CHAPTER 4

2DOF QUARTER CAR PNEUMATIC (AIR DAMPED)
SEMI-ACTIVE ROAD VEHICLE SUSPENSION SYSTEM

4.1 INTRODUCTION
According to R. A. Williams of Jaguar Cars, automotive suspension design is compromise brought about by the conflicting demands of ride and handling[2]. The focus of the automotive designers is predominantly on providing ride comfort to the occupants of the ground vehicle by isolating them from the road surface unevenness. Contemporary ground vehicle suspensions have passive control elements, namely springs and dampers. These designs have evolved to the point at which it seems reasonable to suppose that they will not improve much without changes in the principle. Such changes in principle have become commercially viable with the introduction of semi-active and active suspensions. Semi-active Suspensions which operate under a control system generated from measurements of the sprung mass, unsprung mass motion and road surface amplitudes are only capable of dissipating energy. The necessary dissipation of energy can be effected from a variable damping coefficient of a damper placed across sprung and unsprung mass of the vehicle. As explained in Chapter 3, the variable damping force can be provided by the damper units such as hydraulic fluid damper, hydro-pneumatic damper, electro-magnetic damper, MR damper, ER damper, friction damper and air damper etc. The air damper is of two types (i) air enclosed cylinder-piston and air tank type air damper and (ii) air enclosed cylinder-piston type air damper etc. for controlling the resonant response of the sprung mass. The design and development of an air enclosed Cylinder-Piston and Air Tank Type Air Damper has been explained in Chapter 3. This damper provides a variable damping force which has practically no dependence on the working temperature, it has less maintenance and production costs and there are no long term changes in the damping properties.
Having developed such a type of air damper with necessary design equations for air damping ratio $\zeta_a$ and air spring rate ratio $k$, this damper has been included into a 2DOF quarter car pneumatic (air damped) semi-active road vehicle suspension system model to study its effectiveness in controlling motion transmissibility of the sprung mass to provide better ride comfort to the occupants of the vehicle.

As such, in this chapter, the motion transmissibilities $M_{t1}$ of the sprung mass $m_1$ and $M_{t2}$ of the unsprung mass $m_2$ of a 2DOF quarter car pneumatic (air damped) semi-active road vehicle suspension system have been determined (refer fig. 4.3). The effect of variation of the system damping ratio $\zeta_l$, mass ratio $\mu$, air damping ratio $\zeta_a$ and air spring rate ratio $k$ on $M_{t1}$ and $M_{t2}$ has been investigated when the air damper is modeled as a Maxwell type model.

### 4.2 2DOF QUARTER CAR PNEUMATIC (AIR DAMPED) SEMI-ACTIVE ROAD VEHICLE SUSPENSION SYSTEM

The major elements of a mathematical model of a 2DOF quarter car pneumatic (air damped) semi-active road vehicle suspension system are shown in fig. 4.3. These are: sprung mass (i.e. body mass, suspension springs and viscous damper), unsprung mass (comprising wheel, brakes, the part of the mass of the drive shafts and suspension linkages) and the tyre spring. The air damper introduces a variable coefficient of viscous air damping $c_a$ and to some extent a variable air spring rate $k_a$. Also $x_1(t)$ and $x_2(t)$ are the displacement responses of the sprung mass $m_1$, unsprung mass $m_2$; $u(t)$ and $y(t)$ are the base excitation due to the road surface unevenness and the displacement response of the point 'o' respectively. The parameters defined are, the mass ratio $\frac{m_2}{m_1} = \mu$, the suspension spring rate $= k_1$ and tyre spring rate $= k_2$ (refer Fig. 4.3 and Fig. 4.34)
4.3 EQUATIONS OF MOTION

A 2DOF quarter car modeling concept has been taken for analysis. Equations of motion have been derived and are given respectively in Tables 4.1, 4.2 and 4.3 for the following..

i) A 2DOF quarter car vehicle suspension system with system damping only and without air damper (Refer Fig. 4.1 in Table 4.1). Hereafter referred as Case 4.1

ii) A 2DOF air damped quarter car vehicle suspension system with Vigot type model for air damper (Refer Fig. 4.2 in Table 4.2). Hereafter referred as Case 4.2

iii) A 2DOF air damped quarter car vehicle suspension system with Maxwell type model for air damper (Refer Fig. 4.3 in Table 4.3). Hereafter referred as Case 4.3

4.3.1 Motion Transmissibility for Case 4.3

For the theoretical analysis of the motion transmissibilitys Mt1 and Mt2, the air damper has been modeled as a Maxwell type model (Case 4.3). For this case, the equations for Mt1 and Mt2 have been derived as under… (refer Table 4.3 )

Equation of motion of mass \( m_i \) is given as..

\[
\begin{align*}
    m_i \ddot{x}_i & = -k_i (x_i - x_2) - c_i (\dot{x}_i - \dot{x}_2) - c_a (\dot{x}_i - \dot{y}) \\
    m_i \ddot{\dot{x}}_i & = -k_i \dot{x}_1 + k_i x_2 - c_i \dot{x}_1 + c_i \ddot{x}_2 - c_a \dot{x}_1 + c_a \dot{y} \\
    m_i \ddot{x}_1 + k_i x_1 + c_i \dot{x}_1 + c_a \dot{x}_1 & = k_i x_2 + c_i \ddot{x}_2 + c_a \dot{y}
\end{align*}
\]

(4.9)

Using the differential operator form and simplifying , we obtain equation (4.9) as...

\[
[ m_i D^2 + (c_i + c_a)D + k_i ]x_i = (c_i D + k_i) x_2 + c_a D \dot{y}
\]

(4.10)

At the point ‘o’, equation of motion is given as

\[
\begin{align*}
    -c_a (\dot{y} - \dot{x}_i) - k_a (y - x_2) & = 0 \\
    -c_a \dot{y} + c_a \dot{x}_i - k_a y + k_a x_2 & = 0
\end{align*}
\]

(4.11)

Using the differential operator form and simplifying , we obtain equation (4.11) as..

\[
(c_a D + k_a) y = c_a D x_1 + k_a x_2
\]
\[
y = \frac{c_a D}{(c_a D + k_a)} x_1 + \frac{k_a}{(c_a D + k_a)} x_2
\]

(4.12)

Table 4.3 : 2DOF Air Damped Road Vehicle Suspension System using an Air Damper (Maxwell Model), with System Damping Ratios \(\zeta_1, \zeta_2\), Air Damping Ratio \(\zeta_a\) and Air Spring Rate ratio \(k = (k_a/k_1)\)

Case 4.3

Fig 4.3 A 2DOF Semi-Active (Air Damped) Road Vehicle Suspension System \((\mu < 1)\), Air Damper Modeled as a Maxwell Type

Equations of motion

\[
m_1 \ddot{x}_1 = -k_1(x_1 - x_3) - c_1(x_1 - \dot{x}_1) - c_o(x_1 - \dot{y})
\]

(4.9)

\[
- c_o(\dot{y} - \dot{x}_1) - k_o(y - x_2) = 0
\]

(4.10)

\[
m_2 \ddot{x}_2 = -k_1(x_2 - x_1) - c_1(x_2 - \dot{x}_1) - k_s(x_2 - y) - k_o(x_2 - u) - c_o(x_2 - \dot{u})
\]

(4.11)

\[
\begin{align*}
\text{Mtt1} &= X_1 = [\frac{[a_{11} - a_{22} + a_o]}{U} + [a_{12} + a_o]]^2 \ldots (4.12) \\
\text{Mtt2} &= X_2 = [\frac{[a_{11} - a_{22} + a_o]}{U} + [a_{12} + a_o]]^2 \ldots (4.13)
\end{align*}
\]

Where

\[
\begin{align*}
a_4 &= 4 \zeta_1 \zeta_2 v, a_2 = 2 \zeta_1 v^2 + 8 \zeta_1 \zeta_2 v + 2 \zeta_2 v (1 + k) \\
a_2 &= \delta [4 \zeta_1 v^2 + 2 \zeta_2 v + k 2 \zeta_2 v + 2 \zeta_2 v f + 4 \delta^2 \zeta_1 v + v^2 (1 + k)] \\
a_1 &= \delta^2 [2 \zeta_1 v + 2 \zeta_2 v + \delta v (4 \zeta_1 v + 4 \zeta_2 v + 2 \zeta_2 k)] \\
a_{10} &= \delta^2 v^2 = \delta (k v^2 + 2 v^2), \\
b_5 &= 1, b_3 = (2 \delta + 2 \zeta_1 + (2 \zeta_1/\mu) + 2 \zeta_2 v) \\
b_4 &= [1 + k + (1/\mu) + v^2 + (k/\mu) + 4 \zeta_1 \zeta_2 v + \delta [\zeta_1 + (\zeta_1/\mu) + \zeta_2 v] + \delta^2 \\
b_3 &= \delta [2 + 8 \zeta_1 \zeta_2 v (2/\mu) + 2 v + k (k/\mu)] + \delta [2 \zeta_2 v + 2 \zeta_2 v + 4 \zeta_2 v + 2 \zeta_2 v + 2 \zeta_2 v (1 + k)] \\
b_1 &= \delta^2 [2 \zeta_2 v + 4 v^2 \zeta_1] + 2 \delta v^2 \\
b_0 &= \delta^2 v^2
\end{align*}
\]
Substituting equation (4.12) in (4.10) for elimination of \( y \), we obtain

\[
[m_1 \ddot{x}_1 + (c_1 + c_\alpha) D + k_1] x_1 = (c_1 D + k_1) x_2 + c_\alpha D \left[ \frac{c_\alpha D}{(c_\alpha D + k_\alpha)} \right] x_1 + \frac{k_\alpha}{(c_\alpha D + k_\alpha)} x_1
\]

\[
[m_1 \ddot{x}_2 + (c_1 + c_\alpha) D + k_1] x_2 = (c_1 D + k_1) x_2 + \frac{c_\alpha D c_\alpha D}{(c_\alpha D + k_\alpha)} x_2 + \frac{k_\alpha c_\alpha D}{(c_\alpha D + k_\alpha)} x_1
\]

\[
[m_1 \ddot{x}_2 + (c_1 + c_\alpha) D + k_1] x_2 - \frac{c_\alpha D c_\alpha D}{(c_\alpha D + k_\alpha)} x_1 = ((c_1 D + k_1) + \frac{k_\alpha c_\alpha D}{(c_\alpha D + k_\alpha)}) x_2
\]

\[
[(m_1 D^2 + (c_1 + c_\alpha) D + k_1)[c_\alpha D + k_\alpha] - c_\alpha D c_\alpha D] x_1 = \{(c_1 D + k_1)(c_\alpha D + k_\alpha) + k_\alpha c_\alpha D\} x_2
\]

\[
x_2 = \frac{A}{B} x_1 \quad \text{or} \quad x_1 = \frac{B}{A} x_2 \quad \text{(4.13)}
\]

Where

\[
A = \{(m_1 D^2 + (c_1 + c_\alpha) D + k_1)(c_\alpha D + k_\alpha) - (c_\alpha D)^2\}
\]

\[
B = \{(c_1 D + k_1)(c_\alpha D + k_\alpha) + k_\alpha c_\alpha D\}
\]

Equation of motion of mass \( m_2 \) is given as ..

\[
m_2 \ddot{x}_2 = -k_\alpha (x_2 - x_1) - c_1 (\dot{x}_2 - \dot{x}_1) - k_\alpha (x_2 - y) - k_2 (x_2 - u) - c_2 (\dot{x}_2 - \dot{u})
\]

\[
m_2 \ddot{x}_2 = -k_1 x_2 + k_1 x_1 - c_1 \dot{x}_2 + c_1 \dot{x}_1 - k_\alpha y - k_2 x_2 + k_2 u - c_2 \dot{x}_2 + c_2 \dot{u}
\]

\[
m_2 \ddot{x}_2 + c_1 \dot{x}_2 + c_1 \dot{x}_2 + k_\alpha x_2 + k_\alpha x_2 + c_2 \dot{x}_2 = k_1 x_1 + c_1 \dot{x}_1 + k_\alpha y + k_2 u + c_2 \dot{u}
\]

\[
m_2 \ddot{x}_2 + c_1 \dot{x}_2 + c_2 \dot{x}_2 + k_\alpha x_2 + k_\alpha x_2 + c_2 \dot{x}_2 = c_1 \dot{x}_1 + k_1 x_1 + k_\alpha y + c_2 \dot{u} + k_2 u
\]

\[
m_2 \ddot{x}_2 + c_1 \dot{x}_2 + c_2 \dot{x}_2 + k_\alpha x_2 + k_\alpha x_2 + c_2 \dot{x}_2 = c_1 \dot{x}_1 + k_1 x_1 + k_\alpha y + c_2 \dot{u} + k_2 u \quad \text{(4.14)}
\]

Using the differential operator form, we obtain equation (4.14) as …
Substituting equations (4.12) and (4.13) in equation (4.15), we obtain:

\[
[m_2 D^2 + (c_1 + c_2) D + (k_1 + k_2 + k_3)] x_2 = (c_1 D + k_1) x_1 + k_a \left( \frac{c_a D}{c_a D + k_a} \right) x_1 + \frac{k_a}{(c_a D + k_a)} x_2 + (c_2 D + k_2) u
\]

\[
[m_2 D^2 + (c_1 + c_2) D + (k_1 + k_2 + k_3)] (c_a D + k_a) x_2
\]

\[
= (c_1 D + k_1)(c_a D + k_a) x_1 + k_a c_a D x_1 + k_a k_a x_2 + k_a c_a D x_2 + (c_2 D + k_2)(c_a D + k_a) u
\]

\[
[m_2 D^2 + (c_1 + c_2) D + (k_1 + k_2 + k_3)](c_a D + k_a) x_3
\]

\[
= (c_1 D + k_1)(c_a D + k_a) x_1 + k_a c_a D x_1 + k_a k_a x_2 + (c_2 D + k_2)(c_a D + k_a) u
\]

\[
[m_2 D^2 + (c_1 + c_2) D + (k_1 + k_2 + k_3)](c_a D + k_a) x_4
\]

\[
= (c_1 D + k_1)(c_a D + k_a) x_1 + k_a c_a D x_1 + k_a k_a x_2 + (c_2 D + k_2)(c_a D + k_a) u
\]

\[
[m_2 D^2 + (c_1 + c_2) D + (k_1 + k_2 + k_3)](c_a D + k_a) x_5
\]

\[
= (c_1 D + k_1)(c_a D + k_a) x_1 + k_a c_a D x_1 + k_a k_a x_2 + (c_2 D + k_2)(c_a D + k_a) u
\]

Putting

\[
([m_1 D^2 + (c_1 + c_2) D + k_1](c_a D + k_a) - (c_a D)^2) = A
\]

\[
((c_1 D + k_1)(c_a D + k_a) + k_a c_a D) = B
\]

\[
[m_2 D^2 + (c_1 + c_2) D + (k_1 + k_2 + k_3)] = E
\]

\[
(c_a D + k_a) - k_4 k_a = C
\]

\[
(c_2 D + k_2)(c_a D + k_a) = G
\]

and substituting these expressions in equation (4.6) and after rearranging, we get:

\[
\frac{x_1}{u} = \frac{B G}{(C A - B^2)}
\]

\[
\frac{x_2}{u} = \frac{A G}{(C A - B^2)}
\]

The terms \(B G, A G\) and \((C A - B^2)\) have been calculated as

\[
B G = [(c_1 D + k_1)(c_a D + k_a) + k_a c_a D](c_2 D + k_2)(c_a D + k_a)
\]

\[
A G = [m_2 D^2 + (c_1 + c_2) D + k_1](c_a D + k_a) - (c_a D)^2
\]

\[
(C A - B^2) = E[(c_a D + k_a) - k_4 k_a] \{[m_2 D^2 + (c_1 + c_2) D + k_1](c_a D + k_a) - (c_a D)^2\}
\]
The following terms have been defined.

\[
\begin{align*}
  k_1 & = w_1^2, \quad k_2 = w_2^2, \quad k_a = w_a^2, \quad v = \frac{w_2}{w_1} \quad \text{and} \quad k = \frac{k_a}{k_1}, \\
  c_1 &= 2 \zeta_1 w_1, \quad c_2 = 2 \zeta_2 w_2, \quad \text{and} \quad c_a = 2 \zeta_a w_a.
\end{align*}
\]

we obtain a set of equations as

\[
\begin{align*}
  \frac{c_a}{m_1} &= 2 \zeta_a \sqrt{k} w_1, \quad \frac{c_1}{m_2} = \frac{2 \zeta_1 w_1}{\mu}, \quad \frac{c_a}{m_2} = \frac{2 \zeta_a \sqrt{k} w_1}{\mu}, \\
  \frac{k_a}{m_1} &= k w_1^2 \quad \text{and} \quad \delta = \frac{k_a}{c_a} = \frac{\sqrt{k}}{2 \zeta_a}. \\
\end{align*}
\]

(4.20)

Using the parameters just defined and substituting the operator \( D = j w \) and frequency ratio \( \frac{w}{w_1} = \lambda \) in equation (4.19), it is converted in the complex form as ...

\[
\begin{align*}
  BG &= [ a_4 \lambda^4 - a_2 \lambda^2 + a_0 ] + j[-a_1 \lambda^3 + a_1 \lambda] \\
  AG &= [ a_{44} \lambda^4 - a_{22} \lambda^2 + a_{00} ] + j[a_{55} \lambda^5 - a_{33} \lambda^3 + a_{11} \lambda] \\
  (CA - B^2) &= [-b_6 \lambda^6 + b_4 \lambda^4 - b_2 \lambda^2 + b_0 ] + j[b_5 \lambda^5 - b_3 \lambda^3 + b_1 \lambda]
\end{align*}
\]

where

\[
\begin{align*}
  a_4 &= 4 \zeta_1 \zeta_2 v, a_3 = 2 \zeta_1 v^2 + 8 \delta \zeta_1 \zeta_2 v + 2 \zeta_2 v (1 + k) \\
  a_2 &= \delta [4 \zeta_1 v^2 + 2 \zeta_2 v + k (2 \zeta_2 v + 2 \zeta_2 v ) + 4 \delta^2 \zeta_1 \zeta_2 v + v^2 (1 + k) ] \\
  a_1 &= \delta^2 [2 \zeta_1 v + \zeta_2 ] + \delta v^2 [ 2 + k ], a_0 = \delta^2 v^2
\end{align*}
\]

and

\[
\begin{align*}
  a_{55} &= 2 \zeta_2 v, a_{44} = [ v^2 + 4 \delta \zeta_2 v + 4 \zeta_1 \zeta_2 v ] \\
  a_{33} &= [2 \delta v^2 + 8 \delta \zeta_1 \zeta_2 v + 2 \zeta_1 v^2 + 2 v \delta^2 (\zeta_1 + \zeta_2) + \zeta_2 v (1 + 2 k) ] \\
  a_{22} &= [\delta^2 v (v + 4 \zeta_1 \zeta_2) + \delta v (4 \zeta_1 v + 4 \zeta_2 v + 2 \zeta_2 k ) + v^2 (1 + k) ] \\
  a_{11} &= \delta^2 (4 \zeta_1 v^2 + 2 \zeta_2 v ) + \delta (k v^2 + 2 v^2) \\
  a_{00} &= \delta^2 v^2
\end{align*}
\]
and

\[ b_6 = 1 \]

\[ b_5 = (2 \delta + 2 \zeta_1 + (2 \zeta_1 / \mu) + 2 \zeta_2 v) \]

\[ b_4 = [1 + k + (1 / \mu) + v^2 + (k / \mu) + 4 \zeta_1 \zeta_2 v] + 4 \delta [\zeta_1 + (\zeta_1 / \mu) + \zeta_2 v] + \delta^2 \]

\[ b_3 = \delta[2 + 8 \zeta_1 \zeta_2 v + (2 / \mu) + 2 v^2 + k + (k / \mu)] \]

\[ + [2 \zeta_2 v + 2 \zeta_1 \zeta_2 v k + 4 \zeta_1 \zeta_2 v^2] + \delta^2 [2 \zeta_1 + (2 \zeta_1 / \mu) + 2 \zeta_2 v] \]

\[ b_2 = \delta^2 [1 + 4 \zeta_1 \zeta_2 v + (1 / \mu) + v^2] + \delta [4 \zeta_2 v + 4 v^2 \zeta_1 + 2 \zeta_2 v k] + v^2 (1 + k) \]

\[ b_1 = \delta^2 [2 \zeta_2 v + 4 v^2 \zeta_1] + 2 \delta v^2, b_0 = \delta^2 v^2 \]

Using the above expressions for \( B G, A G \) and \( C A - B^2 \), the transmissibility ratio \( M_{t1} \) is obtained as…

\[ M_{t1} = \frac{X_1}{U} = \frac{[a_{y stub} - a_{y stub} + a_y] + j[-a_{y stub} + a_y]}{[-b_{y stub} + b_{y stub} - b_{y stub} + b_y] + j[b_{y stub} - b_{y stub} + b_y]} \]

or

\[ M_{t1} = \frac{X_1}{U} = \left[ \frac{[a_{y stub} - a_{y stub} + a_y]^2 + [-a_{y stub} + a_y]^2}{[-b_{y stub} + b_{y stub} - b_{y stub} + b_y]^2 + [b_{y stub} - b_{y stub} + b_y]^2} \right]^{1/2} \] (4.21)

Similarly the transmissibility ratio \( M_{t2} \) is obtained as…

\[ M_{t2} = \frac{X_2}{U} = \frac{[a_{y stub} - a_{y stub} + a_y] + j[a_{y stub} - a_{y stub} + a_y]}{[-b_{y stub} + b_{y stub} - b_{y stub} + b_y] + j[b_{y stub} - b_{y stub} + b_y]} \]

or

\[ M_{t2} = \frac{X_2}{U} = \left[ \frac{[a_{y stub} - a_{y stub} + a_y]^2 + [a_{y stub} - a_{y stub} + a_y]^2}{[-b_{y stub} + b_{y stub} - b_{y stub} + b_y]^2 + [b_{y stub} - b_{y stub} + b_y]^2} \right]^{1/2} \] (4.22)

These expressions have been given in Table 4.3
4.3.2 Motion Transmissibility for Case 4.1

For this case, the air damper has been removed from fig. 4.3. (also refer Fig. 4.1 of Table 4.1)

Table 4.1 : A 2DOF Viscously Damped Road Vehicle Suspension System with System Damping Coefficients \( c_1 \) and \( c_2 \) only ( Corresponding System Damping Ratios \( \zeta_1 \) and \( \zeta_2 \) )

<table>
<thead>
<tr>
<th>Case 4.1</th>
<th>Equations of motion</th>
</tr>
</thead>
</table>
| \[
\begin{align*}
    m_1 \ddot{x}_1 &= -k_1(x_1 - x_2) - c_1(\dot{x}_1 - \dot{x}_2) \\
    m_2 \ddot{x}_2 &= -k_2(x_2 - \dot{u}) - c_2(\dot{x}_2 - \dot{u})
\end{align*}
\] |

(refer equation (4.1) in Table 4.1)

| \[
\begin{align*}
    m_1 \ddot{x}_1 &= -k_1(x_1 - x_2) - c_1(\dot{x}_1 - \dot{x}_2) \\
    m_2 \ddot{x}_2 &= -k_2(x_2 - \dot{u}) - c_2(\dot{x}_2 - \dot{u})
\end{align*}
\] |

(refer equation (4.2) in Table 4.1)

Using the procedure similar to that explained in Section 4.2.1, the transmissibility ratios \( M_{t1} \) and \( M_{t2} \) have been obtained and are given as

\[
M_{t1} = \frac{X_1}{U} = \left[ \frac{[-A_2 \lambda^2 + A_5 \lambda^2] + [A_{11} \lambda^2]}{[B_4 \lambda^2 - B_2 \lambda^2 + B_6]^2 + [-B_5 \lambda^2 + B_4 \lambda]^2} \right]^{\frac{1}{2}}
\]  

(equation (4.3) in Table 4.1)

\[
M_{t2} = \frac{X_2}{U} = \left[ \frac{[-A_2 \lambda^2 + A_{10} \lambda^2] + [A_{11} \lambda^2]}{[B_4 \lambda^2 - B_2 \lambda^2 + B_6]^2 + [-B_5 \lambda^2 + B_4 \lambda]^2} \right]^{\frac{1}{2}}
\]  

(equation (4.4) in Table 4.1)

The values of \( A_2, A_{11}, A_{10}, A_{22}, A_{11l}, A_{10b}, B_4, B_2, B_1 \) and \( B_0 \) are have been given in Table 4.1. 

In this case, the equations of motion are ...

\[
\begin{align*}
    m_1 \ddot{x}_1 &= -k_1(x_1 - x_2) - c_1(\dot{x}_1 - \dot{x}_2) \\
    m_2 \ddot{x}_2 &= -k_2(x_2 - \dot{u}) - c_2(\dot{x}_2 - \dot{u})
\end{align*}
\]  

(refer equation (4.1) in Table 4.1)

\[
\begin{align*}
    m_1 \ddot{x}_1 &= -k_1(x_1 - x_2) - c_1(\dot{x}_1 - \dot{x}_2) \\
    m_2 \ddot{x}_2 &= -k_2(x_2 - \dot{u}) - c_2(\dot{x}_2 - \dot{u})
\end{align*}
\]  

(refer equation (4.2) in Table 4.1)
4.2.3 Motion Transmissibility for Case 4.2

For this case, the air damper has been modeled as a Vigot type model as shown in Fig. 4.2.

Table 4.2: 2DOF Air Damped Road Vehicle Suspension System using an Air Damper (Vigot Model), with System Damping Ratios $\zeta_1$, $\zeta_2$.

**Damping Ratio $\zeta_a$ and Air Spring Rate ratio $k = (k_a / k_1)$**

![Fig. 4.2 A 2DOF Semi-Active (Air Damped) Road Vehicle Suspension System ($\mu <1$), Air Damper Modeled as a Vigot type](image)

The equations of motion are ...

\[
m_1 \ddot{x}_1 = -(k_1 + k_a)(x_1 - x_2) - (c_1 + c_a)(\dot{x}_1 - \dot{x}_2)
\]

\[
m_2 \ddot{x}_2 = -(k_1 + k_a)(x_2 - x_1) - (c_1 + c_a)(\dot{x}_2 - \dot{x}_1) - k_2 (x_2 - u) - c_2 (\dot{x}_2 - \dot{u})
\]

Using the procedure similar to that explained in section 4.2.1, the transmissibility ratios Mt1 and Mt2 have been obtained respectively as

\[
Mt1 = \frac{X_1}{U} = \left[ \frac{[-a_2 \dot{x}_2^2 + a_0]^2 + [a_1 \dot{x}_1]^2}{[b_2 \dot{x}_2^2 - b_2 \dot{x}_2 + b_0]^2 + [-b_1 \dot{x}_1^2 + b_1 \dot{x}_1]^2} \right]^{\frac{1}{2}}
\]

(refer equation (4.5) in Table 4.2)

\[
Mt2 = \frac{X_2}{U} = \left[ \frac{[-a_2 \dot{x}_2^2 + a_0]^2 + [a_1 \dot{x}_1]^2}{[b_2 \dot{x}_2^2 - b_2 \dot{x}_2 + b_0]^2 + [-b_1 \dot{x}_1^2 + b_1 \dot{x}_1]^2} \right]^{\frac{1}{2}}
\]

(refer equation (4.6) in Table 4.2)

\[
\text{Where:}
\]

\[
a_2 = (4 \zeta_1 \zeta_2 v + 4 \zeta_2 \zeta_a k^{0.5} v), a_1 = 2 \left( \zeta_1 v^2 + \zeta_a k^{0.5} v^2 + \zeta_2 v + k \zeta_2 v \right)
\]

\[
a_0 = v^2 (1 + k)
\]

\[
a_{22} = 2 \zeta_2 v, a_{11} = (v^2 + 4 \zeta_1 \zeta_2 v + 4 \zeta_a k^{0.5} \zeta_2 v)
\]

\[
a_{00} = 2(\zeta_1 v^2 + \zeta_a k^{0.5} v^2 + \zeta_2 v + k \zeta_2 v)
\]

\[
b_4 = 1, b_3 = 2 (\zeta_1 + (\zeta_1 / \mu) + \zeta_2 v + \zeta_a k^{0.5} + (k^{0.5} / \mu))
\]

\[
b_2 = (1 + k)(4 \zeta_1 v \zeta_2 + 4 \zeta_a k^{0.5} v \zeta_2 + (1 / \mu) + v^2 + (k / \mu))
\]

\[
b_1 = 2(\zeta_2 v + k \zeta_2 v + \zeta_1 v^2 + \zeta_a k^{0.5} v^2)
\]

\[
b_0 = v^2 (1 + k)
\]
\[
M_{t2} = \frac{X_2}{U} = \left[ \frac{-a_2 \lambda^2 + a_0}{[b_2 \lambda^2 + b_0}] + [a_0 \lambda]^2 \right]^{\frac{1}{2}}
\]

(equation (4.8) in Table 4.2)

The values of \(a_2, a_1, a_0, b_{42}, a_{11}, a_{00}, b_4, b_3, b_2, b_1 \) and \(b_0\) are given in Table 4.2

4.4 MOTION TRANSMISSIBILITY \(M_{t1} \quad (\mu<<1)\)

The primary aim of the suspension system is to isolate the occupants of the road vehicle from the excitations due to the road unevenness. As such, transmissibility ratio \(M_{t1} = \frac{X_1}{U}\) plays an important roll in determining the sprung mass response in the neighborhood of the resonance.

For this suspension system, mass ratio \(\mu\) has been varied in the range of 0.075 to 0.205, for the bounce motion analysis of typical road vehicle suspension systems. The curves of \(M_{t1} \) vs \(\lambda\) (where \(\lambda\) is the ratio of excitation frequency \(w\) to the undamped natural frequency \(w_1\) of the system \((m_1, k_1, c_1)\) ), have been plotted for Case 4.1, Case 4.2 and Case 4.3. The peak values of \(M_{t1}\) (at resonance) are given in Tables 4.4, 4.5 and 4.6. The figures 4.4 to 4.12 show the frequency response curves \(M_{t1} \) vs \(\lambda\) for the theoretical results given in Table 4.2, Table 4.3 and Table 4.4 showing the effect of the system damping ratio \(\zeta_1\) and the mass ratio \(\mu\) on \(M_{t1}\). The figures 4.13 to 4.15 show the frequency response curves \(M_{t1} \) vs \(\lambda\) for the theoretical results given in Table 4.5 indicating the effect of the air spring rate ratio \(k\) and the figures 4.16 to 4.18 show the frequency response curves \(M_{t1} \) vs \(\lambda\) for the theoretical results given in Table 4.6 indicating the effect of the air damping ratio \(\zeta_a\) on \(M_{t1}\). All figures show, only the first peak of the \(M_{t1} \) vs \(\lambda\) curves (second peak has been omitted from the figures). Each figure shows variation of \(M_{t1} \) vs \(\lambda\) with a marker as for

Case 4.1 : Without Air Damper as

Case 4.2 : Air Damper Modeled as a Vigot Type Model as

Case 4.3 : Air Damper Modeled as a Maxwell Type Model as
4.4.1 Effect of Variation of System Damping Ratio $\zeta_i$

Figures 4.4, 4.5 and 4.6 show respectively, the effect of variation of system damping ratio $\zeta_i$ on $M_{t1}$ at the resonant frequencies for the air damped 2DOF vibrating systems of Case 4.1, Case 4.2 and Case 4.3.

The values of $\zeta_i$ are varied as $\zeta_1 = 0.10$, $\zeta_1 = 0.133$ and $\zeta_1 = 0.15$ when $\zeta_2 = 0.0$, $\mu = 0.075$ and air damper spring rate ratio $k = 0.10$, air damping ratio $\zeta_a = 0.05$ with spring rate ratio $(k_2/k_1)$ as 6.49. Table 4.4 gives respectively the values of $M_{t1}$ at resonant frequencies obtained from the Figures 4.4, 4.5 and 4.6 for Case 4.1, Case 4.2 and Case 4.3.

It is seen that as the value of system damping ratio $\zeta_i$ increases, the value of $M_{t1}$ decreases substantially in the case where the air damper is modeled as a Maxwell type.

4.4.2 Effect of Variation of Mass Ratio $\mu$

Figures 4.4, 4.7 and 4.10 show the effect of variation of mass ratio $\mu$ on $M_{t1}$ at resonant frequencies.

The values of $\mu$ are varied as $\mu = 0.075$, $\mu = 0.10$ and $\mu = 0.205$ when $\zeta_1 = 0.10$, $\zeta_2 = 0.0$, $k = 0.10$ and $\zeta_a = 0.05$ with spring rate ratio $(k_2/k_1)$ as 6.49. Tables 4.4, 4.5 and 4.6 give respectively the values of $M_{t1}$ at resonant frequencies obtained from the Figures 4.4, 4.7 and 4.10 for Case 4.1, Case 4.2 and Case 4.3.

It is seen that, as the value of $\mu$ increases, there is no substantial change in the value of $M_{t1}$ at resonant frequencies for all the cases taken for analysis.
Table 4.4 Peak Values of $Mt_1$ for $\mu = 0.075$, $\zeta_2 = 0.0$, $k = 0.1$ and $\zeta_a = 0.05$ with $\zeta_1 = 0.1$, 0.133 and 0.15

<table>
<thead>
<tr>
<th>Frequency Response Parameters</th>
<th>$\zeta_1 = 0.100$</th>
<th>$\zeta_1 = 0.133$</th>
<th>$\zeta_1 = 0.15$</th>
</tr>
</thead>
<tbody>
<tr>
<td>With system damping only Case 4.1</td>
<td>Air damper modeled as a</td>
<td>With system damping only Case 4.1</td>
<td>Air damper modeled as a</td>
</tr>
<tr>
<td>$1^{st}$ peak</td>
<td>$Mt_1$</td>
<td>6.3822</td>
<td>5.921</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.93</td>
<td>0.96</td>
<td>1.07</td>
</tr>
<tr>
<td>$2^{nd}$ peak</td>
<td>$Mt_1$</td>
<td>0.0757</td>
<td>0.0751</td>
</tr>
</tbody>
</table>

Fig. 4.4 $Mt_1$ vs $\lambda$

Fig. 4.5 $Mt_1$ vs $\lambda$

Fig. 4.6 $Mt_1$ vs $\lambda$
Table 4.5 Peak Values of \( M_{t1} \) for \( \mu = 0.10 \), \( \zeta_2 = 0.0 \), \( k = 0.1 \) and \( \zeta_n = 0.05 \) with \( \zeta_1 = 0.1 \), 0.133 and 0.15

<table>
<thead>
<tr>
<th>Frequency Response Parameters</th>
<th>( \zeta_1 = 0.100 )</th>
<th>( \zeta_1 = 0.133 )</th>
<th>( \zeta_1 = 0.15 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>With system damping only</td>
<td>Vigot Model Case 4.2</td>
<td>Maxwell Model Case 4.3</td>
<td>Vigot Model Case 4.2</td>
</tr>
<tr>
<td>With system damping only</td>
<td>Vigot Model Case 4.3</td>
<td>Maxwell Model Case 4.3</td>
<td>Maxwell Model Case 4.3</td>
</tr>
<tr>
<td>Case 4.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1st peak Mt1</td>
<td>6.400</td>
<td>5.95</td>
<td>4.888</td>
</tr>
<tr>
<td>Case 4.2</td>
<td>4.6977</td>
<td>1.4308</td>
<td>4.37</td>
</tr>
<tr>
<td>Case 4.3</td>
<td>4.2509</td>
<td>1.3969</td>
<td>4.07</td>
</tr>
<tr>
<td>2nd peak Mt1</td>
<td>0.1029</td>
<td>0.1013</td>
<td>0.100</td>
</tr>
<tr>
<td>Case 4.2</td>
<td>0.100</td>
<td>0.1021</td>
<td>0.1002</td>
</tr>
<tr>
<td>Case 4.3</td>
<td>0.100</td>
<td>0.1021</td>
<td>0.1008</td>
</tr>
<tr>
<td>Fig. No.</td>
<td>4.7</td>
<td>4.8</td>
<td>4.9</td>
</tr>
</tbody>
</table>

Fig. 4.7 Mt1 vs \( \lambda \)

Fig. 4.8 Mt1 vs \( \lambda \)

Fig. 4.9 Mt1 vs \( \lambda \)
Table 4.6 Peak Values of Mt1 for $\mu = 0.205$, $\zeta_2 = 0.0$, $k = 0.1$ and $\zeta_a = 0.05$

with $\zeta_1 = 0.1$, 0.133 and 0.15

<table>
<thead>
<tr>
<th>Frequency Response Parameters</th>
<th>$\zeta_1 = 0.100$</th>
<th>$\zeta_1 = 0.133$</th>
<th>$\zeta_1 = 0.15$</th>
</tr>
</thead>
<tbody>
<tr>
<td>With system damping only Case 4.1</td>
<td>Vigot Model Case 4.2</td>
<td>Maxwell Model Case 4.3</td>
<td>Vigot Model Case 4.2</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.930</td>
<td>0.960</td>
<td>1.080</td>
</tr>
<tr>
<td>2nd peak Mt1</td>
<td>0.2295</td>
<td>0.2226</td>
<td>0.2312</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>5.930</td>
<td>5.930</td>
<td>5.970</td>
</tr>
</tbody>
</table>

Fig. 4.10 Mt1 vs $\lambda$

Fig. 4.11 Mt1 vs $\lambda$

Fig. 4.12 Mt1 vs $\lambda$
4.4.3 Effect of Variation of Air Damper Spring Rate Ratio $k$

Figures 4.13, 4.14 and 4.15 show the effect of variation of air damper spring rate ratio $k$ on $M_{t1}$ at the resonant frequencies. The values of $k$ are varied as $k = 0.075, k = 0.10$ and $k = 0.15$ when $\zeta_1 = 0.133, \zeta_2 = 0.0, \mu = 0.10$ and $\zeta_a = 0.05$ with spring rate ratio $(k_2/k_1)$ as 6.49. Table 4.7 gives respectively the values of $M_{t1}$ at resonant frequencies obtained from the Figures 4.13, 4.14 and 4.15 for Case 4.1 ($k = 0$ and $\zeta_a = 0$), Case 4.2 and Case 4.3. It is seen that, as the value of air damper spring rate ratio $k$ increases, there is a substantial decrease in the value of $M_{t1}$ at the resonant frequencies for the case where the air damper is modeled as a Maxwell type.

Table 4.7 Peak Values of $M_{t1}$ for $\mu = 0.10$, $\zeta_1 = 0.133$, $\zeta_2 = 0.0$ and $\zeta_a = 0.05$ with $k = 0.075, 0.10$ and 0.15

<table>
<thead>
<tr>
<th>Frequency Response Parameters</th>
<th>$k = 0.075$</th>
<th>$k = 0.100$</th>
<th>$k = 0.150$</th>
</tr>
</thead>
<tbody>
<tr>
<td>With system damping only</td>
<td>Air damper modeled as a Vigot Model Case 4.2</td>
<td>Air damper modeled as a Vigot Model Case 4.2</td>
<td>Air damper modeled as a Vigot Model Case 4.2</td>
</tr>
<tr>
<td>With system damping only</td>
<td>Air damper modeled as a Maxwell Model Case 4.3</td>
<td>Air damper modeled as a Maxwell Model Case 4.3</td>
<td>Air damper modeled as a Maxwell Model Case 4.3</td>
</tr>
<tr>
<td>$1^{st}$ peak</td>
<td>$M_{t1}$</td>
<td>4.885</td>
<td>4.697</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.920</td>
<td>0.950</td>
<td>0.92</td>
</tr>
<tr>
<td>$2^{nd}$ peak</td>
<td>$M_{t1}$</td>
<td>0.100</td>
<td>0.100</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>8.100</td>
<td>8.800</td>
<td>8.10</td>
</tr>
<tr>
<td>Fig. No.</td>
<td>4.13</td>
<td>4.14</td>
<td>4.15</td>
</tr>
</tbody>
</table>

Fig. 4.13 $M_{t1}$ vs $\lambda$

Fig. 4.14 $M_{t1}$ vs $\lambda$
Effect of Variation of Air Damping Ratio $\zeta_a$

Figures 4.16, 4.17 and 4.18 show the effect of variation of air damping ratio $\zeta_a$ on $Mt_1$ at resonant frequencies. The values of air damping ratio $\zeta_a$ are varied as $\zeta_a = 0.075$, $\zeta_a = 0.10$ and $\zeta_a = 0.15$ when $\mu = 0.10$, $\zeta_1 = 0.133$, $\zeta_2 = 0.0$ and $k = 0.10$ with spring rate ratio ($k_2/k_1$) as 6.49. Table 4.8 gives respectively the values of $Mt_1$ at resonant frequencies obtained from the Figures 4.16, 4.17 and 4.18 for Case 4.1 ($k = 0$ and $\zeta_a = 0$), Case 4.2 and Case 4.3. It is seen that, as the value of air damping ratio $\zeta_a$ increases there is a substantial decrease in the value of $Mt_1$ at the resonant frequencies in the case where the air damper is modeled as a Maxwell type.

Table 4.8 Peak Values of $Mt_1$ for $\mu = 0.10$, $\zeta_1 = 0.133$, $\zeta_2 = 0.0$ and $k = 0.100$ with $\zeta_a = 0.075$, 0.10 and 0.15

<table>
<thead>
<tr>
<th>Frequency Response Parameters</th>
<th>$\zeta_a = 0.075$</th>
<th>$\zeta_a = 0.10$</th>
<th>$\zeta_a = 0.15$</th>
</tr>
</thead>
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<td>Mt1</td>
<td>Vigot Model Case 4.2</td>
<td>Maxwell Model Case 4.3</td>
<td>Vigot Model Case 4.2</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.920</td>
<td>0.960</td>
<td>0.920</td>
</tr>
<tr>
<td>$k_1$</td>
<td>4.885</td>
<td>4.9431</td>
<td>4.885</td>
</tr>
<tr>
<td>$k_2$</td>
<td>2.010</td>
<td>0.9250</td>
<td>1.4308</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.902</td>
<td>0.902</td>
<td>0.902</td>
</tr>
<tr>
<td>$k_1$</td>
<td>4.885</td>
<td>4.699</td>
<td>4.885</td>
</tr>
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<td>$k_2$</td>
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<td>1.4308</td>
<td>1.070</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.902</td>
<td>0.902</td>
<td>0.902</td>
</tr>
<tr>
<td>$k_1$</td>
<td>4.885</td>
<td>4.699</td>
<td>4.885</td>
</tr>
<tr>
<td>$k_2$</td>
<td>2.010</td>
<td>1.4308</td>
<td>1.070</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.902</td>
<td>0.902</td>
<td>0.902</td>
</tr>
<tr>
<td>$k_1$</td>
<td>4.885</td>
<td>4.699</td>
<td>4.885</td>
</tr>
<tr>
<td>$k_2$</td>
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<td>1.4308</td>
<td>1.070</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.902</td>
<td>0.902</td>
<td>0.902</td>
</tr>
</tbody>
</table>

Fig. No. 4.16 4.17 4.18
4.5 MOTION TRANSMISSIBILITY $Mt_2$ ($\mu<<1$)

Transmissibility ratio $Mt_2 = \frac{X_2}{U}$ for the unsprung mass $m_2$ also plays an important role in determining the unsprung mass response in the neighborhood of the resonance.

For this suspension system, mass ratio $\mu$ has been varied in the range of 0.075 to 0.205, for the bounce motion analysis of typical road vehicle suspension systems. The curves of $Mt_2$ vs $\lambda$ (where $\lambda$ is the ratio of excitation frequency $w$ to the undamped natural frequency $w_1$ of the system $(m_1,k_1,c_1$ trio $)$), have been plotted for Case 4.1, Case 4.2 and Case 4.3. The peak values of $Mt_2$ (at resonance) are given in Tables 4.9, 4.10 and 4.11. The figures 4.19 to 4.27 show the frequency response curves $Mt_2$ vs $\lambda$ for the theoretical results given in Table 4.9, Table 4.10 and Table 4.11 showing the effect
of the system damping ratio $\zeta_1$ and the mass ratio $\mu$ on $M_{t2}$. The figures 4.28 to 4.30 show the frequency response curves $M_{t2}$ vs $\lambda$ for the theoretical results given in Table 4.12 indicating the effect of the air spring rate ratio $k$ and the figures 4.31 to 4.33 show the frequency response curves $M_{t2}$ vs $\lambda$ for the theoretical results given in Table 4.13 indicating the effect of the air damping ratio $\zeta_a$ on $M_{t1}$. Each figure shows curves of $M_{t1}$ vs $\lambda$ with a marker as for

1. Case 4.1: Without Air Damper as 
2. Case 4.2: Air Damper Modeled as a Vigot Type Model as 
3. Case 4.3: Air Damper Modeled as a Maxwell Type Model as

### 4.5.1 Effect of Variation of System Damping Ratio $\zeta_1$

Figures 4.19, 4.20 and 4.21 show respectively, the effect of variation of system damping ratio $\zeta_1$ on $M_{t2}$ at the resonant frequencies for the air damped 2DOF air damped semi-active road vehicle suspension systems of Case 4.1, Case 4.2 and Case 4.3. The values of $\zeta_1$ are varied as $\zeta_1 = 0.10$, $\zeta_1 = 0.133$ and $\zeta_1 = 0.15$ when $\zeta_2 = 0.0$, $\mu = 0.075$ and air damper spring rate ratio $k = 0.10$, air damping ratio $\zeta_a = 0.05$ with spring rate ratio $(k_2/k_1)$ as 6.49. Table 4.9 gives respectively the values of $M_{t1}$ at resonant frequencies obtained from the Figures 4.19, 4.20 and 4.21 for Case 4.1, Case 4.2 and Case 4.3. It is seen that as the value of system damping ratio $\zeta_1$ increases, the value of $M_{t2}$ decreases in the case where the air damper is modeled as a Maxwell type.

### 4.5.2 Effect of Variation of Mass Ratio $\mu$

Figures 4.16, 4.19 and 4.22 show the effect of variation of mass ratio $\mu$ on $M_{t1}$ at resonant frequencies. The values of $\mu$ are varied as $\mu = 0.075$, $\mu = 0.10$ and $\mu = 0.205$ when $\zeta_1 = 0.10$, $\zeta_2 = 0.0$, $k = 0.10$ and $\zeta_a = 0.05$ with spring rate ratio $(k_2/k_1)$ as 6.49. Tables 4.9, 4.10 and 4.11 give respectively the values of $M_{t2}$ at resonant frequencies obtained from the Figures 4.16, 4.19 and 4.22 for Case 4.1, Case 4.2 and Case 4.3. It is seen that, as the value of $\mu$ increases, there is no substantial change in the value of
1st peak as well as 2nd of Mt2 at resonant frequencies for all the cases taken for analysis.

Table 4.9 Peak Values of Mt2 for $\mu = 0.075$, $\zeta_2 = 0.0$, $k = 0.1$ and $\zeta_a = 0.05$ with $\zeta_1 = 0.100$, 0.133 and 0.15

<table>
<thead>
<tr>
<th>Frequency Response Parameters</th>
<th>$\zeta_1 = 0.100$</th>
<th>$\zeta_1 = 0.133$</th>
<th>$\zeta_1 = 0.15$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Vigot Model</td>
<td>Maxwell Model</td>
<td>Vigot Model</td>
</tr>
<tr>
<td>1st peak Mt2</td>
<td>6.382</td>
<td>1.526</td>
<td>4.865</td>
</tr>
<tr>
<td>2nd peak Mt2</td>
<td>0.0754</td>
<td>2.99</td>
<td>0.0752</td>
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</table>

Fig. 4.19 Mt2 vs $\lambda$

Fig. 4.20 Mt2 vs $\lambda$

Fig. 4.21 Mt2 vs $\lambda$
Table 4.10 Peak Value of Mt2 for $\mu = 0.10$, $\zeta_2 = 0.0$, $k = 0.1$ and $\zeta_a = 0.05$
with $\zeta_1$ = 0.100, 0.133 and 0.15

<table>
<thead>
<tr>
<th>Frequency Response Parameters</th>
<th>$\zeta_1 = 0.100$</th>
<th>$\zeta_1 = 0.133$</th>
<th>$\zeta_1 = 0.15$</th>
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</thead>
<tbody>
<tr>
<td>With out Air Damper</td>
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<tr>
<td>Vigot Model</td>
<td>6.40</td>
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<td>4.370</td>
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<tr>
<td>Maxwell Model</td>
<td>1.548</td>
<td>1.026</td>
<td>1.016</td>
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<tr>
<td>With air damper modeled as</td>
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<tr>
<td>Vigot Model</td>
<td>4.886</td>
<td>4.370</td>
<td>1.395</td>
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<tr>
<td>Maxwell Model</td>
<td>1.050</td>
<td>1.026</td>
<td>1.016</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>1st peak Mt2</th>
<th>$\zeta_1 = 0.10$</th>
<th>$\mu = 0.10$</th>
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<tbody>
<tr>
<td>2nd peak Mt2</td>
<td>$\zeta_1 = 0.133$</td>
<td>$\mu = 0.10$</td>
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</table>

Fig. 4.22 Mt2 vs $\lambda$

Fig. 4.23 Mt2 vs $\lambda$

Fig. 4.24 Mt2 vs $\lambda$
Table 4.11 Peak Values of $Mt_2$ for $\mu = 0.205$, $\zeta = 0.0$, $k = 0.1$ and $\zeta_a = 0.05$ with $\zeta_1 = 0.100$, 0.133 and 0.15

<table>
<thead>
<tr>
<th>Frequency Response Parameters</th>
<th>$\zeta_1 = 0.100$</th>
<th>$\zeta_1 = 0.133$</th>
<th>$\zeta_1 = 0.15$</th>
</tr>
</thead>
<tbody>
<tr>
<td>With out Air Damper</td>
<td>Vigot Model</td>
<td>Maxwell Model</td>
<td>Vigot Model</td>
</tr>
<tr>
<td>1st peak Mt2</td>
<td>6.513</td>
<td>1.569</td>
<td>4.964</td>
</tr>
<tr>
<td>1st peak fr</td>
<td>0.930</td>
<td>0.930</td>
<td>0.92</td>
</tr>
<tr>
<td>1st peak $Mt_2$</td>
<td>0.2295</td>
<td>4.414</td>
<td>4.3839</td>
</tr>
<tr>
<td>1st peak $fr$</td>
<td>5.93</td>
<td>6.022</td>
<td>6.036</td>
</tr>
<tr>
<td>Fig. No.</td>
<td>4.25</td>
<td>4.26</td>
<td>4.27</td>
</tr>
</tbody>
</table>

Fig. 4.25 $Mt_2$ vs $\lambda$

Fig. 4.26 $Mt_2$ vs $\lambda$

Fig. 4.27 $Mt_2$ vs $\lambda$
4.5.3 Effect of Variation of Air Damper Spring Rate Ratio $k$

Figures 4.28, 4.29 and 4.30 show the effect of variation of air damper spring rate ratio $k$ on $M_{t2}$ at the resonant frequencies. The values of $k$ are varied as $k = 0.075$, $k = 0.10$ and $k = 0.15$ when $\zeta_1 = 0.133$, $\zeta_2 = 0.0$, $\mu = 0.10$ and $\zeta_a = 0.05$ with spring rate ratio $(k_2 / k_1)$ as 6.49. Table 4.12 gives respectively the values of $M_{t2}$ at resonant frequencies obtained from the Figures 4.28, 4.29 and 4.30 for Case 4.1 ($k = 0$ and $\zeta_a = 0$), Case 4.2 and Case 4.3. It is seen that, as the value of air damper spring rate ratio $k$ increases, there is no substantial change in the value of $M_{t2}$ at the resonant frequencies for the first peak and there is a slight decrease in the value of $M_{t2}$ at the second peak at the resonant frequencies in the case where the air damper is modeled as a Maxwell type.

Table 4.12 Peak Values of $M_{t2}$ for $\mu = 0.10$, $\zeta_1 = 0.133$, $\zeta_2 = 0.0$ and $\zeta_a = 0.05$ with $k = 0.075$, 0.10 and 0.15

<table>
<thead>
<tr>
<th>Frequency Response Parameters</th>
<th>$k = 0.075$</th>
<th>$k = 0.100$</th>
<th>$k = 0.150$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>With out Air Damper</td>
<td>With air damper modeled as a Vigot Model</td>
<td>With air damper modeled as a Vigot Model</td>
</tr>
<tr>
<td>1st peak $M_{t2}$</td>
<td>4.885</td>
<td>1.421</td>
<td>4.886</td>
</tr>
<tr>
<td>fr</td>
<td>0.92</td>
<td>0.91</td>
<td>0.92</td>
</tr>
<tr>
<td>2nd peak $M_{t2}$</td>
<td>0.10</td>
<td>2.54</td>
<td>0.10</td>
</tr>
<tr>
<td>fr</td>
<td>8.10</td>
<td>8.4</td>
<td>8.10</td>
</tr>
<tr>
<td>Fig. No.</td>
<td>4.28</td>
<td>4.29</td>
<td>4.30</td>
</tr>
</tbody>
</table>

![Fig. 4.28 Mt2 vs $\lambda$](image1)

![Fig. 4.29 Mt2 vs $\lambda$](image2)
4.5.4 Effect of Variation of Air Damping Ratio $\zeta_a$

Figures 4.31, 4.32 and 4.33 show the effect of variation of air damping ratio $\zeta_a$ on $M_{t1}$ at resonant frequencies. The values of air damping ratio $\zeta_a$ are varied as $\zeta_a = 0.025$, $\zeta_a = 0.050$ and $\zeta_a = 0.075$ when $\mu = 0.10$, $\zeta_1 = 0.133$, $\zeta_2 = 0.0$ and $k = 0.10$ with spring rate ratio ($k_2/k_1$) as 6.49. Table 4.13 gives respectively the values of $M_{t1}$ at resonant frequencies obtained from the Figures 4.31, 4.32 and 4.33 for Case 4.1 ($k = 0$ and $\zeta_a = 0$), Case 4.2 and Case 4.3. It is seen that, as the value of air damping ratio $\zeta_a$ increases there is a substantial decrease in the value of $M_{t2}$ at the resonant frequencies in the case where the air damper is modeled as a Maxwell type.

Table 4.13 Peak Values of $M_{t2}$ for $\mu = 0.10$, $\zeta_1 = 0.133$, $\zeta_2 = 0.0$, $k = 0.100$ with $\zeta_a = 0.025$, 0.05 and 0.10

<table>
<thead>
<tr>
<th>Frequency Response Parameters</th>
<th>$\zeta_a = 0.025$</th>
<th>$\zeta_a = 0.05$</th>
<th>$\zeta_a = 0.1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>With out Air Damper</td>
<td>Vigot Model</td>
<td>Maxwell Model</td>
<td>Vigot Model</td>
</tr>
<tr>
<td>With air damper modeled as a</td>
<td>4.886</td>
<td>1.4619</td>
<td>1.00</td>
</tr>
<tr>
<td>Mt2</td>
<td>0.92</td>
<td>0.92</td>
<td>2.7458</td>
</tr>
<tr>
<td>fr</td>
<td>2.625</td>
<td>2.483</td>
<td>0.1</td>
</tr>
<tr>
<td>1st peak</td>
<td>4.1</td>
<td>8.1</td>
<td>8.45</td>
</tr>
<tr>
<td>Mt2</td>
<td>4.31</td>
<td>4.32</td>
<td>4.33</td>
</tr>
</tbody>
</table>

Fig. 4.30 $M_{t2}$ vs $\lambda$

Fig. 4.31 $M_{t2}$ vs $\lambda$

Fig. 4.32 $M_{t2}$ vs $\lambda$

Fig. 4.33 $M_{t2}$ vs $\lambda$
**DISCUSSION OF RESULTS**

In view of providing better ride comfort to the occupants in a road vehicle, motion transmissibility $M_{t1}$ and $M_{t2}$ have important role to play. Hence a typical 2DOF Semi-active (Air Damped) Road Vehicle Suspension System using an Air Damper (when air damper is modeled as a the Maxwell type Model) has been analysed for the transmissibility $M_{t1}$ and $M_{t2}$ when the system is subjected to base excitation.

The parameters affecting the values of $M_{t1}$ and $M_{t2}$ are system parameters such as the system damping ratios $\zeta_1$, $\zeta_2$ (in most of the cases $\zeta_2 = 0$), the mass ratio $\mu$ and air damper characteristics such as air damping ratio $\zeta_a$, air spring rate ratio $k$.

When the values of $\zeta_1$ are varied as $\zeta_1 = 0.10$, $\zeta_1 = 0.133$ and $\zeta_1 = 0.15$ with $\zeta_2 = 0.0$, $\mu = 0.075$ and air damper spring rate ratio $k = 0.10$, air damping ratio $\zeta_a = 0.05$ with spring rate ratio $(k_2/k_1)$ as $6.49$. It is seen that as the value of system damping ratio $\zeta_1$ increases, the values of $M_{t1}$ and $M_{t2}$ at resonant frequencies, decrease substantially in the case where the air damper is modeled as a Maxwell type.

When the values of $\mu$ are varied as $\mu = 0.075$, $\mu = 0.10$ and $\mu = 0.205$ with $\zeta_1 = 0.10$, $\zeta_2 = 0.0$, $k = 0.10$ and $\zeta_a = 0.05$ with spring rate ratio $(k_2/k_1)$ as $6.49$. It is seen that, as the value of $\mu$ increases, there is no substantial change in the values of $M_{t1}$ and $M_{t2}$, at resonant frequencies, in case when the air damper is modeled as a Maxwell type, this result is particularly useful when the value of the sprung mass is likely to vary in a given application.
When the values of $k$ are varied as $k = 0.075$, $k = 0.10$ and $k = 0.15$ with $\zeta_1 = 0.133$, $\zeta_2 = 0.0$, $\mu = 0.10$ and $\zeta_a = 0.05$ with spring rate ratio $(k_2/k_1)$ as 6.49. It is seen that, as the value of air damper spring rate ratio $k$ increases, there is a substantial decrease in the value of $M_t1$ at the resonant frequencies, however, there is a no substantial change in the value of $M_t2$ at the resonant frequencies in the case where the air damper is modeled as a Maxwell type, this result is particularly useful when the value of sprung mass is likely to vary in a given application.

When the values of air damping ratio $\zeta_a$ are varied as $\zeta_a = 0.025$, $\zeta_a = 0.050$ and $\zeta_a = 0.075$ with $\mu = 0.10$, $\zeta_1 = 0.133$, $\zeta_2 = 0.0$ and $k = 0.10$ with spring rate ratio $(k_2/k_1)$ as 6.49, it is seen that, as the value of air damping ratio $\zeta_a$ increases there is a substantial decrease in the value of $M_t1$ and $M_t2$ at the resonant frequencies, in the case where the air damper is modeled as a Maxwell type.

In Chapter 6, the experimental analysis of the air damper of Section 3.2 of Chapter 3, 2DOF air damped vibrating system of Section 3.7 of Chapter 3 and, 2DOF quarter car pneumatic (air damped) semi-active road vehicle suspension system of Section 4.2 of Chapter 4 has been carried out and the experimental results have been correlated with those obtained from theoretical analysis.
Fig. 4.34  2DOF Quarter Car Suspension System with Air Damper  
(Air Tank System not shown)

01 Sprung Mass  
02 Air Cylinder  
03 Piston  
04 Spring Positioning Lock Nut  
05 Suspension Spring  
06 Slider Plate  
07 Slider Guide Bars  
08 Cylinder Locking Nut  
09 Tyre Spring