Chapter 3*

SURFACE ROUGHNESS EFFECTS ON SQUEEZE FILM POROELASTIC BEARINGS

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3.1. Introduction

In the previous Chapter, the squeeze film lubrication characteristics between rigid spherical indenter and flat poroelastic cartilage are presented. Modified Reynolds equation is a mathematical model used to describe the pressure distribution in the squeeze film between two parallel plates with Newtonian fluid as lubricant. In this Chapter we study the modelling of the articular cartilage as poroelastic matrix which includes roughness of the cartilage and synovial fluid as a linearly viscous fluid (Newtonian fluid) between two parallel plates.

When fluid permeates into a poroelastic material, the drag force between the fluid and the solid may cause the solid medium to deform and also a force applied to solid matrix induces fluid flow. This deformation of biological soft tissues such as cartilage (Lai and Mow, (1980)), arterial wall (Jayaraman, (1983)), aortic tissue (Kenyon, (1976)) is due to poroelasticity of the solid matrix. Sachs et al. (1994) and Jensen et al. (1994) have studied the linear and non-linear deformation in a poroelastic disk with a free surface which have applications in the study of biological tissues.

Diarthrodial joints provide frictionless motion under high loading conditions with almost negligible wear and are the effective articulating joints of the body. The ability of joints to provide ideal performance inspite of the severe operating conditions is the result of interaction between the bearing components such as articular cartilage and synovial fluid. Articular cartilage is a soft tissue, which surrounds the articulating bone ends of synovial joints. The schematic
diagram of synovial knee joint is shown in Fig. 3.1(a) (Hollinshead and Rose, (1985)). The bone ends are covered by articular cartilage to prevent natural abrasion which are in sac containing fluid for lubricating the two surfaces. The normal joint cavity is enclosed by a tough fibrous capsule together with the muscles, ligaments and intr-articular structures etc. The inner lining of this capsule, the synovial membrane secretes viscous and highly lubricating fluid called synovial fluid. This fluid bathes both articular surfaces and intr-articular structures. Synovial fluid forms an interface with the articular cartilage in synovial joints. Maroudas (1969) and Walker et al. (1968) have shown that during squeeze film action a small amount of a hyaluronic acid (HA) protein macromolecular complex is added through synovial membrane into the joint cavity which increases viscosity of synovial fluid and leads to delay in approach of the cartilages. Hou et al. (1992) have analyzed the squeeze film lubrication aspects of articular cartilage by assuming synovial fluid to be a linearly viscous fluid (Newtonian fluid) and compared the results with experimental findings. Hlavacek (1990, 1993) gave series of papers on the role of synovial fluid on cartilage in squeeze film lubrication of synovial joints.

Bujurke et al. (1990) have studied squeeze film phenomena of articular cartilage modeling them as poroelastic bearings by assuming bearing surfaces to be smooth. But electronic microscopic study of Sayles et al. (1979) revealed that the surfaces of articular cartilage are rough and roughness height distribution is Gaussian in nature. This has motivated us to study the effect of roughness on cartilage surfaces. Christensen (1969) developed the stochastic theory to
understand the effect of surface roughness in hydrodynamic lubrication of bearings. Many researchers have used this theory to analyse the effect of surface roughness of various types of bearings (Naduvinamani at el. (2004), Gururajan and Prakash (2000)).

To optimize contact performance of two articular cartilages, a fluid film separating the two surfaces is required. Such a fluid film gap is thin. But generally it is believed that, surface roughness asperity height of two surfaces is smaller than that of thin fluid film. In order to model the influence of the surface roughness a very fine grid with many points describing the roughness geometry will be needed in difference schemes. Using classical solution techniques solution of large number of unknowns would take much of the computer time and hence storage and sometimes multigrid techniques are used to accelerate the convergence. Notable property of multigrid methods is that the convergence rate does not deteriorate as grid size goes to zero.

The purpose of this Chapter is to investigate the effect of roughness on squeeze film poroelastic bearings with viscous fluid as lubricant in the joint cavity. A simplified mathematical formulation of the problem is given in section 2. In section 3, the stochastic modified Reynolds equation for film pressure of poroelastic bearing is derived. Reynolds equation is discretized using finite differences and solved using multigrid method for fluid film pressure and load carrying capacity. In the subsequent sections results of these predictions on cartilage surface are presented.
3.2. Formulation of the Problem

Fluid film region

The physical configuration of the problem is shown in Fig. 3.1(b), which is the simplified form of synovial knee joint (shown in Fig. 3.1(a)). Following Walker and Erkman (1972), as the load bearing area of the synovial knee joint is small, two articular surfaces may be considered to be parallel under high loading conditions and for mathematical simplicity the average of three layers of the cartilages is modeled as a single poroelastic layer. So, the problem considered is that of squeeze film lubrication between two rectangular surfaces with finite dimensions. The lubricant in the joint cavity is assumed to be Newtonian fluid i.e. linearly viscous and incompressible fluid. The viscous Newtonian fluid is formed by the hyaluronic acid-protein complex, water and all other low molecular weight substances. The upper rough poroelastic cartilage surface is approaching the lower rough poroelastic matrix normally with a constant velocity $dH/dt$. The film thickness has two parts and is given by

$$H = h(t) + h_s(x, y, \xi),$$

where $h(t)$ represents the nominal smooth part of the film geometry and $h_s$ is part due to the surface asperities measured from the nominal level and is a randomly varying quantity of zero mean. Under fluid film lubrication, all articulations of knee joints involve cartilage-viscous fluid-cartilage interactions. With the usual assumptions of hydrodynamic lubrication applicable to thin films, the Navier-Stokes equations in cartesian coordinates reduce to
Fig. 3.1 (a). A schematic diagram of a synovial knee joint.

Fig. 3.1 (b). A simplified model for a synovial knee joint.
where $u, v$ and $w$ are the velocity components in $x, y$ and $z$ directions respectively, $p$ is the pressure and $\mu$ is the viscosity of the fluid.

**Boundary Conditions**

The relevant boundary conditions for the velocity field are

$$
\begin{align*}
\frac{\partial u}{\partial x} (x, y, 0) &= u(x, y, H) = v(x, y, 0) = v(x, y, H) = 0, \\
w(x, y, 0) &= w_n, \\
w(x, y, H) &= w_n - \frac{dH}{dt},
\end{align*}
$$

where $w_n$ represents the normal component of the relative velocity of the fluid at the cartilage surface. Conditions (3.2.6) are no-slip velocity conditions.

**3.3. Solution Procedure**

Integrating equation (2.2.15) with respect to $z$ over the porous layer thickness ($-\delta, 0$) and using the solid backing boundary condition $\frac{\partial P}{\partial z} = 0$ at $z = -\delta$, we get

$$\frac{\partial P}{\partial z} \bigg|_{z=0} = -\int_{-\delta}^{0} \left( \frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} \right) dz.$$
Using the Cameron-Morgan approximation (Morgan and Cameron, 1957), we get

\[
\frac{\partial P}{\partial z} \bigg|_{r=0} = -\delta \left( \frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} \right).
\]  

(3.3.2)

After neglecting inertia terms, equation (2.2.10) may be arranged in terms of relative velocity in the form

\[
\left( V - \frac{dU}{dt} \right) = -\frac{k}{\mu} \left( \nabla P - \frac{E}{K} \nabla \varepsilon \right).
\]  

(3.3.3)

Elimination of \( \varepsilon \) through (2.2.14) and (3.3.3) gives

\[
\left( V - \frac{dU}{dt} \right) = -\nabla P \frac{k}{\mu} \left( 1 - \frac{E}{K} \right).
\]  

(3.3.4)

The normal component of the relative fluid velocity at the cartilage surface is

\[
w_n = -\left( V - \frac{dU}{dt} \right)_n = -\frac{k}{\mu} \left( \frac{E}{K} - 1 \right) \frac{\partial P}{\partial z} \bigg|_{r=0}.
\]  

(3.3.5)

Using equation (3.3.2) in equation (3.3.5), we get

\[
w_n = \frac{k\delta}{\mu} \left( \frac{E}{K} - 1 \right) \left( \frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} \right).
\]  

(3.3.6)

Equations (3.2.3) and (3.2.4) can be integrated for \( u \) and \( v \) with respect to \( z \) using boundary conditions (3.2.6). Substituting \( u \) and \( v \) in the continuity equation (3.2.2) and integrating across the film thickness from \( z = 0 \) to \( z = H \) with respect to \( z \) using boundary conditions (3.2.7), we obtain modified Reynolds equation

\[
\frac{\partial}{\partial x} \left[ H^3 - 24\delta \left( \frac{E}{K} - 1 \right) \frac{\partial P}{\partial x} \right] + \frac{\partial}{\partial y} \left[ H^3 - 24\delta \left( \frac{E}{K} - 1 \right) \frac{\partial P}{\partial y} \right] = -12\mu \frac{dH}{dt},
\]  

(3.3.7)
for the pressure distribution in the joint cavity. For including roughness features, taking the stochastic average of (3.3.7) with respect to $h_s$, we get

$$\frac{\partial}{\partial x} \left\{ E \left[ H^3 - 24\delta k \left( \frac{E}{K} - 1 \right) \frac{\partial p}{\partial x} \right] \right\} + \frac{\partial}{\partial y} \left\{ E \left[ H^3 - 24\delta k \left( \frac{E}{K} - 1 \right) \frac{\partial p}{\partial y} \right] \right\} = -12\mu \frac{dE(H)}{dt}, \quad (3.3.8)$$

where expectancy operator $E(\bullet)$ is defined by

$$E(\bullet) = \int f(h) \, dh, \quad (3.3.9)$$

$f$ is the probability density function of the stochastic film thickness $h_s$. According to the study of Sayles et al. (1979), the cartilage surfaces are rough and roughness height distribution is Gaussian in nature. Therefore, polynomial form which approximates the Gaussian is chosen in the present study. Such a probability density function is (Christensen, 1969)

$$f(h_s) = \begin{cases} \frac{35}{32c^3} (c^2 - h_s^2)^3, & -c < h_s < c \\ 0, & \text{elsewhere} \end{cases}$$

where $c = 3\sigma_1$ and $\sigma_1$ being the standard deviation.

Using the linearity of $E$, equation (3.3.8) reduces to

$$\frac{\partial}{\partial x} \left\{ E \left( H^3 \frac{\partial p}{\partial x} \right) - 24\delta k \left( \frac{E}{K} - 1 \right) \frac{\partial}{\partial x} \left\{ E \left( \frac{\partial p}{\partial x} \right) \right\} \right\} +$$

$$\frac{\partial}{\partial y} \left\{ E \left( H^3 \frac{\partial p}{\partial y} \right) - 24\delta k \left( \frac{E}{K} - 1 \right) \frac{\partial}{\partial y} \left\{ E \left( \frac{\partial p}{\partial y} \right) \right\} \right\} = -12\mu \frac{dE(H)}{dt}. \quad (3.3.10)$$
The modified Reynolds equation (3.3.10) applicable to two types of one
dimensional roughness structure, although similar to the usual Reynolds equation
for a smooth bearing are not easily amenable to mathematical analysis for a
variable film thickness geometry. Therefore many studies (Christensen, (1969),
Prakash and Tiwari, (1982)) were confined to constant film thickness case
pertaining to parallel squeeze film between two plates. With this view, we can
now evaluate the first and third terms of the left hand side of equation (3.3.10)
subject to a particular specific model of the roughness of the surfaces. To evaluate
the above said terms, Christensen postulated some assumptions which are based
on physical consideration of the problem,

i. The magnitude of the pressure ripples associated with the surface
roughness is small compared to the general pressure in the bearing. The
pressure gradient in the roughness direction will be assumed to be a
stochastic variable with a negligible variance.

ii. In the direction perpendicular to the roughness direction, the
component of unit flow will be assumed to be a stochastic variable with
a negligible variance.

Rohde and Whicker (1977) have shown that using perturbation approach, the
Christensen model can be viewed as asymptotic limits in texture frequency and
predicts some accurate results even for low texture frequency. Phan-Thein (1981)
used Keller’s method to obtain a mean Reynolds equation which is correct upto
second order in amplitude of the surface roughness and shown that the theory of
Christensen gives the load enhancement in two-dimensional slider bearing with exponential film thickness. Thus, Christensen stochastic theory has resulted for many researchers to conjecture that the assumptions made above are correct up to second order for two-dimensional bearings with longitudinal or transverse roughness.

In the context of rough surfaces, there are two types of roughness patterns which are of special interest.

**Longitudinal Roughness**

The one-dimensional longitudinal structure where the roughness has the form of long narrow ridges and valleys running in the $x$-direction

$$H = h(t) + h_r(x, \xi).$$

By assumption (i), $\frac{\partial p}{\partial x}$ is a variable with zero or negligible variance. Therefore, $\frac{\partial p}{\partial x}$ and $H^3$ can be considered to be approximately stochastically independent quantities. The first term of equation (3.3.10) takes the form

$$\frac{\partial}{\partial x} \left[ E \left( H^3 \frac{\partial p}{\partial x} \right) \right] = \frac{\partial}{\partial x} \left[ E(H^3) \frac{\partial E(p)}{\partial x} \right].$$

(3.3.11)

To evaluate the third term of the equation (3.3.10), we can make use of assumption (ii). Accordingly, the unit flow of lubricant in $y$-direction is a stochastic variable with zero or negligible variance. Thus, the unit flow is given by
Dividing both sides of equation (3.3.12) by $H^3$ and taking expectation

$$E(q_x(x,y)) = -rac{1}{12\mu} \frac{\partial E(p)}{\partial y}.$$  

(3.3.13)

Also, from equation (3.3.12)

$$E((q_x(x,y)) = -rac{1}{12\mu} E\left( H^3 \frac{\partial p}{\partial y} \right).$$  

(3.3.14)

Comparing equations (3.3.13) and (3.3.14),

$$E\left( H^3 \frac{\partial p}{\partial y} \right) = \frac{1}{E\left( \frac{1}{H^3} \right)} \frac{\partial E(p)}{\partial y}.$$  

(3.3.15)

Thus, substituting equations (3.3.11) and (3.3.15) into equation (3.3.10), we get

$$\frac{\partial}{\partial x} \left[ E(H^3) - 24\delta_k \left( \frac{E}{K} - 1 \right) \frac{\partial E(p)}{\partial x} \right] +$$

$$\frac{\partial}{\partial y} \left[ \frac{1}{E\left( \frac{1}{H^3} \right)} - 24\delta_k \left( \frac{E}{K} - 1 \right) \frac{\partial E(p)}{\partial y} \right] = -12\mu \frac{dE(H)}{dt}.$$  

(3.3.16)

**Transverse roughness**

In this model, the roughness is assumed to have the form of long, narrow ridges and furrows running in a direction perpendicular to the $x$-direction. The film thickness in this model takes the form

$$H = h(t) + h_{x}(y,\xi).$$

Adopting the assumptions (i) and (ii) made above and performing an argument similar to that in the previous case, we obtain stochastic Reynolds equation as
For the distribution function given by (3.3.9), we have

\[ E(H) = h. \tag{3.3.18} \]

\[ E(H^3) = h^3 + \frac{h c^2}{3}. \tag{3.3.19} \]

\[ E(\frac{l}{H^3}) = \frac{35}{32c^7} \left[ 3(5h^2 - c^2)(c^2 - h^2) \log \left( \frac{h + c}{h - c} \right) + 2ch(15h^2 - 13c^2) \right]. \tag{3.3.20} \]

However, the present study is restricted to one-dimensional longitudinal roughness since the one roughness structure can be obtained from other by just rotation of coordinate axes. Therefore, for the one-dimensional longitudinal roughness pattern, Reynolds equation (3.3.16) takes the form

\[ \frac{\partial}{\partial x} \left[ \frac{1}{E(H^3)} - 24\delta k \left( \frac{E}{K} - 1 \right) \right] \frac{\partial E(p)}{\partial x} + \frac{\partial}{\partial y} \left[ E(H^3) - 24\delta k \left( \frac{E}{K} - 1 \right) \right] \frac{\partial E(p)}{\partial y} = -12\mu \frac{dE(H)}{dt}. \tag{3.3.17} \]

where

\[ A = \left[ \frac{1}{E(H^3)} - 24\delta k \left( \frac{E}{K} - 1 \right) \right]^{-1}. \tag{3.3.21} \]

The relevant boundary conditions for the pressure region are

\[ E(p) = 0 \text{ at } x = 0, L_1 \text{ and } y = 0, L_2, \tag{3.3.22} \]

where \( L_1 \) and \( L_2 \) are dimensions of plate in \( x \) and \( y \) directions respectively.
Introduce following non-dimensional parameters and variables

\[ \bar{x} = \frac{x}{L_1}, \bar{y} = \frac{y}{L_2}, \lambda = \frac{L_1}{L_2}, \bar{H} = \frac{H}{h_0} = \frac{\bar{h}}{h_0} + \bar{h}, \bar{h} = \frac{h}{h_0}, \bar{h}_1 = \frac{h_1}{h_0}, \bar{k} = kh_0^2, \]

\[ \bar{\delta} = \delta h_0, C = \frac{c}{h_0}, \text{ and } \bar{\rho} = \frac{E(p)h_0^3}{\mu L_1^3 dh/dt} \]

where \( h_0 \) is the initial film thickness, \( \lambda \) is the aspect ratio, \( \bar{k} \) is the non-dimensional permeability parameter, \( \bar{\delta} \) is the non-dimensional thickness of the poroelastic layer, \( C \) is the non-dimensional roughness parameter and \( \bar{\rho} \) is the non-dimensional fluid film pressure, then equation (3.3.21) and (3.3.22) become

\[ \frac{\partial^2 \bar{\rho}}{\bar{x}^2} + \frac{\bar{A}}{\bar{\lambda}^2} \frac{\partial^2 \bar{\rho}}{\bar{y}^2} = \frac{12}{E(\bar{H}^3) - 24\bar{\delta} \bar{k}} \left( \frac{E}{K} - 1 \right) \]

where

\[ \bar{A} = \left[ \frac{1}{E(\bar{H}^3)} - 24\bar{\delta} \bar{k} \left( \frac{E}{K} - 1 \right) \right]^{-1} \]

\[ E(\bar{H}^3) = \bar{h}^3 + \frac{\bar{h} C^2}{3}, \]

\[ E(\bar{H}^3) = \frac{35}{32C^7} \left[ 3(5\bar{h}^2 - C^2)(C^2 - \bar{h}^2) \log \left( \frac{\bar{h} + C}{\bar{h} - C} \right) + 2C\bar{h}(15\bar{h}^2 - 13C^2) \right], \]

and the relevant boundary conditions for the pressure field are

\[ \bar{p} = 0 \text{ at } \bar{x} = 0, 1 \text{ and } \bar{y} = 0, 1. \]

(3.3.24)

**Numerical Solution**

Since, the modified Reynolds equation (3.3.23) is of elliptic type which is too complicated to be solved analytically, we solve it numerically using multigrid
method. So, using standard second order finite difference scheme for derivative
terms in equation (3.3.23), we get following discretised equation

\[ \overline{p}_{i+1,j} + \overline{p}_{i-1,j} + G_{i} \overline{p}_{i,j+1} + G_{i} \overline{p}_{i,j-1} - G_{i} \overline{p}_{i,j} = (h_{i})^{2} F_{i,j} \]  

(3.3.25)

where coefficients are given by

\[ G_{i} = \frac{A}{\lambda^{2}} G^{2}, \quad G_{i} = 2 + 2G_{i}, \quad F_{i,j} = -\frac{12}{E(h^{4}) - 24\delta k \left( \frac{E}{K} - 1 \right)}, \quad G = \frac{h_{i}}{\Delta x}, h_{i} = \Delta x \]

and boundary conditions are

\[ \overline{p}_{0,j} = \overline{p}_{N,j} = \overline{p}_{i,0} = \overline{p}_{i,N} = 0. \]  

(3.3.26)

**Multigrid Method**

Multigrid method with half-weighting and bilinear interpolation operator is used to solve the finite differenced Reynolds equation (3.3.25) with boundary conditions (3.3.26). In the process of computation, few Gauss-Seidel iterations are applied for smoothing the errors; half weighting restriction operator is used for transferring the calculated residual to the coarser grid level. The procedure is repeated till coarsest level is reached with just single point, and solve it exactly. Next, bilinear interpolation operator is used to prolongate the solution from coarsest level to next finer grid level and then apply few Gauss-Seidel iterations. Repeat this till original finest level is reached. This is one iteration and it is referred to as one V-cycle. The convergence criteria of the scheme used is

\[ |\overline{p}_{i,j}^{n+1} - \overline{p}_{i,j}^{n}| < 10^{-5}, \]

where \( \gamma \) corresponds to number of V-cycles required to achieve this accuracy.
Once, fluid film pressure is obtained, the load carrying capacity can be evaluated. The non-dimensional load carrying capacity $\bar{W}$ per unit area of the joint surface is

$$\bar{W} = \int_0^1 \int_0^1 \bar{p}(\bar{x}, \bar{y}) \, d\bar{x} \, d\bar{y}.$$  \hspace{1cm} (3.3.27)
3.4. Results and Discussion

A simplified mathematical model has been developed for understanding the effect of surface roughness and elasticity on squeeze film lubrication aspects of rough poroelastic bearings in general and that of synovial joints in particular. The pressure distribution $\bar{p}$ and load carrying capacity $\bar{W}$ are functions of non-dimensional parameters $H, C, \bar{k}, \frac{E'}{K}$ and $\lambda$. In the graphs, dotted lines correspond to smooth case ($C = 0$). In the limiting case, when $C = 0$, the analysis corresponds to smooth case.

The fluid film pressure distribution $\bar{p}$ with rectangular co-ordinates $\bar{x}$ and $\bar{y}$ for different roughness parameters is shown Figs. 3. 2(a) and 3. 2(b). From these figures, it is observed that the pressure distribution with roughness parameter say $C = 0.4$ is higher than that with $C = 0.1$. Due to the presence of hyaluronic acid complex in fluid, thick dense substance is being formed on the cartilage surfaces during the squeezing process and presence of surface asperities on the articular cartilage reduces this fluid flow and thus, large fluid is retained in the film region which enhances pressure built-up.

Fig. 3.3 shows the variation of non-dimensional load carrying capacity $\bar{W}$ with articular cartilage permeability $\bar{k}$ for different values of roughness parameter $C$. It is observed that, the effect of surface roughness is to enhance the load carrying capacity compared to smooth case ($C = 0$). Further, the load carrying capacity $\bar{W}$ decreases with increase in $\bar{k}$ for all $C$. This is because, the large...
permeability value means there are more voids available in the poroelastic surface, which permits the quick escape of the fluid, then, the poroelastic surface becomes the main channel for fluid discharge. Therefore, when permeability is large, the modified film thickness caused by the presence of surface roughness has negligible effect.

The variation of non-dimensional load carrying capacity $\overline{W}$ with $\log_{10}(\lambda)$ (aspect ratio) for various values of $C$ is plotted in Fig. 3.4. It is observed that, there exists a critical value $\lambda_c$ of the aspect ratio $\lambda$, at which effect of roughness vanishes. For $\lambda < \lambda_c = 2.1$, the effect of roughness is to increase the load carrying capacity whereas for $\lambda > \lambda_c$, the reverse trend is observed compared to smooth case. Also, it is of interest to note that, for larger values of $C^+ \to 1$, or for smaller values of film thickness $\overline{h}$, the load carrying capacity increases.

In order to observe the significance of roughness of poroelastic material as compared to smooth articular surface the relative load difference

$$R_{\overline{W}} = \left(\frac{(\overline{W}_{\text{Rough}} - \overline{W}_{\text{Smooth}})}{\overline{W}_{\text{Smooth}}}\right) \times 100$$

for various values of $C$, $\frac{E'}{K}$ and $k$ are listed in Table 3.1. It is found that the relative load difference $R_{\overline{W}}$ increases with increase in $\frac{E'}{K}$ and $C$. Whereas for decreasing $k$ it results in the increase of relative load difference $R_{\overline{W}}$. The effect of elasticity is to enhance the $R_{\overline{W}}$ for the
joints with normal cartilage \( \text{(say with } \frac{E'}{K} = 0.6) \) compared to degenerate cartilage \( \text{(with } \frac{E'}{K} = 0.2) \). Similar predictions are also made by Hori and Mockers (1976) in their studies on the role of cartilage in the lubrication of joints.

3.5. Conclusions

Finite difference based multigrid method is found to be accurate for the solution of Reynolds equation which incorporates surface roughness and poroelastic nature of articular cartilage. The fifth place decimal convergent solution for pressure distribution and load carrying capacity is obtained. Whereas, other conventional schemes fail to provide such accurate solution. It is observed that, the effect of roughness is to increase the pressure built up (and hence load carrying capacity) compared to classical case. Also, the role of elasticity is to increase the relative load difference of normal joint compared with diseased ones. The governing equations describing complex structure of cartilage and synovial fluid are complicated because of non-linearity, and also, joints have wide range of articulating features. However, the proposed model do predict some of the salient features of bearing characteristics which would enable in selecting suitable design parameters.
Fig. 3.2(a). Pressure distribution for $C = 0.4$, $k = 7.65 \times 10^{-5}$, $h = 0.5$, and $E'/K = 0.6$. 
Fig. 3.2(b). Pressure distribution for $C = 0.1$, $\bar{K} = 7.65 \times 10^{-4}$, $\bar{h} = 0.5$. 

and $E'/K = 0.6$. 
Fig. 3.3. Variation of load capacity $W$ with $k$ for different roughness parameter $C$ with $E' / K = 0.6$. 
Fig. 3.4. Variation of load capacity $W$ with aspect ratio $\log_{10}(A)$ for different roughness parameter $C$ with $J_c = 7.65 \times 10^5$ and $E'/K = 0.6$. 

$7.0 = \gamma$ 

$5.0 = \gamma$
Table 3.1. Relative load differences $R_{W}$ for $\lambda = 1$, $\bar{h} = 0.5$

<table>
<thead>
<tr>
<th>$k$</th>
<th>$C$</th>
<th>$E'/K=0$</th>
<th>$E'/K=0.2$</th>
<th>$E'/K=0.4$</th>
<th>$E'/K=0.6$</th>
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<td>$7.65 \times 10^{-5}$</td>
<td>0.1</td>
<td>0.27135</td>
<td>0.30816</td>
<td>0.35643</td>
<td>0.42273</td>
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<td>5.55507</td>
<td>6.49624</td>
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