Chapter 3

Cause of Earthquakes

Let us dedicate this chapter to RUAUMOKO\(^3.1\) (Fig. 3.1), the Maori God of Volcanoes and Earthquakes.

**Introduction:** The earthquakes are caused due to a cascade of reasons one among which is the rupture of active faults occurring in shallow parts of the inland regions\(^3.2\). But, the prominent among the causes which in general is not thought of is the "instability of convective motion" of molten lava at the interior of the earth.

Convection currents develop in the earth’s mantle due to difference temperatures between the crust and core of the earth. The convection causes subduction boundary movements of the tectonic plates which ultimately is the cause for earthquakes. In this chapter is discussed a general treatment and the cause for instability of convection which is mathematically analysed by making use of Navier-Stoke’s Equation\(^3.3\) of Plasma Physics to the problem and finally arrives at a Rayleigh number which determines the possibility of development of convection. The method followed is a more effective mechanism of heat transport, new types of convective motion develop for large
Rayleigh numbers. The ordered convective flow gets disturbed and leads to turbulent flow. The heat travels from the core and mantle to the crust creating ‘P’(Pressure) and ‘S’(Shear) waves.

**REVIEW OF LITERATURE**

**General:** “The Earthquake” a verse written in about 1750 summarizes this view.³⁴

> “What powerful hand with force unknown,
> Can these repeated tremblings make?
> Or do the imprison’d vapors groan?
> Or do the shores with fabled Tridents shake?
> Ah no! the tread of impious feet,
> The conscious Earth impatient bears;
> And shuddering with the guilty weight,
> One common grave for her bad race prepares.”

– Anon

When the earth got cooled, the lithosphere cracked and split into seven large and twelve small floating islands with ragged edges: the ‘Tectonic Plate’ which is a large relatively rigid segment of earth’s lithosphere that moves in relation to other plates over the deep interior.

Plates meet in convergent zones and gets separated at divergent zones. The plates move continuously over the viscous mantle rubbing and

![Fig. 3.2 Various Tectonic Plates](image)

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pushing against each other and even trying to mount one

![Fig. 3.3 Earthquake Zones: The Rings or Circle of Fire](image)

[Fig. 3.3 Earthquake Zones: The Rings or Circle of Fire]
Most earthquakes occur near plate margins. According to geologists, these global geological or tectonic forces, though not understood in detail, are the consequences of temperature differences in the earth.

The differences are due to loss of heat by radiation into space and gain of heat from decay of radioactive elements in the rocks. About two thousand years ago, Strabo, the Greek geographer, historian and philosopher, first observed that active seismic regions lay along coastal bands called “Rings of Fire” or “Circle of Fire,” a name given as earthquakes were attributed to volcanic activity. (Fig. 3.3)

Recently, two primary bands of seismic activity have been identified by geologists and Seismologists.

The ‘Circum-Pacific belt’ that rings the coasts of the Pacific Ocean and the ‘Alpide’ belt along the southern boundary of the Eurasian plate, that cuts from the Atlantic Ocean across the Mediterranean into Asia. As the ancient western world was centered in the Mediterranean region along the Alpide belt, it was easy for the Greek philosophers to understand and explain the origin of earthquakes.

The convection currents drive the tectonic plates which on an average move about 5 cm per year over the mantle (Fig. 3.4). The upward movement of the heated particles rising through the mantle from the molten core of the earth.

As one plate is driven against its neighbour, it will slip along the boundary (Fig. 3.5a). It may even slide down (Fig. 3.5b). Sometimes, one plate, usually a continental plate will thrust up over the oceanic plate (Fig. 3.5c).

Fig. 3.4 Tectonic Plates driven by convection currents These three sections occur at convergent or subduction boundaries, regions where volcanic eruptions and earthquakes frequently occur.
At those tectonic boundaries where plates tend instead to separate mainly along the mid-ocean rifts (Fig. 3.6), red hot magma flows up from the mantle to fill the void left by the spreading plates and cools rapidly when meeting the frigid sea water, which also causes cracking in this new crustal material of basalt.

The process gets repeated, new material is constantly being added to the crust to make up for that which plunges back into the mantle and is re-melted along a subduction boundary (Fig. 3.7).

The constant movement of the tectonic plates is resisted along their convergent boundaries by friction between their rough edges, akin to two bricks being rubbed against each other. Over a period of time this creates stress and strain along the boundary. This will continue till a stress level is reached sufficient to overcome the frictional resistance of the plates and ultimately causing a sudden slip (Fig. 3.8). This is how an earthquake occurs.
Generally, many earthquakes occur along the convergent boundaries of the continental coastlines and their magnitudes are very high. Minor earthquakes occur along mid-ocean separating boundaries. Approximately 70% of the continental earthquakes take place along the perimeter of the Pacific plate and 20% along the alpide belt, with the remaining 10% scattered around the globe.

A country which is frequently struck by earthquakes and experiencing heavy damages almost every year is Japan. The earthquakes there are mostly caused by the rupture of active faults occurring in shallow part of the inland regions. An ‘active fault’ is a fault created by repeated earthquakes of the past in the same region and paving way for another major quake in that region. Toshikazu Yoshioka has evaluated rupture probabilities of active faults using the Cascade Earthquake Model based on behavioral segmentation. In order to critically study the possibility of occurrence of earthquakes of large magnitudes that are dependent on the active faults, the author of the paper has divided active faults into behavioral segments and making use of the Earthquake Cascade Model. By cascade model is meant that the earthquake is caused by a single segment or multiple segments. The rupture probability of active faults can be very well determined by such method independent of the field data.

Irrespective of the value of the slippage of a fault, which if repeatedly happens to be in the same direction, a displacement of several meters might accumulate. If the displacement is in the vertical direction, the up thrust side will be converted into a mountain and the bottom side becomes a plain or basin. If there is side-way displacement, the valleys and peaks get bent. As great earthquakes are caused by rupture of active faults, an evaluation rupture probability is very important.

Now, the rupture of active faults or the movement of the tectonic plates is an after-effect and as a physicist, following “Cause and Effect Formalism”, if the movement of the tectonic plates is the effect so as to cause an earthquake, then from the theoretical considerations described above, the cause for the movement of the tectonic plates is convection. More than convection, it is the ‘Instability of Convection’ that is responsible for the motion of tectonic plates. The molten material such as the semi-fluid magma or lava from the core and mantle is akin to weakly ionized plasma, the motion of which falls in the realm of Fluid Mechanics the laws of which can be applied to both liquids and
gases (fluids). Convection currents develop in the earth’s mantle due to difference in temperatures between the crust and the core. The convection causes subduction boundary movements of the tectonic plates which ultimately is the cause for earthquakes. The question now is, “what is the cause for convection and for its instability?” For an answer, we have to deal with some mathematical treatment.

For a generalized mathematical treatment given below, it is legitimate to use the word, ‘Gas’ for the material of the molten fluid under consideration thereby falling in line with the usual treatment found elsewhere in which terms, ‘Gas’ and ‘Fluid’ are synonymously used.

We follow the theory given by B.M. Smirnov and start with an equation first developed by French engineer, C.L.M.H. Navier (1823) and Irish scientist, George. G. Stokes (1845) known as the Navier-Stoke’s equation:

$$\frac{\partial v}{\partial t} + (v \nabla) = - \frac{\text{grad} \, p}{\rho} + \frac{\eta}{\rho} \nabla^2 v + \frac{\eta}{3\rho} \text{grad div} \, v + \frac{F}{M} \quad \ldots (1)$$

Along with the equation of continuity:

$$\frac{\partial N}{\partial t} + \text{div} \,(Nv) = 0 \quad \ldots (2)$$

And the equation of heat transport;

$$\frac{\partial \theta}{\partial t} + v \text{grad} \, \theta = \frac{\chi}{Nc_v} \nabla^2 \theta \quad \ldots (3)$$

Where

- $N$ is the number density of gas particles
- $p$ is the gas pressure
- $\theta$ is the temperature
- $C_v$ is the specific heat per molecule
- $\eta$ is the viscosity
- $\chi$ is the thermal conductivity
- $M$ is the mass of a gas molecule
- $F$ is the force acting on one particle
- $v$ is the mean velocity referred to as the drift velocity
and \( p = MN \) is the mass density

When the temperature gradient is large in a gas which is in a field of external forces, there may appear a more effective mechanism of heat transport than thermal conduction and that is nothing but ‘Convection’. This process consists in the movement of the warmer gas into the cooler regions such as the crust of the earth and the cooler gas into the hot regions such as the mantle of earth.

We have to analyse the stability of gas at rest with the possibility of development of convection. Consider a gas at rest in which a temperature gradient is maintained in the field of external forces. The parameters of the gas are subjected to a small perturbation which is due to the slow motion of the gas and corresponds to convection. If this perturbation proves to be possible, convective instability can develop in such a gas. We have to find the conditions needed for the convective instability to occur so as to have convective heat transport.

The simplest problem can be studied by looking upon the so-called ‘Rayleigh problem’ of the weakly ionized plasma. The mantle lies between the crust and the core. The lower boundary of the crust and the upper boundary of the core can be taken as two infinite parallel walls and the hot fluid fills between them. The temperature of the lower wall is \( \theta_1 \), the temperature of the upper wall is \( \theta_2 \) with \( \theta_1 \) higher than \( \theta_2 \) (Fig. 3.9). The force of the external field is directed downwards and perpendicular to the walls. Let \( D \) be the distance between the walls. We shall find out the conditions for convective instability.

Let us represent the parameters of the gas as sums of two terms: the first term is the parameter for the gas at rest and the second term is a small perturbation of the parameter due to the convective motion of the gas. Thus, the gas density is \( N + N' \), the gas pressure is \( p_0 + p' \), the gas temperature is \( \theta + \theta' \) and the gas velocity is \( v \), which is zero in the absence of convection. Now, insert these parameters into the stationary equations of continuity (2), of the Navier-Stoke’s equation (1) and of heat transport (3). The zero order approximation is,

\[
\text{grad } p_0 = -FN, \ \nabla^2 \theta = 0
\]

In the first small-parameter approximation, these equations yield

\[
\text{div } v = 0,
\]

\[
\text{div } \text{grad } g = 0.
\]
\[-\frac{\text{grad} (p_0 + p')}{(N + N')} + \frac{\eta \nabla^2 v}{(N + N')} + F = 0 \quad \ldots (4)\]

\[v_z \left[ \frac{(\theta_2 - \theta_1)}{D} \right] = \frac{\chi}{Nc_v} \nabla^2 \theta'\]

The parameters of the above problem are used in the last equation. Here, the z-axis is perpendicular to the walls.

Transform the first term in the second equation of (4). Upto the first order of approximation, the term is

\[\frac{\text{grad} (p_0 + p')}{(N + N')} = \frac{\text{grad} p_0}{N} + \frac{\text{grad} p'}{N} \quad \ldots (4)\]

According to the equation of state, \( p = N\theta \), \( N = p/\theta \) and hence we find that

\[N' = \frac{N}{\theta} \theta' = -\frac{N\theta'}{\theta}\]

Inserting this relation into the second equation of (4), we can write the system of equations (4) as

\[
\begin{aligned}
\text{div} \ v &= 0, \\
\frac{\text{grad} p'}{N} - F \left( \frac{\theta'}{\theta} \right) - \frac{\eta}{N} \nabla^2 v &= 0 \\
v_z &= \frac{\chi D}{Nc_v (\theta_2 - \theta_1)} \nabla^2 \theta'
\end{aligned}
\]

\[\ldots (5)\]

Let us reduce the system of equations (5) which connect the parameters of the gas, to an equation for one parameter. First, we apply to the second equation of (5) the operator ‘div’ and take into account the first equation of (5). We find that

\[\frac{\nabla^2 p'}{N} - \frac{F}{\theta} \frac{\partial \theta'}{\partial z} = 0 \quad \ldots (6)\]

Hence, we assume that \( \frac{(\theta_1 - \theta_2)}{\theta_1} << 1 \). Therefore, the undisturbed parameters of the gas do not vary much inside the volume being considered. We shall neglect their variation and assume that the unperturbed gas parameters are spatially constant.

Inserting \( w_z \) from the third equation of (5) into the \( z \)th component of the second equation and

\[
\frac{1}{N} \frac{\partial}{\partial z} (\nabla^2 p') - F \left( \frac{\nabla^2 \theta'}{\theta} \right) + \left[ \frac{\eta \chi D}{N^2 c_v (\theta_2 - \theta_1)} \right] (\nabla^2)^3 \theta' = 0
\]

Using the relation (6) between \( p' \) and \( \theta' \), we obtain finally
\[(\nabla^2)^2 \theta' = -\frac{R}{D^3} \left( \nabla^2 - \frac{\partial^2}{\partial z^2} \right) \theta' \] \quad \ldots (7)

Where the dimensionless combination of parameters

\[ R = \frac{(\theta_1 - \theta_2) C_v F N^2 D^3}{\eta \chi \theta} \] \quad \ldots (8)

R is called as the Rayleigh Number.

Equation (7) shows that the Rayleigh number determines the possibility of development of convection. For example, in the Rayleigh problem, the boundary conditions at the walls are \( \theta' = 0 \), \( v_z = 0 \). Also the tangential forces, \( \eta \left( \frac{\partial v_x}{\partial z} \right) \) and \( \left( \frac{\partial v_y}{\partial z} \right) \) are zero at the walls. Differentiating the equation, div \( v \) = 0 with respect to z and using the conditions for the tangential forces, we find that at the walls \( \left( \frac{\partial^2 v_z}{\partial z^2} \right) = 0 \). Hence, we have the following boundary conditions.

\[ \theta' = 0, \quad v_z = 0 \quad \text{and} \quad \left( \frac{\partial v_z}{\partial z} \right) = 0 \]

Denote by \( z = 0 \) the coordinate of the lower wall. The general solution of equation (7) with the boundary condition, \( \theta' = 0 \) at \( z = 0 \) can be expressed as

\[ \theta' = C \exp \left[ i(\kappa_x x + \kappa_y y) \right] \sin \kappa_z z \] \quad \ldots (9)

The boundary condition, \( \theta' = 0 \) at \( z = D \) yields \( \kappa_z D = \pi n \) where \( n \) is an integer. Inserting the solution given by equation (9) into equation (7), we get

\[ R = \frac{(\kappa^2 D^2 + \kappa^2 n^2)^3}{K D^3} \] \quad \ldots (10)

Where \( \kappa^2 = \kappa_x^2 + \kappa_y^2 \). The solution given by equation (9) satisfies all boundary conditions.

Equation (10) shows that convection can occur for Rayleigh numbers not less than a minimum number, \( R_{\min} \) corresponding to \( n = 1 \) and \( \kappa_{\min} = \frac{\pi}{(D \sqrt{2})} \); Thus

\[ R_{\min} = \frac{27 \pi^4}{4} = 658 \]

The magnitude of \( R_{\min} \) varies according to the geometry of the problem, but in all cases it is the Rayleigh number that characterizes the possibility of convection.
Let us now analyse the simple case of motion of gas in the x-z plane. Making use of the solution given by equation (9), we get:

\[
\text{div } \mathbf{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z} = 0
\]

We find the following expressions for the components of the velocity of the gas:

\[
v_x = -\frac{\pi n}{kD} v_0 \sin kx \cos \left(\frac{\pi nx}{D}\right)
\]

and

\[
v_z = v_0 \cos kx \sin \left(\frac{\pi nz}{D}\right)
\]

where \( n \) is an integer, and the characteristic velocity of the gas \( v_0 \) is assumed to be small compared to the respective parameters in the gas at rest; for instance, \( v_0 \) is small compared to the thermal velocity of the particles.

Let us trace the motion of a gas element. The equations of motion for this element are:

\[
v_x = \frac{dx}{dt}; \text{ and } v_z = \frac{dz}{dt}
\]

Using equations (11) for the components of the gas velocity, we obtain

\[
\frac{dx}{dz} = -\frac{\pi n}{kD} \tan kx \cot \left(\frac{\pi nz}{D}\right)
\]

This equation describes the path of the gas element.

The equation yields

\[
\sin kx \sin \left(\frac{\pi nz}{D}\right) = C \quad \ldots (12)
\]

Where \( C \) is a constant determined by the initial conditions. It varies from -1 to +1; its value depends upon the initial positions of the gas elements under consideration. Of special significance are the lines at \( C = 0 \). Those lines are given by the following equations:

\[
z = \left(\frac{D}{n}\right) m_1 \quad \text{and} \quad x = \left(\frac{\pi}{k}\right) m_2 \quad \ldots (13)
\]

where \( m_1 \) and \( m_2 \) are integers and both \( m_1 \) and \( m_2 \geq 0 \). The straight lines given by equations (13) divide the gas into cells. Equation (12) shows that inside each cell the gas elements travel along closed paths around the centre where the gas is resting. Fig.
3.10 shows the paths of the gas element for \( n = 1 \) and \( k = -\frac{\pi}{D} \), i.e. for the Rayleigh number, \( R = 8\pi^4 = 780 \) which is greater than \( R_{\text{min}} \) by about 100 units or say about 1.2 times \( R_{\text{min}} \).

Instability of convective motion will occur only for Rayleigh numbers of very high values which can disturb the ordered convective flow and finally disrupts the stability of convective motion of gas giving rise to disordered or turbulent flow of gas even if it is contained in a resting closed system.

As convection leading to turbulence is ideal for tectonic plate movements, let us analyse the development of turbulent gas flow.

Reviewing the Rayleigh problem once again, we analyse the convective motion of the gas given by equation (11) corresponding to sufficiently high Rayleigh numbers, say \( R = 108\pi^4 = 10520 \) which is some 16 times \( R_{\text{min}} \). In this case there can simultaneously develop two different types of convective motion. Fig. 3.11 shows two types of convective motion for the Rayleigh number \( 108\pi^4 \) corresponding to the wave number \( k_1 = \frac{9.4}{D} \) for \( n = 1 \) and \( k_2 = \frac{4.7}{D} \) for \( n = 2 \). Fig. 3.11 shows the types of convective motion. The mixing of the gas flows travelling in opposite directions finally results in a random gas motion or turbulence.

![Fig. 3.10 The paths of the gas elements](image1)

![Fig. 3.11 The types of convective motion for very Large Rayleigh numbers](image2)
The fact that *increasing the Rayleigh number* gives rise to new type of solutions implies that the convective flow can become turbulent. We can assume that in the system there is an ordered convective flow corresponding to one of the solutions. But, a small perturbation somewhere in one of the regions of the gas volume gives rise to another type of flow. At the boundary wall of the region two opposite gas flows meet such that the kinetic energy of motion of the gas flows gets transformed into the thermal energy of the gas. This results in a disordered motion of the gas. The development of turbulence not only changes the character of heat transport but facilitates tectonic plate movements.

**Conclusion:** The cascade process of increase of Rayleigh number, development of convection creating turbulence resulting in the movement of tectonic plates which ultimately is the cause for earthquakes, remains a mystery of nature. The earthquakes and volcanoes on various parts of the earth stand as a proof for the physical process that takes place inside the earth. The molten lava that comes out of the volcanoes which can be visually seen is the material of the molten fluid in the earth’s mantle the convection of which we were dealing with in the above treatment. The formation of earthquakes and volcanoes stands as the only experimental proof for the mathematical treatment we have been dealing with.

Let us pray ‘Ruaumoko’ the Maori God of earthquakes and volcanoes in particular and the *nature almighty* in general to control such phenomena for the peace and welfare of mankind.

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**REFERENCES :**


