CHAPTER V

FUZZY MULTINOMIAL CONTROL CHART WITH VARIABLE SAMPLE SIZE

1. Introduction:

Statistical Process Control (SPC) is used to monitor the process stability which ensures the predictability of the process. The power of control charts lies in their ability to detect process shift and to identify abnormal conditions in the process. In 1924, Walter Shewhart designed the first control chart. According to him, if \( w \) be a sample statistic that measures some quality characteristic of interest the mean of \( w \) is \( \mu_w \), and the standard deviation of \( w \) is \( \sigma_w \), then the center line (CL), the upper control limit (UCL) and the lower control limit (LCL) are defined as

\[
\text{UCL} = \mu_w + d \sigma_w \\
\text{CL} = \mu_w \\
\text{LCL} = \mu_w - d \sigma_w
\]

where \( d \) is the “distance” of the control limits from the center line, expressed in standard deviation units.

A single measurable quality characteristic such as dimension, weight or volume is called a variable. In such cases, control charts for variables are used to monitor the process. These include the \( \overline{X} \)-chart for controlling the process average and the \( R \)-chart (or \( S \)-chart) for controlling the process variability. For the quality-related characteristics such as characteristics for appearance, softness, color, taste, etc., attribute control charts such as \( p \)-chart, \( c \)-chart are used to monitor the production process. Some times the product units are classified as either "conforming" or "nonconforming", depending upon whether or not product units meet some specifications. The \( p \)-chart is used to monitor the process based upon the fraction of nonconforming units.

2. Fuzzy logic and Linguistic variables

The concept of fuzzy logic plays a fundamental role in formulating quantitative fuzzy variables. These are variables whose states are fuzzy members. The members represent linguistic concepts, such as very small, small, medium and so on, as interpreted in a particular context. The resulting constructs are usually called linguistic variables. The linguistic terms are commonly used in industry to express properties or characteristics of a particular product.
The conformity to specifications of a quality standard is evaluated on a two-state scale, for example, acceptable or unacceptable, good or bad, and so on. In some situations the binary classification might not be suitable, where product quality can assume more intermediate states. For each state weight is assigned depending upon the importance. The assignment of weights, to reflect the degree of severity of product nonconformity, has been adopted in many circumstances. When the products are classified into mutually exclusive linguistic categories, fuzzy control charts are used. Different procedures are proposed to construct these charts. Raz and Wang [1990] have developed fuzzy control charts for linguistic data based on membership and probabilistic approaches, Kanagawa et al. [1993] developed the control charts for process average and variability based on linguistic data and Gulbay and Kagraman [2004] proposed \( \alpha \) cut control charts for linguistic data.

3. Proposed methodology:

In this chapter, a fuzzy multinomial control chart (FM chart) for linguistic variables with variable sample size is proposed. The FM – chart deals with a linguistic variable which is classified into more than two categories. The FM – chart with VSS and the traditional \( p \) – chart for studying the shift in process mean have been discussed with numerical examples.

4. Methodology

Based upon Fuzzy set theory, a linguistic variable \( \tilde{L} \) is characterized by the set of \( k \) mutually exclusive members \( \{l_1, l_2, ... l_k\} \). Attach a weight \( m_i \) to each term \( l_i \) that reflects the degree of membership in the set. Then it can be written by a fuzzy set as

\[
\tilde{L} = \{(l_1, m_1), (l_2, m_2), ..., (l_k, m_k)\} \quad \text{........................... (1)}
\]

To monitor the production process, take independent samples of different sizes. The size of the sample to be drawn each time is decided by choosing a member randomly from \( \{n_1, n_2, ..., n_s\} \).
5. Fuzzy Multinomial Control Chart

In this section a new approach for construction of Fuzzy multinomial control chart based on variable sample size is proposed. The statistical principles underlying the fuzzy multinomial control chart (FM -chart) with variable sample size are based on the multinomial distribution.

As defined in (1), \( \bar{L} \) is a linguistic variable which can take \( k \) mutually exclusive members \( \{l_1,l_2,...l_k\} \). Assume that the production process is operating in a stable manner and \( p_i \) is the probability that an item is \( l_i \), \( i = 1, 2 \ldots k \), and successive items produced are independent. Suppose that a random sample of size \( n_r \) units of the product is selected and let \( X_i, i = 1, 2 \ldots k \), be the number of items of the product that are \( l_i \), \( i = 1, 2 \ldots k \), then \( \{X_1,X_2...X_k\} \) has a multinomial distribution with parameters \( n_r \) and \( p_1,p_2...p_k \). It is known that each \( X_i, i = 1, 2 \ldots k \), marginally has a binomial distribution with the mean \( n_r p_i \) and variance \( n_r p_i (1-p_i) \), \( i = 1, 2 \ldots k \), respectively. The weighted average of the linguistic variable \( \bar{L} \) with sample size \( n_r \) is defined by

\[
\bar{L} = \frac{\sum_{i=1}^{k} X_i m_i}{\sum_{i=1}^{k} X_i} = \frac{\sum_{i=1}^{k} X_i m_i}{n_r}, \quad n_r \in \{n_1,n_2,...n_s\} \quad \text{.................. (2)}
\]

The control limits for FM – chart by the conventional concept are

\[
\text{UCL} = E[\bar{L}] + d \sqrt{\text{var}(\bar{L})} \quad ; \quad \text{CL} = E[\bar{L}] \quad ; \quad \text{and} \quad \text{LCL} = E[\bar{L}] - d \sqrt{\text{var}(\bar{L})},
\]

where \( d \) is the distance of the control limits from the center line. The procedure for computing \( E[\bar{L}] \) and \( \text{var}(\bar{L}) \) for each sample is established from the following theorem.

6. Theorem:

Let \( \bar{L} = \{(l_1,m_1),(l_2,m_2),...,(l_k,m_k)\} \) be a linguistic variable such that \( p_i \) is the probability that an item is \( l_i \), \( i = 1, 2 \ldots k \). If \( X_i, i = 1, 2 \ldots k \) is the number of units of the product that are \( l_i \), \( i = 1, 2 \ldots k \) in a sample of size \( n_r \), then

\[
(i) \quad E[\bar{L}] = \sum_{i=1}^{k} p_i m_i
\]
(ii) \[ \text{var} (\bar{L}) = \frac{1}{n_r} \left[ \sum_{i=1}^{k} m_i^2 p_i (1-p_i) - 2 \sum_{i=1}^{k} \sum_{j=1, j \neq i}^{k} m_i m_j p_i p_j \right], \quad n_r \in \{n_1, n_2, \ldots n_s\} \]

where \( n_1, n_2, \ldots n_s \) are pre-determined sample sizes.

**Proof:**

In a sample of \( n_r \) units, \( X_i \) has a binomial distribution with the mean \( n_r p_i \) and variance \( n_r p_i (1-p_i) \), \( i = 1, 2, \ldots k \) and

\[
\text{Cov} (X_i, X_j) = -n_r p_i p_j, \quad \text{if} \quad i \neq j \quad \text{and then}
\]

(i) The mean is: \[ E[\bar{L}] = \frac{1}{n_r} \left[ \sum_{i=1}^{k} m_i X_i \right] = \frac{1}{n_r} \left[ \sum_{i=1}^{k} m_i E(X_i) \right] = \frac{1}{n_r} \left[ \sum_{i=1}^{k} m_i n_r p_i \right] = \sum_{i=1}^{k} p_i m_i, \quad n_r \in \{n_1, n_2, \ldots n_s\} \ldots \ldots \ldots (3) \]

(ii) The variance is: \[ \text{var} (\bar{L}) = \text{var} \left( \frac{1}{n_r} \left[ \sum_{i=1}^{k} m_i X_i \right] \right) = \frac{1}{n_r^2} \left[ \text{var} \left( \sum_{i=1}^{k} m_i X_i \right) \right], \]

\[
= \frac{1}{n_r^2} \left[ \text{var} (m_1 x_1 + m_2 x_2 + \ldots + m_k x_k) \right]
\]

\[
= \frac{1}{n_r^2} \left[ \sum_{i=1}^{k} m_i^2 \text{var}(X_i) + 2 \sum_{i=1}^{k} \sum_{j=1, j \neq i}^{k} m_i m_j \text{cov}(X_i, X_j) \right]
\]

\[
= \frac{1}{n_r^2} \left[ \sum_{i=1}^{k} m_i^2 \text{var}(X_i) + 2 \sum_{i=1}^{k} \sum_{j=1}^{k} m_i m_j \text{cov}(X_i, X_j) \right]
\]

\[
= \frac{1}{n_r^2} \left[ \sum_{i=1}^{k} m_i^2 n_r p_i (1-p_i) + 2 \sum_{i=1}^{k} \sum_{j=1}^{k} m_i m_j (-n_r p_i p_j) \right]
\]

\[
= \frac{1}{n_r} \left[ \sum_{i=1}^{k} m_i^2 p_i (1-p_i) - 2 \sum_{i=1}^{k} \sum_{j=1, j \neq i}^{k} m_i m_j p_i p_j \right], \quad \text{where} \quad n_r \in \{n_1, n_2, \ldots n_s\} \ldots \ldots (4)
\]

The results for \( E[\bar{L}] \) and \( \text{var} (\bar{L}) \) become the same as that of the results due to Amirzadeh et al. [2008], when the sample size is fixed. That is, when \( n_r = n \) for all \( r \), the mean and variance...
derived in (3) and (4) for a linguistic variable with variable sample size will be equal to the mean and variance of the linguistic variable for fixed sample size.

7. Choice of Sample size:

Many authors, for example Costa [1994], Sawlapurkar et al. [1990] have recommended variable sample sizes (VSS) for the construction of control charts for variables as well as attributes. Later, the Markov dependent sample size (MDSS) was proposed by Sivasamy et al. [2000], and Pandurangan [2002] for the advantage of economic sampling inspection. To construct FM control chart, the sample size for each draw can be randomly chosen from the pre – determined set \( \{n_1, n_2, \ldots, n_s\} \). The advantage of taking variable sample size lies in ASN and consequently the costs of sampling inspection.

8. Numerical Example:

On a production line, a visual control of the aluminum die-cast of a lighting component might have the following assessment possibilities

1. "reject" if the aluminum die-cast does not work;
2. "poor quality" if the aluminum die-cast works but has some defects;
3. "medium quality" if the aluminum die-cast works and has no defects, but it has some aesthetic flaws;
4. "good quality" if the aluminum die-cast works and has no defects, but has few aesthetic flaws;
5. "excellent quality" if the aluminum die-cast works and has neither defects nor aesthetic flaws of any kind.

To monitor the quality of this product, 25 samples of different sizes are selected. The degrees of membership for the above assessment are taken as 1, 0.75, 0.5, 0.25 and 0 respectively. The data with \( \overline{L}_i \) and \( \hat{p}_i \) are given in Table – 1.

The values of \( \overline{L}_i \) are calculated in the following ways

\[
\overline{L}_i = \frac{\sum_{i=1}^{k} X_i m_i}{\sum_{i=1}^{k} X_i} = \frac{\sum_{i=1}^{k} X_i m_i}{n_r}, \quad n_r \in \{n_1, n_2, \ldots, n_s\}
\]
A Study on Process Variability using CUSUM and Fuzzy Control Charts - Ph.D Thesis

and so on. The value of \( \hat{p}_i \), the control limits for \( p \) – charts can be calculated as , 

\[
\hat{p}_i = \frac{D_i}{n_r},
\]

\( n_r \in \{ n_1, n_2, \ldots n_s \} \), \( \hat{p}_1 = \frac{D_1}{n_1} = \frac{12}{100} = 0.120 \), \( \hat{p}_2 = \frac{D_2}{n_2} = \frac{8}{80} = 0.100 \), \( \hat{p}_3 = \frac{D_3}{n_3} = \frac{6}{80} = 0.075 \);

and so on.
### Table – 1

The data of various sample size and the values of $L_i$ and $\hat{p}_i$

<table>
<thead>
<tr>
<th>Sample No</th>
<th>Sample size</th>
<th>Reject (R)</th>
<th>Poor Quality (PQ)</th>
<th>Medium Quality (MQ)</th>
<th>Good Quality (GQ)</th>
<th>Excellent Quality (EQ)</th>
<th>$L_i$</th>
<th>$\hat{p}_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>12</td>
<td>10</td>
<td>12</td>
<td>54</td>
<td>12</td>
<td>0.390</td>
<td>0.120</td>
</tr>
<tr>
<td>2</td>
<td>80</td>
<td>8</td>
<td>7</td>
<td>9</td>
<td>48</td>
<td>8</td>
<td>0.372</td>
<td>0.100</td>
</tr>
<tr>
<td>3</td>
<td>80</td>
<td>6</td>
<td>11</td>
<td>12</td>
<td>43</td>
<td>8</td>
<td>0.388</td>
<td>0.075</td>
</tr>
<tr>
<td>4</td>
<td>100</td>
<td>9</td>
<td>7</td>
<td>13</td>
<td>53</td>
<td>18</td>
<td>0.340</td>
<td>0.090</td>
</tr>
<tr>
<td>5</td>
<td>110</td>
<td>10</td>
<td>16</td>
<td>12</td>
<td>54</td>
<td>12</td>
<td>0.405</td>
<td>0.091</td>
</tr>
<tr>
<td>6</td>
<td>110</td>
<td>12</td>
<td>5</td>
<td>17</td>
<td>60</td>
<td>16</td>
<td>0.357</td>
<td>0.109</td>
</tr>
<tr>
<td>7</td>
<td>100</td>
<td>11</td>
<td>12</td>
<td>13</td>
<td>50</td>
<td>14</td>
<td>0.390</td>
<td>0.110</td>
</tr>
<tr>
<td>8</td>
<td>100</td>
<td>10</td>
<td>22</td>
<td>18</td>
<td>45</td>
<td>5</td>
<td>0.468</td>
<td>0.100</td>
</tr>
<tr>
<td>9</td>
<td>90</td>
<td>10</td>
<td>8</td>
<td>13</td>
<td>50</td>
<td>9</td>
<td>0.389</td>
<td>0.111</td>
</tr>
<tr>
<td>10</td>
<td>90</td>
<td>6</td>
<td>5</td>
<td>14</td>
<td>51</td>
<td>14</td>
<td>0.328</td>
<td>0.067</td>
</tr>
<tr>
<td>11</td>
<td>110</td>
<td>20</td>
<td>13</td>
<td>23</td>
<td>47</td>
<td>7</td>
<td>0.482</td>
<td>0.182</td>
</tr>
<tr>
<td>12</td>
<td>120</td>
<td>15</td>
<td>13</td>
<td>20</td>
<td>58</td>
<td>14</td>
<td>0.410</td>
<td>0.125</td>
</tr>
<tr>
<td>13</td>
<td>120</td>
<td>9</td>
<td>12</td>
<td>22</td>
<td>64</td>
<td>13</td>
<td>0.375</td>
<td>0.075</td>
</tr>
<tr>
<td>14</td>
<td>120</td>
<td>8</td>
<td>9</td>
<td>20</td>
<td>61</td>
<td>22</td>
<td>0.333</td>
<td>0.067</td>
</tr>
<tr>
<td>15</td>
<td>110</td>
<td>6</td>
<td>10</td>
<td>19</td>
<td>61</td>
<td>14</td>
<td>0.348</td>
<td>0.055</td>
</tr>
<tr>
<td>16</td>
<td>80</td>
<td>8</td>
<td>5</td>
<td>12</td>
<td>47</td>
<td>8</td>
<td>0.369</td>
<td>0.100</td>
</tr>
<tr>
<td>17</td>
<td>80</td>
<td>10</td>
<td>8</td>
<td>12</td>
<td>40</td>
<td>10</td>
<td>0.400</td>
<td>0.125</td>
</tr>
<tr>
<td>18</td>
<td>80</td>
<td>7</td>
<td>13</td>
<td>10</td>
<td>42</td>
<td>8</td>
<td>0.403</td>
<td>0.088</td>
</tr>
<tr>
<td>19</td>
<td>90</td>
<td>5</td>
<td>7</td>
<td>14</td>
<td>54</td>
<td>10</td>
<td>0.342</td>
<td>0.056</td>
</tr>
<tr>
<td>20</td>
<td>100</td>
<td>8</td>
<td>11</td>
<td>14</td>
<td>50</td>
<td>17</td>
<td>0.358</td>
<td>0.080</td>
</tr>
<tr>
<td>21</td>
<td>100</td>
<td>5</td>
<td>8</td>
<td>16</td>
<td>58</td>
<td>13</td>
<td>0.335</td>
<td>0.050</td>
</tr>
<tr>
<td>22</td>
<td>100</td>
<td>8</td>
<td>9</td>
<td>15</td>
<td>51</td>
<td>17</td>
<td>0.350</td>
<td>0.080</td>
</tr>
<tr>
<td>23</td>
<td>100</td>
<td>10</td>
<td>12</td>
<td>14</td>
<td>50</td>
<td>14</td>
<td>0.385</td>
<td>0.100</td>
</tr>
<tr>
<td>24</td>
<td>90</td>
<td>6</td>
<td>13</td>
<td>17</td>
<td>45</td>
<td>9</td>
<td>0.394</td>
<td>0.067</td>
</tr>
<tr>
<td>25</td>
<td>90</td>
<td>9</td>
<td>10</td>
<td>14</td>
<td>46</td>
<td>11</td>
<td>0.389</td>
<td>0.100</td>
</tr>
</tbody>
</table>
8.1. Conventional p – chart with VSS:

The control limits for p – chart is obtained as

\[
\bar{p} = \frac{\sum_{i=1}^{s} D_i}{\sum_{i=1}^{s} n_i} = \frac{234}{2450} = 0.096
\]

For Sample 1:

\[
UCL_1 = \bar{p} + d \sqrt{\frac{\bar{p}(1-\bar{p})}{n_r}} = 0.096 + 3 \sqrt{\frac{0.096(1-0.096)}{100}} = 0.184
\]

\[
CL_1 = \bar{p} = 0.096;
\]

\[
LCL_1 = \bar{p} - d \sqrt{\frac{\bar{p}(1-\bar{p})}{n_r}} = 0.096 - 3 \sqrt{\frac{0.096(1-0.096)}{100}} = 0.008,
\]

For Sample 2:

\[
UCL_2 = \bar{p} + d \sqrt{\frac{\bar{p}(1-\bar{p})}{n_r}} = 0.096 + 3 \sqrt{\frac{0.096(1-0.096)}{80}} = 0.195
\]

\[
CL_2 = \bar{p} = 0.096;
\]

\[
LCL_2 = \bar{p} - d \sqrt{\frac{\bar{p}(1-\bar{p})}{n_r}} = 0.096 - 3 \sqrt{\frac{0.096(1-0.096)}{80}} = -0.003 \approx 0,
\]

and so on.

The chart given below depicts the conventional p – chart for 25 samples.
In Figure 1, the out of control signal is seen corresponding to 11\textsuperscript{th} sample, for which the sample size is 110. From the p – chart, out of 2450 sample observations, 1070 sample observations were needed to get the signal. The corresponding center line and control limits are as under

\[ \text{CL} = 0.096, \text{UCL} = 0.180 \text{ and } \text{LCL} = 0.012 \]

8.2. FM – chart with VSS:

To construct the FM – chart, the UCL and LCL values are computed for each sample as under

For Sample 1: \[ UCL = E[\bar{L}_1] + d \sqrt{\text{var}(\bar{L}_1)} = \]

\[
\sum_{i=1}^{k} p_i m_i + d \sqrt{\frac{1}{n_1} \left[ \sum_{i=1}^{k} m_i^2 p_i (1 - p_i) - 2 \sum_{i=1}^{k} \sum_{j=1}^{k} m_i m_j p_i p_j \right]} \]

\[ = 0.3750 + 3\sqrt{0.0008214} = 0.4609 \]
\[ CL_1 = E[\bar{L}_1] = \sum_{i=1}^{k} p_i m_i = 0.3750 \]

\[ LCL_1 = E[\bar{L}_1] - d \sqrt{\text{var}(\bar{L}_1)} = \sum_{i=1}^{k} p_i m_i - d \left[ \frac{1}{n_1} \sum_{i=1}^{k} m_i^2 p_i (1 - p_i) - 2 \sum_{i=1}^{k} \sum_{j=1}^{k} m_i m_j p_i p_j \right] \]

\[ = 0.3750 - 3\sqrt{0.0008214} = 0.2891 \]

For Sample 2: \[ UCL_2 = E[\bar{L}_2] + d \sqrt{\text{var}(\bar{L}_2)} = \sum_{i=1}^{k} p_i m_i + d \left[ \frac{1}{n_2} \sum_{i=1}^{k} m_i^2 p_i (1 - p_i) - 2 \sum_{i=1}^{k} \sum_{j=1}^{k} m_i m_j p_i p_j \right] \]

\[ = 0.3863 + 3\sqrt{0.001234} = 0.4917 \]

\[ CL_2 = E[\bar{L}_2] = \sum_{i=1}^{k} p_i m_i = 0.3863 \]

\[ LCL_2 = E[\bar{L}_2] - d \sqrt{\text{var}(\bar{L}_2)} = \sum_{i=1}^{k} p_i m_i - d \left[ \frac{1}{n_2} \sum_{i=1}^{k} m_i^2 p_i (1 - p_i) - 2 \sum_{i=1}^{k} \sum_{j=1}^{k} m_i m_j p_i p_j \right] \]

\[ = 0.3863 - 3\sqrt{0.001234} = 0.2809, \text{ and so on.} \]

Various values of UCL and LCL for all the 25 samples are tabulated below.
Table – 2
Calculation of Mean, Variance of \( \bar{L}_i \) and Control Limits for various sample size

<table>
<thead>
<tr>
<th>Sample No.</th>
<th>Sample size</th>
<th>( \bar{L}_i )</th>
<th>( E(\bar{L}_i) )</th>
<th>( V(\bar{L}_i) )</th>
<th>LCL</th>
<th>UCL</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>0.390</td>
<td>0.3750</td>
<td>0.0008214</td>
<td>0.2891</td>
<td>0.4609</td>
</tr>
<tr>
<td>2</td>
<td>80</td>
<td>0.372</td>
<td>0.3863</td>
<td>0.0012340</td>
<td>0.2809</td>
<td>0.4917</td>
</tr>
<tr>
<td>3</td>
<td>80</td>
<td>0.388</td>
<td>0.3863</td>
<td>0.0012340</td>
<td>0.2809</td>
<td>0.4917</td>
</tr>
<tr>
<td>4</td>
<td>100</td>
<td>0.340</td>
<td>0.3750</td>
<td>0.0008214</td>
<td>0.2891</td>
<td>0.4609</td>
</tr>
<tr>
<td>5</td>
<td>110</td>
<td>0.405</td>
<td>0.3898</td>
<td>0.0007330</td>
<td>0.3086</td>
<td>0.4710</td>
</tr>
<tr>
<td>6</td>
<td>110</td>
<td>0.357</td>
<td>0.3898</td>
<td>0.0007330</td>
<td>0.3086</td>
<td>0.4710</td>
</tr>
<tr>
<td>7</td>
<td>100</td>
<td>0.390</td>
<td>0.3750</td>
<td>0.0008214</td>
<td>0.2891</td>
<td>0.4609</td>
</tr>
<tr>
<td>8</td>
<td>100</td>
<td>0.468</td>
<td>0.3750</td>
<td>0.0008214</td>
<td>0.2891</td>
<td>0.4609</td>
</tr>
<tr>
<td>9</td>
<td>90</td>
<td>0.389</td>
<td>0.3683</td>
<td>0.0004014</td>
<td>0.3082</td>
<td>0.4284</td>
</tr>
<tr>
<td>10</td>
<td>90</td>
<td>0.328</td>
<td>0.3683</td>
<td>0.0004014</td>
<td>0.3082</td>
<td>0.4284</td>
</tr>
<tr>
<td>11</td>
<td>110</td>
<td>0.482</td>
<td>0.3898</td>
<td>0.0007330</td>
<td>0.3086</td>
<td>0.4710</td>
</tr>
<tr>
<td>12</td>
<td>120</td>
<td>0.410</td>
<td>0.3729</td>
<td>0.0006950</td>
<td>0.2865</td>
<td>0.4593</td>
</tr>
<tr>
<td>13</td>
<td>120</td>
<td>0.375</td>
<td>0.3729</td>
<td>0.0006950</td>
<td>0.2865</td>
<td>0.4593</td>
</tr>
<tr>
<td>14</td>
<td>120</td>
<td>0.333</td>
<td>0.3729</td>
<td>0.0006950</td>
<td>0.2865</td>
<td>0.4593</td>
</tr>
<tr>
<td>15</td>
<td>110</td>
<td>0.348</td>
<td>0.3898</td>
<td>0.0007330</td>
<td>0.3086</td>
<td>0.4710</td>
</tr>
<tr>
<td>16</td>
<td>80</td>
<td>0.369</td>
<td>0.3863</td>
<td>0.0012340</td>
<td>0.2809</td>
<td>0.4917</td>
</tr>
<tr>
<td>17</td>
<td>80</td>
<td>0.400</td>
<td>0.3863</td>
<td>0.0012340</td>
<td>0.2809</td>
<td>0.4917</td>
</tr>
<tr>
<td>18</td>
<td>80</td>
<td>0.403</td>
<td>0.3863</td>
<td>0.0012340</td>
<td>0.2809</td>
<td>0.4917</td>
</tr>
<tr>
<td>19</td>
<td>90</td>
<td>0.342</td>
<td>0.3683</td>
<td>0.0004014</td>
<td>0.3082</td>
<td>0.4284</td>
</tr>
<tr>
<td>20</td>
<td>100</td>
<td>0.358</td>
<td>0.3750</td>
<td>0.0008214</td>
<td>0.2891</td>
<td>0.4609</td>
</tr>
<tr>
<td>21</td>
<td>100</td>
<td>0.335</td>
<td>0.3750</td>
<td>0.0008214</td>
<td>0.2891</td>
<td>0.4609</td>
</tr>
<tr>
<td>22</td>
<td>100</td>
<td>0.350</td>
<td>0.3750</td>
<td>0.0008214</td>
<td>0.2891</td>
<td>0.4609</td>
</tr>
<tr>
<td>23</td>
<td>100</td>
<td>0.385</td>
<td>0.3750</td>
<td>0.0008214</td>
<td>0.2891</td>
<td>0.4609</td>
</tr>
<tr>
<td>24</td>
<td>90</td>
<td>0.394</td>
<td>0.3683</td>
<td>0.0004014</td>
<td>0.3082</td>
<td>0.4284</td>
</tr>
<tr>
<td>25</td>
<td>90</td>
<td>0.389</td>
<td>0.3683</td>
<td>0.0004014</td>
<td>0.3082</td>
<td>0.4284</td>
</tr>
</tbody>
</table>

The above table is used to draw the FM chart with VSS. The chart is given below.
Figure 2 shows that the process is out of control at samples 8 and 11, with the respective cumulative sample sizes are 100 and 110 and the corresponding center lines and control limits are

(i) \[ \text{CL} = 0.3750, \text{UCL} = 0.4609 \text{ and } \text{LCL} = 0.2891 \text{ for sample size 100. } \]
\[ (P_R = 0.10, \ P_{PQ} = 0.22, \ P_{MQ} = 0.18, \ P_{GQ} = 0.45, \ P_{EQ} = 0.05) \]

(ii) \[ \text{CL} = 0.3898, \text{UCL} = 0.4710 \text{ and } \text{LCL} = 0.3086 \text{ for sample size 110. } \]
\[ (P_R = 0.1818, \ P_{PQ} = 0.1181, \ P_{MQ} = 0.2091, \ P_{GQ} = 0.4273, \ P_{EQ} = 0.0636) \]

From figure 2, it is seen that the FM chart gives the first signal corresponding to 8\textsuperscript{th} sample. Whereas, in p – chart the first signal for the existence of assignable causes is seen only at the 11\textsuperscript{th} sample. That is, in the case of FM – chart, only 780 samples are inspected to get the first out of control signal. But, 1070 samples are to be inspected to get the alarm with the help of a p – chart. Thus, the FM is more economical and more sensitive in identifying any shift in the specified quality level.
9. Conclusion:

FM – chart has been proposed for linguistic data set. To draw the chart, samples of varying sizes are chosen randomly from a pre–determined set. The FM-chart has been compared with the conventional p–chart with VSS. It is found that the FM – chart is more economical and more sensitive in giving the alarm for shift in the specified quality level than the conventional p – chart with VSS. This work can be extended for Markov dependent sample sizes.