Chapter 3  Reliability Characteristics Analysis of Complex Systems with Multiple Redundancy and Multiple Failure

Three different complex industrial models with different redundancy and failure rates are discussed in this chapter. These are:

3.1 Introduction

(a) Model I - Performability Analysis of a System under 1-out-of-2: G Scheme with Perfect Reworking

In safety critical applications, it becomes common to improve the reliability through $k$-out-of-$n$ redundancy. In this model we have presented an analytical approach to compute the reliability measures of a system, which consist a mixed configuration. The system consists of three subsystems, namely $A$, $B$ and $C$, connected in mixed configurations (i.e. combination of series and parallel configuration). Subsystem $A$ is of the type 1-out-of-2: $G$ and it is connected to subsystem $B$ in a parallel configuration and these two subsystems, $A$ and $B$, are connected in series configuration with subsystem $C$. Subsystem $B$ and $C$ has $n$ units in series configuration. The system has faced three states throughout the comprehensive task, namely good, degraded and failed state.

(b) Model II - Performance of a Structure, which Consists of 2-out-of-3: F Substructure, Under Human Failure

In recent years, the studies of reliability properties of consecutive $k$-out-of-$n$ structures have attracted a great attention from both theoretical and practical approaches. In this model, we have studied the reliability measures of a complex structure, which consists of two substructures namely, $A$ and $B$, handled by a human operator. The substructure $A$ is of the type 2-out-of-3: $F$ (with unequal components), which is connected with the substructure $B$ in series configuration. The complete structure can be failed due to the failure of any two components of substructure $A$, any component of $A$ with the failure of substructure $B$ and due to human error.

(c) Model III - System Reliability Measures in Presence of Common Cause Failure

Standby and $k$-out-of-$n$ redundancies are one of the most famous ways to improve the reliability of a system. $k$-out-of-$n$: $G, n > k$, redundancy implies that at least $k$ component are
required to work properly for successful working of the system. In this model, we deal with the mathematical modeling of a system in which both types of the redundancies i.e. standby and \( k \)-out-of-\( n \) are considered. The considered system consists of two subsystems, namely \( A \) and \( B \) connected in series configuration. Subsystem \( A \) has two units in parallel configuration with a standby unit and it is connected to subsystem \( B \), which is of 2-out-of-3: F type.

### 3.2 Assumptions and Nomenclature

The following assumptions are considered throughout the chapter:

(a) Initially the system is in perfect state and all units and subunits are in working conditions.
(b) A repaired unit works as a new one.
(c) At every time sufficient repair facilities are available.
(d) Average failure rates are taken to be constants.
(e) Each defect is either absent or present.

Including the assumptions mentioned in section 3.2, there are some more assumptions associated with the models:

(a) **Model-I**
   (i) Average failure and repair rate of the system are taken to be constant.

(b) **Model-II**
   (i) The substructure \( A \) works under 2-out-of-3: F policy.
   (ii) The structure may work in a degraded state.

(c) **Model-III**
   (i) The considered system may work with reduced efficiency.
   (ii) Changeover time from active to standby unit is negligible.

Including the nomenclatures mentioned in abbreviations, there are some more nomenclatures associated with the models. These are:
(a) Model-I

\( P_{ABC}(t) \)  
The probability of the state at time \( t \) when the system is working with full capacity.

\( P_{ABc}(t) \)  
The probability of the state at time \( t \) when the system is working with one failed unit of subsystem \( A \).

\( P_{AAbc}(t) \)  
The probability of the state at time \( t \) when the system is working with two failed units of subsystem \( A \).

\( P_{AAbc}(t) \)  
The probability of the state at time \( t \) when the system is working with one failed unit of subsystem \( A \) and failed subsystem \( B \).

\( P_{AAbc}(t) \)  
The probability of the state at time \( t \) when the system is working with the failed subsystem \( B \).

\( P_{AAbc}(x,t) \)  
The probability of the state at time \( t \) when the system is failed due to failure of subsystem \( A \) and subsystem \( C \).

\( P_{AAbc}(x,t) \)  
The probability of the state at time \( t \) when the system is failed due to failure of subsystem \( A \) and subsystem \( B \).

\( P_{AAbc}(x,t) \)  
The probability of the state at time \( t \) when the system is failed due to failure of one unit of subsystem \( A \) and subsystem \( C \).

\( P_{AAbc}(x,t) \)  
The probability of the state at time \( t \) when the system is failed due to failure of subsystems \( B \) and \( C \).

\( P_{AAbc}(x,t) \)  
The probability of the state at time \( t \) when the system is failed due to failure of subsystem \( B \), \( C \) and one unit of subsystem \( A \).

\( P_{AAbc}(x,t) \)  
The probability of the state at time \( t \) when the system is failed due to failure of subsystem \( C \).

\( \lambda_A / \lambda_B / \lambda_C \)  
Failure rate of subsystem \( A/B/C \).

\( \phi_A / \phi_B / \phi_C \)  
Repair rate of subsystem \( A/B/C \).

\( \phi_{AB} / \phi_{BC} / \phi_{CA} \)  
Simultaneous repair rate of subsystem \( A \) and \( B/B \) and \( C/C \) and \( A \) respectively.

\( \phi_{ABC} \)  
Simultaneous repair rate of subsystems \( A, B \) and \( C \).
(b) Model-II

\[ P_0(t) \] The probability that the structure is in good working condition at time \( t \).

\[ P_1(t) \] The probability that the structure is working with the first failed component of substructure \( A \) at any time \( t \).

\[ P_2(t) \] The probability that the structure is working with the second failed component of substructure \( A \) at any time \( t \).

\[ P_3(t) \] The probability that the structure is working with the third failed component of substructure \( A \) at any time \( t \).

\[ P_4(x,t) \] The probability that the structure is failed due to the failure of substructure \( B \), at epoch \( t \) and has an elapsed repair time of \( x \).

\[ P_5(x,t) \] The probability that the structure is failed due to failure of the first and second component of substructure \( A \), at epoch \( t \) and has an elapsed repair time of \( x \).

\[ P_6(x,t) \] The probability that the structure is failed due to failure of second and third component of substructure \( A \), at epoch \( t \) and has an elapsed repair time of \( x \).

\[ P_7(x,t) \] The probability that the structure is failed due to failure of the first and third component of substructure \( A \), at epoch \( t \) and has an elapsed repair time of \( x \).

\[ P_8(x,t) \] The probability that the structure is failed due to the human error, at epoch \( t \) and has an elapsed repair time of \( x \).

\[ P_9(x,t) \] The probability that the structure is failed due to failure of third component of substructure \( A \) with failure of substructure \( B \), at epoch \( t \) and has an elapsed repair time of \( x \).

\[ P_{10}(x,t) \] The probability that the structure is failed due to the failure of first component of substructure \( A \) with failure of substructure \( B \), at epoch \( t \) and has an elapsed repair time of \( x \).

\[ P_{11}(x,t) \] The probability that the structure is failed due to failure of second component of substructure \( A \) with failure of substructure \( B \), at epoch \( t \) and has an elapsed repair time of \( x \).

\[ \lambda_1 / \lambda_2 / \lambda_3 \] Failure rate of the first/second/third component of the substructure \( A \).

\[ \lambda_B / \lambda_h \] Failure rate of the substructure \( B \)/human error.

\[ \mu_1 / \mu_2 / \mu_3 \] Repair rate of the first/second/third component of the substructure \( A \).
\( \mu \) Simultaneous repair rate of any two components of the substructure \( A \).

\( \mu_B / \mu_h \) Repair rate of substructure \( B \)/human error.

\( \mu_{AB} \) Simultaneous repair rate of any two component of the substructure \( A \) and the substructure \( B \).

(c) Model-III

\( P_{AB}(t) \) The probability that at time \( t \) system is in the perfect functioning state.

\( P_{AB}(t) \) The probability that at time \( t \) system is working with one failed unit of subsystem \( B \).

\( P_{AB}(t) \) The probability that at time \( t \) system is working with one failed unit of subsystem \( A \).

\( P_{ABS}(t) \) The probability that at time \( t \) subsystem \( A \) is failed and system is working with standby unit.

\( P_{AB}(t) \) The probability that at time \( t \) system is working with one failed unit of each of the subsystems \( A \) and \( B \).

\( P_{ABS}(t) \) The probability that at time \( t \) subsystem \( A \) is failed, one unit of subsystem \( B \) is failed and the system is working with standby unit of subsystem \( A \).

\( P_{AB}(x,t) \) The probability that at time \( t \) system is failed due to failure of subsystem \( B \).

\( P_{AB}(x,t) \) The probability that at time \( t \) system is failed due to failure of subsystem \( A \) and its standby unit.

\( P_{AB}(x,t) \) The probability that at time \( t \) system is failed due to failure of subsystem \( B \) and one unit of subsystem \( A \).

\( P_{ABS}(x,t) \) The probability that at time \( t \) system is failed due to failure of both subsystems \( A \) and \( B \).

\( P_{ABS}(x,t) \) The probability that at time \( t \) system is failed due to failure of subsystem \( A \), one unit of subsystem \( B \) and standby unit of subsystem \( A \).

\( P_h(x,t) \) The probability that at time \( t \) system is fails due to human error.

\( P_E(x,t) \) The probability that at time \( t \) system is failed due to electrical failure.

\( P_{CCF}(x,t) \) The probability that at time \( t \) system is failed due to common cause failure.
\( \lambda_A / \lambda_B / \lambda_h / \lambda_E / \lambda_{CCF} \) Failure rate of subsystem \( A/B \)/human failure rate/ electrical failure rate/common cause failure rate.

\( \mu_{AS} \) Simultaneous repair rate of subsystem \( A \) and its standby unit.

\( \mu_B \) Repair rate of subsystem \( B \).

\( \mu_{AB} \) Simultaneous repair rate of subsystem \( A \) and subsystem \( B \).

\( \mu_{ABS} \) Simultaneous repair rate of subsystem \( A \), subsystem \( B \) and standby unit of subsystem \( A \).

\( \mu_h \) Repair rate of human failure.

\( \mu_E \) Repair rate of electrical failure.

\( \mu_{CCF} \) Repair rate of common cause failure.

### 3.3 System Configuration

(a) Model-I

The considered system consists of three subsystems, namely \( A \), \( B \) and \( C \), connected in mixed configurations (i.e. combination of series and parallel configuration). Subsystem \( A \) is of the type 1-out-of-2: G. Subsystem \( A \) is connected to subsystem \( B \) in a parallel configuration and these two subsystems, \( A \) and \( B \), are connected in series configuration with subsystem \( C \). Subsystem \( B \) and \( C \) has \( n \) units in series configuration. The system configuration has been shown in Fig. 3.1. The corresponding state transition diagram is shown in Fig. 3.2.

![System Configuration Diagram](image)

Fig. 3.1. System Configuration
On the basis of system configuration as shown in Fig. 3.1, the state transition diagram Fig. 3.2 has been formed as follows:

Initially system is in perfect working state (good state); this state is shown as $P_{ABC}(t)$. From here when any of the subunit of the subsystem $A$ is failed with the failure rate $\lambda_A$ than the system is goes into a degraded state $P_{ABC}(t)$. The system comes back in the state $P_{ABC}(t)$ from the state $P_{ABC}(t)$ after the repair of subunit of subsystem $A$ with the repair rate $\phi_A$. Furthermore
if the other unit of the subsystem A is failed from $P_{ABC}(t)$ then again the system is goes into a degraded state this state is represented as $P_{ABC}(t)$. The system comes back in the state $P_{ABC}(t)$ from the state $P_{ABC}(t)$ after the repair of subunit of subsystem A with the repair rate $\phi_A$. From the state $P_{ABC}(t)$ if subsystem B/C is failed then the system is goes into a complete failed state $P_{ABC}(t)$. The system comes back in the good state $P_{ABC}(t)$ from the state $P_{ABC}(t)$ when repair is complete with the repair rate $\phi_A / \phi_{AC}$. From the state $P_{ABC}(t)$ if the subsystem C is failed then the system is goes into failed state $P_{ABC}(t)$, from this state the system comes back in the good state $P_{ABC}(t)$ after repair with a repair rate $\phi_{BC}$. From the state $P_{ABC}(t)$ if the subsystem B is failed then system goes into a degraded state and this degraded state is represented as $P_{ABC}(t)$, from this state the system comes back in the state $P_{ABC}(t)$ after repair with a repair rate $\phi_B$. If from the state $P_{ABC}(t)$ the subsystem C is failed than system goes into failed state $P_{ABC}(t)$, from this state the system comes back in the good state $P_{ABC}(t)$ after repair with a repair rate $\phi_{AC}$. From the state $P_{ABC}(t)$ if the subsystem C is failed then the system goes into failed state $P_{ABC}(t)$, from this state the system comes back in the good state $P_{ABC}(t)$ after repair with a repair rate $\phi_{BC}$. From the state $P_{ABC}(t)$ if subsystem B is failed then the system is goes into a degraded state $P_{ABC}(t)$, from this state the system comes back in the good state after repair of subunit B with repair rate $\phi_B$. From the state $P_{ABC}(t)$ if subsystem C is failed then the system goes into failed state $P_{ABC}(t)$, from this state the system comes back in good state $P_{ABC}(t)$ after repair with repair rate $\phi_{BC}$. From the state $P_{ABC}(t)$ if the subunit of subsystem A is failed then the system goes into degraded state $P_{ABC}(t)$, from the state $P_{ABC}(t)$, the system comes back in the state $P_{ABC}(t)$ after repair with repair rate $\phi_A$. From the state $P_{ABC}(t)$ the system goes into a failed state $P_{ABC}(t)$ after the failure of subsystem C with failure rate $\lambda_C$. The system comes back in good state $P_{ABC}(t)$ from the state $P_{ABC}(t)$ after the repair with repair rate $\phi_{ABC}$.

In the similar way, the state transition diagrams for other models are formed.
States Description

$S_0$ The state in which system is working with full capacity..

$S_1$ The state in which system is working with one failed unit of subsystem $A$.

$S_2$ The state in which system is working with two failed units of subsystem $A$.

$S_3$ The state in which system is working with the failed subsystem $B$.

$S_4$ The state in which system is working with one failed unit of subsystem $A$ and with failed subsystem $B$.

$S_5$ The state in which system is failed due to failure of subsystems $B$ and $C$.

$S_6$ The state in which system is failed due to failure of subsystem $B$, $C$ and one unit of subsystem $A$.

$S_7$ The state in which system is failed due to failure of subsystem $C$.

$S_8$ The state in which system is failed due to failure of one unit of subsystem $A$ and failure of subsystem $C$.

$S_9$ The state in which system is failed due to failure of subsystem $A$ and subsystem $C$.

$S_{10}$ The state in which system is failed due to failure of subsystem $A$ and subsystem $B$.

(b) Model II

The considered structure consists of two substructures namely, $A$ and $B$, handled by a human operator. The substructure $A$ is of the type 2-out-of-3: F (with unequal components), which is connected with the substructure $B$ in series configuration as shown in Fig. 3.3. The corresponding state transition diagram is shown in Fig. 3.4.

![Fig. 3.3 System Configuration](image)
(c) Model-III
The considered system consists of two subsystems, namely A and B, connected in series configuration. Subsystem A has two units in parallel configuration with a standby unit and it is connected to subsystem B, which is of 2-out-of-3: F type as shown in the Fig. 3.5. The corresponding state transition diagram is shown in Fig. 3.6.
3.4 Mathematical Formulation and Solution

(a) Model-I

By the probability considerations and continuity arguments, we can obtain the following set of differential equations governing the present mathematical model:
\[
\left( \frac{\partial}{\partial t} + \lambda_A + \lambda_B + \lambda_C \right) P_{ABC}(t) = \sum_{i,j} \phi_i P_j(t) + \sum_{k,l}^\infty \phi_k P_l(x,t) dx ;
\]

where \( i = A, B; \ j = \overline{ABC}, \overline{AB}; k = AB, BC, ABC, C, AC; l = \overline{ABC}, \overline{AB}, \overline{AC}, \overline{BC} \), \( AB\overline{C}, \overline{ABC}, \overline{AB}\overline{C} \) respectively

\[
\left( \frac{\partial}{\partial t} + \lambda_A + \lambda_B + \lambda_C + \phi_A \right) P_{\overline{ABC}}(t) = \phi_A P_{\overline{ABC}}(t) + \phi_B P_{A\overline{BC}}(t) + \lambda_A P_{ABC}(t)
\]

(3.1)

\[
\left( \frac{\partial}{\partial t} + \lambda_B + \lambda_C + \phi_A \right) P_{A\overline{BC}}(t) = \phi_A P_{A\overline{BC}}(t) + \phi_B P_{AB\overline{C}}(t) + \lambda_A P_{ABC}(t)
\]

(3.2)

\[
\left( \frac{\partial}{\partial t} + \lambda_A + \lambda_C + \phi_B \right) P_{AB\overline{C}}(t) = \phi_B P_{AB\overline{C}}(t) + \phi_A P_{A\overline{BC}}(t) + \lambda_B P_{ABC}(t)
\]

(3.3)

\[
\left( \frac{\partial}{\partial t} + \lambda_A + \lambda_C + \phi_B + \phi_A \right) P_{\overline{AB\overline{C}}}(t) = \lambda_A P_{\overline{AB\overline{C}}}(t) + \phi_B P_{AB\overline{C}}(t)
\]

(3.4)

\[
\left( \frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \phi_i \right) P_i(x,t) = 0;
\]

where \( i = BC, ABC, C, AC, AB; j = \overline{ABC}, \overline{AB}, \overline{AC}, \overline{BC}, \overline{ABC}, \overline{AB\overline{C}} \) respectively

\[
P_i(0,t) = \delta_{c} P_j(t)
\]

(3.6)

\[
P_i(t) = \sum_{i,j} \lambda_{i} P_j(t); i = A, B; j = \overline{ABC}, \overline{AB}\overline{C} \text{ respectively}
\]

(3.7)

Boundary Conditions:

\[
P_{\overline{ABC}}(0,t) = \sum_{i,j} \lambda_{i} P_j(t); i = A, B; j = \overline{ABC}, \overline{AB}\overline{C} \text{ respectively}
\]

(3.8)

Initial condition

\[
P_{ABC}(0) = 1 \text{ and all other state probabilities are zero at } t = 0.
\]

(3.9)

Taking Laplace transformation from equation (3.1) to (3.8), one gets:

\[
(s + \lambda_A + \lambda_B + \lambda_C) \overline{P}_{ABC}(s) = 1 + \sum_{i,j} \phi_i \overline{P}_j(s) + \sum_{k,l}^\infty \phi_k \overline{P}_l(x,s) dx
\]

where \( i = A, B; j = \overline{ABC}, \overline{AB}\overline{C}; k = AB, BC, ABC, C, AC; \)

\( l = \overline{ABC}, \overline{AB\overline{C}}, \overline{A\overline{BC}}, \overline{A\overline{BC}}, \overline{AB\overline{C}} \) respectively;

(3.10)
\[(s + \lambda_A + \lambda_B + \lambda_C + \phi_A)\overline{P}_{ABC}(s) = \phi_A \overline{P}_{A\overline{B}C}(s) + \phi_b \overline{P}_{A\overline{B}C}(s) + \lambda_A \overline{P}_{ABC}(s)\] (3.11)

\[(s + \lambda_B + \lambda_C + \phi_A)\overline{P}_{A\overline{B}C}(s) = \lambda_A \overline{P}_{ABC}(s)\] (3.12)

\[(s + \lambda_A + \lambda_C + \phi_B)\overline{P}_{A\overline{B}C}(s) = \lambda_B \overline{P}_{ABC}(s) + \phi_A \overline{P}_{ABC}(s)\] (3.13)

\[(s + \lambda_A + \lambda_C + \phi_B + \phi_A)\overline{P}_{\overline{A}\overline{B}C}(s) = \lambda_B \overline{P}_{ABC}(s) + \lambda_A \overline{P}_{ABC}(s)\] (3.14)

\[
\left(\frac{\partial}{\partial x} + s + \phi_i\right)\overline{P}_j(x, s) = 0, \quad \text{where } i = BC, AB\overline{C}, C, AC, AC, AB;
\]

\[
\overline{P}_i(0, s) = \lambda_C \overline{P}_j(s), \quad \text{where } i = AB\overline{C}, \overline{A}B\overline{C}, AB\overline{C}, AB\overline{C}, \overline{A}BC, \overline{A}BC, \overline{A}BC \text{ respectively};
\] (3.15)

\[
\overline{P}_{\overline{A}\overline{B}C}(0, s) = \sum_{i, j} \lambda_i \overline{P}_j(s); i = A, B; j = \overline{A}BC, AB\overline{C} \text{ respectively};
\] (3.16)

Solving equations from (3.10) to (3.15) with the help of equations (3.16-3.17) and initial condition, one obtains:

\[
\overline{P}_{ABC}(s) = \frac{1}{(s + \lambda_A + \lambda_B + \lambda_C) - H_5 - H_6}
\] (3.18)

\[
\overline{P}_{A\overline{B}C}(s) = H_4 \overline{P}_{ABC}(s)
\] (3.19)

\[
\overline{P}_{\overline{A}\overline{B}C}(s) = \frac{H_3}{H_2} \overline{P}_{ABC}(s)
\] (3.20)

\[
\overline{P}_i(s) = \overline{P}_{ABC}(s)H_2 \lambda_A
\] (3.21)

\[
\overline{P}_{\overline{A}\overline{B}C}(s) = \left[\frac{\lambda_B}{(s + \lambda_A + \lambda_C + \phi_B)} + \frac{\phi_A H_4}{(s + \lambda_A + \lambda_C + \phi_B)}\right] \overline{P}_{ABC}(s)
\] (3.22)
\[
\overline{P}_{\lambda\lambda}(s) = \frac{\overline{P}_{\lambda\lambda}(s) H_{3} \lambda C}{(s + \phi_{ABC})} \tag{3.23}
\]

\[
\overline{P}_{\lambda\lambda}(s) = \frac{\lambda C \overline{P}_{\lambda\lambda}(s)}{(s + \phi_{AC})} \tag{3.24}
\]

\[
\overline{P}_{\lambda\lambda}(s) = \frac{\lambda C \overline{P}_{\lambda\lambda}(s)}{(s + \phi_{AC})} \tag{3.25}
\]

\[
\overline{P}_{\lambda\lambda}(s) = \frac{\lambda C \overline{P}_{\lambda\lambda}(s)}{(s + \phi_{AC})} \tag{3.26}
\]

\[
\overline{P}_{\lambda\lambda}(s) = \frac{\lambda A \overline{P}_{\lambda\lambda}(s) + \lambda B \overline{P}_{\lambda\lambda}(s)}{(s + \phi_{AB})} \tag{3.27}
\]

\[
\overline{P}_{\lambda\lambda}(s) = \frac{\lambda C \overline{P}_{\lambda\lambda}(s)}{(s + \phi_{BC})} \tag{3.28}
\]

Where:

\[H_{1} = (s + \phi_{A} + \phi_{AB} + \lambda_{A} + \lambda_{C})(s + \phi_{B} + \lambda_{A} + \lambda_{C}),\]

\[H_{2} = \frac{H_{7}}{(s + \phi_{A} + \lambda_{B} + \lambda_{C})(H_{1} - \lambda_{A} \phi_{A})}, \quad H_{3} = \frac{(H_{1} - \lambda_{A} \phi_{A}) \lambda_{A} + \phi_{B} \lambda_{A} \lambda_{B}}{(H_{1} - \lambda_{A} \phi_{A})},\]

\[H_{4} = \frac{H_{1} \lambda B(s + \lambda_{A} + \lambda_{C} + \phi_{B})}{(H_{1} - \lambda_{A} \phi_{A})H_{2}} + \frac{\lambda A \lambda B}{(H_{1} - \lambda_{A} \phi_{A})},\]

\[H_{5} = \{\phi_{A} + \lambda C \overline{T}_{1}(s)\} \frac{H_{1}}{H_{2}} + \lambda A \overline{T}_{3}(s)H_{4} + \lambda C \overline{T}_{5}(s)H_{4} + \lambda C \overline{T}_{6}(s),\]

\[H_{6}^{6} = \frac{\{\phi_{B} + \lambda C \overline{T}_{4}(s)\} \lambda B}{(s + \lambda_{A} + \lambda_{C} + \phi_{B})} + \frac{\phi_{B} + \lambda C \overline{T}_{4}(s)\phi_{A} H_{4}}{(s + \lambda_{A} + \lambda_{C} + \phi_{B})} + \frac{\lambda A H_{3} \lambda B \overline{T}_{4}(s) + \lambda C \overline{T}_{4}(s)}{H_{2}(s + \lambda_{B} + \lambda_{C} + \phi_{A})},\]

\[H_{7} = (s + \lambda_{A} + \lambda_{B} + \lambda_{C} + \phi_{A})(s + \lambda_{B} + \lambda_{C} + \phi_{A})(H_{1} - \lambda_{A} \phi_{A}) - \lambda A \phi_{A} (H_{1} - \lambda_{A} \phi_{A})\]

\[-\lambda B \phi_{B}(s + \lambda_{A} + \lambda_{C} + \phi_{B})(s + \lambda_{B} + \lambda_{C} + \phi_{A})\]

From the state transition diagram (Fig. 3.2), the working (good and degraded i.e. up state) and failed (i.e. down state) probability of the system is given as:
\( P_{up}(s) = P_{ABC}(s) + P_{ABc}(s) + P_{ABc}(s) + P_{ABC}(s) \) \hfill (3.29)

\[ P_{down}(s) = P_{ABC}(s) + P_{ABc}(s) + P_{ABC}(s) + P_{ABC}(s) \] \hfill (3.30)

(b) Model-II

With the help of Markov process, the state transition diagram (Fig. 3.4) gives the following set of differential equations for the considered model:

\[
\left( \frac{\partial}{\partial t} + \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 \right) P_0(t) = \sum_i \mu_i P_i(t) + \sum_{k,j} \mu_k P_k(x,t)dx + \sum_j \mu_j P_j(x,t)dx
\]

where \( i = 1, 2, 3 \);

\( j = 5, 6, 7 \);

\( k = B, h, AB, AB, AB \);

\( l = 4, 8, 9, 10, 11 \) respectively;

\[
\left( \frac{\partial}{\partial t} + \lambda_B + \lambda_2 + \lambda_3 + \lambda_4 + \mu_1 \right) P_1(t) = \lambda_1 P_0(t) \] \hfill (3.32)

\[
\left( \frac{\partial}{\partial t} + \lambda_B + \lambda_1 + \lambda_3 + \lambda_4 + \mu_2 \right) P_2(t) = \lambda_2 P_0(t) \] \hfill (3.33)

\[
\left( \frac{\partial}{\partial t} + \lambda_B + \lambda_1 + \lambda_2 + \lambda_3 + \mu_3 \right) P_3(t) = \lambda_3 P_0(t) \] \hfill (3.34)

\[
\left( \frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_B \right) P_4(x,t) = 0 \] \hfill (3.35)

\[
\left( \frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu \right) P_5(x,t) = 0 \] \hfill (3.36)

\[
\left( \frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu \right) P_6(x,t) = 0 \] \hfill (3.37)

\[
\left( \frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu \right) P_7(x,t) = 0 \] \hfill (3.38)

\[
\left( \frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu \right) P_8(x,t) = 0 \] \hfill (3.39)

\[
\left( \frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_{AB} \right) P_9(x,t) = 0 \] \hfill (3.40)

\[
\left( \frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_{AB} \right) P_{10}(x,t) = 0 \] \hfill (3.41)
\[
\left( \frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_{AB} \right) P_{11}(x,t) = 0
\]  
(3.42)

\[ P_2(0,t) = \lambda_b P_0(t) \]  
(3.43)

\[ P_3(0,t) = \lambda_1 P_2(t) + \lambda_2 P_1(t) \]  
(3.44)

\[ P_6(0,t) = \lambda_2 P_5(t) + \lambda_3 P_4(t) \]  
(3.45)

\[ P_7(0,t) = \lambda_1 P_6(t) + \lambda_3 P_4(t) \]  
(3.46)

\[ P_8(0,t) = \lambda_1 \{ P_0(t) + P_1(t) + P_2(t) + P_3(t) \} \]  
(3.47)

\[ P_9(0,t) = \lambda_b P_5(t) \]  
(3.48)

\[ P_{10}(0,t) = \lambda_b P_1(t) \]  
(3.49)

\[ P_{11}(0,t) = \lambda_b P_2(t) \]  
(3.50)

Initial condition

\[ P_0(0) = 1 \]  
and all other state probabilities are zero at \( t = 0 \).  
(3.51)

Taking the Laplace transformation from equation (3.31) to (3.50), one gets:

\[
(\text{s} + \lambda_1 + \lambda_2 + \lambda_3 + \lambda_B + \lambda_h) \bar{P}_0(s) = \sum_i \mu_i \bar{P}_i(s) + \sum_{i=0}^{\infty} \mu_k \bar{P}_l(x,s)dx + \sum_{j=0}^{\infty} \mu \bar{P}_j(x,s)dx
\]  
(3.52)

where \( i = 1,2,3; \) \( j = 5,6,7; \)

\[ j = B, h, AB, AB, AB; \]

\[ l = 4,8,9,10,11 \]  
respectively;

\[
(\text{s} + \lambda_B + \lambda_2 + \lambda_3 + \lambda_h + \mu_i) \bar{P}_1(s) = \lambda_1 \bar{P}_0(s)
\]  
(3.53)

\[
(\text{s} + \lambda_B + \lambda_1 + \lambda_3 + \lambda_h + \mu_2) \bar{P}_2(s) = \lambda_2 \bar{P}_0(s)
\]  
(3.54)

\[
(\text{s} + \lambda_B + \lambda_1 + \lambda_2 + \lambda_h + \mu_3) \bar{P}_3(s) = \lambda_3 \bar{P}_0(s)
\]  
(3.55)

\[
\left( \frac{\partial}{\partial x} + s + \mu_B \right) \bar{P}_4(x,s) = 0
\]  
(3.56)

\[
\left( \frac{\partial}{\partial x} + s + \mu \right) \bar{P}_5(x,s) = 0
\]  
(3.57)
\[
\left( \frac{\partial}{\partial x} + s + \mu \right) \overline{P}_6(x, s) = 0 \quad (3.58)
\]

\[
\left( \frac{\partial}{\partial x} + s + \mu \right) \overline{P}_7(x, s) = 0 \quad (3.59)
\]

\[
\left( \frac{\partial}{\partial x} + s + \mu \right) \overline{P}_8(x, s) = 0 \quad (3.60)
\]

\[
\left( \frac{\partial}{\partial x} + s + \mu_{AB} \right) \overline{P}_9(x, s) = 0 \quad (3.61)
\]

\[
\left( \frac{\partial}{\partial x} + s + \mu_{AB} \right) \overline{P}_{10}(x, s) = 0 \quad (3.62)
\]

\[
\left( \frac{\partial}{\partial x} + s + \mu_{AB} \right) \overline{P}_{11}(x, s) = 0 \quad (3.63)
\]

\[
\overline{P}_4(0, s) = \lambda_\beta \overline{P}_0(s) \quad (3.64)
\]

\[
\overline{P}_5(0, s) = \lambda_\beta \overline{P}_2(s) + \lambda_\beta \overline{P}_1(s) \quad (3.65)
\]

\[
\overline{P}_6(0, s) = \lambda_\beta \overline{P}_2(s) + \lambda_\beta \overline{P}_3(s) \quad (3.66)
\]

\[
\overline{P}_7(0, s) = \lambda_\beta \overline{P}_3(s) + \lambda_\beta \overline{P}_1(s) \quad (3.67)
\]

\[
\overline{P}_8(0, s) = \lambda_\beta \left\{ \overline{P}_0(s) + \overline{P}_1(s) + \overline{P}_2(s) + \overline{P}_3(s) \right\} \quad (3.68)
\]

\[
\overline{P}_9(0, s) = \lambda_\beta \overline{P}_3(s) \quad (3.69)
\]

\[
\overline{P}_{10}(0, s) = \lambda_\beta \overline{P}_1(s) \quad (3.70)
\]

\[
\overline{P}_{11}(0, s) = \lambda_\beta \overline{P}_2(s) \quad (3.71)
\]

Solving equations from (3.52) to (3.63) with the help (3.64)-(3.71) and initial condition, one can obtain:

\[
\overline{P}_0(s) = \frac{1}{(H_5 - H_1 - H_2 - H_3 - H_4)} \quad (3.72)
\]

\[
\overline{P}_1(s) = \frac{\overline{P}_0(s) \lambda_1}{(s + \lambda_\beta + \lambda_\beta + \lambda_\beta + \lambda_\beta + \mu_1)} \quad (3.73)
\]

\[
\overline{P}_2(s) = \frac{\overline{P}_0(s) \lambda_2}{(s + \lambda_\beta + \lambda_\beta + \lambda_\beta + \lambda_\beta + \mu_2)} \quad (3.74)
\]
\[ \overline{P}_3(s) = \frac{\overline{P}_0(s)\lambda_1}{(s + \lambda_B + \lambda_1 + \lambda_2 + \lambda_n + \mu_3)} \]  
\hspace{2cm} (3.75)

\[ \overline{P}_4(s) = \frac{\overline{P}_0(s)\lambda_B}{(s + \lambda_B)} \]  
\hspace{2cm} (3.76)

\[ \overline{P}_5(s) = \frac{\overline{P}_0(s)\lambda_1\lambda_2}{(s + \mu)} \left[ \frac{1}{(s + \lambda_B + \lambda_1 + \lambda_2 + \lambda_n + \mu_2)} + \frac{1}{(s + \lambda_B + \lambda_2 + \lambda_3 + \lambda_n + \mu_3)} \right] \]  
\hspace{2cm} (3.77)

\[ \overline{P}_6(s) = \frac{\overline{P}_0(s)\lambda_2\lambda_3}{(s + \mu)} \left[ \frac{1}{(s + \lambda_B + \lambda_1 + \lambda_3 + \lambda_n + \mu_3)} + \frac{1}{(s + \lambda_B + \lambda_2 + \lambda_3 + \lambda_n + \mu_3)} \right] \]  
\hspace{2cm} (3.78)

\[ \overline{P}_7(s) = \frac{\overline{P}_0(s)\lambda_1\lambda_3}{(s + \mu)} \left[ \frac{1}{(s + \lambda_B + \lambda_1 + \lambda_2 + \lambda_n + \mu_2)} + \frac{1}{(s + \lambda_B + \lambda_1 + \lambda_3 + \lambda_n + \mu_3)} \right] \]  
\hspace{2cm} (3.79)

\[ \overline{P}_8(s) = \frac{\overline{P}_0(s)\lambda_n}{(s + \mu_n)} \left[ 1 + \frac{\lambda_1}{(s + \lambda_B + \lambda_1 + \lambda_3 + \lambda_n + \mu_1)} + \frac{1}{(s + \lambda_B + \lambda_2 + \lambda_3 + \lambda_n + \mu_2)} \right] \]  
\hspace{2cm} (3.80)

\[ \overline{P}_9(s) = \frac{\overline{P}_0(s)\lambda_B\lambda_1}{(s + \mu_B)(s + \lambda_B + \lambda_1 + \lambda_2 + \lambda_n + \mu_3)} \]  
\hspace{2cm} (3.81)

\[ \overline{P}_{10}(s) = \frac{\overline{P}_0(s)\lambda_B\lambda_1}{(s + \mu_B)(s + \lambda_B + \lambda_2 + \lambda_3 + \lambda_n + \mu_1)} \]  
\hspace{2cm} (3.82)

\[ \overline{P}_{11}(s) = \frac{\overline{P}_0(s)\lambda_B\lambda_2}{(s + \mu_B)(s + \lambda_B + \lambda_1 + \lambda_3 + \lambda_n + \mu_2)} \]  
\hspace{2cm} (3.83)
Where \[ H_1 = \frac{\lambda_1}{\left( s + \lambda_g + \lambda_c + \lambda_b + \lambda_m + \mu_1 \right) + \left( s + \lambda_g + \lambda_c + \lambda_b + \lambda_m + \mu_1 \right)} \]
\[ H_2 = \frac{\lambda_2}{\left( s + \lambda_g + \lambda_c + \lambda_b + \lambda_m + \mu_1 \right) + \left( s + \lambda_g + \lambda_c + \lambda_b + \lambda_m + \mu_1 \right)} \]
\[ H_3 = \frac{\lambda_3}{\left( s + \lambda_g + \lambda_c + \lambda_b + \lambda_m + \mu_1 \right) + \left( s + \lambda_g + \lambda_c + \lambda_b + \lambda_m + \mu_1 \right)} \]
\[ H_4 = \left[ \frac{\lambda_b}{s + \mu_b} + \frac{\lambda_m}{s + \mu_m} \right] \]

\[ H_5 = (s + \lambda_1 + \lambda_2 + \lambda_3 + \lambda_b + \lambda_m) \]

From the state transition diagram (Fig. 3.4), the working (good and degraded i.e. up state) and failed (i.e. down state) probability of the system is given as:
\[ \bar{P}_{up}(s) = \bar{P}_0(s) + \bar{P}_1(s) + \bar{P}_2(s) + \bar{P}_3(s) \]  
(3.84)
\[ \bar{P}_{down}(s) = \bar{P}_4(s) + \bar{P}_5(s) + \bar{P}_6(s) + \bar{P}_7(s) + \bar{P}_8(s) + \bar{P}_9(s) + \bar{P}_{10}(s) + \bar{P}_{11}(s) \]  
(3.85)

(c) Model-III

With the help of Markov process, the following set of differential equations, which lead the present mathematical model is obtained:
\[ \left( \frac{\partial}{\partial t} + 2\lambda_{A} + \lambda_{B} + \lambda_{C} + \lambda_{E} + \lambda_{CCF} \right) \mathcal{P}_{AB}(t) = \int_{0}^{\infty} \mu_{E} \mathcal{P}_{E}(x,t)dx + \int_{0}^{\infty} \mu_{B} \mathcal{P}_{E}(x,t)dx + \int_{0}^{\infty} \mu_{B} \mathcal{P}_{AB}(x,t)dx \]
\[ + \int_{0}^{\infty} \mu_{CCF} \mathcal{P}_{CCF}(x,t)dx + \int_{0}^{\infty} \mu_{AB} \mathcal{P}_{AB}(x,t)dx + \int_{0}^{\infty} \mu_{ABS} \mathcal{P}_{ABS}(x,t)dx + \int_{0}^{\infty} \mu_{AB} \mathcal{P}_{AB}(x,t)dx \]
\[ + \int_{0}^{\infty} \mu_{ABS} \mathcal{P}_{ABS}(x,t)dx \]  
(3.86)

\[ \left( \frac{\partial}{\partial t} + \lambda_{A} + \lambda_{B} + \lambda_{C} + \lambda_{E} + \lambda_{CCF} \right) \mathcal{P}_{EB}(t) = 2\lambda_{A} \mathcal{P}_{AB}(t) \]  
(3.87)

\[ \left( \frac{\partial}{\partial t} + \lambda_{B} + \lambda_{C} + \lambda_{E} + \lambda_{CCF} \right) \mathcal{P}_{AB}(t) = \lambda_{A} \mathcal{P}_{AB}(t) \]  
(3.88)
\[
\left( \frac{\partial}{\partial t} + 2\lambda_A + \lambda_B + \lambda_h + \lambda_E + \lambda_{CCF} \right) P_{AB}(t) = 2\lambda_B P_{AB}(t) \quad (3.89)
\]

\[
\left( \frac{\partial}{\partial t} + \lambda_A + \lambda_B + \lambda_h + \lambda_E + \lambda_{CCF} \right) P_{AB}(t) = \lambda_B P_{AB}(t) + 2\lambda_A P_{AB}(t) \quad (3.90)
\]

\[
\left( \frac{\partial}{\partial t} + \lambda_B + \lambda_S + \lambda_h + \lambda_E + \lambda_{CCF} \right) P_{AB}(t) = \lambda_B P_{AB}(t) + \lambda_A P_{AB}(t) \quad (3.91)
\]

\[
\left( \frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_E \right) P_E(x,t) = 0 \quad (3.92)
\]

\[
\left( \frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_{CCF} \right) P_{CCF}(x,t) = 0 \quad (3.93)
\]

\[
\left( \frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_B \right) P_{AB}(x,t) = 0 \quad (3.94)
\]

\[
\left( \frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_h \right) P_b(x,t) = 0 \quad (3.95)
\]

\[
\left( \frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_{AB} \right) P_{AB}(x,t) = 0 \quad (3.96)
\]

\[
\left( \frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_{AB} \right) P_{AB}(x,t) = 0 \quad (3.97)
\]

\[
\left( \frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_{ABS} \right) P_{ABS}(x,t) = 0 \quad (3.98)
\]

\[
\left( \frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_{AS} \right) P_{AB}(x,t) = 0 \quad (3.99)
\]

\[
P_{AB}(0,t) = \lambda_B P_{AB}(t) \quad (3.100)
\]

\[
P_{ABS}(0,t) = \lambda_S P_{ABS}(t) \quad (3.101)
\]
\[ P_{\overline{AB}}(0, t) = \lambda_B P_{\overline{AB}}(t) \]  
(3.102)

\[ P_{\overline{ABS}}(0, t) = \lambda_B P_{\overline{ABS}}(t) \]  
(3.103)

\[ P_{\overline{ABS}}(0, t) = \lambda_S P_{\overline{ABS}}(t) \]  
(3.104)

\[ P_E(0, t) = \lambda_E [P_{AB}(t) + P_{\overline{AB}}(t) + P_{\overline{A}}(t) + P_{\overline{ABS}}(t) + P_{\overline{ABS}}(t)] \]  
(3.105)

\[ P_B(0, t) = \lambda_B [P_{AB}(t) + P_{\overline{AB}}(t) + P_{\overline{B}}(t) + P_{\overline{ABS}}(t) + P_{\overline{ABS}}(t)] \]  
(3.106)

\[ P_{CCF}(0, t) = \lambda_{CCF} [P_{AB}(t) + P_{\overline{AB}}(t) + P_{\overline{B}}(t) + P_{\overline{ABS}}(t) + P_{\overline{ABS}}(t)] \]  
(3.107)

Initial condition

\[ P_{AB}(t) = \begin{cases} 1, & t = 0 \\ 0, & \text{otherwise} \end{cases} \quad \text{and all other state probabilities are zero at } t = 0. \]  
(3.108)

Taking Laplace transformation from equation (3.86) to (3.107), one gets:

\[
\left( s + 2\lambda_A + \lambda_B + \lambda_S + \lambda_E + \lambda_{CCF} \right) \overline{P}_{AB}(s) = \int_0^\infty \mu_E \overline{P}_E(x, s)dx + \int_0^\infty \mu_B \overline{P}_B(x, s)dx + \int_0^\infty \mu_S \overline{P}_{\overline{ABS}}(x, s)dx
\]

\[
+ \int_0^\infty \mu_{CCF} \overline{P}_{CCF}(x, s)dx + \int_0^\infty \mu_{ABS} \overline{P}_{\overline{ABS}}(x, s)dx + \int_0^\infty \mu_{AB} \overline{P}_{\overline{AB}}(x, s)dx
\]

\[
+ \int_0^\infty \mu_{ABS} \overline{P}_{\overline{ABS}}(x, s)dx \tag{3.109}
\]

\[
\left( s + \lambda_A + \lambda_B + \lambda_S + \lambda_E + \lambda_{CCF} \right) \overline{P}_{\overline{AB}}(s) = 2\lambda_A \overline{P}_{AB}(s) \tag{3.110}
\]

\[
\left( s + \lambda_B + \lambda_S + \lambda_E + \lambda_{CCF} \right) \overline{P}_{\overline{ABS}}(s) = \lambda_A \overline{P}_{\overline{AB}}(s) \tag{3.111}
\]

\[
\left( s + 2\lambda_A + \lambda_B + \lambda_S + \lambda_E + \lambda_{CCF} \right) \overline{P}_{\overline{AB}}(s) = 2\lambda_B \overline{P}_{AB}(s) \tag{3.112}
\]

\[
\left( s + \lambda_A + \lambda_B + \lambda_S + \lambda_E + \lambda_{CCF} \right) \overline{P}_{\overline{ABS}}(s) = \lambda_B \overline{P}_{\overline{AB}}(s) + 2\lambda_A \overline{P}_{\overline{AB}}(s) \tag{3.113}
\]
\[(s + \lambda_B + \lambda_S + \lambda_h + \lambda_E + \lambda_{CCF})\overline{P}_{\overline{\eta}S}(s) = \lambda_B \overline{P}_{\overline{\eta}S}(s) + \lambda_A \overline{P}_{\overline{\lambda}B}(s)\] (3.114)

\[\left(\frac{\partial}{\partial x} + s + \mu_E\right) \overline{P}_B(x, s) = 0\] (3.115)

\[\left(\frac{\partial}{\partial x} + s + \mu_{CCF}\right) \overline{P}_{CCF}(x, s) = 0\] (3.116)

\[\left(\frac{\partial}{\partial x} + s + \mu_B\right) \overline{P}_{\overline{\eta}B}(x, s) = 0\] (3.117)

\[\left(\frac{\partial}{\partial x} + s + \mu_h\right) \overline{P}_h(x, s) = 0\] (3.118)

\[\left(\frac{\partial}{\partial x} + s + \mu_{AB}\right) \overline{P}_{\overline{\eta}AB}(x, s) = 0\] (3.119)

\[\left(\frac{\partial}{\partial x} + s + \mu_{AB}\right) \overline{P}_{\overline{\eta}AB}(x, s) = 0\] (3.120)

\[\left(\frac{\partial}{\partial x} + s + \mu_{ABS}\right) \overline{P}_{\overline{\eta}ABS}(x, s) = 0\] (3.121)

\[\left(\frac{\partial}{\partial x} + s + \mu_{AS}\right) \overline{P}_{\overline{\eta}AS}(x, s) = 0\] (3.122)

\[\overline{P}_{\overline{\eta}B}(0, s) = \lambda_h \overline{P}_{\overline{\eta}B}(s)\] (3.123)

\[\overline{P}_{\overline{\eta}BS}(0, s) = \lambda_S \overline{P}_{\overline{\eta}BS}(s)\] (3.124)

\[\overline{P}_{\overline{\eta}B}(0, s) = \lambda_h \overline{P}_{\overline{\eta}B}(s)\] (3.125)

\[\overline{P}_{\overline{\eta}AB}(0, s) = \lambda_h \overline{P}_{\overline{\eta}AB}(s)\] (3.126)

\[\overline{P}_{\overline{\eta}BS}(0, s) = \lambda_S \overline{P}_{\overline{\eta}BS}(s)\] (3.127)
\[ P_E(0, s) = \lambda_E [P_{AB}(s) + P_{A\bar{B}}(s) + P_{\bar{A}B}(s) + P_{\bar{A}\bar{B}}(s) + P_{ABS}(s) + P_{\bar{A}B\bar{S}}(s)] \] (3.128)

\[ P_h(0, s) = \lambda_h [P_{AB}(s) + P_{A\bar{B}}(s) + P_{\bar{A}B}(s) + P_{\bar{A}\bar{B}}(s) + P_{ABS}(s) + P_{\bar{A}B\bar{S}}(s)] \] (3.129)

\[ P_{CCF}(0, s) = \lambda_{CCF} [P_{AB}(s) + P_{A\bar{B}}(s) + P_{\bar{A}B}(s) + P_{\bar{A}\bar{B}}(s) + P_{ABS}(s) + P_{\bar{A}B\bar{S}}(s)] \] (3.130)

Solving equations from (3.109) to (3.122) with the help of equations (3.123-3.130) and initial condition, one gets the steady state probabilities of the system as:

\[ P_{AB}(s) = \frac{1}{D(s)} \] (3.131)

\[ P_{A\bar{B}}(s) = \frac{2\lambda_A}{H_1} P_{AB}(s) \] (3.132)

\[ P_{\bar{A}B}(s) = \frac{\lambda_B}{H_2} P_{AB}(s) \] (3.133)

\[ P_{ABS}(s) = \frac{2\lambda_A^2}{H_1H_2} P_{AB}(s) \] (3.134)

\[ P_{AB}(s) = 2\lambda_A \lambda_B \left\{ \frac{1}{H_1^2} + \frac{1}{H_1H_3} \right\} P_{AB}(s) \] (3.135)

\[ P_{ABS}(s) = 2\lambda_A^2 \lambda_B \left\{ \frac{1}{H_1^2H_2} + \frac{1}{H_1H_2H_3} + \frac{1}{H_1H_2^2} \right\} P_{AB}(s) \] (3.136)

\[ P_{\bar{A}B}(s) = \frac{\lambda_B^2}{(s + \mu_\theta)H_3} P_{AB}(s) \] (3.137)

\[ P_{AB}(s) = \frac{2\lambda_A \lambda_B}{(s + \mu_\theta)} \left\{ \frac{1}{H_1^2} + \frac{1}{H_1H_3} \right\} P_{AB}(s) \] (3.138)

\[ P_{ABS}(s) = \frac{2\lambda_A^2 \lambda_B}{(s + \mu_\theta)} \left\{ \frac{1}{H_1^2H_2} + \frac{1}{H_1H_2H_3} + \frac{1}{H_1H_2^2} \right\} P_{AB}(s) \] (3.139)

\[ P_{\bar{A}B\bar{S}}(s) = \frac{2\lambda_A^2 \lambda_B^2 \lambda_S}{(s + \mu_{\bar{A}B\bar{S}})} \left\{ \frac{1}{H_1^2H_2} + \frac{1}{H_1H_2H_3} + \frac{1}{H_1H_2^2} \right\} P_{AB}(s) \] (3.140)
\[ \bar{P}_{ABS}(s) = \frac{2\lambda_A^2 \lambda_s}{(s + \mu_s)H_1H_2} \bar{P}_{AB}(s) \]  
(3.141)

\[ \bar{P}_{E}(s) = \frac{\lambda_E}{(s + \mu_E)} \left[ 1 + \frac{2\lambda_A}{H_1} + \frac{\lambda_B}{H_3} + \frac{2\lambda_A \lambda_B}{H_2^2} + \frac{2\lambda_A^2 \lambda_B}{H_1H_3} + \frac{2\lambda_A^2 \lambda_B}{H_1H_2} + \frac{2\lambda_A^2 \lambda_B}{H_1H_2H_3} \right] \bar{P}_{AB}(s) \]  
(3.142)

\[ \bar{P}_{CCF}(s) = \frac{\lambda_{CCF}}{(s + \mu_{CCF})} \left[ 1 + \frac{2\lambda_A}{H_1} + \frac{\lambda_B}{H_3} + \frac{2\lambda_A \lambda_B}{H_2^2} + \frac{2\lambda_A^2 \lambda_B}{H_1H_3} + \frac{2\lambda_A^2 \lambda_B}{H_1H_2} + \frac{2\lambda_A^2 \lambda_B}{H_1H_2H_3} \right] \bar{P}_{AB}(s) \]  
(3.143)

\[ \bar{P}_{h}(s) = \frac{\lambda_h}{(s + \mu_h)} \left[ 1 + \frac{2\lambda_A}{H_1} + \frac{\lambda_B}{H_3} + \frac{2\lambda_A \lambda_B}{H_2^2} + \frac{2\lambda_A^2 \lambda_B}{H_1H_3} + \frac{2\lambda_A^2 \lambda_B}{H_1H_2} + \frac{2\lambda_A^2 \lambda_B}{H_1H_2H_3} \right] \bar{P}_{AB}(s) \]  
(3.144)

Where

\[ H_1 = (s + \lambda_A + \lambda_B + \lambda_h + \lambda_E + \lambda_{CCF}) \]  
\[ H_2 = (s + \lambda_B + \lambda_h + \lambda_E + \lambda_{CCF} + \lambda_s) \]  
\[ H_3 = (s + 2\lambda_A + \lambda_B + \lambda_h + \lambda_E + \lambda_{CCF}) \]
\[
D(s) = H_3 \left[ 1 + \frac{\lambda_B}{H_3} + \frac{2\lambda_A\lambda_B}{H_4} + \frac{2\lambda_A\lambda_B}{H_1H_3} + \frac{2\lambda_A + 2\lambda_A^2}{H_1} H_1H_2 \right] + \frac{2\lambda_A^2\lambda_B}{H_1^2H_2} + \frac{2\lambda_A^2\lambda_B}{H_1H_2H_3} + \frac{2\lambda_A^2\lambda_B}{H_1H_2} \left[ \frac{\lambda_{E,E}}{(s + \mu_E)} + \frac{\lambda_{A,D}}{(s + \mu_A)} \right] \\
+ \frac{\lambda_{CCF,CCF}}{(s + \mu_{CCF})} - \frac{\lambda_B^2\mu_B}{H_3(s + \mu_B)} - \frac{2\lambda_A^2\lambda_A\mu_A}{H_1H_2(s + \mu_A)} \\
- \frac{\lambda_B\mu_{AB}}{(s + \mu_{AB})} \left[ \frac{2\lambda_A\lambda_B}{H_1^2} + \frac{2\lambda_A\lambda_B}{H_1H_3} - \frac{2\lambda_A^2\lambda_B}{H_1H_2} \right] \left[ \frac{1}{(s + \mu_{AB})} \right] \left[ \frac{1}{H_1H_2} \right] \\
+ \frac{1}{H_1H_2H_3} + \frac{1}{H_1H_2^2} \left[ \frac{\lambda_{HH,\phi_{HH}}}{(s + \phi_{HH})} - \frac{\lambda_{CCF,\phi_{CCF}}}{(s + \phi_{CCF})} \right] + \frac{\lambda_{PSF,\phi_{PSF}}}{(s + \phi_{PSF})} \left[ s + \lambda_{PSF} \right] \left[ \phi_{PSF} + \phi_{SPS} \right] \\
- \frac{\lambda_p(\phi_p)}{(s + \phi_p)} - \frac{\lambda_p(\phi_D)}{(s + \phi_D)} - \frac{\lambda_{WP,\phi_{WP}}}{(s + \phi_{WP})}
\]

From the state transition diagram (Fig. 7.6), one can see that the working (good and degraded i.e. up state) and failed (i.e. down state) probability of the system is given as:

\[
\overline{P}_{up}(s) = \overline{P}_{AB}(s) + \overline{P}_{A\overline{B}}(s) + \overline{P}_{\overline{A}B}(s) + \overline{P}_{\overline{A}}(s) + \overline{P}_{\overline{A}BS}(s) + \overline{P}_{\overline{A}\overline{B}S}(s) + \overline{P}_{\overline{A}\overline{B}}(s)
\]

\[
\overline{P}_{down}(s) = \overline{P}_{\overline{A}B}(s) + \overline{P}_{\overline{A}}(s) + \overline{P}_{AB}(s) + \overline{P}_{CCF}(s) + \overline{P}_{AB}(s) + \overline{P}_{AB}(s) + \overline{P}_{AB}(s) + \overline{P}_{AB}(s)
\]

3.5 Particular Cases and Numerical Computations

3.5.1 Availability Analysis

(a) Model-I

Substituting \( \lambda_A = 0.05, \lambda_B = 0.10, \lambda_C = 0.02 \) and \( \phi_A = \phi_B = \phi_C = \phi_{AB} = \phi_{BC} = \phi_{AC} = \phi_{ABC} = 1 \) in equation (3.29), then using inverse Laplace transform, one gets availability of the system as:

\[
P_{up}(t) = 0.02043203861e^{(-1.117465993t)} - 0.001188476145e^{(-1.32349842t)} + 0.02591449733e^{(-1.32349842t)} + 1.035223471e^{(0.0412785604t)} + 0.05529057868e^{(0.945556176t)} + 0.077280978890e^{(-1.035223471t)} \\
\sin(0.06234965874t) + 0.002223700456e^{(-2.22520335t)} - 0.137895951e^{(1.03745805t)} \cos(0.06234965874t)
\]
Now varying time unit $t$ from 0 to 15 in the equation (3.147), one gets Table 3.1 and Fig. 3.7 for availability of the considered system as:

<table>
<thead>
<tr>
<th>Time ($t$)</th>
<th>Availability $P_{up}(t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.000000</td>
</tr>
<tr>
<td>1</td>
<td>0.9795783</td>
</tr>
<tr>
<td>2</td>
<td>0.9492224</td>
</tr>
<tr>
<td>3</td>
<td>0.9137586</td>
</tr>
<tr>
<td>4</td>
<td>0.8775636</td>
</tr>
<tr>
<td>5</td>
<td>0.8422206</td>
</tr>
<tr>
<td>6</td>
<td>0.8081646</td>
</tr>
<tr>
<td>7</td>
<td>0.7754656</td>
</tr>
<tr>
<td>8</td>
<td>0.7440935</td>
</tr>
<tr>
<td>9</td>
<td>0.7139962</td>
</tr>
<tr>
<td>10</td>
<td>0.6851198</td>
</tr>
<tr>
<td>11</td>
<td>0.6574132</td>
</tr>
<tr>
<td>12</td>
<td>0.6308280</td>
</tr>
<tr>
<td>13</td>
<td>0.6053182</td>
</tr>
<tr>
<td>14</td>
<td>0.5808402</td>
</tr>
<tr>
<td>15</td>
<td>0.5573521</td>
</tr>
</tbody>
</table>

Table 3.1. Availability vs. Time
(b) Model-II

Substituting $\lambda_1 = 0.12, \lambda_2 = 0.15, \lambda_3 = 0.20, \lambda_B = 0.09, \lambda_b = 0.05$ and repair rates as $\mu_1 = \mu_2 = \mu_3 = \mu_{AB} = \mu_B = \mu_b = \mu = 1$ in the equation (3.84) then taking the inverse Laplace transform, one get the availability of the structure as:

$$P_{up}(t) = e^{(-1.14r)} + 0.1148138506 e^{(-1.49r)} - 0.4754860423 e^{(-1.643215657r)} - 0.4662786643 e^{(-1.42250624t)} + 0.8269508557 e^{(-0.004278103386t)}$$

(3.148)

Now varying $t$ from 0 to 10 unit of time in equation (3.148), one gets Table 3.2 and Fig. 3.8 for availability of the considered structure as:
Substituting the values of different parameters as $\lambda_a = 0.048, \lambda_b = 0.025, \lambda_n = 0.009, \lambda_E = 0.10, \\ \lambda_d = 0.03, \lambda_{CCF} = 0.01$, $\mu_{AS} = \mu_B = \mu_h = \mu_{AB} = \mu_{ARS} = \mu_{CCF} = \mu_E = 1$ in the equation (3.145), and then using inverse Laplace transform, one gets the availability of the system as:
$$P_{up}(t) = -0.1099021489e^{(-1.125385378 t)} + 0.009225448228e^{(-0.1806217066 t)} + 0.1105625061e^{(-0.245381066 t)}$$
$$\sin(0.06215644662t) - 0.4015895790e^{(-0.1548747573 t)} + 1.204047747e^{(-0.0358704258 t)}$$
$$+ 0.09222504930e^{(-0.245381066 t)} \cos(0.06215644662t) - 0.0138108318e^{(-0.2244856 t)}$$

(3.149)

Now changing $t$ in the equation (3.149), one obtained the following Table 3.3 and Fig. 3.9 for availability of the considered system as:

<table>
<thead>
<tr>
<th>Time $(t)$</th>
<th>Availability $P_{up}(t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.00000</td>
</tr>
<tr>
<td>1</td>
<td>0.92737</td>
</tr>
<tr>
<td>2</td>
<td>0.89967</td>
</tr>
<tr>
<td>3</td>
<td>0.88416</td>
</tr>
<tr>
<td>4</td>
<td>0.87073</td>
</tr>
<tr>
<td>5</td>
<td>0.85652</td>
</tr>
<tr>
<td>6</td>
<td>0.84093</td>
</tr>
<tr>
<td>7</td>
<td>0.82402</td>
</tr>
<tr>
<td>8</td>
<td>0.80603</td>
</tr>
<tr>
<td>9</td>
<td>0.78722</td>
</tr>
<tr>
<td>10</td>
<td>0.76781</td>
</tr>
<tr>
<td>11</td>
<td>0.74799</td>
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<td>12</td>
<td>0.72792</td>
</tr>
<tr>
<td>13</td>
<td>0.70773</td>
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<tr>
<td>14</td>
<td>0.68754</td>
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<tr>
<td>15</td>
<td>0.66744</td>
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<td>16</td>
<td>0.64750</td>
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<tr>
<td>17</td>
<td>0.62779</td>
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<td>18</td>
<td>0.60837</td>
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<tr>
<td>19</td>
<td>0.58927</td>
</tr>
<tr>
<td>20</td>
<td>0.57054</td>
</tr>
</tbody>
</table>

Table 3.3. Availability vs. Time
3.5.2 Reliability Analysis

(a) Model-I

Putting all repairs equal to zero in the equation (3.29) and setting the values of different failure rates as \( \lambda_a = 0.05, \lambda_b = 0.10, \lambda_c = 0.02 \), then using inverse Laplace transform, the reliability of the system is given as:

\[
R(t) = \left[ 1000(0.0002t + 0.0115)e^{0.17t} + 400000(-0.3125\times10^{-7}t^2 + 0.125\times10^{-6}t) \\
+ 0.00002125e^{0.07t} - 19e^{0.12t} \right] 
\]

(3.150)

Now varying time unit \( t \) from 0 to 15 units of time in the equation (3.150), one gets Table 3.4 and Fig. 3.10 respectively for reliability of system.
<table>
<thead>
<tr>
<th>Time $(t)$</th>
<th>Reliability $R(t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.000000</td>
</tr>
<tr>
<td>1</td>
<td>0.9797022</td>
</tr>
<tr>
<td>2</td>
<td>0.9571523</td>
</tr>
<tr>
<td>3</td>
<td>0.9305093</td>
</tr>
<tr>
<td>4</td>
<td>0.8986663</td>
</tr>
<tr>
<td>5</td>
<td>0.8610713</td>
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<tr>
<td>6</td>
<td>0.8176038</td>
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<td>0.7140847</td>
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<td>9</td>
<td>0.6550568</td>
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<td>10</td>
<td>0.5920736</td>
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<td>0.5258844</td>
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<td>12</td>
<td>0.4572584</td>
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<td>0.3869580</td>
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<td>0.3157176</td>
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<td>0.2442295</td>
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</table>

Table 3.4. Reliability vs. Time

Fig. 3.10. Reliability vs. Time
(b) Model-II

Substituting all repairs equal to zero and $\lambda_1 = 0.12, \lambda_2 = 0.15, \lambda_3 = 0.20, \lambda_B = 0.09, \lambda_H = 0.05$ in equation (3.84), and then using inverse Laplace transform, the reliability of the structure is given as:

$$R(t) = \exp\{-0.61 \cdot t\}$$  \hspace{1cm} (3.151)

Now changing time $t$ from 0 to 10 in the equation (3.151), Table 3.5 and Fig. 3.11 for reliability of the considered system is obtained as:

<table>
<thead>
<tr>
<th>Time ($t$)</th>
<th>Reliability $R(t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.000000</td>
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<tr>
<td>1</td>
<td>0.5433508</td>
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<td>0.0473589</td>
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<td>8</td>
<td>0.0075970</td>
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<tr>
<td>9</td>
<td>0.0041278</td>
</tr>
<tr>
<td>10</td>
<td>0.0022428</td>
</tr>
</tbody>
</table>

Table 3.5: Reliability vs. Time

Fig. 3.11: Reliability vs. Time
(c) Model-III
Substituting all repairs equal to zero and $\lambda_s = 0.03$, $\lambda_{CCF} = 0.01$, $\lambda_A = 0.048$, $\lambda_B = 0.025$, $\lambda_B = 0.009$, $\lambda_E = 0.10$, in equation (3.145) and taking the inverse Laplace transform, the reliability of the system is given as:

$$R(t) = 0.7575757576 \, e^{0.207t} \sinh(0.033t) + 0.2066115702 \, e^{0.240t} + 10.66666 \, e^{-0.183t} \sinh(0.009t) + (0.352969697 \, t - 30.40414143 + 0.0032 \, t^2) \, e^{-0.172t} + (31.19753086 + 0.222222) \, e^{-0.192t}$$

(3.152)

Now changing time unit $t$ in the equation (3.152), one gets Table 3.6 and Fig. 3.12 for reliability of the considered system as:

<table>
<thead>
<tr>
<th>Time ($t$)</th>
<th>Reliability $R(t)$</th>
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<tbody>
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<tr>
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<td>0.94452</td>
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<td>0.88559</td>
</tr>
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<td>3</td>
<td>0.82510</td>
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<td>4</td>
<td>0.76451</td>
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<td>5</td>
<td>0.70493</td>
</tr>
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<td>6</td>
<td>0.64718</td>
</tr>
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<td>7</td>
<td>0.59186</td>
</tr>
<tr>
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<td>0.53937</td>
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<td>0.48996</td>
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<td>0.44379</td>
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<td>14</td>
<td>0.29134</td>
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<tr>
<td>15</td>
<td>0.26085</td>
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<td>16</td>
<td>0.23311</td>
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<tr>
<td>17</td>
<td>0.20795</td>
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<td>18</td>
<td>0.18521</td>
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<tr>
<td>19</td>
<td>0.16469</td>
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<tr>
<td>20</td>
<td>0.14624</td>
</tr>
</tbody>
</table>

Table 3.6. Reliability vs. Time
3.5.3 MTTF Analysis

(a) Model-I

Putting all repairs equal to zero in equation (3.29) and \( s \) tends to zero, one can obtain the MTTF of the system as:

\[
\text{MTTF} = \left\{ \frac{1}{(\lambda_A + \lambda_B + \lambda_C)} + \frac{\{ (\lambda_A + \lambda_C)^2 - \lambda_A \} \lambda_A}{(\lambda_A + \lambda_B + \lambda_C)^2} + \frac{\lambda_A^2 \{ (\lambda_A + \lambda_C)^2 - \lambda_A \}}{(\lambda_A + \lambda_B + \lambda_C)^2} \right\}
\]

\[
+ \frac{\lambda_B \{ (\lambda_A + \lambda_C)^2 - \lambda_A \} \lambda_A}{(\lambda_A + \lambda_B + \lambda_C)^2} + \frac{\lambda_A \lambda_B}{(\lambda_A + \lambda_B + \lambda_C)^2} + \frac{\lambda_B}{(\lambda_A + \lambda_B + \lambda_C)(\lambda_A + \lambda_B + \lambda_C)}
\]

(3.153)

Setting \( \lambda_A = 0.05, \lambda_B = 0.10, \lambda_C = 0.02 \) and varying \( \lambda_A, \lambda_B \) and \( \lambda_C \) from 0.01 to 0.09 one by one respectively in equation (3.153), one gets the MTTF of the considered system with respect to failure rates which is shown in Table 3.7 and Fig. 3.13.
Variations in failure rates & MTTF

<table>
<thead>
<tr>
<th>Variations in failure rates</th>
<th>$\hat{\lambda}_A$</th>
<th>$\hat{\lambda}_B$</th>
<th>$\hat{\lambda}_C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>15.4558404</td>
<td>-186.4628886</td>
<td>-53.2249579</td>
</tr>
<tr>
<td>0.02</td>
<td>-9.0986394</td>
<td>-127.5141273</td>
<td>-25.0195036</td>
</tr>
<tr>
<td>0.03</td>
<td>-19.6666666</td>
<td>-94.4169096</td>
<td>-10.8577427</td>
</tr>
<tr>
<td>0.04</td>
<td>-23.7847222</td>
<td>-73.4649704</td>
<td>-3.3240725</td>
</tr>
<tr>
<td>0.05</td>
<td>-25.0195036</td>
<td>-59.0763686</td>
<td>0.8333333</td>
</tr>
<tr>
<td>0.06</td>
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<td>3.1691408</td>
</tr>
<tr>
<td>0.07</td>
<td>-24.2673909</td>
<td>-40.6161784</td>
<td>4.4782157</td>
</tr>
<tr>
<td>0.08</td>
<td>-23.3333333</td>
<td>-34.3278263</td>
<td>5.1892046</td>
</tr>
<tr>
<td>0.09</td>
<td>-22.3044664</td>
<td>-29.2353771</td>
<td>5.5434078</td>
</tr>
</tbody>
</table>

Table 3.7. MTTF vs. Failure rates

Fig. 3.13. MTTF vs. Failure rates

(b) Model-II

Taking all repairs equal to zero in (3.84) and $s$ tends to zero, one can obtain the MTTF of the structure as:

$$\text{MTTF} = \frac{1}{(\hat{\lambda}_1 + \hat{\lambda}_2 + \hat{\lambda}_3 + \hat{\lambda}_B + \hat{\lambda}_n)} \quad (3.154)$$
Setting $\lambda_1 = 0.12, \lambda_2 = 0.15, \lambda_3 = 0.20, \lambda_B = 0.09, \lambda_h = 0.05$ and varying these failure rates one by one from 0.01 to 0.09 in equation (3.154), one gets the MTTF of the considered structure with respect to failure rates as given in Table 3.8 and corresponding graph has shown in Fig. 3.14.

<table>
<thead>
<tr>
<th>Variations in $\lambda_1, \lambda_2, \lambda_3, \lambda_B, \lambda_h$</th>
<th>MTTF</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_1$</td>
<td>$\lambda_2$</td>
</tr>
<tr>
<td>0.01</td>
<td>2.0000000</td>
</tr>
<tr>
<td>0.02</td>
<td>1.9607843</td>
</tr>
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<td>0.03</td>
<td>1.9230769</td>
</tr>
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<td>0.04</td>
<td>1.8867924</td>
</tr>
<tr>
<td>0.05</td>
<td>1.8518518</td>
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<td>0.06</td>
<td>1.8181818</td>
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<td>0.07</td>
<td>1.7857142</td>
</tr>
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<td>0.08</td>
<td>1.7543859</td>
</tr>
<tr>
<td>0.09</td>
<td>1.7241379</td>
</tr>
</tbody>
</table>

Table 3.8. MTTF vs. Failure rates

![Fig. 3.14. MTTF vs. Failure rates](image-url)
(c) Model-III

Setting all repairs equal to zero in equation (3.145) and $s$ tends to zero, one can obtain the MTTF of the system and putting $\lambda_A = 0.048$, $\lambda_B = 0.025$, $\lambda_h = 0.009$, $\lambda_E = 0.10$, $\lambda_{CCF} = 0.01$, $\lambda_s = 0.03$ and varying failure rates from 0.01 to 0.09 one by one in MTTF expression, one gets the MTTF with respect to failure rates as tabulated in Table 3.9 and Fig. 3.15.

| Variations in $\lambda_A$, $\lambda_B$, $\lambda_h$, $\lambda_E$, $\lambda_{CCF}$ | MTTF |
|---|---|---|---|---|---|---|
| $\lambda_A$ | $\lambda_B$ | $\lambda_h$ | $\lambda_E$ | $\lambda_{CCF}$ | $\lambda_s$ |
| 0.01 | 7.66709 | 11.52280 | 10.88526 | 40.65141 | 10.98523 | 12.59106 |
| 0.02 | 8.56457 | 11.17242 | 9.97007 | 32.54077 | 10.05518 | 11.73355 |
| 0.03 | 9.44254 | 10.79394 | 9.18653 | 26.81942 | 9.25973 | 10.98523 |
| 0.04 | 10.30479 | 10.40709 | 8.50931 | 22.62116 | 8.57284 | 10.32653 |
| 0.05 | 11.15419 | 10.02372 | 7.91899 | 19.44018 | 7.97457 | 9.74228 |
| 0.06 | 11.99295 | 9.65079 | 7.40049 | 16.96522 | 7.44947 | 9.22055 |
| 0.07 | 12.82277 | 9.29220 | 6.94194 | 14.99624 | 6.98539 | 8.75182 |
| 0.08 | 13.64502 | 8.94995 | 6.53388 | 13.39991 | 6.57265 | 8.32842 |
| 0.09 | 14.46081 | 8.62485 | 6.16869 | 12.08451 | 6.20347 | 7.94407 |
| 0.10 | 15.27102 | 8.31697 | 5.84018 | 10.98523 | 5.87153 | 7.59363 |

Table 3.9. MTTF vs. Failure rates
3.5.4 Sensitivity Analysis

1) Sensitivity of Reliability

(a) Model-I

We carry out the sensitivity analysis of reliability by differentiating partially the reliability expression of the system, with respect to failure rates \( \lambda_A, \lambda_B, \lambda_C \) respectively and then putting \( \lambda_A = 0.05, \lambda_B = 0.010, \lambda_C = 0.02 \), the values of \( \frac{\partial R(t)}{\partial \lambda_A}, \frac{\partial R(t)}{\partial \lambda_B}, \frac{\partial R(t)}{\partial \lambda_C} \) is obtained.

Now, taking \( t = 0 \) to 10 units of time in these partial derivatives of reliability with respect to different failure rates, one can obtain the Table 3.10 and Fig. 3.16 for sensitivity analysis with respect to reliability.
(b) Model-II

We carry out the sensitivity analysis of reliability by differentiating the reliability expression with respect to various failure rates, then putting \( \lambda_1 = 0.12, \lambda_2 = 0.15, \lambda_3 = 0.20, \lambda_B = 0.09, \lambda_n = 0.05 \). One gets the values of \( \frac{\partial R(t)}{\partial \lambda_1}, \frac{\partial R(t)}{\partial \lambda_2}, \frac{\partial R(t)}{\partial \lambda_3}, \frac{\partial R(t)}{\partial \lambda_B}, \frac{\partial R(t)}{\partial \lambda_n} \). Taking \( t=0 \) to 10 units...
of time in these partial derivatives, we get the sensitivity of reliability of the considered structure as shown in Table 3.11 and Fig. 3.17.

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Table 3.11. Sensitivity of Reliability vs. Time

Fig. 3.17. Sensitivity of Reliability vs. Time
(c) Model-III

The sensitivity analysis of reliability is carried out by differentiating the reliability expression with respect to various failure rates, and then setting $\lambda_A = 0.048, \lambda_B = 0.025, \lambda_h = 0.009, \lambda_E = 0.10, \lambda_{CCF} = 0.01, \lambda_s = 0.03$, the values of $\frac{\partial R(t)}{\partial \lambda_A}, \frac{\partial R(t)}{\partial \lambda_B}, \frac{\partial R(t)}{\partial \lambda_h}, \frac{\partial R(t)}{\partial \lambda_E}, \frac{\partial R(t)}{\partial \lambda_{CCF}}, \frac{\partial R(t)}{\partial \lambda_s}$ is obtained. Now, taking $t = 0$ to 10 units of time in these partial derivatives, Table 3.12 and Fig. 3.18 obtained respectively.

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<th>$\frac{\partial R(t)}{\partial \lambda_h}$</th>
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Table 3.12. Sensitivity of Reliability vs. Time

79
2) Sensitivity of MTTF

(a) Model-I

By differentiating equation (3.153) with respect to failure rates and then putting the values of different failure rate as $\lambda_A = 0.05, \lambda_B = 0.10, \lambda_C = 0.02$, the values of $\frac{\partial (MTTF)}{\partial \lambda_A}, \frac{\partial (MTTF)}{\partial \lambda_B}$, $\frac{\partial (MTTF)}{\partial \lambda_C}$ is obtained. Varying the failure rates one by one respectively as 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09 in these partial derivatives, one can obtain Table 3.13 and Fig. 3.19 correspondingly.
### Variations in failure rates

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<tr>
<th>Variations in failure rates</th>
<th>( \frac{\partial (\text{MTTF})}{\partial \lambda_A} )</th>
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**Table 3.13. Sensitivity of MTTF vs. Failure rates**

![Graph](image-url)  

**Fig. 3.19. Sensitivity of MTTF vs. Failure rates**

**(b) Model-II**

By differentiating (3.154) with respect to failure rates and then putting the values of failure rates as \( \lambda_1 = 0.12, \lambda_2 = 0.15, \lambda_3 = 0.20, \lambda_B = 0.09, \lambda_A = 0.05 \), the values of \( \frac{\partial (\text{MTTF})}{\partial \lambda_i} \).
\frac{\partial (\text{MTTF})}{\partial \lambda_1}, \frac{\partial (\text{MTTF})}{\partial \lambda_2}, \frac{\partial (\text{MTTF})}{\partial \lambda_3}, \frac{\partial (\text{MTTF})}{\partial \lambda_h} is obtained. Varying the failure rates one by one as 0.01 to 0.09 with interval 0.01 in these partial derivatives, we get the sensitivity of MTTF of the considered structure as shown in Table 3.14 and Fig. 3.20.

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Table 3.14. Sensitivity of MTTF vs. Failure rates

Fig. 3.20. Sensitivity of MTTF vs. Failure rates
(c) Model-III

By differentiating MTTF expression with respect to failure rates and then putting the values of various failure rates as $\lambda_A = 0.048$, $\lambda_B = 0.025$, $\lambda_h = 0.009$, $\lambda_E = 0.10$, $\lambda_{CCF} = 0.01$, $\lambda_S = 0.03$, one can obtained the values of

$$\frac{\partial(MTTF)}{\partial \lambda_A}, \frac{\partial(MTTF)}{\partial \lambda_B}, \frac{\partial(MTTF)}{\partial \lambda_h}, \frac{\partial(MTTF)}{\partial \lambda_E}, \frac{\partial(MTTF)}{\partial \lambda_{CCF}}, \frac{\partial(MTTF)}{\partial \lambda_S}.$$ Varying the failure rates one by one respectively as 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09 in these partial derivatives, one can obtain Table 3.15 and Fig. 3.21 correspondingly.

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Table 3.15. Sensitivity of MTTF vs. Failure rates
3.5.5 Expected Profit

The expected profit throughout \([0, t)\) is specified as:

\[
E_p(t) = K_1 \int_{0}^{t} P_{up}(t) \, dt - tK_2 \tag{3.155}
\]

(a) Model-I

Using equation (3.147), in equation (3.155), the expected profit for the considered system obtained as:

\[
E_p(t) = \begin{cases}
K_1 \left[ 0.000897806983 e^{(-1.323498421)} - 0.009649290457 e^{(2.117465933)} - 0.02560506407 \\
0.1279781301 e^{(-1.03745805)} \cos(0.06234965874 t) - 0.08218199794 e^{(-1.03745805)} - 0.05847448407 e^{(-0.945558176)} + 0.009649290457 e^{(-0.0412785604)} - 0.02560506407 \\
0.02 e^{(-0.06234965874)} - 0.04 e^{(-0.06234965874)} - 0.06 e^{(-0.06234965874)} - 0.08 e^{(-0.06234965874)} - 0.1 e^{(-0.06234965874)} - 0.12 e^{(-0.06234965874)} \right] \\
0.1279781301 e^{(-1.03745805)} \sin(0.06234965874 t) - 0.08218199794 e^{(-1.03745805)} + 0.009649290457 e^{(-0.06234965874)} + 0.02560506407 e^{(-0.0412785604)} + 0.05847448407 e^{(-0.945558176)} - 0.02560506407 \\
0.02 e^{(-0.06234965874)} - 0.04 e^{(-0.06234965874)} - 0.06 e^{(-0.06234965874)} - 0.08 e^{(-0.06234965874)} - 0.1 e^{(-0.06234965874)} - 0.12 e^{(-0.06234965874)} \right] \\
0.1279781301 e^{(-1.03745805)} \end{cases}
\]

Setting \(K_1 = 1\) and varying service cost \(K_2\) as 0.1, 0.2, 0.3, 0.4, 0.5 respectively in (3.156), one can get Table 3.16 and corresponding Fig. 3.22 for expected profit of the system as:

84
Time $(t)$ | Expected Profits $E_p(t)$
--- | ---
$K_2 = 0.1$ | $K_2 = 0.2$ | $K_2 = 0.3$ | $K_2 = 0.4$ | $K_2 = 0.5$
0 | 0 | 0 | 0 | 0 | 0
1 | 0.8903954 | 0.7903954 | 0.6903954 | 0.5903954 | 0.4903954
2 | 1.7555056 | 1.5555056 | 1.3555056 | 1.1555056 | 0.9555056
3 | 2.5871815 | 2.2871815 | 1.9871815 | 1.6871815 | 1.3871815
4 | 3.3828102 | 2.9828102 | 2.5828102 | 2.1828102 | 1.7828102
5 | 4.1426041 | 3.6426041 | 3.1426041 | 2.6426041 | 2.1426041
6 | 4.8676840 | 4.2676840 | 3.6676840 | 3.0676840 | 2.4676840
7 | 5.5593867 | 4.8593867 | 4.1593867 | 3.4593867 | 2.7593867
8 | 6.2190577 | 5.4190577 | 4.6190577 | 3.8190577 | 3.0190577
9 | 6.8499986 | 5.9479986 | 5.0479986 | 4.1479986 | 3.2479986
10 | 7.4474571 | 6.4474571 | 5.4474571 | 4.4474571 | 3.4474571

Table 3.16. Expected profit vs. Time

![Graph showing expected profit vs. time for different values of $K_2$.]
(b) Model-II

Using equation (3.148), in equation (3.155), the expected profit for the considered structure is obtained as:

\[
E_p(t) = K_1 \left\{ -0.7092198582 \ e^{(-1.41t)} - 0.0770562755 \ e^{(-1.49t)} + 0.2893631401 \ e^{(-1.643215657t)} \
+ 0.3277867268 \ e^{(-1.422506248t)} - 193.2984828 \ e^{(-0.00427810388t)} + 193.4676091 \right\} - t \ K_2
\]

(3.157)

Setting \( K_1 = 1 \) and service cost \( K_2 \) as 0.1, 0.2, 0.3, 0.4, 0.5 respectively and varying \( t \) from 0 to 10 in (3.157), one can get the Table 3.17 and corresponding Fig. 3.23 for expected profit of considered structure as:

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<td>0</td>
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</table>

Table 3.17. Expected profit vs. Time
Using equation (3.149), in equation (3.155), the expected profit for the considered structure is obtained as:

\[
E_p(t) = \left\{K_1\left[-0.4604341670 e^{(0.2453886608 t)} \cos(0.06215644662 t) - 0.033944080 2 e^{(-0.2453810660 t)}
\sin(0.0621564466 2t) - 0.0976573448 \left(1 + 10^{-1.1253953780 t}\right) - 0.0510760771 4 e^{(-0.1062170660 0.0615221285 t)} e^{(-0.2244856000 t)} - 33.56658629 e^{(-0.03587042587 t)} + 2.592995695 e^{(-0.1548745736 t)} + 31.52123606 \right] - iK_2 \right\}
\]

Setting \(K_1= 1\) and varying service cost \(K_2\) as 0.1, 0.2, 0.3, 0.4, 0.5 respectively and then varying \(t\) in equation (3.158), Table 3.18 and corresponding Fig. 3.24 for expected profit of the considered system is obtained as.
<table>
<thead>
<tr>
<th>Time (t)</th>
<th>Expected Profits $E_p(t)$</th>
</tr>
</thead>
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<tr>
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</tr>
<tr>
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<td>10</td>
<td>7.57253</td>
</tr>
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</table>

Table 3.18. Expected profit vs. Time

Fig. 3.24. Expected profit vs. Time

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### 3.6 Results Discussion

(a) **Model-I**

On the basis of the above calculation, the following points are observed.

- From Fig. 3.7, reflects that the availability of the system decreases swiftly (as a straight line) as the time increases.
- From Fig. 3.10, the reliability of the system decreases in curved shape with time.
- From Fig. 3.13, the MTTF of the system increases with respect to the failure rate of subsystem $B$ and subsystem $C$, but it is surprising that the **MTTF with respect to the failure rate of subsystem A, first decreases and then becomes approximate a constant**.
- From Fig. 3.16, the sensitivity of the system reliability with respect to failure rates, of subsystems $A$ and $C$, decreases as time passes and with respect to the failure rate of subsystem $B$, it is approximately constant. It indicates that the system reliability is more sensitive with respect to the failure rates of subsystem $A$ and $C$ than the other. So, the system can be made less sensitive by controlling the failure rates of subsystems $A$ and $C$.
- From Fig. 3.19, sensitivity of MTTF decreases with respect to failure rate of subsystem $B$ and $C$ and increases with respect to failure rate of subsystem $A$. The graph points out that MTTF of the system is more sensitive with failure rate of subsystem $B$.
- Keeping the revenue per unit time fixed at one and varying service cost as 0.1, 0.2, 0.3, 0.4, and 0.5, one can obtain Fig. 3.22. It is very clear from the graph that the profit decreases as the service cost increases with the passage of time.

(b) **Model-II**

In this work, the various reliability measures of a $k$-out-of-$n$ subsystem are analyzed. From Fig. 3.8, it is observed that the availability decreases smoothly with respect to time. Fig. 3.11 reveals that the reliability of the structure firstly decreases efficiently and with the passes of time, it becomes constant. MTTF of the structure decreases with respect to the input parameters. From Fig. 3.14, one can observe that the MTTF is lowest with respect to human failure and highest with respect to the failure of third component of the substructure $A$. 

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From Fig. 3.17, it is observed that the sensitivity of reliability is first decreases rapidly, as the time passes and later on, it increases along a curvilinear path. The reliability of given structure is equally sensitive with respect to each failure rate. It is very surprising to see the Fig. 3.17; reliability of the structure is equally sensitive with respect to all types of failure for the designed structure. Further, Fig. 3.20 conveys that sensitivity of MTTF increases with respect to each failure rate, as failure rates vary from 0.01 to 0.09 one by one. It can be seen easily from the graph, the MTTF of the structure is more sensitive with respect to failure rate of third component of substructure A. From Fig. 3.23, we study that expected profit of the structure decreases as service cost increases. So for finding the maximum profit, controlling of service cost is necessary.

(c) Model-III

It is concluded that the graph of availability vs. time (Fig. 3.9) yields that the availability of the system decreases continuously with increment in time. The graph of reliability vs. time (Fig. 3.12) yields that the reliability of the system decreases in a straight line with increment in time. The graph of MTTF (Fig. 3.15) shows that MTTF of the system decreases with respect to all type of failures, which reflect that the failure rates of all units are increasing as time passes. We can see that MTTF is lowest with respect to common cause failure and human failure and highest with respect to electrical failure. The sensitivity of the system reliability with respect to different failure rates are shown in Fig. 3.18. It is surprising that the system is equally sensitive with respect to common cause failure, electrical failure and human failure and highly sensitive with respect to failure rate of subsystem A. Fig. 3.21 shows the sensitivity of MTTF with respect to different failure rates of the system. Critical observation of the graph point out that the MTTF of the system is more sensitive with respect to electrical failure. Keeping the revenue per unit time fixed as one and varying service cost as 0.1, 0.2, 0.3, 0.4, 0.5, one can obtain Fig. 3.24. It is very clear from the Fig. 3. 24 that the profit decreases as the service cost increases.

3.7 Conclusions

(a) Model-I

This work shows the importance of k-out-of-n system in context of unit failure. Availability and reliability of system decreases smoothly as time unit is increased. From the graphs, one can observe that availability is much higher with respect to reliability for the same time unit.
Although in beginning, availability and reliability of the system are just about the same, but later on, availability decreases very slowly in comparison to reliability. This shows the importance of repair facility. So, for maintaining the high reliability of a system, we have to control its failure-rate and make available sufficient repair facility to make the system less sensitive. It is also clear that the sensitivity of the system depends much more upon system failure rates i.e. to say that the system can be made less sensitive by controlling its failures.

(b) Model-II

This work is very useful in real engineering and industrial applications, having $k$-out-of-$n$ structures. $k$-out-of-$n$ structures are widely investigated in the history of reliability. As Erylimaz [45, 46], Erylimaz et al. [47] investigated about $k$-out-of-$n$ system for finding its various reliability measures but they did not consider human error in that work which is an essential factor in context to the failure of any system but here this model investigates reliability measures of $k$-out-of-$n$ system with human error as an extension of that work and concluded that the MTTF of the system is lowest with respect to human error. The finding of this model provided a better understanding in context of human error in $k$-out-of-$n$ systems.

(c) Model-III

As discussed in literature review many other authors included Ram and Singh [49], Erylimaz [45, 46] investigated $k$-out-of-$n$ system but they did not consider human error and common cause failure simultaneously. Here this model investigated simultaneous effect of common cause failure and human error in context of $k$-out-of-$n$ system and found improved results for various reliability measures for the system. It is found that the system is sensitive with respect to these failure rates.