ABSTRACT

The phenomenon of the solitary wave, which was discovered by the famous British scientist Scott Russell as early as in 1834, has been greatly concerned with the development of the physics and mathematics. Now it has been proved that a large number of nonlinear evolution equations have the soliton solution by the numerical calculations and the theoretical analysis. Solitary waves have the striking property of particles. So Kruskal and Zabusky named them “solitons”. The solitary waves not only have been observed in nature, some of them have also been produces in laboratories now. Also, when two or three of the solitary waves interact it appears that solitary waves retains shape after interaction, despite the fact that some studies show the appearance of small tail after the collision as shown latter in the thesis. However, these properties lead scientist to do more research work in this area during the past decades through both numerical and analytical solutions of their equations. In fact, finding the analytical solutions of the nonlinear evolutionary equations generally is difficult and probably impossible in some cases; as in the case of interaction of two and three solitary waves or in case of development of Maxwellian initial condition into solitary waves. So finding the accurate approximate solution for these equations is the main aim of the researchers in order to study solitary waves on a wide range and to investigate their properties.

The nonlinear waves in shallow water are of great interest of many researchers. Its importance comes from its capability for describing many physical phenomena in the fields of physics, mathematics and engineering. These nonlinear waves were mathematically modeled by the Korteweg-de Vries. The derived equation is well known Korteweg-de Vries (KdV) equation. Such a powerful equation could successfully simulate the red spots at Jupiter. Moreover, the giant ocean waves known
as Tsunami are also described by the KdV equation. The giant internal waves in the interior of the ocean arising from the temperature difference which my destruct marine vessels could also be described by such a power KdV equation. The Regularized Long Wave (RLW) equation is one of the model partial differential equation of the nonlinear dispersive waves which has many applications in many areas e.g. ion-acoustic waves in plasma, magneto-hydrodynamic waves in plasma, longitudinal dispersive waves in elastic rods, pressure waves in liquid-gas bubble mixtures and rotating flow down a tube. Peregrine [76] was first to use the RLW equation for modeling the development of an undular bore. Later on Benjamine et al. [9] proposed the use of the RLW equation as a preferred alternative to the more classical Korteweg de Vries (KdV) equation to model a large class of physical phenomena. The Equal Width (EW) equation, which is less well-known and proposed by Morrison et al. [73], is an alternative description to the more usual KdV equation and RLW equation. The solutions of the EW equation are solitary waves, such that the width of each solitary wave is same irrespective of their amplitude. So, this equation is named as Equal Width equation. Thus the KdV, RLW, EW equations are evolutionary equations giving solitary wave solutions.

In fact, we aim here to develop this work in which the present inclusive study of the properties of solitons specially the solitons represented by the generalized Korteweg-de Vries (gKdV) equation $u_t + \varepsilon u_x u_{xx} + \mu u_{xxx} = 0$, generalized Equal Width (GEW) equation $u_t + \varepsilon u_x u_{xx} - \delta u_{xxx} = 0$, the generalized Relgularized Long Wave (GRLW) equation $u_t + u_x + p(p + 1)u_x u_{xx} - \mu u_{xxx} = 0$, Rosenau-Hyman $K(2,2)$ equation $u_t + (u^n)_x + (u^n)_{xxx} = 0$ etc. The present thesis: “Numerical Study of Some Aspects of Solitary Wave” is an attempt to study the different properties (conservation laws and interactions) of solitary waves by a variety of efficient
numerical techniques. The thesis is arranged in seven chapters and lastly a conclusion. The outline of these chapters is presented in the following sections.

In Chapter I, we briefly described the history of solitary wave and soliton. Also we try to give various evolutionary equations giving soliton solutions. In this chapter, we also described various techniques of finite element methods. Furthermore, the Von Neumann stability analysis has also been described. We also mention the selection Mathematica software for the simulation of various numerical schemes developed in this thesis.

In Chapter II, we developed the Petrov-Galerkin method using the linear hat function and cubic B-spline function as the trial and test functions respectively for solving the gKdV equation. We also give a review of the numerical methods for solving the gKdV equation. The analytical solutions of the gKdV equation using classical method have also been presented. A linear stability analysis shows the scheme so developed is unconditionally stable. The test problems taken up are the motion of the single solitary wave, interaction of two solitary waves, splitting of solitons from a single initial pulse, for various degrees of nonlinearity. The KdV and mKdV equations have a large number of conservation laws. But for the gKdV equation i.e. the KdV equation with the degree of nonlinearity is more than three, there is only three conservation laws. So, we consider the three conservation laws of the gKdV equation to test the efficiency of our method. It will also be seen that our scheme so developed is more efficient for handling the nonlinear term involved in the equation. Further, it will observe the appearance of a small tail when two solitary waves interact in those cases where the degree of nonlinearity is more than three.

In Chapter III, the Petrov-Galerkin method is developed using the linear hat function as the trial function and cubic B-spline function as the test function for solving the one dimensional Korteweg-de Vries Burgers’ (KdVB) equation. The
Linear stability analysis shows the scheme to be unconditionally stable. Four test problems have been taken up and these test problems show the accuracy and efficiency of the scheme. We also compared the numerical solution obtained by our scheme with the analytical solution of the KdVB equation for different cases whenever possible. We also consider three conservation laws for the KdV equation, which is a special case of the KdVB equation to validate our scheme.

In Chapter IV, we discussed the Forced Korteweg-de Vries (fKdV) equation. Here, we developed the quintic B-spline collocation method for the numerical solution of the fKdV equation. The generation of the surface waves when the fluid flow is disturbed by a small bump for various values of the Froude number is shown in this chapter. The results obtained showed good agreement with the theoretical results.

In Chapter V, a new numerical method for the Rosenau-Hyman $K(2,2)$ equation is presented based on a Galerkin method with quintic B-splines for both trial and test functions. No product approximation has been used. The motion of the solitary waves of the $K(2,2)$ equation, having compact support, and called compactons, free of exponential tails will be observed. We will also observe several common features of compactons and solitons. For example, a single compacton moves with velocity that is proportional to its amplitude; several compactons moving with different velocities collide, will go through a nonlinear interaction from which they emerge with a phase shift; also, general initial data can break into a train of compactons. So, in our test problems it will include the motion of single compacton, interaction of two and three compactons and the splitting of compactons from an initial pulse to show the accuracy and efficiency of the proposed method in this chapter. Our results are compared with the analytical solution.
In Chapter VI, the Petrov-Galerkin method using the linear hat function and quadratic B-spline function as the trial and test function have been developed for solving the GEW equation. Here also, we find the analytic solution of the GEW equation using the classical method. A review of the numerical methods for solving the GEW equation has been given. A linear stability analysis shows the method so developed is conditionally stable. The numerical scheme so developed is validated by the simulation of the motion of single solitary wave, interaction of two solitary waves and development of solitons from the Maxwellian initial pulse for various degrees of the nonlinearity. We will observe the remains of the residue when two solitary waves are interacted in the cases where the degree of nonlinearity is high. Comparisons of the numerical results obtained by our scheme with the analytical results as well as with the results of other methods have been made and it shows that our scheme is more accurate and efficient.

In Chapter VII, we developed the Petrov-Galerkin method using the linear hat function as the trial function and the quintic B-spline function as the test function for solving the GRLW equation. A review of the numerical methods for finding the solutions of GRLW equation has been given. We find the analytic solution of the GRLW equation by the classical method. A linear stability analysis shows the scheme so developed is conditionally stable. The method so developed is validated by the simulation of the motion of the single solitary wave, interaction of two solitary waves and the development of solitons of the Maxwellian initial pulse for various degrees of the nonlinearity. In this case also it will be observed the remains of residue when two solitary waves are interacted in the cases where the degree of nonlinearity is high. We make various comparisons for the numerical solutions obtained by our scheme with the analytical results as well as the results of other methods and it is found that our method is more accurate and efficient.
In the last chapter we give a detailed conclusion. Here, we mentioned the advantages of using product approximation technique. The product approximation technique has been used to solve gKdV, KdVB, GEW and GRLW equations. The Petrov-Galerkin method with the used of product approximation technique, gKdV and KdVB equations reduced to a system of nonlinear equations and it is solved by Newtons’ method with the help of Mathematica function, FindRoot[]. On the other hand the method with the used of product approximation the GEW and GRLW equations reduced to systems of linear equations and these systems are solved by Mathematica function Solve[]. The numerical results obtained by our methods are compared with various analytical solutions as well as other results found in published papers. It is found that our methods are better than the other methods that are cited in the relevant papers.

Most of the results presented in this thesis have been published in national and international journals and some of them are presented in national and international conferences/seminars. Some of the published papers are placed at the end of this thesis as an appendix.