Chapter 7

Reversed percentile residual life and related concepts*

7.1 Introduction

In Section 2.2.4, we have given a brief review on percentile residual life function (PRL). A compact review provided there help to know the developments of PRL in different periods. Theoretically there is analogy in the works relating to residual and reversed residual life functions, the properties and models relating to them differ substantially to merit the study of the latter. The relevance of various existing concepts in reversed time and the enormous literature on percentile residual lifetime mentioned above motivate us to study the properties of the reversed version of the percentile residual life function in the present chapter. Such a study along with the relationships that reversed percentile residual life has with other concepts used in this connection, does not appear to have been discussed in literature.

In this chapter we discuss the properties of the reversed percentile residual life function (RPRL) and its relationship with the reversed hazard function (RHR). Some models with simple functional forms for both RHR and RPRL are proposed. A method of distinguishing

*The discussions in this chapter is based on Nair and Vineshkumar (2011) appeared in the Journal of Korean Statistical Society (see reference no. 94)
7.2. Definition and properties

Let $F$ be the distribution function of a lifetime random variable (such that $F(0^-) = 0$) with quantile function $Q(u)$. We have from (2.65) that the reversed residual life $tX = (t - X | X \leq t)$ has distribution function

$$tF(x) = 1 - \frac{F(t-x)}{F(t)}, \quad 0 \leq x \leq t.$$  \hspace{1cm} (7.1)

Accordingly for $0 < \alpha < 1$, the $\alpha$th reversed percentile residual life function of $X$ is defined as

$$q_\alpha(t) = F_t^{-1}(\alpha) = \inf \left( x : F(x) \geq \alpha \right),$$

$$= \inf \left( x : F(t-x) \leq (1-\alpha)F(t) \right),$$

$$= t - F^{-1}\left((1-\alpha)F(t)\right), \quad 0 \leq t < Q(1), \hspace{1cm} (7.2)$$

with $Q(1) = \sup\{x : F(x) < 1\}$ as the right hand end point of the support of $F$. From (7.2) we see that the functional equation that solves for $q_\alpha(t)$ is

$$t - q_\alpha(t) = F^{-1}\left((1-\alpha)F(t)\right)$$

or

$$F(t - q_\alpha(t)) = (1-\alpha)F(t). \hspace{1cm} (7.3)$$

In terms of the quantile function $Q(u)$, the $\alpha$-th RPRL, as a function of $u$, is obtained as

$$q_\alpha^*(u) = q_\alpha(Q(u)) = Q(u) - Q((1-\alpha)u). \hspace{1cm} (7.4)$$

This definition is quite useful in situations where the quantile functions exist in the simple forms but whose distribution functions do not have closed forms to utilize (7.1) and (7.2). We have seen several such models
in Chapter 3 and its application in analyzing lifetime data. As a simple example, for power Pareto distribution given in Section 3.1.3, specified by

\[ Q(u) = cu^\lambda (1-u)^{-\lambda_2}, \ c, \lambda_1, \lambda_2 > 0, \ 0 < u < 1, \]

has

\[ q_\alpha^*(u) = cu^\lambda \left[ (1-u)^{-\lambda_2} - (1-\alpha)^\lambda (1-(1-\alpha)u)^{-\lambda_2} \right]. \quad (7.5) \]

We now discuss some properties of RPRL. First is the problem of characterizing \( F \) by the functional form of \( q_\alpha(t) \). We demonstrate through the following example that the RPRL for a given \( \alpha \) does not determine \( F \) uniquely. In other words the functional equation (7.3) is satisfied by more than one distribution function for a given \( q_\alpha(t) \). See the following example.

**Example 7.1** Assuming that \( X \) follows the power distribution

\[ F(t) = t^\alpha, \quad 0 \leq t \leq 1, \quad \alpha > 0. \]

From (7.2), for the choice of \( \alpha = 1 - e^{-2\pi a} \)

\[ q_\alpha(t) = \left[ 1 - (1-\alpha)^{1/\alpha} \right] t = (1-e^{-2\pi})t. \]

Now consider the distribution specified by

\[ G(t) = t^\alpha \left[ 1 + \frac{1}{2} \sin \log t \right], \quad 0 \leq t \leq 1, \quad \alpha > 0, \]

so that \( G(t) \neq F(t) \). Then for the above choice of \( q_\alpha(t) \) and \( \alpha \),

\[ G(t - q_\alpha(t)) = (t - q_\alpha(t))^\alpha \left[ 1 + \frac{1}{2} \sin \log (t - q_\alpha(t)) \right] \]

\[ = (te^{-2\pi})^\alpha \left[ 1 + \frac{1}{2} \sin \left( \log t - 2\pi \right) \right] \]

\[ = t^\alpha e^{-2\alpha} \left[ 1 + \frac{1}{2} \sin (\log t - 2\pi) \right] \]
so that both $G$ and $F$ have the same RPRL satisfying (7.3).

Thus we are lead to the search for some general conditions under which $F$ is determined uniquely. Equation (7.3) is a particular case of Schroder’s functional equation

$$S(\phi(t)) = uS(t), \quad 0 \leq t \leq \infty,$$

(7.6)
discussed in Gupta and Langford (1984), where $0 < u < 1$ and $\phi(t)$ is a continuous and strictly increasing function on $(0, \infty)$ which satisfies $\phi(t) > t$ for all $t$. The general solution of the equation is

$$S(t) = S_0(t)K(\log S_0(t)),$$

where $K(.)$ is a periodic function with period $-\log u$ and $S_0(.)$ is a particular solution which is continuous and strictly decreasing and satisfies $S_0(0) = 1$. In our case, in analogy with (7.6), $\phi(t) = t - q_\alpha(t)$, which does not satisfy the requirement $\phi(t) > t$ for the above solution. Therefore we seek the conditions for two distributions to have the same RPRL for a given $\alpha$, which is presented in the following theorem.

**Theorem 7.1.**

Let $F$ and $G$ be two continuous and strictly increasing distribution functions with corresponding RPRL’s $q_\alpha(t)$ and $r_\alpha(t)$. Then a necessary and sufficient condition that $q_\alpha(t) = r_\alpha(t)$ for all $t \geq 0$ is that

$$F(t) = G(t)K(-\log G(t)),$$

(7.7)

where $K(.)$ is a periodic function with period $-\log(1-\alpha)$, $0 < \alpha < 1$. 

Proof: Assume that for a given \( 0 < \alpha < 1 \), \( q_\alpha(t) = r_\alpha(t) \) for all \( t \). Then from (7.2)

\[
F^{-1}((1-\alpha)F(t)) = G^{-1}((1-\alpha)G(t)). \tag{7.8}
\]

Setting \( G(t) = u \), \( 0 < u < 1 \),

\[
(1-\alpha)F(G^{-1}(u)) = F(G^{-1}(1-\alpha)u), \quad 0 < u < 1. \tag{7.9}
\]

For the function

\[
K(t) = e^{FG^{-1}(e^{-t})}, \tag{7.10}
\]

\[
G(t)K(-\log G(t)) = F(G^{-1}G(t)) = F(t),
\]

showing that (7.10) solves (7.7). Further

\[
K(t - \log(1-\alpha)) = e^{t - \log(1-\alpha)}FG^{-1}(e^{\log(1-\alpha)-t})
\]

\[
= (1-\alpha)^{-1}e^{t}FG^{-1}(1-\alpha)e^{-t}
\]

\[
= (1-\alpha)^{-1}e^{t}FG^{-1}(e^{-t}) \quad \text{(by (7.9))}
\]

\[
= K(t).
\]

Thus \( K(.) \) is periodic with period \(-\log(1-\alpha)\) and therefore, the condition is necessary. Conversely if (7.7) holds for all \( t \),

\[
FG^{-1}((1-\alpha)G(t)) = G(G^{-1}(1-\alpha)G(t))K(-\log GG^{-1}((1-\alpha)G(t)))
\]

\[
= (1-\alpha)G(t)K(-\log(1-\alpha)G(t))
\]

\[
= (1-\alpha)G(t)K(-\log G(t))
\]

\[
= (1-\alpha)F(t).
\]

Since \( F \) is strictly increasing,

\[
G^{-1}((1-\alpha)G(t)) = F^{-1}((1-\alpha)F(t))
\]

and hence \( q_\alpha(t) = r_\alpha(t) \) as desired.

Remark 7.1 In equation (7.9), if we set \( A(u) = FG^{-1}(u) \), we have

\[
A((1-\alpha)u) = (1-\alpha)A(u),
\]

which is a particular case of the Schroder’s equation.
In the next theorem we seek conditions for the distribution to be determined by two RPRLs.

**Theorem 7.2**

If $F$ is strictly increasing and continuous and \( \frac{\log(1-\alpha)}{\log(1-\beta)} \) is irrational, then $F$ is uniquely determined by the RPRL's $q_\alpha(t)$ and $q_\beta(t)$.

*Proof: Since $F(t)$ is a particular solution, another distribution function satisfying (7.7) can be expressed as

\[
G(t) = F(t)K(-\log F(t)).
\]

Thus from the condition that $q_\alpha(t)$ and $q_\beta(t)$ are RPRL's we have

\[
G(t) = F(t)K_1(-\log F(t)) = F(t)K_2(-\log F(t))
\]

where $K_1$ and $K_2$ are periodic functions with periods $-\log(1-\alpha)$ and $-\log(1-\beta)$ respectively. The condition of the irrationality of the periods ensures that $G(t) = cF(t)$ where $c$ is a constant. As $t \to Q(1), c = 1$ and hence $G(t) = F(t)$.

Now we look at some more properties of RPRL. For deducing further features of RPRL, we find a relationship it has with RHR. From (7.3),

\[
F(t-q_\alpha(t)) = (1-\alpha)F(t).
\]

Now we recall the definition of the reversed hazard rate, $\lambda(x)$ given in Section 2.3.1. We have

\[
\lambda(x) = \frac{f(x)}{F(x)} = \frac{d}{dx} \log F(x).
\]

Now
Reversed percentile residual life and related concepts

\[
\int_{t-q_{\alpha}(t)}^{t} \lambda(x)dx = \log F(t) - \log F(t-q_{\alpha}(t)) \\
= \log F(t) - \log ((1-\alpha)F(t)) \\
= -\log(1-\alpha). \tag{7.11}
\]

Differentiating with respect to \( t \), when \( q_{\alpha}(t) \) is differentiable, we have

\[
\lambda(t-q_{\alpha}(t))(1-q_{\alpha}'(t)) = \lambda(t)
\]

or

\[
q_{\alpha}'(t) = 1 - \frac{\lambda(t)}{\lambda(t-q_{\alpha}(t))}. \tag{7.12}
\]

Since \((1-\alpha)F(t) < F(t) \Rightarrow F^{-1}((1-\alpha)F(t)) < t\),

we conclude that \( 0 \leq q_{\alpha}(t) \leq t \). Hence as \( t \) tends to zero from above

\( q_{\alpha}(t) = 0 \). If we assume that \( q_{\alpha}(t) \) is decreasing we should have

\( q_{\alpha}(t) \leq q_{\alpha}(0) = 0 \) for all \( t > 0 \) which is impossible. Thus we have

(a) there is no strictly decreasing RPRL on the whole positive real line;

(b) whenever \( \lambda(t) \) is decreasing, \( q_{\alpha}(t) \) is increasing and

(c) the function \( q_{\alpha}(t) \) cannot be a constant on \((0, \infty)\).

To prove (c), we note that the class of distributions with same \( q_{\alpha}(t) \)

has the form

\[
F(t) = G(t)K(-\log G(t)),
\]

where \( G(t) \) is a distribution with RPRL \( q_{\alpha}(t) \). Taking

\[
G(t) = \exp[c(t-b)], \quad -\infty < t < \infty
\]

we note that \( q_{\alpha}(t) = -\frac{1}{c}\log(1-\alpha) \), which is a constant. Thus the class of distributions characterized by constant RPRL is

\[
F(t) = K(c(b-t))\exp[c(t-b)], \quad -\infty < t < b.
\]
Hence there is no distribution on \((0, \infty)\) with constant RPRL.

These results show that unlike the percentile residual life functions \(q_n(t)\) has limited use in describing ageing classes among various life distributions.

7.3. Models

Equation (7.12) provides a simple identity that relates RPRL with RHR. For many of the standard lifetime models like the exponential, Weibull, Pareto, etc. which have simple forms for the hazard rate, the expression for RHR is more complicated. Even for such models with simple forms for failure rate it is difficult to deduce properties of RHR from them. Hence it is desirable to have models that have simple functional forms for RHR. In the present section, we discuss a general method for obtaining such models from the following theorem.

**Theorem 7.3.**

For a nonnegative random variable \(X\) with hazard rate \(h(t)\), its reciprocal \(X^{-1}\) has RHR \(\lambda(t)\) that satisfies

\[
t^2 h(t) = \lambda \left( \frac{1}{t} \right).
\]

**Proof:** Let \(G(t)\) be the distribution of \(X^{-1}\). Then

\[
F(t) = 1 - G \left( \frac{1}{t} \right).
\]

Now

\[
h(t) = -\frac{d}{dt} \log(1 - F(t)) = -\frac{d}{dt} \log G \left( \frac{1}{t} \right),
\]

\[
= t^{-2} \lambda \left( \frac{1}{t} \right).
\]

Hence the proof.
Remark 7.2. Suppose RHR of \( \frac{1}{X} \), \( \lambda(t) \) is decreasing. Then

\[
\frac{d}{dt} t^{-2} \lambda \left( \frac{1}{t} \right) = \frac{-\lambda' \left( \frac{1}{t} \right) - 2t \lambda \left( \frac{1}{t} \right)}{t^4} < 0, \]

which means that \( h'(t) < 0 \) \( \Rightarrow \) \( h(t) \) is decreasing. It follows that if \( \frac{1}{X} \) has increasing RHR, \( X \) has decreasing failure rate.

Example 7.2. Consider the form

\[
\lambda(t) = (at + bt^2)^{-1}
\]

From (2.63), we have

\[
F(t) = \exp \left\{ - \int_{t}^{\infty} \lambda(x)dx \right\}
\]

\[
= \exp \left\{ - \int_{t}^{\infty} (ax + bx^2)^{-1} dx \right\}
\]

\[
= \left\{ \frac{bt}{a + bt} \right\}^{-1}, \quad t > 0.
\]

For \( b<0 \), there is no proper distribution function. When \( a>0, b>0 \) we have with appropriate reparametrization,

\[
F(t) = \left\{ \frac{pt}{1 + pt} \right\}^{c}, \quad t > 0, p, c > 0
\]

while for \( a<0, b>0 \)

\[
F(t) = \left( \frac{Rt - 1}{Rt} \right)^{c}, \quad \frac{1}{R} < t < \infty, R, c > 0
\]

and as \( a \to 0, b > 0 \)

\[
F(t) = e^{-\frac{\lambda}{t}} \quad t, \lambda > 0, \quad \lambda = b^{-1}.
\]

The distribution defined in (7.13) is called the reciprocal exponential distribution. The reciprocal random variable \( X^{-1} \) has the generalized

**Example 7.3** Consider the Weibull distribution with survival function

\[ F(t) = \exp \left\{ -\left( \frac{t}{\sigma} \right)^\lambda \right\}, \quad \sigma, \lambda > 0 \]

The hazard rate function is

\[ h(t) = \frac{\lambda}{\sigma} \left( \frac{t}{\sigma} \right)^{\lambda-1}. \]

Using Theorem 7.3, \( Y = \frac{1}{X} \) has reversed hazard rate

\[ \lambda(t) = \frac{1}{t^2} h \left( \frac{1}{t} \right) = \lambda \sigma (\sigma t)^{-\lambda-1}. \]

Now

\[ F(t) = \exp \left\{ - \int_t^\infty \lambda(x) dx \right\} = \exp \left\{ - \int_t^\infty \lambda \sigma (\sigma x)^{-\lambda-1} dx \right\} = \exp \left\{ - \left( \frac{1}{\sigma t} \right)^\lambda \right\}, \]

which defines a distribution function for \( \sigma, \lambda > 0 \). We call this distribution as reciprocal Weibull distribution. The RPRL of \( Y \) has the expression

\[ q_n(t) = \frac{t^{\lambda+1} \sigma^\lambda \log(1 - \alpha)}{t^\lambda \sigma^\lambda \log((1 - \alpha) - 1)}. \]

The RHR and RPRL of the above distributions and those of others obtained by the same method from some standard life distributions are given in Table 7.1, at the end of the chapter. The hazard rate properties
of these distributions and other reliability aspects are documented in Marshal and Olkin (2007).

7.4 Classification of distributions

The fact that \( \lambda(t) \left( q_n(t) \right) \) is non-increasing (non-decreasing) on the entire positive real line leaves little scope for classification or identification of life distributions on the basis of their monotonicity as with the cases of ordinary hazard rate function and percentile residual life. One way of resolving this problem is to compare their growth rates. For the reversed hazard rate we define its growth rate as

\[
g(t) = \frac{1}{\lambda(t)} \frac{d\lambda(t)}{dt}, \quad t \geq 0. \tag{7.14}
\]

It is easy to see that \( g(t) \) determines \( \lambda(t) \) up to a constant. Hence the function \( g(t) \) is an appropriate quantity to distinguish a suitable model among the class of decreasing reversed hazard rate distributions. From the expressions of \( \lambda(t) \) of different distributions given in Table 7.1, we can easily find out the growth rates. For example the power distribution has

\[
\lambda(t) = at^{-1}
\]

and hence the growth rate

\[
g(t) = \frac{-1}{t}.
\]

Similarly the growth rate of reciprocal Weibull distribution is

\[
g(t) = (-\lambda - 1)t^{-1}.
\]

Table 7.2 exhibits the growth rate and behaviour of some important models.

It seems desirable to compare the relative growth rates of one distribution with respect to another distribution to see the extent to
which changes are taking place in their reversed hazard rates. Here we study the relative growth rate by comparing the rate of a given distribution with that of the reciprocal exponential. This is motivated by

(i) there is no distribution on \((0, \infty)\) with constant growth rate, which would have been the natural choice if one existed,

(ii) the RHR of reciprocal exponential has a simple form,

(iii) there are many distribution that have growth rate less or more than that of reciprocal exponential and

(iv) \(\frac{1}{X}\) has exponential distribution.

**Definition 7.1** A life distribution \(F\) is said to have higher growth rate – HGR (lower growth rate - LGR) in reversed hazard rate compared to the reciprocal exponential if

\[ g_f(t) \geq (\leq) g_{RE}(t) \quad \text{for all } t > 0, \]

where RE stands for reciprocal exponential distribution.

Using the above definition, we can see that there exist classes of distributions, in the same way as get classes using the monotonicity of failure rates or mean residual life functions.

**Example 7.3** Consider the expressions of \(g(t)\) presented in Table 7.2. For example \(g(t)\) of power distribution is

\[ g(t) = \frac{-1}{t}, \]

\[ g_{RE}(t) = \frac{-1}{2t}. \]

Now
\[
\frac{g(t)}{g_{RE}(t)} = \frac{1}{2} < 1,
\]
means that the power distribution is LGR. For reciprocal Weibull distribution
\[
\frac{g(t)}{g_{RE}(t)} = \frac{\lambda + 1}{2},
\]
which is >1, \(\lambda > 1\) and <1 for \(0 < \lambda < 1\). Hence the distribution is HGR for \(\lambda > 1\) and LGR for \(0 < \lambda < 1\). In the case of reciprocal beta
\[
\frac{g(t)}{g_{RE}(t)} = 1 - \frac{1}{2(1 - Rt)}.
\]
Since \(\frac{1}{R} < t < \infty, Rt > 1, 1 - \frac{1}{2(1 - Rt)} > 1\), means the distribution is HGR. While the generalized power law is initially HGR and then LGR with a change point at \(t = \left(\frac{2}{2 - \beta}\right)^\frac{1}{\beta}\). To show this note that
\[
\frac{g(t)}{g_{RE}(t)} < 1 \Rightarrow \frac{\beta t^\beta}{2(t^\beta - 1)} < 1
\]
\[
\Rightarrow t > \left(\frac{2}{2 - \beta}\right)^\frac{1}{\beta}.
\]
Behaviour of some other distribution is exhibited in Table 7.2. In a similar manner, we define growth rate for the reversed percentile residual life.

**Definition 7.2** \(F\) is said to have higher growth rate in reversed percentile life –HGP (lower growth rate-LGP) if
\[
a_F(t) \geq (\leq) a_{RE}(t) \quad \text{for all} \quad t \geq 0,
\]
where
\[
a(t) = \frac{1}{P_\alpha(t)} \frac{dP_\alpha(t)}{dt}.
\]
Although \( a(t) \) determines \( q_{a}(t) \) up to a constant, the latter does not characterize the corresponding distribution.

**Example 7.4** The growth rate in reversed percentile residual life of the reciprocal exponential distribution is

\[
a_{RE}(t) = \frac{2\lambda - t \log(1 - \alpha)}{t\left(\lambda - t \log(1 - \alpha)\right)}.\]

For the power distribution the growth rate is

\[
a_{p}(t) = t^{-1}.
\]

Now

\[
\frac{a_{p}(t)}{a_{RE}(t)} = \frac{\lambda - t \log(1 - \alpha)}{2\lambda - t \log(1 - \alpha)}
\]

\[
= 1 - \frac{\lambda}{2\lambda - t \log(1 - \alpha)} < 1
\]

Hence power distribution has LGP. On the other hand for the reciprocal beta we have

\[
a_{B}(t) = \frac{2Rt - 1}{t(Rt - 1)}.
\]

Hence \( a_{B}(t) > a_{RE}(t) \) for all \( t < 2\lambda \left(R\lambda + \log \alpha \right)^{-1} \) and hence for this range of \( t \), reciprocal beta has HGP. Thus the classification is well defined.
<table>
<thead>
<tr>
<th>Distribution</th>
<th>$F(t)$</th>
<th>RPRL</th>
<th>RHR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power</td>
<td>$\left(\frac{t}{b}\right)^{\alpha}$, $0 \leq t \leq b$</td>
<td>$\left(1-(1-\alpha)\frac{\lambda}{\beta}\right)t$</td>
<td>$\alpha t^{-1}$</td>
</tr>
<tr>
<td>Reciprocal exponential</td>
<td>$e^{-\lambda t}$, $t &gt; 0$</td>
<td>$t^2 \log(1-\alpha) \sqrt{(t \log(1-\alpha) - \lambda)}$</td>
<td>$\lambda t^{-2}$</td>
</tr>
<tr>
<td>Reciprocal beta</td>
<td>$\left(\frac{Rt-1}{Rt}\right)^{\alpha}$, $\frac{1}{R} &lt; t &lt; \infty$</td>
<td>$(1-(1-\alpha)\frac{\lambda}{\beta})t(Rt-1)$</td>
<td>$\frac{c}{t(Rt-1)}$</td>
</tr>
<tr>
<td>Reciprocal Lomax</td>
<td>$\left(\frac{pt}{1+pt}\right)^{\alpha}$, $t &gt; 0$</td>
<td>$(1-\alpha)\frac{\lambda}{\beta} - 1)t(pt + 1)$</td>
<td>$\frac{c}{t(pt + 1)}$</td>
</tr>
<tr>
<td>Reciprocal Weibull</td>
<td>$\exp\left[-\left(\frac{1}{\sigma t}\right)^{\alpha}\right]$, $t &gt; 0$</td>
<td>$t^{\lambda+1} \sigma^{\lambda} \log(1-\alpha) \sqrt{(t^{\lambda+1} \sigma^{\lambda} \log(1-\alpha) - 1)}$</td>
<td>$\lambda e^{\lambda+1} t^{-2}$</td>
</tr>
<tr>
<td>Reciprocal Gompertz</td>
<td>$\exp\left[-\theta\left(e^{\lambda t} - 1\right)\right]$, $t &gt; 0$</td>
<td>$t - \lambda \left</td>
<td>\log\left(e^{\lambda t} - \theta^{-1} \log(1-\alpha)\right)\right</td>
</tr>
<tr>
<td>Power exponential</td>
<td>$(1-\alpha)^{\theta}$, $t &gt; 0$</td>
<td>$t + \lambda^{-1} \log\left(1-(1-\alpha)\frac{\lambda}{\beta}(1-e^{-\lambda t})\right)$</td>
<td>$\theta \lambda e^{\lambda t} - 1$</td>
</tr>
<tr>
<td>Burr</td>
<td>$(1+t^{-\alpha})^{\theta}$, $t &gt; 0$</td>
<td>$t^\left[1-(1-\alpha)\frac{\lambda}{\beta}\left(1+t^{-\alpha}\right)^{-1}\right]$</td>
<td>$\theta \lambda e^{\lambda t} - 1$</td>
</tr>
<tr>
<td>Generalized Power</td>
<td>$(1-t^{-\alpha})^{\theta}$, $t &gt; 0$</td>
<td>$t^\left[1-(1-\alpha)\frac{\lambda}{\beta}\left(1-t^{-\alpha}\right)^{-1}\right]$</td>
<td>$\beta \theta (t^{\beta} - 1)^{-1}$</td>
</tr>
<tr>
<td>Negative Weibull</td>
<td>$\exp\left[-\theta\left(t^{-\alpha} - 1\right)\right]$, $0 &lt; t &lt; 1$</td>
<td>$t - \left(t^{-\alpha} - \theta^{-1} \log(1-\alpha)\right)^{-\frac{1}{\beta}}$</td>
<td>$\theta \beta t^{-\beta-1}$</td>
</tr>
</tbody>
</table>
Table 7.2 Growth rate of reversed hazard rate

<table>
<thead>
<tr>
<th>Distribution</th>
<th>( g(t) )</th>
<th>( g(t) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power</td>
<td>(-t^{-1})</td>
<td>LGR</td>
</tr>
<tr>
<td>Reciprocal Exponential</td>
<td>(-2t^{-1})</td>
<td>HGR/LGR</td>
</tr>
<tr>
<td>Reciprocal beta</td>
<td>( \frac{(1−2Rt)}{t(Rt−1)} )</td>
<td>HGR</td>
</tr>
<tr>
<td>Reciprocal Lomax</td>
<td>( -\frac{(1+pt)}{t(pt+1)} )</td>
<td>HGR</td>
</tr>
<tr>
<td>Reciprocal Weibull</td>
<td>( -(\lambda+1)t^{-1} )</td>
<td>LGR for (0 \leq \lambda \leq 1), HGR for (\lambda \geq 1)</td>
</tr>
<tr>
<td>Power Exponential</td>
<td>( \lambda e^{\lambda t}(e^{\lambda t}−1)^{-1} )</td>
<td>LGR</td>
</tr>
<tr>
<td>Reciprocal Gompertz</td>
<td>( -t^{-1}(\lambda t^{-1} + 2) )</td>
<td>LGR</td>
</tr>
<tr>
<td>Generalized Power</td>
<td>( -\beta t^{\beta-1}(t^{\beta}−1)^{-1} )</td>
<td>LGR for ( t \geq \left(\frac{2}{2-\beta}\right)^{\frac{1}{\beta}} ) and HGR for ( t \leq \left(\frac{2}{2-\beta}\right)^{\frac{1}{\beta}} )</td>
</tr>
<tr>
<td>Negative Weibull</td>
<td>( -(\beta+1)t^{-1} )</td>
<td>HGR for ( 0&lt;\beta \leq 1 ), LGR for ( \beta \geq 1 )</td>
</tr>
</tbody>
</table>