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Identification Of Nonlinear Static systems
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6.1 Introduction

System identification is an important problem which finds extensive applications in the area of speech and signal processing. The problem of identification of systems is a classical problem from the days of Gauss and Legendre and was in terms of curve-fitting. The recent advancement in the nonlinear control theory has encouraged the development of adaptive control schemes for nonlinear plants with feedback [1]. In these schemes it has been assumed that all or the part of the system dynamics are known. Practically it is quite difficult as it imposes many constraints. In the past three decades major advances have been made in adaptive identification and control for identifying linear time-invariant plants with unknown parameters. The choice of the identifier and controller structures is based on well established results in linear systems theory [2].

The systems can be static or dynamic. The static systems are described by the algebraic equations whereas the dynamic systems are described by the difference equations or the differential equations. In the present study we have focused mainly on the identification of nonlinear static systems. This property of ANN has facilitated the identification of nonlinear systems. The ANNs are quite competent of performing nonlinear, complex functions because of their inherent nodes and their nonlinearities. Whenever a signal is passed through a nonlinear system or channel, it is contaminated with noise [3-7]. This noise is generated by the background thermal effect of the system or due to the different channel characteristics. Thus the desired output is a combination of the output of the system and the associated noise. The problem of system identification in this case is to match the output of the proposed model with this desired output obtained from the ideal nonlinear system.
6.2 Characterization and the identification of systems

The fundamental problems in the system theory are system characterization and identification[8]. A model of a system is expressed as an operator $F$ from an input space $X$ into an output space $Y$ and the objective is to characterize the class $f$ to which $F$ belongs. Given a class $f$ and the fact that $F \in f$, the problem of identification is to determine a class $\hat{f} \subset f$ and an element $\hat{F} \in \hat{f}$ so that $\hat{F}$ approximates $F$ in some desired sense. The operator $F$ is defined implicitly by the specified input-output pair.

A typical example of identification of static systems is the problem of pattern recognition. Compact sets $X_i \subset \mathbb{R}^n$ are mapped into elements $y_i \in \mathbb{R}^m; (i = 1, 2, ...,)$ in the output space by a decision function $F$. The elements of $X_i$ denote the pattern vectors corresponding to class $y_i$. The objective is to determine $\hat{F}$ so that

$$\left\| y - \hat{y} \right\| = \left\| \hat{F}(x) - F(x) \right\| \leq \epsilon,$$

where $x \in X$ and $\epsilon$ is some desired small value. The norm $\| \|$ is the defined norm in the output space. In Eq. (6.1) $\hat{F}(x) = \hat{y}$ denotes the output of the proposed model and $F(x) = y$ denotes the output of the plant or the desired output. The difference $e = y - \hat{y}$ is the error between desired(plant) and the estimated (model) output.

6.3 Proposed Technique

There are basically two categories of neural networks. The multilayer networks which have been extremely proved successful in pattern recognition

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problems [9-12] and the recurrent networks which are useful in associative memories and the solution of optimization problems [13-16].

The problem of system identification for both static as well as dynamic systems was originally proposed by Narendra and Parthasarathy [8]. They had used a multilayer perceptron (MLP) structure for identification of static and dynamic systems. With extensive computer simulations they have shown that the MLP structure is capable of identifying a wide class of nonlinear systems quite efficiently.

Here we have proposed a functionally link single layer structure for identification of nonlinear static systems. More over the noise of different strengths have also been poured with the immediate output of the system and then identification is achieved. This proposed structure contains a generalized set of expansions of the same input. These expansions are based on the powers of the input, the sine of the input and the cosine of the input. Chen and Billings [17] have utilized a functional link structure with polynomial expansion in terms of the outer product of the elements of the input vector and the output node is considered to have linear characteristics. Pao et. al. [18] have utilized the products of a random vector with the input vector for functional expansion in the system identification problem. Here we have used functional expansion with a power series of the input and its trigonometric expansions. The output node of the network contains a nonlinearity. This nonlinearity is a tan-hyperbolic nonlinearity which is also known as sigmoid.

Fig. 6.1 shows a system identification scheme in general without noise. Here \( x(k) \) is the input at instant \( k \), \( \hat{y}_e(k) \) is the estimated output at the \( k \)-th instant and \( y_e(k) \) is the desired output at that instant. \( e(k) \) is the error due to difference between the desired and the observed output.
Fig. 6.2 depicts a system identification scheme with noise taken into account. The noise is added at the output of the system when the input signal $x(k)$ passes through it. The strength of noise is expressed in dB.

The proposed technique of system identification using functional expansion scheme has been shown in Fig. 6.3. The input signal $x(k)$ at instant $k$, undergoes a functional expansion in terms of the powers of the original input $x(k)$, the sines and cosines of the same input.
6.4 Learning Algorithm

Consider the network shown in Fig.6.3. The learning procedure is mentioned below.

Apply an input to the system. The input is a random number \( x(k) \) at \( k \)-th iteration and its value is randomly taken between +0.5 to -0.5. Functionally expand the input \( x(k) \) to generate a larger set of input patterns. Let these expansions be \( x(k), x^2(k), ..., x^r(k), \sin \pi x(k), \sin 2\pi x(k), ..., \sin m\pi x(k), \cos \pi x(k), \cos 2\pi x(k), ..., \cos m\pi x(k) \).

Let \( M = 1 + m + n \) so that the number of expansions at \( k \)-th instant are \( M \). For simplicity we represent these inputs by a common term \( X_i(k) \), for \( i = 1, 2, ..., M \). The zero mean random input is passed through the nonlinear system \( F \). Its output will be \( F(x) \) which is added with uncorrelated white noise of strength \( N(k) \) to produce the desired output \( y_a(k) \)

\[ y_a(k) = F(x) + N(k) \]

This output is used for the comparison purpose. The various steps involved in training the functionally expanded network are listed in Section 2.5.4 of Chapter 2.

![Fig. 6.3 ANN technique used in identification of nonlinear systems](image-url)
6.5 Simulation Studies

To assess the performance of the proposed functional expansion based ANN model, simulation study was carried out using different nonlinear systems. Four different static nonlinear systems used for simulation are as follows:

(a) $F_1(x) = x^3 + 0.3x^2 - 0.4x$

(b) $F_2(x) = 0.6\sin(\Pi x) + 0.3\sin(3\Pi x) + 0.1\sin(5\Pi x)$

(c) $F_3(x) = \frac{4.0x^3 - 1.2x^2 - 0.3x + 1.2}{0.4x^5 + 0.8x^4 - 1.2x^3 + 0.2x^2 - 3.0}$

\[ \text{(6.2)} \]

(d) $F_4(x) = 0.5\sin^3(\Pi x) - \frac{2.0}{x^3 + 2.0} - 0.1\cos(4\Pi x) + 1.125$

Identifications of these systems have been studied [3] using 3-layer MLP structure. The structure of the MLP used by the authors was 1,20,10,1 \textit{i.e.} 1 at the input, 20, 10 in the first and second hidden layers respectively and 1 neuron in the final output layer. The threshold weights attached with neuron of the first, second and output layers were 20, 10 and 1 respectively. Thus training of the network required updating of 261 weights per iteration. Moreover the effect of noise had not been taken into account by them. On the other hand the present study has been considered as a realistic situation of the problem. Here the structure proposed is a single layer ANN with functional expansions at the input which is simple and is different from the MLP structure. As mentioned earlier the expansions are a combination of power series of the input, trigonometric expansions of the input. Fig.6.3 shows the adaptive nonlinear structure simulated for identification study. We have proposed only one ANN structure which is used for identifying four different nonlinear systems. These are $x, x^2, x^3, x^4, \sin(\pi x), \sin(2\pi x) \ldots, \sin(5\pi x), \cos(\pi x), \cos(2\pi x), \ldots, \cos(5\pi x)$. There are 14 connecting weights such that one weight is connected to each of the above expansions. Another weight called the bias weight $w_b$ is used as extra input. Thus there are 15 weights to be updated at every iteration during training.
The learning parameter $\mu$ and the momentum term $\alpha$ are set to 0.55 and 0.1 respectively.

The learning characteristics curve is obtained from simulation for each of different four systems. Fig. 6.4 (a) shows learning characteristics for the nonlinear system $f_i$ i.e. (a) of Eq.(6.2) with additive noise -20 dB. Similar characteristics were obtained for other three nonlinear systems under the same noise condition and have been displayed by Fig. 6.4 (b),(c) and (d). The observation obtained from these characteristics are discussed in the next section. After modeling the nonlinear system $f_i$ using functional expansion based ANN, the steady state weights are frozen and the response of the modeled system for a white input was obtained and has been compared with the corresponding results obtained from the actual nonlinear system $f_i$. The zero mean random input used for testing and the corresponding desired and simulated outputs obtained from the first nonlinear system have been displayed in Figs. 6.5 (a) and (b). Further simulation was carried out for nonlinear system $f_i$ under varying noise conditions. The effect of additive noise in the learning characteristics is shown in Fig. 6.6 by taking additive noises -20 dB in (a), -30 dB in (b), -40 dB in (c). Different random test inputs were also generated and were applied to other three NN models of the nonlinear system. The desired and the simulated responses corresponding to the test inputs for -20 dB noise were also obtained. These are shown in Figs. 6.7, 6.8 and 6.9 for nonlinear systems $f_2$, $f_3$ and $f_4$ respectively.
(a) $f_1$

(b) $f_2$
Fig. 6.4 Learning characteristics for four nonlinear functions $f_1$, $f_2$, $f_3$ and $f_4$ with noise $= -20$ dB
Fig. 6.5 Comparison of desired and the simulated responses of the ANN model for nonlinear system $f_1$ with noise $= -20$
Fig. 6.6 Effect of different additive noises on the learning characteristics of system $f_1$. 

(a) Noise $= -20$ dB

(b) Noise $= -30$ dB

(c) Noise $= -40$ dB
Fig. 6.7 Comparison of desired and the simulated responses of the ANN model for nonlinear system $f_2$ with noise = -20
Fig. 6.8 Comparison of desired and the simulated responses of the ANN model for nonlinear system $f_3$ with noise $= -20$
Fig. 6.9 Comparison of desired and the simulated responses of the ANN model for nonlinear system $f_4$ with noise $= -20$

6.6 Conclusions

Four typical nonlinear systems are chosen and using a single ANN structure all these systems were modeled. The learning characteristics of these systems
shown in Fig. 6.4 reveal that under the same noise condition of -20 dB, the system \( f_4 \) takes the minimum time i.e. 200 iterations. Further Fig. 6.6 indicates that the MSE floor of a nonlinear system after training settles to a level corresponding to the additive noise of the actual system. MSE characteristics corresponding to the additive noise of -20 dB settles faster than the other additive noise cases which is achieved at 150 iterations. Referring to Figs. 6.5, 6.7, 6.8 and 6.9, it is observed that the simulated responses obtained from the ANN model has an excellent with the desired responses of the actual systems. Thus we can conclude that the functional expansion based generalized model presented in the investigation is capable of identifying various nonlinear systems. Besides the proposed model involves less number of connecting weights to be learnt per iteration as compared to the multilayer ANN structure. Thus the proposed single layer structure is simple and economical for identifying nonlinear systems.
References


