Chapter 1

Introduction

Understanding the formation of the large scale structure is one of the frontiers of modern cosmological research. According to the standard scenario, the large scale structure of the universe that we recognize today under the form of galaxies, filaments and sheets is the result of evolution and gravitational growth of small density fluctuations generated at very early times. The study of large scale structure in the expanding universe is currently a challenging problem for cosmologists. As universe expands, galaxies cluster under the influence of motivating gravitational force. Recent observations indicate that while, the large-scale distribution of galaxies appears to be uniform, however, small scale distribution is appreciably influenced by well known tendency towards clustering. Based on a few physically motivated assumptions, Saslaw and Hamilton (1984) arrived at a simple gravitational distribution function, based on the gravitational thermodynamic approximations which we call as gravitational quasi-equilibrium distribution (GQED). Applicability of thermo-
dynamics to cosmological many-body problem suggests that statistical mechanics should also apply (Ahmad et al 2002).

The discovery of fundamental statistical mechanical description of the cosmological many-body problem has provided a more rigorous basis for its earlier thermodynamic description (summarized e.g. in Saslaw (2000)). The statistical mechanical approach agrees with thermodynamics and is also more powerful for some purposes such as determining the velocity and energy distribution functions, along with the associated probabilities that clusters are bound and virialized (Leong and Saslaw 2004). It is also more general, allowing the particles to have softened gravitational potentials, corresponding to the models of galaxies with dark matter halos. Furthermore, it can be generalized to systems in which the particles, such as galaxies, have different masses (Ahmad et al 2006).

These results agree very well with N-body simulations (Itoh 1988, 1990; Inagaki 1991). They also describe the observed spatial distribution functions of galaxies with increasing accuracy as catalogs became larger and more complete. The recent example is the analysis of extended sources in the 2 MASS (Two Micron All Sky Survey) catalog, where the theory, which has no free parameters, agrees with the observations to better than 97 percent (Sivakoff and Saslaw 2005; Saslaw & Fang 1996).

Recent study by Saslaw and Ahmad (2010) reveal that gravitational many-body clustering of galaxies in expanding universe may be regarded as a form
Chapter: 1

of phase transition, which differ in several ways from laboratory phase transitions. The study of large scale structure is based on certain assumptions which have their observational support and background. The well known assumptions are as:

1. galaxies are the fundamental units of universe.

2. the galaxies interact gravitationally.

3. the universe is homogeneous on large scales.

4. galaxies and clusters of galaxies follow Hubble expansion law.

The large scale homogeneity of the universe was predicted by Einstein and observational developments agreed well with this prediction. The universe though considered homogeneous on large scales do have inhomogeneities due to density contrasts and presence of small irregularities in the distribution of matter (Lemaitre 1933a, 1934; Shapley 1933 and Saslaw 1985). It is also believed that during expansion of the universe instabilities arose which produced the clusters and super-clusters of galaxies. Islands of over-density in the early universe are widely believed to be the origin of the large structures we observe today. The more practical point of view is that sufficient perturbations were present in the initial conditions (Peebles 1980).

Galaxies on very large scales interact gravitationally as particles with a pairwise potential and the density of the universe determines the strength of gravity on a global scale (Saslaw 1985). It is the mutual gravity that causes
the galaxies to aggregate for the formation of clusters. Gravitational clustering have the advantage that rules do not change as the system evolves and is understood better. Gravitational galaxy clustering has played a central and important role in the cosmology. The strength and amount of galaxy clustering in the expanding universe tells cosmologists a lot about how matter was laid down in the early universe and what physical processes have evolved the clustering since that time.

Although the discovery of large scale structure have revealed many complexities, but the understanding of many physical processes is still a challenge in the modern cosmology. The advent of more advanced telescopes and supercomputers have revealed the physics of large scale structures in the universe but the mysteries of homogeneity and inhomogeneity are still the mysteries. A number of statistical techniques have been used to study the large scale distribution of galaxies. Some of these techniques are percolation (Zeldovich, Einesto and Shandarin 1982; Grimmet 1989), minimal spanning trees (Pearson & Coles 1995; Bhavsar & Splinter 1996), fractals (Mandelbrot 1982; Feder 1988 and Itoh 1990), correlation functions (Totsuji & Kihara 1969; Peebles 1980), distribution functions (Saslaw & Hamilton 1984; Ahmad et al 2002) have been used to study the large scale structure in the universe. There are many more variety of theoretical approaches to large systems of gravitating particles and no one technique encompasses the full richness of the problem (Saslaw 1987). Early robust methods in the analysis of galaxy clustering are based on two point correlation functions. The two point correlation describes one way in
which the actual distribution of galaxies deviates from simple Poisson. The use of correlation functions and related statistics have been favourite measure of the distribution of matter in recent years both in theoretical and observational analysis (Peebles 2001). The popularity of two point correlation function to express the statistical properties of galaxies is because this contains information about clustering on all higher scales (Zhan 1989; Zhan and Dyer 1989 and Hamilton 1993). Distribution function have been developed mainly from a thermodynamic approach to galaxy clustering. The prediction of this theory as given by Saslaw and Hamilton (1984) agree well with the observations of galaxy counts. This theory is based on the assumption of gravitational quasi-equilibrium approximation. In this approach the starting point is the general forms of equations of state based on first two laws of thermodynamics and the quasi-equilibrium evolution in the context of expanding universe (Saslaw & Hamilton 1984). Thermodynamics is applicable to gravitating systems in the same way as applied to ordinary gases, but the results are surprising. Specific heat being negative and equilibrium a distant ideal are among the surprising results of gravithermodynamics or GTD. These differences are because of the long range and unsaturated nature of gravitational forces. Direct introduction of gravity into thermodynamics leads to the Jeans instability, it also links up with linear kinetic theory and provides new insight into non-linear clustering. The gravity being a binary interaction shows that the particle one-one correlations are needed for thermodynamics (Saslaw 1985, 2000). In galaxy clustering the initial correlations may not have had enough time to grow to larger scales. As correlations clumps and clusters evolve and grow in scale, the ther-
modynamic quantities are averaged over larger scales. Cosmological many body galaxy distribution function have been obtained from thermodynamics (Saslaw & Hamilton 1984). Gravitational thermodynamics have been compared to the cosmological many body problem using N-body computer simulation results (Saslaw et al 1990; Itoh et al 1988, 1993), observed galaxy clustering (Sheth et al 1994; Fang and Zhou 1994 and Raychoudhary and Saslaw 1996) and theoretical arguments (Zhan 1989; Zhan and Dyer 1989). N-body simulations (Itoh, Inagaki and Saslaw 1988) showed that homogeneous gravitational clustering in an expanding universe evolves slowly through a series of quasi-equilibrium states. The thermodynamic theory of gravitational clustering in an expanding universe has been extended to incorporate the volume dependence of the ratio, $b$, of gravitational correlation energy to thermal energy and the resulting form of the gravitational quasi-equilibrium distribution function remains unchanged by scale dependence. This showed improved agreement between simulations, observations, and theory and the new results when compared to N-body simulations showed accuracy of 98 percent (Sheth & Saslaw 1996). The resulting consequences of the volume scale dependence of $b$ has added to our understanding of observational and theoretical aspects of gravitational clustering. Most of the earlier works (Saslaw & Hamilton 1984; Saslaw 1986; Itoh, Inagaki and Saslaw 1988) were mainly based on the Poisson initial conditions and its conclusion does not show any dependence on power spectra of the primordial density fluctuations by considering scale free models (Suto, Itoh and Inagaki 1984). Itoh et al (1988) and Suto et al (1990) have discussed the scale dependence for N-body simulations and therefore have opened a track
for making theoretical predictions of a scale dependent distribution function in a non-linear gravitational clustering.

Galaxy clustering is mainly concerned with infinite systems. Partition function (Huang 1987) and thermodynamics of an infinite system (Saslaw 2000) will need grand canonical ensemble. This is because in grand canonical ensemble, there is exchange of both galaxies as well as energy between the cells representing galaxy clustering. Thermodynamics and galaxy clustering as described by Saslaw & Hamilton (1984) is based on the well known result (Hill 1956) that in a system whose members interact in pairs as with the gravitational potential, the two particle correlation function completely determines the thermodynamic properties. Among these properties are equations of state which include gravitation. These equations of state determines the equilibrium fluctuations. From the fluctuations the distribution functions for the probability that a volume $V$ of arbitrary shape contains just $N$ galaxies (Saslaw & Hamilton 1984). A useful way of measuring correlation of galaxies on large scales is the distribution of galaxy counts in cells which is measured by the probability $P_N(V)$ of finding $N$ galaxies in randomly placed volume $V$ and is given (Saslaw & Hamilton 1984, Ahmad et al 2002)

$$F(N) = \frac{\bar{N}(1-b)}{N!}[Nb + \bar{N}(1-b)]^{N-1}e^{-Nb}$$

The distribution function so obtained depends on a parameter $b$ which is interpreted as being the ratio $b = -\frac{W}{\frac{1}{2}K}$ of gravitational correlation energy, $W$ to the kinetic energy in the peculiar motions, $K$. 

$$F(N) = \frac{\bar{N}(1-b)}{N!}[Nb + \bar{N}(1-b)]^{N-1}e^{-Nb}$$
Our understanding of the structure of the universe is also based on the distribution of galaxies. The distribution functions of galaxy counts lead naturally to the void probability with observations provides a test of gravitational clustering theories (White 1979; Ryden & Melott 1996; Arseth & Saslaw 1982). The conditions under which statistical mechanics is applied to study cosmological many body problem are very similar to those necessary for the applicability of thermodynamics (Saslaw & Hamilton 1984; Saslaw & Fang 1996; Saslaw 2000). Thus both the theories are based on simple general physical assumption i.e, local dynamical time scale \((G\rho)^{-1/2}\) in an over dense region is faster than global gravitational time scale \((G\bar{\rho})^{-1/2}\). This assumption makes it possible for gravitational clustering to evolve through a sequence of quasi-equilibrium state. This assumption according to statistical mechanics point of view indicates that a system can undergo rapid microscopic transition which sample all its accessible states with approximately equal a priori probability. The statistical mechanics of N-body system is based on the N-body Hamiltonian partition function as a function of N-dimensional integral is evaluated. All the thermodynamic properties including the equations of state are evaluated.

Gravitational many body clustering of galaxies in an expanding universe have been regarded as a form of phase transition. Its properties have been calculated and it was found that they differ from the usual laboratory phase transitions because of their non occurrence over all scales in short time. Phase
transitions are a fascinating and fundamental property of many body systems and can take place when microscopic interactions among the galaxies propagate over large scales resulting in the macroscopic changes in the properties of the system (Saslaw & Ahmad 2010). The properties of cosmological many body clustering have been developed at high redshifts and the resulting theoretical distributions have been compared with the observations of groups of galaxies at redshift $z = 3$. The observed variance of counts in cells and the probability for forming strongly over dense regions have been analysed. It has been shown that future observations of the galaxy distribution function can test cosmological many body clustering and can determine $\Omega_0$ more accurately (Saslaw & Edgar 1999). The peculiar velocity distribution function of galaxies in cosmological many body gravitational clustering have been examined. The statistical mechanical approach generalises the earlier results to galaxies with haloes. Comparison of the results with the peculiar velocity distributions shows that individual massive galaxies are usually surrounded by their own haloes but not embedded in common haloes. The peculiar velocities of galaxies i.e, their departure from the global cosmic expansion, facilitates a basis for the discussion of binding and virialization and contain valuable information about the history of galaxy clustering and the geography of dark matter (Saslaw & Edgar 1999). The theory in which galaxies are supposed to be surrounded by individual haloes, is in excellent agreement with relevant N-body simulations and agrees more accurately with the observations (Crane & Saslaw 1986; Fang & Zhou 1994; Sheth et al. 1998; Leong & Saslaw 2004).
The main advantage of statistical mechanical approach is that it can be applied to non-point mass systems. For extended mass model, each galaxy is considered to be finite size and the finite size of a galaxy is represented by introducing softening parameter $\epsilon$ (Ahmad 1987). The softening parameter $\epsilon$, introduces a correction term that lowers the correlation energy in the modified cosmic energy equation. Different approaches have been explored to understand the evolutionary effects of $\epsilon$ both for linear and non-linear theories of evolution of $b$ (Sheth & Saslaw 1996). This leads to an understanding of the nature of extensivity in infinite statistically homogeneous gravitating systems.

Distribution of voids have been studied and the void probability function is linked to the hierarchy of correlation functions of all orders (White 1979). Voids arise naturally as an initially homogeneous system of galaxies cluster gravitationally. It was found that comparing void probability with observations provides a strong proof of gravitational clustering theories. The other important technique used for studying the large gravitating gas clouds is the Riemannian geometric approach to thermodynamic fluctuation theory (Ruppeiner 1996). The major motivating factor behind this theory is the connection between the Riemannian curvature and interactions. Thermodynamic fluctuation theory was originated by Einstein in 1907. This approach views the problem from the macroscopic perspective. It particularly stresses on the notions of covariance and consistency, which is expressed using the language of Riemannian geometry. In addition to this it also describes the basic structure of thermodynamic fluctuation theory (Ruppeiner 1979, 1981) beyond the classical
Chapter: 1

one (Einstein 1907, 1910). The main application of this approach is to galaxy clustering, where the overall size of the universe appears to exceed the correlation length. The equations of state which are found by this approach are analytic. These equations of state are obtained first by starting at the ideal gas state then extending to the states having pressure about one quarter those of the corresponding ideal gas. The end of this analytical approach corresponds to galaxy clustering (Ruppeiner 1996). The distribution functions obtained by thermodynamical approach (Saslaw & Hamilton 1984) and statistical mechanical approach (Ahmad et al 2002) depends on a parameter \( b \) which is interpreted physically as being the ratio of gravitational correlation to the kinetic energy in the peculiar motions. Different forms of \( b \) were developed by Ruppeiner by using Riemannian Geometric approach to thermodynamic fluctuations of density and temperature.

The aim of the present work is to investigate cosmological many-body problem on the basis of Riemannian geometric approach to thermodynamic fluctuation theory. A variety of theoretical approaches have been applied to study the properties of very large gaseous systems of particles interacting through their mutual gravity, since large gravitating systems are central to several issues in astrophysics. One of the important approach was introduced by Ruppeiner (1996) to study the gravitating system i.e. Riemannian geometric approach to thermodynamics. This approach allows us to work out thermodynamic equations of state, and when applied to the clustering of galaxies in an expanding universe, it confirms and extends Saslaw & Hamilton’s (1984) work
which did not allow an explicit calculation of the equations of state. These equations of state became the basis for calculation of thermodynamic fluctuations for any thermodynamic state (Ruppeiner 1996). Riemannian geometric approach determines the interaction parameter $b$ explicitly. Ruppeiner (1996) made use of the geometric relation between thermodynamic curvature and free energy per volume at critical point and expressed it as partial differential equations for $b$. Solutions of these differential equations depend upon some singular points in thermodynamic space. These singular points are the points where the existence and uniqueness theorems break down. Five types of singularities were identified. Thus we see a number of solutions were obtained which gave relation between $b$ and $\beta T^{-3}$ and one of the solutions resembles Saslaw & Hamilton (1984) and Ahmad et al (2002). The explanations of other solutions was not given in detail. In present thesis our main motive is to interpret the other solutions. We made use of the power spectrum analysis to study these solutions.

We also aimed to study the effect of higher order corrections on the thermodynamic properties and distribution functions. The additional higher order terms do have the effect on these properties but the effect decreases as the number of particles increases. Earlier, the system was assumed to be dilute and less dense and only the first term of expansion was included in the partition function (Ahmad et al 2002). Later two terms were included to study the phase transitions (Saslaw & Ahmad 2010). the aim of the present work is to study the thermodynamic properties and the distribution functions of moder-
The present thesis is organised as: in second chapter we determine the exact functional form of the clustering parameter $b(x)$. It is the clustering parameter which contains the complete information of clustering. Earlier a number of methods have been applied to study the functional dependence of clustering parameter and it was found that clustering parameter depends on a special combination of density and temperature (Saslaw & Fang 1996; Saslaw & Hamilton 1984). The functional dependence of clustering parameter was determined by combining first and second laws of thermodynamics (Ahmad 1996) and separately by using Maxwell’s relation (Ruppeiner 1996). Then the exact functional form of clustering parameter was suggested by Saslaw & Hamilton (1984) as a hypothesis. It was only after the discovery of fundamental statistical mechanical description of the cosmological many body problem (Ahmad et al 2002) that the exact functional form of clustering parameter appeared naturally from the equations of state. Now, in this chapter we derive the exact functional form of clustering parameter. By using the fact that the thermodynamic curvature has the dimensions of volume a second order differential equation is developed, the solution of which gives the exact functional form of $b(x)$. The same form of interaction parameter is obtained with the help of Riemannian geometric approach to thermodynamic fluctuation theory by using the assumption that temperature fluctuations on a given scale are proportional to the density fluctuations. We first calculate the curvature and then compare the curvature with that of the curvature at critical point and the clus-
tering parameter appears spontaneously from the relation.

In third chapter we examine cosmic energy equation for extended galaxy structures on the basis of different models of universe and by using the cosmic energy equation for extended masses we study the evolution of clustering parameter. We find the critical values of clustering parameter for non-point masses. We made an attempt to investigate different numerical values of clustering parameter which were earlier found by Ruppeiner (1996) using Riemannian geometric approach to thermodynamic fluctuation theory. We further study the evolution of this interaction parameter using the assumption that mean square number fluctuations are equal to mass fluctuations. This study is done for both linear and non-linear regimes. We solve the linear perturbation equations for point mass systems numerically and study the evolution of clustering parameter as a function of redshifts. The comparative evolution of the clustering strength with redshift is done for point and non-point mass systems. We also made an attempt to solve the perturbation equation analytically by series solution method. We also extend the power spectrum and density fluctuations for extended structure by introducing softening parameter for both linear and non-linear regimes. The results are compared with earlier results of point mass structures. It is found that softening parameters introduced in the theory has some influence on the thermodynamic fluctuation theory. Results obtained with spectrum analysis are also compared with Riemannian geometric approach (Ruppeiner 1996) to galaxy clustering. The singular solutions of thermodynamic fluctuation results can be interpreted on the basis of power
index of two point correlation function.

In fourth chapter we determine the functional form of clustering parameter on a special combination of temperature and pressure. Assuming temperature and pressure as independent coordinates of the thermodynamic space, we evaluate the non-diagonal line element and the general expression for curvature with the help of Riemannian geometric approach. We apply this expression to calculate the curvature of an ideal gas and a gravitationally interacting system. Thus we use the curvature to study the gravitational clustering of galaxies and further we attempted to explore the possibility for evaluating the thermodynamic curvature using free energy.

In fifth chapter we study the gravitational clustering of strongly degenerate systems of galaxies, by including higher order terms in the gravitational partition function. We study the effect of softening parameter on higher order clustering. Using the more accurate form of partition function, we calculate the exact equations of state and the distribution function for moderately and strongly degenerate systems. Comparative study of distribution function is done to see the effect of higher order terms on clustering. We also calculate the number fluctuation moments for a strongly interacting systems.

Finally in chapter sixth, we discuss and summarize the results of the present study.