}{\overset{>}{\overset{\sim}{\overset{\approx}{=}}} \ V_2(t)+ Q_{2,10}(t) \overset{<}{\overset{>}{\overset{\sim}{\overset{\approx}{=}}} \ V_{10}(t) \]
\[ V_4(t) = Q_{4,11}(t) \overset{<}{\overset{>}{\overset{\sim}{\overset{\approx}{=}}} \ V_{11}(t) \]
\[ V_5(t) = Q_{5,12}(t) \overset{<}{\overset{>}{\overset{\sim}{\overset{\approx}{=}}} \ V_{12}(t) \]
\[ V_8(t) = Q_{8,13}(t) \overset{<}{\overset{>}{\overset{\sim}{\overset{\approx}{=}}} \ V_{13}(t) \]
\[ V_{10}(t) = Q_{10,14}(t) \overset{<}{\overset{>}{\overset{\sim}{\overset{\approx}{=}}} \ V_{14}(t) \]
\[ V_{11}(t) = Q_{11,1}(t) \overset{<}{\overset{>}{\overset{\sim}{\overset{\approx}{=}}} \ V_1(t) \]
\[ V_{12}(t) = Q_{12,2}(t) \odot V_2(t) \]
\[ V_{13}(t) = Q_{13,1}(t) \odot V_1(t) \]
\[ V_{14}(t) = Q_{14,2}(t) \odot V_2(t) \]

where

\[ V_0^{**}(s) = \frac{N_4(s)}{D_1(s)} \]

\[ N_4(s) = (Q_{12}^{(6)**}(s) + Q_{15}^{**}(s))Q_{20}^{**}(s) + Q_{10}^{**}(s)(1 - Q_{22}^{(9)**}(s)Q_{2,10}^{**}(s)) \]

and \( D_1(s) \) is already specified.

In steady-state, the expected number of visits per unit time of the expert repairman is given by

\[ V_0 = \lim_{s \to 0} s V_0^{**}(s) = \frac{N_4}{D_1} \]

where

\[ N_4 = (p_{12}^{(6)} + p_{15}) p_{20} + p_{10} (1 - p_{22}^{(9)} - p_{2,10}) \]

and \( D_1 \) is already specified.

**Expected Number of Replacements**

Let \( R_i(t) \) be the probability of number of replacement at instant \( t \). By probabilistic arguments we have the following recursive relations:

\[ R_0(t) = Q_{01}(t) \odot R_1(t) + Q_{02}(t) \odot R_2(t) \]
\[ R_1(t) = Q_{10}(t) \odot R_0(t) + Q_{11}^{(3)}(t) \odot R_1(t) + Q_{14}(t) \odot R_{14}(t) \odot R_{15}(t) \odot R_5(t) + Q_{12}^{(6)}(t) \odot R_2(t) \]
\[ R_2(t) = Q_{20}(t) \odot [1 + R_0(t)] + Q_{21}^{(7)}(t) \odot [1 + R_1(t)] + Q_{28}(t) \odot V_8(t) + Q_{22}^{(9)}(t) \]
\[ \odot [1 + R_2(t)] + Q_{2,10}(t) \odot R_{10}(t) \]
\[ R_4(t) = Q_{4,11}(t) \odot R_{11}(t) \]
\[ R_5(t) = Q_{5,12}(t) \odot R_{12}(t) \]
\[ R_8(t) = Q_{8,13}(t) \odot R_{13}(t) \]
\[ R_{10}(t) = Q_{10,14}(t) \odot R_{14}(t) \]
\[ R_{11}(t) = Q_{11,1}(t) \odot R_1(t) \]
\[ R_{12}(t) = Q_{12,2}(t) \odot R_2(t) \]
\[ R_{13}(t) = Q_{13,1}(t) \$ [1+R_1(t)] \]
\[ R_{14}(t) = Q_{14,2}(t) \$ [1+R_2(t)] \]

Taking L.S.T. of the above equations and solving them for \( R_0^{**}(s) \), we get

\[ R_0^{**}(s) = \frac{N_5(s)}{D_1(s)} \]

where

\[ N_5(s) = 1 - Q_{11}^{(3)*}(s) - Q_{14}^{**}(s) - Q_{01}^{**}(s) Q_{10}^{**}(s) \]

and \( D_1(s) \) is already specified

In steady-state, the expected number of replacements per unit time is given by

\[ R_0(s) = \lim_{s \to 0} s \ R_0^{**}(s) = \frac{N_5}{D_1} \]

where

\[ N_5 = 1 - p_{11}^{(3)} - p_{14} - p_{01}p_{10} \]

and \( D_1 \) is already specified

**Expected Time During which the Operation is Performed by Some Other System on the Failure of Both the Units**

Let \( AP_i(t) \) be the probability that the operation is performed by some other system on failure of both the units. By probabilistic arguments we have the following recursive relations:

\[ AP_0(t) = q_{01}(t) \odot AP_1(t) + q_{02}(t) \odot AP_2(t) \]
\[ AP_1(t) = q_{10}(t) \odot AP_0(t) + q_{11}^{(3)}(t) \odot AP_1(t) + q_{12}^{(6)}(t) \odot AP_2(t) + q_{14}(t) \odot AP_4(t) \]
\[ + q_{15}(t) \odot AP_5(t) \]
\[ AP_2(t) = q_{20}(t) \odot AP_0(t) + q_{21}^{(7)}(t) \odot AP_1(t) + q_{22}^{(9)}(t) \odot AP_2(t) + q_{28}(t) \odot AP_8(t) \]
\[ + q_{210}(t) \odot AP_{10}(t) \]
\[ AP_3(t) = q_{411}(t) \odot AP_{11}(t) \]
\[ AP_5(t) = q_{512}(t) \odot AP_{12}(t) \]
\[ AP_8(t) = q_{8,13}(t) \odot AP_{13}(t) \]
\[ AP_{10}(t) = q_{10,14}(t) \odot AP_{14}(t) \]
\[ AP_{11}(t) = W_{11}(t) + q_{11,1}(t) \odot AP_{1}(t) \]
\[ AP_{12}(t) = W_{12}(t) + q_{12,2}(t) \odot AP_{2}(t) \]
\[ AP_{13}(t) = W_{13}(t) + q_{13,1}(t) \odot AP_{1}(t) \]
\[ AP_{14}(t) = W_{14}(t) + q_{14,2}(t) \odot AP_{2}(t) \]
\[ AP_{0} = \frac{N_6}{D_1} \]

where
\[ W_{11}(t) = W_{12}(t) = \overline{G_1(t)} \]
\[ W_{13}(t) = W_{14}(t) = \overline{G_2(t)} \]
\[ AP_{0}^{\ast}(s) = \frac{N_6(s)}{D_1(s)} \]

where
\[ N_6(s) = (W_{11}^{\ast}(s) q_{14}^{\ast}(s) + W_{12}^{\ast}(s) q_{15}^{\ast}(s) q_{01}^{\ast}(s) q_{20}^{\ast}(s) + q_{27}^{\ast}(s) + q_{28}^{\ast}(s)) \]
\[ + (W_{13}^{\ast}(s) q_{28}^{\ast}(s) + W_{14}^{\ast}(s) q_{2,10}^{\ast}(s) q_{02}^{\ast}(s) q_{10}^{\ast}(s) + q_{12}^{(6)} + q_{15}^{\ast}(s)) \]

and \( D_1(s) \) is already specified.

In steady-state, the expected time during which operation is performed by some other system

\[ AP_{0} = \lim_{s \to 0} s \ AP_{0}^{\ast}(s) = \frac{N_6}{D_1} \]

where
\[ N_6 = (W_{11} p_{14} + W_{12} p_{15}) (p_{01} p_{20} + p_{27} + p_{28}) + (W_{13} p_{28} + W_{14} p_{2,10}) (p_{02} p_{10} + p_{12}^{(6)} + p_{15}) \]

and \( D_1 \) is already specified

**Profit Analysis**

\[ P_3 = C_0 A_0 - C_1 B_0 - C_2 B R_0 - C_3 V_0 - C_4 R_0 - C_5 A P_0 \]
\[ C_0 = \text{revenue per unit up time} \]
\[ C_1 = \text{cost per unit time for which the repairman is busy for repair} \]
\[ C_2 = \text{cost per unit time for which the repairman is busy for replacement} \]
$C_3 = \text{cost per visit of the repairman}$

$C_4 = \text{cost per unit replacement}$

$C_5 = \text{cost per unit of alternate performance}$

**Particular Case**

The following particular case is considered for graphical interpretation:

\[ g_1(t) = \alpha_1 e^{-\alpha_1 t} \quad g_2(t) = \alpha_2 e^{-\alpha_2 t} \]

Therefore, we have:

\[
\begin{align*}
p_{01} &= p \\
p_{10} &= \frac{1}{\alpha_1 + \lambda} \\
p_{11}^{(3)} &= \frac{q_1 p}{\alpha_1 + \lambda} \\
p_{12}^{(6)} &= \frac{q_1 q}{\alpha_1 + \lambda} \\
p_{14} &= \frac{p_1 p}{\alpha_1 + \lambda} \\
p_{15} &= \frac{p_1 q}{\alpha_1 + \lambda} \\
p_{16} &= \frac{q_1 q}{\alpha_1 + \lambda} \\
p_{2,1}^{(7)} &= \frac{q_1 p}{\alpha_2 + \lambda} \\
p_{2,7} &= \frac{p q_1}{\alpha_2 + \lambda} \\
p_{2,9} &= \frac{q q_1}{\alpha_2 + \lambda} \\
\mu_0 &= \frac{1}{\lambda} \\
\mu_1 &= \frac{1}{(\alpha_1 + \lambda)} \\
\mu_2 &= \frac{1}{(\alpha_2 + \lambda)} \\
\mu_4 &= \mu_5 = \mu_8 = \mu_{10} = \frac{1}{\beta} \\
\mu_{11} = \mu_{12} &= \frac{1}{\alpha_1} \\
\mu_{13} = \mu_{14} &= \frac{1}{\alpha_2}
\end{align*}
\]
\[ K_1 = \frac{p_1}{\lambda + \alpha_1} + \frac{q_1}{\alpha_1} \quad K_2 = \frac{p_1}{\lambda + \alpha_2} + \frac{q_1}{\alpha_2} \]

Using the above particular case, the following values as estimated in Chapter II, i.e.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>Failure Rate ((\lambda))</td>
<td>0.000251784/hr</td>
</tr>
<tr>
<td>Repair Rate ((\alpha_1))</td>
<td>0.090136047/hr</td>
</tr>
<tr>
<td>Replacement Rate ((\alpha_2))</td>
<td>0.023161446/hr</td>
</tr>
<tr>
<td>Probability for repair ((p))</td>
<td>0.282</td>
</tr>
<tr>
<td>Cost for replacement the unit ((C_3))</td>
<td>Rs.86390.196</td>
</tr>
</tbody>
</table>

and the rest of values as assumed values, the values of various measures of system effectiveness are obtained as:

<p>| | |</p>
<table>
<thead>
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</thead>
<tbody>
<tr>
<td>MTSF</td>
<td>40400 hrs</td>
</tr>
<tr>
<td>Availability ((A_0))</td>
<td>0.8626</td>
</tr>
<tr>
<td>Expected busy period for repair ((B_0))</td>
<td>0.0021/hr</td>
</tr>
<tr>
<td>Expected busy period for replacement ((BR_0))</td>
<td>0.0057/hr</td>
</tr>
<tr>
<td>Expected number of visits by the ordinary repairman ((V_0))</td>
<td>2.487 E-3</td>
</tr>
<tr>
<td>Expected number of replacements ((R_0))</td>
<td>0.5265</td>
</tr>
<tr>
<td>Expected number of alternate performance ((AP_0))</td>
<td>0.0074</td>
</tr>
</tbody>
</table>

**Graphical Representation of Results**

Fig.3.2 depicts the behaviour of profit (\(P_3\)) with respect to revenue (\(C_0\)) for different values of cost of alternate performance (\(C_5\)).
It can be interpreted from that the profit increases with the increase in the values of $C_0$ and has lower values for higher values of $C_5$

**Interpretation**

(i) For $C_5 = 6500$, the profit is positive or zero or negative according as $C_0 > \text{ or } = \text{ or } < 136.33$. Hence, for this case the revenue per unit up time should be fixed greater than 136.33.

(ii) For $C_5 = 7000$, the profit is positive or zero or negative according as $C_0 > \text{ or } = \text{ or } < 146.539$. Hence, for this case the revenue per unit up time should be fixed greater than 146.539.

(iii) For $C_5 = 7500$, the profit is positive or zero or negative according as $C_0 > \text{ or } = \text{ or } < 156.7497$. Hence, for this case the revenue per unit up time should be fixed greater than 156.7497.
Fig. 3.3 reveals the behaviour of profit \(P_3\) with respect to cost per visit of repairman \(C_4\) for different values of cost of busy period of repairman \(C_1\).

**Interpretation**

It can be interpreted from the graph that profit decreases with the increase in cost per visit of repairman \(C_4\) and has higher values for lower values of \(C_1\).

(i) For \(C_1 = 880\), the profit is positive or zero or negative according as \(C_4 < \text{ or } 336.8\). Hence, not more than an amount of \(336.8\) per visit should be paid to the repairman.

(ii) For \(C_1 = 885\), the profit is positive or zero or negative according as \(C_4 < \text{ or } 293.6\). Hence, not more than an amount of \(293.6\) per visit should be paid to the repairman.

(iii) For \(C_1 = 890\), the profit is positive or zero or negative according as \(C_4 < \text{ or } 250.4\). Hence, not more than an amount of \(250.4\) per visit should be paid to the repairman.