Orthogonal frequency division multiplexing is highly sensitive to CFO which introduce ICI at the OFDM receiver. The CFO is induced either due to Doppler spreading or by oscillator discrepancies between the transmitter and receiver. Due to ICI, the performance of OFDM system deteriorates. Therefore, it is required to mitigate ICI at the receiver. The receiver windowing is suggested as one of the best ICI reduction techniques in the literature. In this chapter, a window is proposed first time for ICI reduction using receiver windowing.

The proposed window is a modified version of the MBH window family. It is used in an OFDM system to reduce the effect of frequency offset on both the parameters i.e., the ICI and BER. The performance of MBH window family is compared with other available Nyquist pulse shapes, like RC, Bartlett, BTRC, SOCW, FRANK and PMSP. These pulse shapes are also found as members of the MBH pulse shape family, exactly or equivalently. It is found that the ICI and SIR of MBH pulse shapes are better than these pulse shapes except PMSP. The BER performance of proposed window family is better than all these pulse shapes.

6.1 INTRODUCTION

The ICI among sub-carriers is one of the major problems of the OFDM system. The main cause of ICI is the CFO, induced either due to Doppler shift arises by channel or due to the mismatching of transmitter and receiver oscillator. The ICI may also be caused by the phase noise and timing offset.

However, the ICI induced by CFO, phase noise and due to timing offset can be compensated or corrected. Since the Doppler spread or shift is random, its impact can only be reduced or mitigate. Not only due to Doppler spread, the ICI induced due residual frequency offset, which is the result of estimation error, is also required to be mitigated separately. Therefore, a separate method besides of CFO estimation and correction at the receiver is required to reduce the ICI. Several techniques have been introduced in the past to reduce ICI. These include frequency domain equalization [53, 85], self cancellation scheme [84 and 140], and windowing technique (pulse shaping at the transmitter and receiver windowing) [25, 26, 65, 86, 87, 94, 99, 111, and 129]. The receiver windowing is one of the best ICI reduction techniques available in the literature [86]. This chapter deals with the study and analysis of receiver windowing techniques and proposed a new window for ICI reduction.
The windowing techniques can be used at both the transmitter and receiver sides. The use of RC window at transmitter side has also been suggested by IEEE 802.11 standards. Some other Nyquist pulse shapes like BTRC, PMSP, ISP, Frank’s window and SOCW are also available in the literature \([25, 86, 87, 94, 99, \text{ and } 129]\). The windowing at the transmitter side is used to reduce the sensitivity to linear distortions \([70 \text{ and } 106]\). In transmitter windowing method, the window function shapes the cyclic extension part of OFDM symbol and leaving original part of the symbol unchanged. This form of windowing has no effect on the system performance when there is only frequency distortion.

To reduce the sensitivity to frequency errors or distortions, the receiver windowing has been used by many authors. The concept of windowing at the receiver end for the reduction of ICI is very well reported in \([25, 86, \text{ and } 111]\). All these works demonstrated that, the use of a proper Nyquist window will reduce the ICI to the greater extent. The window is applied to the estimated ISI-free part of a received OFDM symbol. The time-limited window which reduces the side lobes and conserves the carrier orthogonality is called Nyquist window \([25]\). The work reported in \([86]\) demonstrate the effects of several Nyquist windows, including RC, BTRC, SOCW, the frank window, and the double jump window on the performance of OFDM system. The evaluation was based on both the parameters like BER and SIR.

### 6.2 OFDM SYSTEM MODEL WITH RECEIVER WINDOWING

The discrete time baseband OFDM system model with N subcarriers using receiver windowing is shown in Figure-6.2.1. It consists of transmitter, channel and receiver blocks which are described below. This system model is same as given in Figure-2.2.1, except the window function block at the receiver (shown as shaded block).

#### 6.2.1 Transmitter Model

In this system, a block of \(\log_2 M\) input bits is mapped into a symbol constellation point \(X(k)\) by an \(M\)-ary data encoder, and then \(N\) such symbols are transferred by the serial-to-parallel converter. Data encoder can use any types of modulation techniques (e.g. BPSK, QPSK, QAM etc). These complex parallel data symbols \(\{X(1), X(2), \ldots, X(N)\}\) are then fed to the IFFT block. After taking \(N\)-point IFFT, the last \(G\) samples are appended at the front (the CP addition) to form \(x(n)\). The discrete time baseband OFDM signal with CP, transmitted during \(i^{th}\) block can be written as –
Figure-6.2.1: Block diagram of baseband OFDM system with receiver windowing

\[ x(i, n) = \begin{cases} 
  x(i, n + N), & \text{for } n = 0, 1, \ldots, G - 1 \\
  \frac{1}{N} \sum_{k=0}^{N-1} X(i, k) e^{j \frac{2\pi}{N} k (n-G)}, & \text{for } n = G, G + 1, \ldots, G + N - 1 
\end{cases} \]

(6.2.1.1)

by assuming that data symbols are uncorrelated. That is –

\[ P[X(i, k) X^*(i, m)] = \begin{cases} 
  1, & k = m \\
  0, & k \neq m 
\end{cases} \]

(6.2.1.2)

where, \(X^*(i, m)\) represents the complex conjugate of \(X(i, m)\), \(X(i, k)\) is the complex data symbol obtained after M-PSK or M-QAM modulation, \(N\) is total number of sub-carriers and \(G\) is total number of CP samples appended during \(i^{th}\) block transmission.
6.2.2 Channel Model

The multipath fading channel model used in this analysis has already been discussed in Chapter-2 (Section-2.2.3).

6.2.3 Receiver Model

A receiver model with time domain windowing, as given in [25, 86, and 111], has been considered. This model uses the ISI free part of guard interval for windowing. The \( V \)-samples of ISI free duration (usable guard interval \( T'_{g} \)) together with \( N \)-samples of usable period \( T_{u} \) are symmetrically multiplied with Nyquist window. After that, the symmetrical zero padding is done to make total number of samples to \( 2N \). Subsequently, the \( 2N \) point FFT is calculated for the resultant signal. After considering the effects of multipath fading channel \( \{ h(\tau, t) \} \) (2.2.3.1) and AWGN, the \( n^{th} \) sample of \( i^{th} \) OFDM symbol in the presence of CFO (\( \Delta f \)), can be expressed as

\[
r(i,n) = \sum_{i=0}^{L-1} h_l(\tau) x(i,n - \tau_l) e^{j \frac{2 \pi}{N} n - \tau_l} + w(i,n), \text{ for } n = 0,1,...,G + N - 1
\]

(6.2.3.1)

where '\( \varepsilon \)' is the carrier-frequency offset normalized by the sub-carrier spacing \( (1/T_u = 1/NT_s) \), \( T_u (= NT_s) \) is the useful duration of one OFDM symbol, \( T_s \) is the sampling interval, \( h_l(\tau) \) is a tap coefficient, \( \tau_l \) is a propagation delay of the \( l^{th} \) path, respectively and \( w(i,n) \) is a zero-mean, complex Gaussian noise process with variance \( \sigma_w^2 \) per dimension. For time domain windowing, only \( V + N \) samples of received signal are taken –

\[
r(i,n + G - V) = \sum_{i=0}^{L-1} h_l(\tau) x(i,n + G - V - \tau_l) e^{j \frac{2 \pi}{N} n + G - V - \tau_l} + w(i,n + G - V), \text{ for } n = 0,1,...,V + N - 1
\]

(6.2.3.2)

where, \( x(i,n + G - V) \) is given as –

\[
x(i,n + G - V) = \frac{1}{N} \sum_{k=0}^{N-1} X(i,k) e^{j \frac{2 \pi}{N} k(n-V)}, \text{ for } n = 0,1,...,V + N - 1
\]

(6.2.3.3)
After substituting the value of \( x(i, n + G - V) \) in (6.2.3.2) –

\[
 r(i, n + G - V) = \sum_{l=0}^{L-1} h_l(\tau) \left\{ \frac{1}{N} \sum_{k=0}^{N-1} X(i, k) e^{j\frac{2\pi k (n-V-\tau_l)}{N}} \right\} e^{j\frac{2\pi \epsilon (n+G-V-\tau_l)}{N}} + w(i, n + G - V), \quad \text{for } n = 0, 1, \ldots, V + N - 1
\] (6.2.3.4)

After interchanging the summation term –

\[
 r(i, n + G - V) = \frac{1}{N} \sum_{k=0}^{N-1} X(i, k) e^{j\frac{2\pi n (k+\epsilon)}{N}} e^{-j\frac{2\pi k V}{N}} \times \left\{ \frac{1}{N} \sum_{l=0}^{L-1} h_l(\tau) e^{-j\frac{2\pi \tau_l (k+\epsilon)}{N}}, e^{j\frac{2\pi \epsilon (G-V)}{N}} \right\} + w(i, n + G - V)
\] (6.2.3.5)

Replacing the inner summation terms by \( H(k, \Delta f) \) –

\[
 r(i, n + G - V) = \frac{1}{N} \sum_{k=0}^{N-1} X(i, k) e^{j\frac{2\pi n (k+\epsilon)}{N}} e^{-j\frac{2\pi k V}{N}} H(k, \Delta f) e^{j\frac{2\pi \epsilon (G-V)}{N}} + w(i, n + G - V), \quad \text{for } n = 0, 1, \ldots, V + N - 1
\] (6.2.3.6)

where, the term \( H(k, \Delta f) \) is the channel response to the frequency \( (k + \epsilon) \) at the \( i^\text{th} \) OFDM symbol defined as -

\[
 H(k, \Delta f) = \sum_{l=0}^{L-1} h_l(\tau) e^{-j\frac{2\pi \tau_l (k+\epsilon)}{N}}
\] (6.2.3.7)

The correlation between channel response \( H(k, \Delta f) \) and \( H(q, \Delta f) \) by assuming exponential distribution of the multipath time delay is given in [86] as –

\[
 E[H(k, \Delta f) H^*(q, \Delta f)] = \sigma^2_{\tilde{\tau}} / (1 - j\pi (k - q)\eta)
\] (6.2.3.8)

where, \( \eta = \frac{\bar{\tau}}{N} \) is the normalized mean time delay, \( \bar{\tau} \) is the mean time delay measure, and

\[
 \sigma^2_{\tilde{\tau}} = E[ |H(k, \Delta f)|^2 ] = \sum_{l_1}^{L-1} \sum_{l_2}^{L-1} E[h_{l_1}(\tau_1) h_{l_2}^*(\tau_2)] e^{j\frac{2\pi \epsilon (k+\epsilon)}{N}(\tau_2-\tau_1)} = \sum_{l=0}^{L-1} E[ |h_l(\tau)|^2 ]
\] (6.2.3.9)
Since, the complex amplitudes of different paths are independent and identically Gaussian distributed. Now, the signal \( r(i, n + G - V) \) is symmetrically multiplied with the window function \( p(n) \) and then, the resultant signal \( y(i, n) = r(i, n + G - V) \times p(n) \) is extended to \( 2N \) points by inserting zeros at both the sides symmetrically, so that –

\[
\hat{y}(i, n) = \begin{cases} 
0, & 0 \leq n < \frac{N(1 - \alpha)}{2} \\
y(i, n), & \frac{N(1 - \alpha)}{2} \leq n < \frac{3N(1 + \alpha/3)}{2} \\
0, & \frac{3N(1 + \alpha/3)}{2} \leq n < 2N - 1
\end{cases}
\]

where, \( \alpha = V/N \) is a roll-off factor

The \( 2N \)-point FFT of resultant signal \( \hat{y}(i, n) \) is then taken, which is denoted by \( \hat{X}(i, p) \) –

\[
\hat{X}(i, p) = \sum_{n=0}^{2N-1} \hat{y}(i, n) e^{-j \frac{2\pi p n}{2N}}, \quad \text{for } p = 0, 1, ..., 2N - 1
\]

After substituting the value of \( \hat{y}(i, n) \) in (6.2.3.11),

\[
\hat{X}(i, p) = \sum_{n=0}^{2N-1} \left\{ \frac{1}{N} \sum_{k=0}^{N-1} X(i, k) e^{j \frac{2\pi n (k+1)}{N}} e^{-j \frac{2\pi k V}{N}} H(k, \Delta f) e^{j \frac{2\pi \epsilon (G-V)}{N}} p(n) \right\} e^{-j \frac{2\pi p n}{2N}}, \\
+ \sum_{n=0}^{2N-1} p(n) w(i, n + G - V) e^{-j \frac{2\pi p n}{2N}},
\]

\[
\text{for } p = 0, 1, ..., 2N - 1
\]

After interchanging the summation –

\[
\hat{X}(i, p) = \frac{1}{N} \sum_{k=0}^{N-1} X(i, k) H(k, \Delta f) \sum_{n=0}^{2N-1} p(n) e^{j \frac{2\pi n}{2N} (2k-p+2\epsilon)} e^{-j \frac{2\pi k V}{N}} e^{j \frac{2\pi \epsilon (G-V)}{N}}, \\
+ \sum_{n=0}^{2N-1} p(n) w(i, n + G - V) e^{-j \frac{2\pi p n}{2N}}, \quad \text{for } p = 0, 1, ..., 2N - 1
\]
After simplification, it becomes –

\[ \hat{X}(i,p) = \sum_{k=0}^{N-1} X(i,k) H(k, \Delta f) S(k,p, \Delta f) + W(i,p), \text{ for } p = 0,1, ..., 2N - 1 \]

where,

\[ W(i,p) = \sum_{n=0}^{2N-1} p(n) w(i,n + G - V) e^{j \frac{2 \pi p n}{2N}}, \]

and, \( S(k,p, \Delta f) \) is the ICI coefficient [86], defined as –

\[ S(k,p, \Delta f) = \frac{1}{N} e^{j \frac{2 \pi k V}{N}} e^{j \frac{2 \pi (G - V)}{N}} \sum_{n=0}^{2N-1} p(n) e^{j \frac{2 \pi n}{2N} (2k - p + 2\epsilon)} \]

If, \( \mu = G/N \) and \( \alpha = V/N \), then \( S(k,p, \Delta f) \) can be rewritten as

\[ S(k,p, \Delta f) = \frac{1}{N} e^{j \frac{2 \pi (\mu - \alpha) k}{N}} \sum_{n=0}^{2N-1} p(n) e^{j \frac{2 \pi n}{2N} (2k - p + 2\epsilon)} \]

As it is known, that either even or odd terms contain the desired information; therefore the transmitted data sequence \( X(i,q), \text{ for } q = 0,1, ..., N - 1, \) can be recover from \( \hat{X}(i,p) \) by choosing only even terms at \( p = 2q \). That is –

\[ \hat{X}(i,2q) = \sum_{k=0}^{N-1} X(i,k) H(k, \Delta f) S(k,2q, \Delta f) + W(i,2q), \text{ for } q = 0,1, ..., N - 1 \]

After breaking the summation in two terms -

\[ \hat{X}(i,2q) = X(i,q) H(q, \Delta f) S(q,2q, \Delta f) + \sum_{k=0, k \neq q}^{N-1} X(i,k) H(k, \Delta f) S(k,2q, \Delta f) + W(i,2q), \]

for \( q = 0,1, ..., N - 1 \)

The first term in the above expression (6.2.3.19) is the desired signal, the second term is the ICI component, and the last term is AWGN.
6.3 EXPRESSION OF ICI POWER AND SIR

In the previous section, it has been shown that in the presence of CFO, the signal after FFT at the receiver consists of desired signal, white Gaussian noise and ICI term. To reduce this ICI term, it is necessary to analyze the ICI term in detail. Therefore, the expression of ICI power and SIR are given in this section. The ICI power can be determined from (6.2.3.18) as-

\[
\sigma_{IClq}^2 = \sum_{k_1=q}^{N-1} \sum_{k_2=q}^{N-1} E[X(i, k_1) X^*(i, k_2)] E[(H(k_1, \Delta f) S(k_1, 2q, \Delta f))(H(k_2, \Delta f) S(k_2, 2q, \Delta f))^*]
\]

(6.3.1)

\[
\sigma_{IClq}^2 = \frac{\sigma_f^2}{N} \sum_{k=0}^{N-1} \sum_{n_1=0}^{N-1} \sum_{n_2=0}^{N-1} p(n_1) p(n_2) e^{j \frac{2 \pi (n_1-n_2)(k-q+\epsilon)}{N}}
\]

(6.3.2)

On rearranging the above equation and substituting \(\sigma_f^2 = 1\), the ICI power in terms of Fourier transform can be written as –

\[
\sigma_{IClq}^2 = \sum_{k=0, k \neq q}^{N-1} |P(k-q+\epsilon)|^2
\]

(6.3.3)

From (6.3.3), it is clear that the average ICI power for the \(q^{th}\) symbol depends on the number of subcarriers and on the spectral magnitudes of the pulse-shaping function at the frequencies \((k-q+\epsilon)\), for \(k \neq q\) and \(k = 0, 1, 2, \ldots, N-1\). For the window function which follows Nyquist criteria have nulls at the frequency points \((k-q)/N\), and hence no ICI occurs when \(\Delta f = 0\). The Power of desired signal is-

\[
\sigma_S^2 = E[|X(i,q)|^2] E[|H(q, \Delta f) S(q, 2q, \Delta f)|^2] = |P(\epsilon)|^2
\]

(6.3.4)

Now, signal to interference ratio \(SIR_q\) is defined as the ratio of the average desired signal power \(\sigma_S^2\) to the average ICI power \(\sigma_{IClq}^2\).

\[
SIR_q = \frac{\sigma_S^2}{\sigma_{IClq}^2} = \frac{|P(\epsilon)|^2}{\sum_{k=0, k \neq q}^{N-1} |P(k-q+\epsilon)|^2}
\]

(6.3.5)
6.4 PROPOSED COMBINATIONAL WINDOW FAMILY

It has been shown in the last section that the ICI power depends on the Fourier transform of window function used at the receiver. Several window functions which have been used for ICI reduction are already presented in Chapter-2. Here, the proposed window function used at the receiver is described. The definition of proposed pulse shape is derived from [58] with appropriate modification with regards to OFDM system. The original definition of proposed window family, as given by J. K. Gautam *et al.* in [58] is –

\[
p(t, \beta) = \begin{cases} 
\beta - (4 \beta - 2) \left( \frac{|t|}{T} \right) + (1 - \beta) \cos \left( \frac{2\pi t}{T} \right), & \text{for } 0 \leq |t| \leq \frac{T}{2} \\
0, & \text{otherwise}
\end{cases} \tag{6.4.1}
\]

where, \( \beta \) is a window shape parameter defined in the range of \( 0.5 \leq \beta \leq 1.88 \). The function \( p(t, \beta) \) does not satisfy the properties of window outside this range of \( \beta \) as described in [58]. The better parameters, viz. maximum side-lobe level, decay rates of side lobes and main lobe width, of MBH family actually inspired its use in the OFDM system. It is found that MBH window family provides all the pulse shapes given in [86] which are useful in improving an OFDM system. For the application of this pulse shape in an OFDM system, the window family given in (6.4.1) has to be rearranged as –

\[
p_{MBH}(t, \beta) = \begin{cases} 
\frac{1}{T}, & \text{for } 0 \leq |t| \leq \frac{(1 - \alpha) T}{2} \\
\beta - \frac{(4 \beta - 2)}{2 \alpha T} \left( |t| - \frac{(1 - \alpha) T}{2} \right) + (1 - \beta) \cos \left( \frac{\pi}{\alpha T} \left( |t| - \frac{(1 - \alpha) T}{2} \right) \right), & \text{for } \frac{(1 - \alpha) T}{2} \leq |t| \leq \frac{(1 + \alpha) T}{2} \\
0, & \text{otherwise}
\end{cases} \tag{6.4.2}
\]

The Fourier transform of proposed window family for \( T=1 \) is given as –

\[
P_{MBH}(f) = \text{Sinc}(f) \left\{ \frac{2(1 - \beta) \cos(\alpha \pi f)}{1 - 4 f^2 \alpha^2} - \frac{(1 - 2 \beta) \sin(\alpha \pi f)}{\alpha \pi f} \right\} \tag{6.4.3}
\]
There are two parameters (‘β’ and ‘α’; which are window shape parameter and roll-off factor respectively) which decide the shape of window and its performance in an OFDM system. The range of ‘β’ is same as given above and ‘α’ can take any value between 0 and 1. Some well known pulse shapes are the members of this family under following condition:

a) At α = 0 and for any value of β; this will become rectangular pulse shape.
b) At β = 1.2 and for any value of α; this behaves equivalent to BTRC pulse shape.
c) At β = 0.5 and for any value of α; this behaves as RC pulse shape.
d) At β = 1 and for any value of α; this will become Bartlett pulse shape.
e) At β = 1 and α = 1 this will become Frank and SOCW [99: a₁ = -0.5].

The time domain and frequency domain plots of proposed pulse shape for different values of ‘β’ at α = 1.0 and 0.3 are shown in Figure- 6.4.1 and Figure- 6.4.2 respectively.

![Figure-6.4.1: Time domain plots of proposed pulse shape for α = 1 and 0.3 with different β](image-url)
Figure-6.4.2: Frequency domain plots of proposed pulse shape for $\alpha = 1$ and $0.3$ with different $\beta$

It is clearly visible from Figure- 6.4.1 and Figure- 6.4.2, that the plots of BTRC pulse shapes and the MBH pulse shape at $\beta = 1.2$ are same for both $\alpha = 1$ and $0.3$. The solid and dashed lines represents plots for MBH window family pulse shapes at different $\beta$ and the symbol (O) represents the plots for BTRC pulse shape. This proves that many existing pulse shapes are the member of MBH window family.

6.5 OPTIMIZATION OF SIR

The $SIR_q$ given in (6.3.5) depends on the desired sub-carrier location ($q$), frequency offset ($\varepsilon$), window function $p(n)$ and the window parameters ('$\alpha$' and ' $\beta$'). In this section, a method is proposed to determine the optimum value of pulse shape parameter ($\beta$) which maximizes the SIR performance at the receiver which in turn minimizes the ICI in the OFDM system.
For this analysis, the average SIR over all the sub-carriers has been introduced as-

\[
\bar{\text{SIR}} = \alpha_s^2 \left/ \left(1 / N \right) \sum_{q=0}^{N-1} \sigma_{Iq}^2 \right.
\]

(6.5.1)

Now, rewriting this expression of SIR in terms of pulse shape parameter (\(\beta\)) –

\[
\bar{\text{SIR}} = \left( A \beta^2 + B \beta + C \right) \left/ \left( D \beta^2 + E \beta + F \right) \right.
\]

(6.5.2)

where,

\[
A = P_1(\varepsilon); \quad B = P_2(\varepsilon); \quad C = P_3(\varepsilon); \quad D = \left( \frac{1}{N} \right) \sum_{q=0}^{N-1} \sum_{k=0}^{N-1} P_1(k-q+\varepsilon);
\]

(6.5.3)

\[
E = \left( \frac{1}{N} \right) \sum_{q=0}^{N-1} \sum_{k=0}^{N-1} P_2(k-q+\varepsilon); \quad F = \left( \frac{1}{N} \right) \sum_{q=0}^{N-1} \sum_{k=0}^{N-1} P_3(k-q+\varepsilon);
\]

(6.5.4)

\[
P_1(f) = 4 \text{Sinc}^2(f) \left\{ \text{Sinc}(\alpha f) - \frac{\cos(\alpha \pi f)}{(1 - 4 f^2 \alpha^2)} \right\}^2
\]

(6.5.5)

\[
P_2(f) = -4 \text{Sinc}^2(f) \left\{ \left( \text{Sinc}(\alpha f) - \frac{\sqrt{2} \cos(\alpha \pi f)}{(1 - 4 f^2 \alpha^2)} \right)^2 - \frac{0.54}{\pi} \frac{\cos(\alpha \pi f)}{(1 - 4 f^2 \alpha^2)} \text{Sinc}(\alpha f) \right\};
\]

(6.5.6)

\[
P_3(f) = \text{Sinc}^2(f) \left\{ \text{Sinc}(\alpha f) - \frac{2 \cos(\alpha \pi f)}{(1 - 4 f^2 \alpha^2)} \right\}^2;
\]

(6.5.7)

To maximize \(\bar{\text{SIR}}\) with respect to \(\beta\), set \(\partial\bar{\text{SIR}}/\partial \beta = 0\). The optimum \(\beta_{\text{opt}}\) is thus obtained as –

\[
\beta_{\text{opt}} = \begin{cases} 
\frac{-2(AF - CD) - \sqrt{(AF - CD)^2 - (AE - BD)(BF - EC)}}{AE - BD}, & \text{for } AE \neq BD \\
BF - EC & 2(AF - CD), \quad \text{for } AE = BD
\end{cases}
\]

(6.5.8)

Figure- 6.5.1 demonstrates the relationship given in (6.5.8) between \(\beta_{\text{opt}}\) and \(\varepsilon\) for various values of \(\alpha\).
It is clearly visible from Figure 6.5.1 that if normalized frequency offset is greater than 0.42, then $\beta_{opt}$ comes out to be 1.88 irrespective of the roll off factor ($\alpha$). The different values of $\beta_{opt}$ can be obtained for various combinations of normalized frequency offsets and roll-off factors.

### 6.6 PERFORMANCE EVALUATION

In this section the performance of proposed window function is analyzed and compared with the other existing window function. The performance metric consist of ICI power, SIR, and BER in frequency selective fading channel.
6.6.1 Performance Evaluation in Terms of ICI and SIR

For the evaluation of proposed window function and for the comparison with other window functions, the ICI power for different values of frequency offset as given in (6.3.2) has been determined using MATLAB and plotted in dB as shown in different Figures (from Figure-6.6.1.1 to Figure- 6.6.1.5).

Figure-6.6.1.1 compares the average ICI power for a 64 sub-carrier OFDM system with $\alpha = 1$ by varying the normalized frequency offset $\varepsilon$. It is clearly evident from the graphs that the MBH pulse-shape with $\beta = 1.06$ is better than BTRC in the range of 0 to 0.18 of normalized frequency offset ($\varepsilon$). Out of this range, the MBH pulse shape with $\beta = 1.5$ is better than BTRC pulse shape. The SIR as expressed by (6.3.5) is also evaluated and plotted in Figure- 6.6.1.2. These graphs are showing the SIR for 64 sub-carrier OFDM system with roll off factor, i.e., $\alpha = 1$ in the range of 0 to 0.5 of normalized frequency offset. It is observed that the MBH pulse shape with $\beta = 1.06$ have better SIR than BTRC pulse shape in the range of 0 to 0.18 of normalized frequency offset ($\varepsilon$). Out of this range, the MBH pulse shape with $\beta = 1.5$ is better than BTRC pulse shape.

It is mentioned in [86] that a rectangular pulse shaped OFDM system has always greater ICI power than a RC and BTRC pulse shaped OFDM system. In Figure-6.6.1.1, the ICI power for MBH pulse shape with $\beta=1.06$ is 8.24 dB better than BTRC at $\varepsilon = 0.04$ and $\alpha = 1$. This is good enough to claim the superiority of MBH pulse shape over BTRC, RC and rectangular pulse shapes. These results are in conformity with the theoretical expressions given above.

Similarly, Figure-6.6.1.3 compares the average ICI power for a 64 sub-carrier OFDM system by taking roll-off factor ($\alpha$) as 0.4, 0.6 and 0.8 respectively. It is clearly evident from the plots that the MBH pulse shape with different values of $\beta$ ( 1.8 and 1.5) are better than BTRC pulse shape in the range of 0 to 0.5 of normalized frequency offset ($\varepsilon$). This further consolidates the finding that the MBH pulse shapes are better candidate for OFDM system in terms of ICI power with any value of roll off factor. Figure-6.6.1.4 again demonstrates the betterment in terms of SIR made by MBH pulse shapes in contrast to BTRC (all the parameters, i.e., $\alpha$, $\beta$, $\varepsilon$ and N, are same as taken in the plots of ICI power shown in Figure-6.6.1.3).
Figure 6.6.1.1: The average ICI power for a 64-subcarrier OFDM system at $\alpha=1$

Figure 6.6.1.2: The SIR with MBH and BTRC for a 64-subcarrier OFDM system at $\alpha=1$
Figure 6.6.1.3: Comparison of ICI power for a 64-subcarrier OFDM system as $\alpha$ varies from 0.4 to 0.8

Figure 6.6.1.4: SIR comparison for a 64-subcarrier OFDM system as $\alpha$ varies from 0.4 to 0.8
As compared with other available pulse shapes like SOCW, Frank and PMSP, it is observed that the ICI (with $\alpha = 0.4$) can be kept in between Frank and BTRC pulse shapes as visible from Figure-6.6.1.5. The ICI is found to be better than BTRC and comparable with SOCW [$99: \alpha = 0.4$]. Similarly, at $\alpha = 1$, the MBH window ($\beta = 1$), SOCW [$99: \alpha = 0.5$] and Frank window becomes the triangular window. Therefore, their ICI performance is same and better than BTRC for lower frequency offset as shown in Figure- 6.6.1.5. The PMSP window is defined only for $\alpha = 1$, which is found better than all other windows in terms of ICI as shown in Figure- 6.6.1.5.

![Graph comparing ICI power for different pulse shapes](image)

**Figure-6.6.1.5.** Comparison of ICI power for a 64-subcarrier OFDM system at $\alpha = 0.4$ and 1

The significance of improvisation with proposed MBH family is quite visible and in conformity with the various graphs shown in different Figures (from Figure- 6.6.1.1 to Figure- 6.6.1.5). The pulse shape parameter ($\beta$) of proposed MBH window family has been taken as 1 and 1.88 for roll-off factor $\alpha = 1.0$ and $\alpha = 0.4$ respectively. The window shape parameter ($a_1$) of SOCW has been taken as 0.5 and -0.4 for roll-off factor $\alpha = 1.0$ and $\alpha = 0.4$ respectively.

The variation in ICI power with respect to pulse shape parameter ($\beta$) of MBH pulse shapes is shown in Figure- 6.6.1.6. It has been shown with the help of various graphs that by increasing the roll-off factor ($\alpha$), the ICI power can be minimized (in the order of about 65 dB). In Figure-6.6.1.6, the change in ICI power has been shown when frequency offset ($\varepsilon$) is reduced from $\varepsilon = 0.22$ to $\varepsilon = 0.16$ and from $\varepsilon =
0.08 to $\varepsilon = 0.02$ (by a similar difference of 0.06) while keeping roll-off factor at the same level $\alpha = 0.4$ and $\alpha = 1.0$ respectively. This is clearly visible from these pair of graphs that while $\varepsilon$ is reduced from 0.08 to 0.02, the ICI power reduces significantly as compared to the case when $\varepsilon$ is reduced from 0.22 to 0.16. This suggests that the effect of roll-off factor is more dominating at low levels of frequency offset ($\varepsilon$). The $\beta = 0.5$ and $\beta = 1.2$ represents RC and BTRC pulse shapes respectively for any value of $\alpha$, as they are also marked on the plots of Figure-6.6.1.6.

Noteworthy, for the same value of $\alpha$ and $\varepsilon$ the MBH pulse shapes provides better ICI power as compared to RC and BTRC pulse shape. For example, if $\alpha = 0.4$, $\varepsilon = 0.16$ or 0.22 and $\alpha = 0.7$, $\varepsilon = 0.18$, the ICI power of MBH pulse shape (with $\beta$ is greater than 1.2) is better than RC and BTRC pulse shapes. Similarly, in the remaining three plots, with $\alpha = 1$; $\varepsilon = 0.02$ or 0.08 or 0.14, the ICI power of MBH pulse shape (with $\beta$ lying between 0.5 and 1.2) is found better than RC and BTRC pulse shapes. The best value of ICI power for high roll-off factor ($\alpha = 1$) can be obtained between the values of 0.8 and 1.2 of $\beta$. On the other hand, if the spectrum of one pulse shaping function has smaller side lobe than the other one, then it is expected that the pulse shaping function will be leading to less ICI power and better bit error rate. By choosing the different values of $\beta$, numerous side lobes level can be generated which are better than RC and BTRC pulse shapes. This feature of proposed MBH pulse shapes makes it superior and spectrally efficient than RC and BTRC pulse shapes. In this way, it is visible that the various pulse shapes of MBH family out performs the BTRC pulse shapes which has been established as one of the best candidate for OFDM system in [86].

![Figure-6.6.1.6: The ICI power for different pulse shape parameter $\beta$ for a 64 sub-carrier OFDM system with different values of roll off factor ($\alpha$) and normalized frequency offset ($\varepsilon$)](image-url)
6.6.2 Performance Evaluation in Terms of BER

In this section, the BER of an OFDM system with windowing at the receiver end is determined over frequency selective fading channel. The expression given in [91] is used to evaluate the BER performance. To evaluate the BER of an OFDM system with carrier frequency offset, the expression for ICI coefficient has always been taken while considering an OFDM system without receiver windowing, as given in [91]. In present analysis, the windowed ICI coefficient has been taken at the receiver to find out the BER.

The recovered signal after 2N point FFT is given in (6.2.3.19). The terms $H(k,\Delta f)$, $S(k,2q,\Delta f)$, and $W(i,2q)$ have already defined in Section 6.2.3. The term $S(k,p,\Delta f)$ as given in (6.2.3.17) is the ICI coefficient for sub-carrier frequency $p$. The ICI coefficient for first subcarrier by assuming equi-probable message symbols can be written as –

$$S(k,1,\Delta f) = \frac{1}{N} e^{j 2\pi \left(\epsilon (\mu - \alpha) - k\alpha\right)} \sum_{n=0}^{2N-1} \rho(n) e^{j 2\pi n \left(\frac{2k-1}{2N} + \Delta f\right)}$$

(6.6.2.1)

The bit error probability for frequency selective fading channel (channel characteristic as already defined in Section- 2.2.3) with BPSK modulation is given as [91, eq.34].

$$P_b(\xi) = 1 - \frac{1}{2N} \sum_{k=1}^{2N-1} \frac{\gamma [\Re(S(1)) + a_k]^2}{1 + \gamma \left(\Re(S(1))^2 + b_k\right)^2} + \frac{\gamma [\Re(S(1)) - a_k]^2}{1 + \gamma \left(\Re(S(1))^2 + b_k\right)^2}$$

(6.6.2.2)

where, $\gamma = E_b / N_0$ and the variables $a_k$ & $b_k$ are defined as –

$$a_k = \Re \left( P_k^T C_{\rho \rho_1} \right)$$

(6.6.2.3)

$$b_k = P_k^T \left( C_{\rho \rho_1} - C_{\rho \rho_1} C_{\rho \rho_1}^H \right) P_k$$

(6.6.2.4)

where, $\Re(.)$ denotes the real term of complex value
The $C_{\rho \rho_1}$ is obtained from the frequency domain channel covariance matrix defined as –

$$C = E \{ (\rho_1 \rho^T) (\bar{\rho}_1 \rho^H) \} = \begin{pmatrix} C_{\rho_1 \rho_1} & C_{\rho \rho_1}^H \\ C_{\rho \rho_1} & C_{\rho \rho} \end{pmatrix}$$  

(6.6.2.5)

where, $C_k$ is the time domain channel covariance matrix, with $(.)^H$ and $E(.)$ denoting the Hermitian transpose operation and the statistical expectation, respectively. The term $P_k$ is a diagonal matrix depending on ICI coefficient and is defined as –

$$P_k = \text{diag}(S(2) \ S(3) \ \cdots \ S(N)) \ e_k$$  

(6.6.2.6)

where, $e_k$ is the $k^{th}$ column of matrix $E_{N-1}$. The dimension of $E$ is $N-1 \times 2^{N-2}$. The $k^{th}$ column of $E_M$ is the binary representation of the number $2^{M-k}$.

The BER performance of the 16-subcarrier BPSK OFDM system over L-tap Rayleigh frequency selective fading channel (L=5) with different receiver windowing options is shown in Figure- 6.6.2.1. The theoretical results are in excellent agreement with the simulation results which has been shown clearly on these plots. The solid and dashed lines represent the theoretical results obtained from the BER analysis, as given in (6.6.2.2), with and without windowing respectively, and the symbols (x) represents the simulation results. For determining the BER using simulation, the Monte Carlo method as given in [39, 73 and 133] has been used. It is clearly visible from Figure-6.6.2.1 that the MBH pulse shape ($\beta = 1.85$) gives a better bit error rate in comparison to BTRC, Frank and SOCW with $\alpha = 0.4$ and $\varepsilon = 0.1$.

Similarly for $\alpha = 1.0$ and $\varepsilon = 0.1$, the MBH pulse shape ($\beta = 1$) performs better than BTRC and PMSP [87: at a=1, c=2, b=0.5 and n=2]. Though the performance of Frank, SOCW [99: at $a_i = -0.5$] and MBH ($\beta=1$) is same due to their equivalence to triangular window at $\alpha = 1$. Figure- 6.6.2.2 demonstrate the BER performance of the 64-subcarrier BPSK OFDM system over Rayleigh frequency selective fading channel (with L = 10). Due to computational complexity of BER expression given in (6.6.2.2), simulation results are shown for 64 sub-carrier OFDM system. However, the theoretical results using (6.6.2.2) by truncating the ICI terms have also been obtained and shown in Figure- 6.6.2.2.
Figure-6.6.2.1: BER for 16- subcarrier BPSK-OFDM over frequency selective Rayleigh fading channel with $L=5$ and $\varepsilon = 0.1$

Figure-6.6.2.2: BER for 64- subcarrier BPSK-OFDM over frequency selective Rayleigh fading channel with $L=10$
In Figure-6.6.2.2 the theoretical results are found again in excellent conformity with little error to simulation results due to 16 ICI terms considered in evaluation. MBH pulse shapes (with $\beta = 1.85$ and $1$) is found better than BTRC pulse shapes in several scenarios. This again consolidates the claim of superiority of MBH family of pulse shapes over BTRC, in terms of BER in different situations.

The combined analysis of ICI and BER of proposed window family and comparison with other available pulse shapes like SOCW, Frank and PMSP is also shown in Table- 6.1. The ICI and BER in an OFDM system is found worst in case of without windowing as it is clearly visible from the Table- 6.1. The proposed window is found the best candidate for improvising the ICI and BER together for every roll-off factor ($\alpha$) and frequency offset ($\varepsilon$). The values of ICI and BER for the proposed (MBH) window family are included as bold in the Table- 6.1. The value of ICI is found best in case of PMSP window with $\alpha = 1.0$, however, the BER is not comparable to all the windows except without windowing case. Though, the PMSP window is defined only for one value of the roll-off factor ($\alpha = 1.0$).
Table– 6.1: Comparative analysis of various window functions in an OFDM system.

<table>
<thead>
<tr>
<th>S.No</th>
<th>Window Function</th>
<th>$\alpha = 1$</th>
<th>$\alpha = 0.4$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\varepsilon = 0.1$</td>
<td>$\varepsilon = 0.2$</td>
</tr>
<tr>
<td>ICI (dB)</td>
<td>BER at 50 dB</td>
<td>ICI (dB)</td>
<td>BER at 50 dB</td>
</tr>
<tr>
<td>SNR</td>
<td></td>
<td>SNR</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>RC</td>
<td>-25.765</td>
<td>$2.739 \times 10^{-5}$</td>
</tr>
<tr>
<td>3</td>
<td>BTRC</td>
<td>-33.35</td>
<td>$6.895 \times 10^{-6}$</td>
</tr>
<tr>
<td>4</td>
<td>SOCW</td>
<td>-36.71</td>
<td>$3.209 \times 10^{-6}$</td>
</tr>
<tr>
<td>5</td>
<td>Frank</td>
<td>-36.71</td>
<td>$3.209 \times 10^{-6}$</td>
</tr>
<tr>
<td>6</td>
<td>PMSP</td>
<td><strong>-92.99</strong></td>
<td>$2.237 \times 10^{-3}$</td>
</tr>
<tr>
<td>7</td>
<td>MBH (Proposed)</td>
<td>-36.71</td>
<td>$3.209 \times 10^{-6}$</td>
</tr>
</tbody>
</table>
6.7 SUMMARY

In this chapter, the ICI and BER over Rayleigh frequency selective fading channel are determined for the receiver windowing based OFDM system given in Figure-6.2.1. The employment of the MBH pulse shapes in the above said system gives substantial improvement in ICI as well as in BER in comparison to BTRC pulse shape which was established as one of the best candidate for OFDM system in [86]. The OFDM system is sensitive to carrier frequency offset which introduces ICI in OFDM receiver, the ICI power and BER have been plotted against normalized frequency offset in Figure 6.7.1. It is clearly visible from the graphs that the performance of the MBH pulse shapes (with different values of window shape parameter ‘β’) is better than BTRC pulse shapes for different roll off factors (α) in 64-subcarrier BPSK OFDM system over Rayleigh frequency selective fading channel (with L=5) which performs windowing at the receiver sides with 50 dB of SNR. Simultaneously, the MBH pulse shape family encompasses all the pulse shapes which are being analyzed in [86] for improvising the performance of an OFDM system with receiver pulse shaping. Finally, the proposed window family (MBH) can be considered as best candidate for improving the performance of OFDM system in terms of both ICI and BER hand-in-hand.

Figure-6.7.1: BER and ICI for 64- subcarrier BPSK-OFDM with receiver windowing over frequency selective Rayleigh fading channel with L=5 and SNR = 50 dB