Chapter 6

MHD LUBRICATION FLOW BETWEEN ROUGH RECTANGULAR PLATES
6.1. Introduction

In this Chapter, the study of effect of surface roughness on the squeeze film characteristics between two rectangular plates in the presence of transverse magnetic field is presented.

Magnetohydrodynamic (MHD) is the study of the interaction of conducting fluids with electromagnetic phenomena. The use of electrically conducting fluid as lubricant is of interest, because, it prevents the unexpected variation of lubricant viscosity with temperature under severe operating conditions. The MHD lubrication in an externally pressurized thrust bearing has been investigated both theoretically and experimentally by Maki et al. (1966). Limited studies of MHD lubrication include- MHD slider bearings (Anwar and Rodkiewicz, (1972), Gupta and Bhat, (1979)), MHD Journal bearings (Kamiyama (1969), Malik and Singh, (1980)) MHD squeeze film bearings (Shukla,(1965), Hamza, (1988)). Hamza (1991) has shown the MHD effects on a fluid film squeezed between two rotating surfaces. Recently, Lin (2001) has studied the MHD squeeze film characteristics pertaining to flow between parallel plates.

All the above MHD lubrication studies were restricted to their smooth bearing surfaces. However, the hydrodynamic lubrication theory of rough surfaces has been studied with considerable interest in recent years, because all bearing surfaces are rough to some extent and the roughness asperity height is of the same order as the mean separation between the lubrication contacts. Bearing surfaces develop roughness after having some run-in and wear. In some cases,
contamination of lubricant is also one of the reasons to generate surface roughness through chemical degradation. Several approaches have been proposed to study the surface roughness effects of bearing surfaces. Devies (1963) proposed a sawtooth curve to model the surface roughness. Burton (1963) modeled roughness by a Fourier series type approximation. Since, the surface roughness distribution is random in nature, a stochastic approach to model the surfaces roughness mathematically has to be adopted. Christensen (1969) developed a stochastic theory for the study of rough surfaces in hydrodynamic lubrication of solid bearings. Prakash and Tiwari (1982) used this theory to study the effect of surface roughness on the porous bearings. Recently, Chiang et al. (2004) have analysed the lubrication performance of rough finite journal bearings.

In this Chapter an attempt has been made to study the effect of surface roughness on magnetohydrodynamic squeeze film lubrication characteristics between two rectangular plates. In section 2, the modified Reynolds equation describing surface roughness is derived. In section 3, Reynolds equation is discretised with finite difference method and solved using multigrid method for fluid film pressure and obtained load carrying capacity and squeeze time. Predictions on bearing characteristics are given for varying roughness, Hartmann number and aspect ratio in the subsequent section.
6.2. *Formulation and Solution of the Problem*

The physical configuration of the problem is shown in Fig. 6.1. The upper rough plate approaches the lower smooth plate with constant velocity $dH/dt$. A uniform transverse magnetic field $M_0$ is applied in the $z$-direction. An isothermal, incompressible electrically conducting fluid is taken as the lubricant. The film thickness is made up of two parts

$$H = h(t) + h_r(x, y, \xi),$$

(6.2.1)

where $h(t)$ represents the nominal smooth part of the film geometry and $h_r$ is part due to the surface asperities measured from the nominal level and is a randomly varying quantity of zero mean and $\xi$ is the index parameter determining a definite roughness structure. It is assumed that, the fluid film is thin, the fluid inertia is small, and the body forces are neglected except the Lorentz force. The governing equations of the lubricant film in Cartesian co-ordinates system are (Lin, 2003)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0,$$

(6.2.2)

$$\frac{\partial p}{\partial x} = \mu \frac{\partial^2 u}{\partial z^2} - \sigma M_0^2 u,$$

(6.2.3)

$$\frac{\partial p}{\partial y} = \mu \frac{\partial^2 v}{\partial z^2} - \sigma M_0^2 v,$$

(6.2.4)

$$\frac{\partial p}{\partial z} = 0,$$

(6.2.5)

where $u, v$ and $w$ are the velocity components in $x, y$ and $z$ directions respectively,
Figure 6.1. The physical configuration of squeeze film rectangular plates in the presence of a transverse magnetic field.
\( p \) is pressure, \( \sigma \) is electrical conductivity of the fluid, \( M_0 \) is the impressed magnetic field and \( \mu \) is viscosity of the fluid.

**Boundary Conditions**

The relevant boundary conditions for the velocity components are

\[
\begin{align*}
    u &= 0, \quad v = 0, \quad w = 0 \quad \text{at} \quad z = 0, \\
    u &= 0, \quad v = 0, \quad w = \frac{dH}{dt} \quad \text{at} \quad z = H.
\end{align*}
\]

Solution of equations of (6.2.3) and (6.2.4) for \( u \) and \( v \) using boundary conditions (6.2.6) and (6.2.7) are given by

\[
\begin{align*}
    u &= \frac{h_0^2}{\mu M^2} \frac{\partial p}{\partial x} \left[ \cos(Mz/h_0) - 1 - \frac{\cosh(MH/h_0) - 1}{\sinh(MH/h_0)} \cdot \sinh(Mz/h_0) \right], \\
    v &= \frac{h_0^2}{\mu M^2} \frac{\partial p}{\partial y} \left[ \cos(Mz/h_0) - 1 - \frac{\cosh(MH/h_0) - 1}{\sinh(MH/h_0)} \cdot \sinh(Mz/h_0) \right],
\end{align*}
\]

where \( M \) denotes the Hartmann number and it is defined by \( M = h_0 M_0 (\sigma/\mu)^2 \), \( h_0 \) is the minimum film thickness of the geometry. Substituting these solution of \( u \) and \( v \) in continuity equation (6.2.2) and integrating across the film thickness with respect to \( z \) using boundary conditions \( w = 0 \) at \( z = 0 \) and \( w = \frac{dH}{dt} \) at \( z = H \), we get following modified Reynolds equation describing pressure distribution in the fluid film region

\[
\frac{\partial}{\partial x} \left( \frac{h_0^2}{M^3 \mu} F(H,M) \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{h_0^2}{M^3 \mu} F(H,M) \frac{\partial p}{\partial y} \right) = \frac{dH}{dt},
\]

where
\[ F(H,M) = \frac{MH}{h_0} - 2 \tanh \left( \frac{MH}{2h_0} \right). \]

For including roughness features, taking the expected values of equation (6.2.10), we get

\[
\frac{\partial}{\partial x} \left[ E \left( \frac{h_0}{M^3 \mu} F(H,M) \frac{\partial}{\partial x} \right) \right] + \frac{\partial}{\partial y} \left[ E \left( \frac{h_0}{M^3 \mu} F(H,M) \frac{\partial}{\partial y} \right) \right] = \frac{dE(H)}{dt}, \quad (6.2.11)
\]

where expectancy operator \( E(\bullet) \) is defined by

\[
E(\bullet) = \int_{-\infty}^{\infty} f(h_t) dh_t, \quad (6.2.12)
\]

\( f \) is the probability density function of the stochastic film thickness \( h_t \). In many engineering problems, sliding surfaces show a roughness height distribution which is Gaussian in nature. Therefore, polynomial form which approximates the Gaussian is chosen in the analysis. Such a probability density function is given by (Christensen, 1969)

\[
f(h_t) = \begin{cases} 
\frac{35}{32c^3} (c^2 - h_t^2)^3, & -c < h_t < c \\
0, & \text{elsewhere}
\end{cases}
\]

where \( c \) is the half total range of random film thickness variable and function terminates at \( c = \pm 3\sigma \) and \( \sigma \) is the standard deviation.

**Longitudinal Roughness**

In this case, the roughness is assumed to have the form of long narrow ridges and furrows running in the \( x \)-direction and the film thickness assumes the form

\[ H = h(t) + h_r(x, \xi) \]

then, equation (6.2.11) becomes
\[
\frac{\partial}{\partial x} \left[ E(F(H,M)) \frac{\partial E(p)}{\partial x} \right] + \frac{\partial}{\partial y} \left[ \frac{1}{E(\sqrt{F(H,M)})} \frac{\partial E(p)}{\partial y} \right] = \frac{\mu M^3}{h_0^3} \frac{dE(H)}{dt}. \quad (6.2.13)
\]

**Transverse Roughness**

The roughness is assumed to have the form of long narrow ridges and furrows running in the z-direction and in this case, the film thickness assumes the form

\[ H = h(t) + h_1(y, \xi) \]

then, equation (6.2.11) takes the form

\[
\frac{\partial}{\partial x} \left[ E(\sqrt{F(H,M)}) \frac{\partial E(p)}{\partial x} \right] + \frac{\partial}{\partial y} \left[ E(F(H,M)) \frac{\partial E(p)}{\partial y} \right] = \frac{\mu M^3}{h_0^3} \frac{dE(H)}{dt}. \quad (6.2.14)
\]

From equation (6.2.12), we have

\[ E(H) = h. \quad (6.2.15) \]

The present study is restricted to only one-dimensional longitudinal roughness, since, the case of other roughness pattern (transverse roughness) can be obtained by just rotating the co-ordinate axes. Using equation (6.2.15), equation (6.2.13) can be written in the form

\[
\frac{\partial^3 E(p)}{\partial x^2} + A \frac{\partial^2 E(p)}{\partial y^2} = \frac{\mu M^3}{h_0^3} \frac{1}{E(F(H,M))} \frac{dE}{dt}, \quad (6.2.16)
\]

where

\[
A = [E(\sqrt{F(H,M)}) \cdot E(F(H,M))]^{-1},
\]

\[
E(F(H,M)) = \frac{35}{32c^7} \int F(H,M) \left( e^2 - h_1^2 \right)^3 dh_1,
\]

\[
E(\sqrt{F(H,M)}) = \frac{35}{32c^7} \int F(H,M) \frac{dh_1}{\sqrt{F(H,M)}}.
\]
In order to solve stochastically averaged Reynolds equation, the following are the required boundary conditions for the pressure

\[ E(p) = 0 \text{ at } x = 0, a \text{ and } y = 0, b, \]  

\[ (6.2.17) \]

where \( a \) and \( b \) are dimensions of plates in \( x \) and \( y \) directions respectively.

Introduce non-dimensional parameters and variables as follows

\[
\bar{x} = \frac{x}{a}, \bar{y} = \frac{y}{b}, \lambda = \frac{h}{H}, \frac{\bar{h}}{h_o} = \bar{h}, \frac{h}{h_o} = \bar{h}, C = \frac{c}{h_o}, \bar{p} = -\frac{E(p)h_o^3}{\mu a^2 \frac{dh}{dt}},
\]

\[
\bar{W} = -\frac{E(W)h_o^3}{\mu a^2 b^2 \frac{dh}{dt}} \text{ and } \bar{T} = -\frac{tE(W)h_o^3}{\mu a^2 b^2},
\]

where \( h_o \) is the initial film thickness, \( C \) is the non-dimensional roughness parameter, \( \bar{p} \) is the non-dimensional fluid film pressure, \( \bar{W} \) is the non-dimensional load and \( \bar{T} \) is the non-dimensional squeeze time, then equations (6.2.16) and (6.2.17) become

\[
\frac{\partial^2 \bar{p}}{\partial \bar{x}^2} + \frac{\lambda^2}{\lambda^2} \frac{\partial^2 \bar{p}}{\partial \bar{y}^2} = -\frac{M^3}{E(F(\bar{H}, M))},
\]

\[ (6.2.18) \]

where

\[
\tilde{\lambda} = \left[ E(|F(\bar{H}, M)|, E(F(\bar{H}, M))) \right]^{-1},
\]

\[
E(F(\bar{H}, M)) = -\frac{35}{32C^2} \int_{-c}^{c} F(\bar{H}, M) \left(C^2 - \bar{h}^2\right)^3 d\bar{h},
\]

\[
E(|F(\bar{H}, M)|) = -\frac{35}{32C^2} \int_{-c}^{c} F(\bar{H}, M) \left(C^2 - \bar{h}^2\right)^3 d\bar{h},
\]

\[
F(\bar{H}, M) = M\bar{H} - 2 \tanh(M\bar{H}/2),
\]
and boundary conditions for the pressure are

\[ \bar{p} = 0 \text{ at } \bar{x} = 0,1 \text{ and } \bar{y} = 0,1. \]  

(6.2.19)

6.3. Multigrid Solution

The modified Reynolds equation (6.2.18) is of elliptic type describing the pressure distribution in the presence of transverse magnetic field between rough rectangular plates, which is too complicated to be solved analytically we solve it numerically using multigrid method. Derivative terms in equation (6.2.18) can be approximated using standard second order finite difference scheme and after substituting into equation (6.2.18), we get the following discretized equation

\[ \bar{p}_{i+1,j} + \bar{p}_{i-1,j} + B_0 \bar{p}_{i+1,j+1} + B_0 \bar{p}_{i,j-1} - B_1 \bar{p}_{i,j} = (\Delta \bar{x})^2 D_{i,j}, \]  

(6.3.1)

where coefficients are given by

\[ B_0 = \frac{\bar{A}}{\epsilon^2}, \quad B_1 = 2 + 2B_0, \quad D_{i,j} = \frac{M^3}{E(F(\bar{H}_{i,j}, M))}, \quad \epsilon = \frac{\Delta \bar{x}}{\Delta \bar{y}}. \]

To enforce the boundary conditions, we set

\[ \bar{p}_{0,j} = \bar{p}_{N,j} = \bar{p}_{i,0} = \bar{p}_{i,N} = 0. \]  

(6.3.2)

Few Gauss-Seidel iterations are applied for smoothing the errors; half weighting restriction operator is used for transferring the calculated residual to the coarser grid level. Repeat this procedure till we reach coarsest level with just single point, and solve it exactly. Next, bilinear interpolation operator is used to prolongate the solution from coarsest level to next finer grid level and then apply
few Gauss-Seidel iterations. Repeat this till original finest level is reached. The convergence criteria of the scheme used is

$$|\bar{p}_{i,j}^{r+1} - \bar{p}_{i,j}^r| < 10^{-5},$$

(6.3.3)

where \( r \) corresponds to number of V-cycles required to achieve this accuracy.

Once, fluid film pressure is obtained, hydrodynamic bearing characteristics, such as load carrying capacity and squeezing time can be evaluated. The load carrying capacity \( \bar{W} \) per unit area of the bearing surface in non-dimensional form is

$$\bar{W} = \int_{-1}^{1} \int_{-1}^{1} \bar{p}(\bar{x}, \bar{y}) \, d\bar{x} \, d\bar{y}.$$

(6.3.4)

The time-height relation \( \bar{T} \) in non-dimensional form is

$$\bar{T} = \int_{-1}^{1} \int_{-1}^{1} \bar{p}(\bar{x}, \bar{y}) \, d\bar{x} \, d\bar{y} \, \bar{h}.$$

(6.3.5)
6.4. Results and Discussion

The study of combined effects of surface roughness and transverse magnetic field on the performance of squeeze film lubrication between two rectangular plates is presented. The characteristics of squeeze film bearing are obtained as functions of non-dimensional roughness parameter $C$, film thickness $\bar{H}$, aspect ratio $\lambda$ and Hartmann number $M$. In the following graphs, the dotted lines correspond to either smooth case ($C = 0$) or electrically non-conducting lubricant case ($M = 0$). In the limiting case, when $C = 0$ the analysis corresponds to smooth case studied by Lin (2003).

The variation of non-dimensional pressure $\bar{p}$ with rectangular co-ordinates $\bar{x}$ and $\bar{y}$ is shown in Figures 6.2(a-d) for different values of roughness parameter $C$ and Hartman number $M$. It is observed from Figures that, for higher values of roughness parameter ($C = 0.4$), the built up pressure is higher than that of $C = 0.1$. Also, the effect of magnetic field is to enhance the pressure distribution. This is so because; the effect of magnetic field is to reduce the flow of lubricant velocity. Besides this the presence of surface asperities further reduces the sidewise leakage of the fluid. Thus, the larger amount of fluid is retained in the film region and this result in an increase in the pressure. Pressure in the film region increases with increasing values of $C$ and $M$.

Fig. 6.3 shows, the non-dimensional load $\bar{W}$ as a function of Hartmann number $M$ for different roughness parameters $C$. It is observed that, as $M$ and $C$...
increase the pressure level in the film region and hence, the load carrying capacity increases compared to smooth case. Fig. 6.4 displays the dimensionless load $\bar{W}$ as function of aspect ratio $\log_{10}(\lambda)$ for different values of Hartmann number $M$. It is observed that, the load carrying capacity increases with increase in Hartmann number $M$ compared to non-conducting lubricant case. For smaller values of film height $\tilde{h}$ the effect of magnetic field provides larger load carrying capacity. The variation of non-dimensional load carrying capacity $\bar{W}$ with $\log_{10}(\lambda)$ (aspect ratio) for different roughness parameters $C$ is shown in Fig. 5. It is of importance to note that, there exists a critical value $\lambda_c$ of the aspect ratio $\lambda$ at which effects of roughness vanishes. For $\lambda < \lambda_c = 2.65$, the load carrying capacity $\bar{W}$ increases and for $\lambda > \lambda_c$, the reverse trend is observed. In order to get the maximum load, it is necessary that dimension in $x$-direction should be greater than that of $y$-direction.

To observe the percentage significance of the roughness and magnetic field effect as compared to classical case, the relative load difference

$$R_W = \left(\frac{\bar{W}_{\text{Rough}} - \bar{W}_{\text{Smooth}}}{\bar{W}_{\text{Smooth}}}\right) \times 100$$

is listed in Table 6.1 for various values of $C$ and $M$ and the above predictions are presented quantitatively. For $\tilde{h} = 0.6, C = 0.5, M = 4$ and $\lambda = 1$, an increase of 18% in load carrying capacity is found compared to classical case (smooth and non conducting lubricant case).

Fig. 6.6 shows, the variation of non-dimensional squeeze time $\bar{T}$ with film thickness $\tilde{h}$ for different values of Hartmann number $M$. In the presence of roughness the effect of magnetic field is to enhance the squeezing time compared
to non-conducting lubricant case. Applied magnetic field and surface roughness both lead in enhancing the squeezing time of upper plate in order to achieve the required film height. This delayed squeezing time reduces the coefficient of friction and results in the negligible rate of wear of rectangular plates.

6.4. Conclusions

On the basis of Christensen stochastic model, the effect of surface roughness on MHD squeeze film between rough finite rectangular plates lubricated with electrically conducting fluid in the presence of transverse magnetic field is presented. Finite difference based multigrid method is found to be accurate for the solution of modified form of Reynolds equation. Fifth place decimal convergent solution for all bearing characteristics is obtained. It is found that, the effect of roughness and magnetic field is to increase the load carrying capacity and hence to lengthen the squeeze time compared to classical case. It is expected that these findings help the designers to choose the appropriate roughness parameters for given magnetic field which enhances the normal functioning of the bearing life.
Fig. 6.2(a). Pressure distribution of the fluid film for $C = 0.1$, $M = 3$ and $R = 0.4$.
Fig. 6.2(b). Pressure distribution of the fluid film for $C = 0.3$, $M = 3$ and $h = 0.4$. 
Fig. 6.2(c). Pressure distribution of the fluid film for $N = 2$, $C = 0.3$ and $\bar{h} = 0.4$
Figure 6.2(d): Pressure distribution of the fluid film for $F = 6$, $C = 0.3$ and $y = 0.4$. 

$\delta \quad 0 \quad 4$
Fig. 6.3. Variation of non-dimensional load capacity $W$ with Hartmann number $M$ for different roughness parameter $C$. 

$W$ 

$M$ 

$C = 0.1$ 

$C = 0.3$ 

$C = 0.5$
Fig. 6.4: Variation of non-dimensional load capacity $W$ with aspect ratio $\log_{10}(A)$ for different values of $M$. For different values of $M$. $W = W(\log_{10}(A))$.

For $\gamma = 0.0$, $0.2$, and $0.6 = \eta$.

$C = 0.3$

$0.0 = W$ ———

$0.2 = W$ ———

$0.6 = W$ ———
Different values of roughness parameter $C$ with $\log^{10}(\gamma)$ for $M=5.5$. Variation of non-dimensional load capacity $m$. $\gamma = 0.1$.

- $\gamma = 0.5$.
- $\gamma = 0.3$.
- $\gamma = 1.0$.
- $\gamma = 0.0$. 

Graph shows the relationship between roughness parameter $C$ and non-dimensional load capacity $m$ for different values of $\gamma$. The curves indicate how the load capacity changes with respect to $\log^{10}(\gamma)$.
Fig. 6.6. Variation of non-dimensional squeeze time $\bar{T}$ with film thickness $\bar{h}$ for different Hartmann number $M$. 

- $M = 0.0$
- $M = 2.0$
- $M = 4.0$
- $M = 6.0$

$C = 0.2$
Table 6.1. Relative load differences $R_w$ for different values of $C$ and $M$.

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<th>$M=2$</th>
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