Chapter 8 Performability Analysis of Various Industrial Systems using Markov Technique

In this chapter two complex industrial systems are discussed. These are:

8.1 Introduction

(a) Model IX-Performance Evaluation of a Marine Power Plant under Reliability Approach

Power generation is one of the most essential sections for any plant for continuous functioning without any interruption. A marine power plant (MPP) is the same section. In this model, the authors have tried to find the various reliability measures of a MPP. A marine power plant is composed of two generators in which one of them is located at the stern and other at the bow, each of these two generators is associated to its main switch board (MSB). The distributive switch board (DSB) receives the supply from the MSB through cables and their respective junctions. The two MSB are interconnected through a cable and junction boxes.

(b) Model X-Performance Assessment and Reliability Analysis of a Sugar Mill

Sugar mill is a factory devoted to make sugar from cane. A sugar mill comprises feeding system, evaporation system and crystallization system which are connected with each other in series configuration. Feeding system consist cutting, crushing, bagasse system and heat generating system. After the feeding system, output goes to evaporation and then crystallization for the final output, which is sugar. This work deals with the mathematical modeling and performance optimization for a sugar mill.

8.2 Assumptions and Nomenclature

Including the assumptions mentioned in section 3.2, there are some more assumptions associated with the model. These are:

(a) Model-IX
   (i) No two units can fail simultaneously.
(b) Model-X

(i) Raw material (i.e. cane) is always available for the production of sugar.

Including the nomenclatures mentioned in abbreviations, there are some more nomenclatures associated with the model. These are:

(a) Model-IX

\[ P_{GMD}(t) \] The probability that at time \( t \), the MPP is working with full efficiency.

\[ P_{GMD}(t) \] The probability that at time \( t \), the MPP is operational in the degraded state with one failed generator.

\[ P_{GMD}(t) \] The probability that at time \( t \), the MPP is operational in the degraded state with one failed MSB.

\[ P_{GMD}(t) \] The probability that at time \( t \), the MPP is operational in the degraded state with one failed generator and one failed MSB.

\[ P_{GMD}(x,t) \] The probability that at time \( t \), the MPP is failed due to the failure of both the generators.

\[ P_{GMD}(x,t) \] The probability that at time \( t \), the MPP is failed due to the failure of both the MSB.

\[ P_{GMD}(x,t) \] The probability that at time \( t \), the MPP is failed due to the failure of DSB.

\[ P_{GMD}(x,t) \] The probability that at time \( t \), the MPP is failed due to the failure of both generators and one MSB.

\[ P_{GMD}(x,t) \] The probability that at time \( t \), the MPP is failed due to the failure of one generator and both the MSB.

\[ P_{GMD}(x,t) \] The probability that at time \( t \), the MPP is failed due to the failure of one generators and DSB.

\[ P_{GMD}(x,t) \] The probability that at time \( t \), the MPP is failed due to the failure of one MSB and DSB.

\[ P_{GMD}(x,t) \] The probability that at time \( t \), the MPP is failed due to the failure of one generator, one MSB and DSB.

\[ \lambda_G / \lambda_M / \lambda_D \] The failure rate of generator/MSB/DSB.
\( \mu_G / \mu_M / \mu_D \)

The repair rate of generator/MSB/DSB.

\( \mu_{GM} / \mu_{GD} / \mu_{MD} / \mu_{GMD} \)

Simultaneous repair rate of the generator and MSB/generator and DSB/MSB and DSB/generator, MSB and DSB.

(b) Model-X

\( P_{ABCDEF}(t) \)

The probability that at time \( t \) system is in Good state.

\( P_{AB\bar{C}\bar{D}\bar{E}\bar{F}}(t) \)

The probability that at time \( t \) sugar mill is working in degraded state with failed bagasse carrying system.

\( P_{\bar{A}BCDEF}(x,t) \)

The probability that at time \( t \) sugar mill is failed due to failure of cutting process.

\( P_{\bar{A}BCDE\bar{F}}(x,t) \)

The probability that at time \( t \) sugar mill is failed due to failure of crushing process.

\( P_{ABC\bar{D}\bar{E}\bar{F}}(x,t) \)

The probability that at time \( t \) sugar mill is failed due to failure of heat generating process.

\( P_{ABCD\bar{E}\bar{F}}(x,t) \)

The probability that at time \( t \) sugar mill is failed due to failure of evaporation process.

\( P_{ABC\bar{D}E\bar{F}}(x,t) \)

The probability that at time \( t \) sugar mill is failed due to failure of crystallization process.

\( P_{\bar{A}BC\bar{D}E\bar{F}}(x,t) \)

The probability that at time \( t \) sugar mill is failed due to failure of cutting process and bagasse carrying process.

\( P_{\bar{A}BC\bar{D}E\bar{F}}(x,t) \)

The probability that at time \( t \) sugar mill is failed due to failure of crushing process and bagasse carrying process.

\( P_{\bar{A}BC\bar{D}E\bar{F}}(x,t) \)

The probability that at time \( t \) sugar mill is failed due to failure of heat generating process and bagasse carrying process.

\( P_{\bar{A}BC\bar{D}E\bar{F}}(x,t) \)

The probability that at time \( t \) sugar mill is failed due to failure of evaporation process and bagasse carrying process.

\( P_{\bar{A}BC\bar{D}E\bar{F}}(x,t) \)

The probability that at time \( t \) sugar mill is failed due to failure of crystallization process and bagasse carrying process.

\( P_{\bar{A}BC\bar{D}E\bar{F}}(x,t) \)

The probability that at time \( t \) sugar mill is failed due to failure of cutting process and bagasse carrying process.
Failure rate of cutting process/ crushing process/bagasse carrying process/ heat generating process/ evaporation process/ crystallization process/human error.

Repair rate of cutting process/crushing process/bagasse carrying process/heat generating process/evaporation process/crystallization process/human failure.

Simultaneous repair rate of cutting process and bagasse carrying process/crushing process and bagasse carrying process/heat generating process and bagasse carrying process/evaporation process and bagasse carrying process/ crystallization process and bagasse carrying process.

8.3 System Description

(a) Model-IX

A MPP is a system which is responsible for power generation in a marine. A MPP consists of two generators: one is located as stern and other at the bow, two main switch boards (MSB) and one distributed switch board (DSB). The two MSBs are interconnected by a cable [1]. The DSB get the power from MSB for further distribution and MSB from generators. The configuration of a MPP is shown in Fig. 8.1. Working based transition state diagram has been generated for various reliability measures of MPP and shown in Fig. 8.2.
(b) Model-X

A sugar mill comprises feeding system, evaporation system and crystallization system, which are connected with each other in series configuration as shown in following Fig. 8.3. The considered system may work in three different states, which are good degraded and failed states, throughout the process of sugar making. Corresponding state transition diagram is shown in Fig. 8.4.

![State Transition Diagram](image)

**Fig. 8.4. State Transition Diagram**

**Fig. 8.3. System Description**

Cane → Feeding system → Evaporation system → Crystallization system → Sugar
8.4 Mathematical Formulation and Solution

(a) Model-IX

On the basis of the state transition diagram as shown in Fig. 8.2 and with the help of Markov process, the following set of differential equations is generated:

\[
\begin{align*}
\left( \frac{\partial}{\partial t} + 2\lambda_G + 2\lambda_M + \lambda_D \right) P_{GMD}(t) &= \mu_G P_{GMD}(t) + \mu_M P_{GMD}(t) + \int_0^\infty \mu_D P_{GMD}(x,t)dx + \int_0^\infty \mu_G P_{GMD}(x,t)dx \\
+ \int_0^\infty \mu_M P_{GMD}(x,t)dx + \int_0^\infty \mu_D P_{GMD}(x,t)dx + \int_0^\infty \mu_G P_{GMD}(x,t)dx + \int_0^\infty \mu_M P_{GMD}(x,t)dx \\
+ \int_0^\infty \mu_G P_{GMD}(x,t)dx + \int_0^\infty \mu_D P_{GMD}(x,t)dx
\end{align*}
\]

(8.1)

\[
\left( \frac{\partial}{\partial t} + \lambda_G + 2\lambda_M + \lambda_D + \mu_G \right) P_{GMD}(t) = 2\lambda_G P_{GMD}(t) + \mu_M P_{GMD}(t)
\]

(8.2)
\[
\left( \frac{\partial}{\partial t} + \lambda_G + \lambda_M + \lambda_D + \mu_G + \mu_M \right) P_{GMD}(t) = 2\lambda_M P_{GMD}(t) + 2\lambda_G P_{GMDB}(t) (8.3)
\]

\[
\left( \frac{\partial}{\partial t} + 2\lambda_G + \lambda_M + \lambda_D + \mu_M \right) P_{GMDB}(t) = \mu_G P_{GMD}(t) + 2\lambda_M P_{GMD}(t) (8.4)
\]

\[
\left( \frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_i \right) P_j(x,t) = 0; \quad i = G, D, MD, M, GMD, GM, GD
\quad j = \bar{G}DM, GM\bar{D}, GM\bar{D}, G\bar{M}D, \bar{G}MD, \bar{G}MD, \bar{G}MD, \bar{G}MD
\]

\[
P_j(0,t) = \lambda_j P_{x}(t); \quad j = G, D, D, M, \bar{D}, G, M, \bar{D}
\quad k = \bar{G}MD, GMD, GM\bar{D}, G\bar{M}D, \bar{G}MD, \bar{G}MD, \bar{G}MD, \bar{G}MD
\]

Initial condition

\[
P_{GMD}(0) = 1 \text{ and all other state probabilities are zero at } t = 0. (8.7)
\]

Taking the Laplace transform from the equation (8.1-8.6), one gets:

\[
(s + 2\lambda_G + 2\lambda_M + \lambda_D) \tilde{P}_{GMD}(s) = 1 + \mu_G \tilde{P}_{GMD}(s) + \mu_M \tilde{P}_{GMDB}(s) + \int_0^\infty \mu_D \tilde{P}_{GMDB}(x,s)dx + \int_0^\infty \mu_G \tilde{P}_{GMDBD}(x,s)dx + \int_0^\infty \mu_M \tilde{P}_{GMDBD}(x,s)dx
\]

\[
\int_0^\infty \mu_G \tilde{P}_{GMDB}(x,s)dx + \int_0^\infty \mu_M \tilde{P}_{GMDB}(x,s)dx + \int_0^\infty \mu_G \tilde{P}_{GMDB}(x,s)dx + \int_0^\infty \mu_M \tilde{P}_{GMDB}(x,s)dx + \int_0^\infty \mu_D \tilde{P}_{GMDB}(x,s)dx + \int_0^\infty \mu_M \tilde{P}_{GMDB}(x,s)dx + \int_0^\infty \mu_G \tilde{P}_{GMDB}(x,s)dx + \int_0^\infty \mu_M \tilde{P}_{GMDB}(x,s)dx
\]

\[
(s + \lambda_G + 2\lambda_M + \lambda_D + \mu_G) \tilde{P}_{GMD}(s) = 2\lambda_G \tilde{P}_{GMD}(s) + \mu_M \tilde{P}_{GMDB}(s) (8.9)
\]

\[
(s + \lambda_G + \lambda_M + \lambda_D + \mu_M) \tilde{P}_{GMDB}(s) = 2\lambda_M \tilde{P}_{GMDB}(s) + 2\lambda_G \tilde{P}_{GMDB}(s) (8.10)
\]

\[
(s + 2\lambda_G + \lambda_M + \lambda_D + \mu_M) \tilde{P}_{GMDB}(s) = \mu_G \tilde{P}_{GMDB}(s) + 2\lambda_M \tilde{P}_{GMD}(s) (8.11)
\]

\[
\left( \frac{\partial}{\partial x} + s + \mu \right) \tilde{P}_j(x,s) = 0; \quad i = G, D, MD, M, GMD, GM, GD
\quad j = \bar{G}DM, GM\bar{D}, GM\bar{D}, G\bar{M}D, \bar{G}MD, \bar{G}MD, \bar{G}MD, \bar{G}MD
\]

\[
\tilde{P}_j(0,s) = \lambda_j \tilde{P}_j(s); \quad j = G, D, D, M, \bar{D}, G, M, \bar{D}
\quad k = \bar{G}MD, GMD, GM\bar{D}, G\bar{M}D, \bar{G}MD, \bar{G}MD, \bar{G}MD, \bar{G}MD
\]

Solving equations (8.8-8.13) with the help of initial conditions, the operative transition state probabilities are given as:
\[
\bar{P}_{GMD}(s) = \frac{1}{s + 2\lambda_g + 2\lambda_M + \lambda_D - \frac{\lambda_D H_D}{(s + \mu_D)} - \left(\frac{\mu_G + \frac{\lambda_G \mu_G}{(s + \mu_G)} + \frac{\lambda_D \mu_{GD}}{(s + \mu_{GD})}}{(H_1 - \mu_M H_5)}\right)(2\lambda_g + \mu_M H_4) - H_6 - H_7}
\]

\[
\bar{P}_{GMD}(s) = \bar{P}_{GMD}(s) \left\{ \frac{2\lambda_g + \mu_M H_4}{H_1 - \mu_M H_5} \right\}, \quad \bar{P}_{GMD}(s) = \bar{P}_{GMD}(s) \left\{ \frac{H_4 + H_5(2\lambda_g + \mu_M H_4)}{(H_1 - \mu_M H_5)} \right\}
\]

\[
\bar{P}_{G\bar{M}D}(s) = \bar{P}_{GMD}(s) \left[ \frac{\mu_G}{H_3} \left\{ H_4 + \frac{H_5(2\lambda_g + \mu_M H_4)}{(H_1 - \mu_M H_5)} \right\} + \frac{2\lambda_M}{H_3} \right]
\]

Where

\[ H_1 = (s + \lambda_g + 2\lambda_M + \lambda_D + \mu_G), \quad H_2 = (s + \lambda_g + \lambda_M + \lambda_D + \mu_G + \mu_M) \]

\[ H_3 = (s + 2\lambda_g + \lambda_M + \lambda_D + \mu_M) \]

\[ H_4 = \frac{4\lambda_g \lambda_M}{H_2 H_3} \]

\[ H_5 = \frac{2\lambda_M}{H_2} \]

\[ H_6 = \left\{ \mu_M + \frac{\lambda_D \mu_{MD}}{(s + \mu_{MD})} + \frac{\lambda_M \mu_M}{(s + \mu_M)} \right\} \left\{ \frac{\mu_G}{H_3} \left\{ H_4 + \frac{H_5(2\lambda_g + \mu_M H_4)}{(H_1 - \mu_M H_5)} \right\} + \frac{2\lambda_M}{H_3} \right\} \]

\[ H_7 = \left\{ \frac{\lambda_D \mu_{GMD}}{(s + \mu_{GMD})} + \frac{\lambda_G \mu_{GM}}{(s + \mu_{GM})} + \frac{\lambda_M \mu_{GM}}{(s + \mu_{GM})} \right\} \left\{ \frac{H_4 + \frac{H_5(2\lambda_g + \mu_M H_4)}{(H_1 - \mu_M H_5)}}{H_3} \right\} \]

The probability that the system is in the up (i.e. in good or degraded state) and down (failed state) state at any time is given as:

\[
\bar{P}_{up}(s) = \bar{P}_{GMD}(s) + \bar{P}_{G\bar{M}D}(s) + \bar{P}_{G\bar{M}D}(s)
\]

\[
\bar{P}_{down}(s) = \bar{P}_{GMD}(x,s) + \bar{P}_{G\bar{M}D}(x,s) + \bar{P}_{GM\bar{D}}(x,s) + \bar{P}_{G\bar{MD}}(x,s) + \bar{P}_{G\bar{MD}}(x,s) + \bar{P}_{G\bar{MD}}(x,s) + \bar{P}_{G\bar{MD}}(x,s) + \bar{P}_{G\bar{MD}}(x,s)
\]

(b) Model X

By Markov birth-death process and continuity arguments, the following set of differential equations formed which lead the present mathematical model:

\[
\left( \frac{\partial}{\partial t} + \lambda_A + \lambda_B + \lambda_C + \lambda_D + \lambda_E + \lambda_F + \lambda_h \right) P_{ABCD}(t) = \sum_{i,j=0}^{\infty} \mu_i P_j(x,t) dx
\]
where $i = A, B, D, E, F, h, CF, AC, BC, CD, CE$;

\[ j = \overline{ABCDEF}, \overline{A\overline{B}CDEF}, ABCD\overline{E}F, ABCDE\overline{F}, h, AB\overline{C}DE\overline{F}, \overline{A\overline{B}CDEF}, A\overline{B}CDEF, AB\overline{C}DEF, ABC\overline{D}EF \]

\[
\left( \frac{\partial}{\partial t} + \lambda_A + \lambda_B + \lambda_C + \lambda_D + \lambda_E + \lambda_F + \lambda_h \right) P_{\overline{ABCDEF}} (t) = \lambda_c P_{ABCDEF} (t) \quad (8.17)
\]

\[
\left( \frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \mu_k \right) P_l (x, t) = 0 \quad (8.18)
\]

where $k = A, B, D, E, F, h, CF, AC, BC, CD, CE$;

\[ l = \overline{ABCDEF}, \overline{A\overline{B}CDEF}, ABCD\overline{E}F, ABCDE\overline{F}, h, AB\overline{C}DE\overline{F}, \overline{A\overline{B}CDEF}, A\overline{B}CDEF, AB\overline{C}DEF, ABC\overline{D}EF \]

\[ P_j (0, t) = \lambda_k \; P_j (t) \quad (8.19) \]

where $j = \overline{ABCDEF}, \overline{A\overline{B}CDEF}, ABCD\overline{E}F, ABCDE\overline{F}, h, AB\overline{C}DE\overline{F}, \overline{A\overline{B}CDEF}, A\overline{B}CDEF, AB\overline{C}DEF, ABC\overline{D}EF$;

\[ h, AB\overline{C}DE\overline{F}, \overline{A\overline{B}CDEF}, A\overline{B}CDEF, AB\overline{C}DEF, ABC\overline{D}EF \]

\[ k = A, B, D, E, F, h, F, A, B, D, E \]

\[ l = ABCDEF, AB\overline{C}DEF, ABCDE\overline{F}, ABCDEF, ABCDEF, ABC\overline{D}EF, ABCD\overline{E}F, ABC\overline{D}EF, AB\overline{C}DEF, \overline{A\overline{B}CDEF}, A\overline{B}CDEF, AB\overline{C}DEF, ABC\overline{D}EF \]

Initial condition

\[
P_{ABCDEF} (t) = \begin{cases} 1 & , t = 0 \\ 0 & , \text{otherwise} \end{cases} \quad \text{and initially all other state probabilities are zero.} \quad (8.20)
\]

Taking Laplace transformation from equation (8.16) to (8.19), one gets:

\[
(s + \lambda_A + \lambda_B + \lambda_C + \lambda_D + \lambda_E + \lambda_F + \lambda_h) \overline{P}_{ABCDEF} (s) = 1 + \sum_{i,j}^{\infty} \int_0^\infty \mu_i \overline{P}_j (x, s) dx 
\]

\[
(s + \lambda_A + \lambda_B + \lambda_D + \lambda_E + \lambda_F + \lambda_h) \overline{P}_{AB\overline{C}DEF} (s) = \lambda_c \overline{P}_{ABCDEF} (s) \quad (8.22)
\]

\[
\left( \frac{\partial}{\partial x} + s + \mu_k \right) \overline{P}_l (x, s) = 0 \quad (8.23)
\]

\[
\overline{P}_j (0, s) = \lambda_k \overline{P}_l (s) \quad (8.24)
\]
Solving equations (8.21) to (8.24) with the help of initial condition, the steady state probabilities of the system is given as:

$$
\overline{P}_{ABCDEF}(s) = \frac{1}{H_1 - H_3 - H_4} 
$$  \hspace{1cm} (8.25)

$$
\overline{P}_{ABCDEF}(s) = \frac{\lambda_C}{H_2} \overline{P}_{ABCDEF}(s) 
$$  \hspace{1cm} (8.26)

$$
\overline{P}_{ABCDEF}(s) = \frac{\lambda_A}{s + \mu_A} \overline{P}_{ABCDEF}(s) 
$$  \hspace{1cm} (8.27)

$$
\overline{P}_{ABCDEF}(s) = \frac{\lambda_B}{s + \mu_B} \overline{P}_{ABCDEF}(s) 
$$  \hspace{1cm} (8.28)

$$
\overline{P}_{ABCDEF}(s) = \frac{\lambda_D}{s + \mu_D} \overline{P}_{ABCDEF}(s) 
$$  \hspace{1cm} (8.29)

$$
\overline{P}_{ABCDEF}(s) = \frac{\lambda_E}{s + \mu_E} \overline{P}_{ABCDEF}(s) 
$$  \hspace{1cm} (8.30)

$$
\overline{P}_{ABCDEF}(s) = \frac{\lambda_F}{s + \mu_F} \overline{P}_{ABCDEF}(s) 
$$  \hspace{1cm} (8.31)

$$
\overline{P}_{ABCDEF}(s) = \frac{\lambda_h}{s + \mu_h} \overline{P}_{ABCDEF}(s) 
$$  \hspace{1cm} (8.32)

$$
\overline{P}_{ABCDEF}(s) = \frac{\lambda_C \lambda_F}{H_2 (s + \mu_{CF})} \overline{P}_{ABCDEF}(s) 
$$  \hspace{1cm} (8.33)

$$
\overline{P}_{ABCDEF}(s) = \frac{\lambda_A \lambda_C}{H_2 (s + \mu_{AC})} \overline{P}_{ABCDEF}(s) 
$$  \hspace{1cm} (8.34)

$$
\overline{P}_{ABCDEF}(s) = \frac{\lambda_B \lambda_C}{H_2 (s + \mu_{BC})} \overline{P}_{ABCDEF}(s) 
$$  \hspace{1cm} (8.35)

$$
\overline{P}_{ABCDEF}(s) = \frac{\lambda_C \lambda_D}{H_2 (s + \mu_{CD})} \overline{P}_{ABCDEF}(s) 
$$  \hspace{1cm} (8.36)

$$
\overline{P}_{ABCDEF}(s) = \frac{\lambda_C \lambda_E}{H_2 (s + \mu_{CE})} \overline{P}_{ABCDEF}(s) 
$$  \hspace{1cm} (8.37)

Where
From the state transition diagram (Fig. 8.4), the probability that the system is in upstate and down state is given as:

\[
\begin{align*}
\bar{P}_{up}(s) &= \bar{P}_{ABCDEF}(s) + \bar{P}_{AB\overline{C}DEF}(s) \\
\bar{P}_{down}(s) &= \sum_j \bar{P}_j(s)
\end{align*}
\]

(8.38)

(8.39)

where \( j = \overline{ABCDEF}, ABC\overline{DEF}, ABC\overline{D}EF, ABCDE\overline{F}, ABCDEF, ABC\overline{C}DEF, ABC\overline{DE}F, ABC\overline{DEF}, ABC\overline{DE}F, h \)

8.5 Particular Cases and Numerical Computations

8.5.1 Availability Analysis

(a) Model-IX

For computing availability of the MPP, setting the values of different parameters as \( \lambda_G = 0.003, \lambda_M = 0.011, \lambda_D = 0.111 \) [114] and \( \mu_G = \mu_M = \mu_D = \mu_{MD} = \mu_{GMD} = \mu_{GM} = \mu_{GD} = 1 \) in equation (8.14), then taking inverse Laplace transform, the availability of the system is given as:

\[
P_{up}(t) = 0.1223517562 \, e^{(-1.117696201)} \cos(0.01482285241 \, t) + 0.9099910714 \, e^{(-0.0141460237 \, t)} \\
+ 0.1477367053 \, e^{(-1.117696201)} \sin(0.01482285241 \, t) - 0.9551003000 \times 10^{-6} \, e^{(-2.152746028)} \\
- 0.01458452197 \, e^{(-1.130558144)} - 0.01775739530 \, e^{(-1.123157403)}
\]

(8.40)

Now changing the time unit \( t \) from 0 to 15 in equation (8.40), one gets Table 8.1 and corresponding Fig. 8.5 for availability of MPP as:
<table>
<thead>
<tr>
<th>Time ( (t) )</th>
<th>Availability ( P_{up}(t) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.000000</td>
</tr>
<tr>
<td>1</td>
<td>0.927449</td>
</tr>
<tr>
<td>2</td>
<td>0.894755</td>
</tr>
<tr>
<td>3</td>
<td>0.875583</td>
</tr>
<tr>
<td>4</td>
<td>0.861069</td>
</tr>
<tr>
<td>5</td>
<td>0.848232</td>
</tr>
<tr>
<td>6</td>
<td>0.836069</td>
</tr>
<tr>
<td>7</td>
<td>0.824242</td>
</tr>
<tr>
<td>8</td>
<td>0.812636</td>
</tr>
<tr>
<td>9</td>
<td>0.801212</td>
</tr>
<tr>
<td>10</td>
<td>0.789955</td>
</tr>
<tr>
<td>11</td>
<td>0.778858</td>
</tr>
<tr>
<td>12</td>
<td>0.767917</td>
</tr>
<tr>
<td>13</td>
<td>0.757131</td>
</tr>
<tr>
<td>14</td>
<td>0.746496</td>
</tr>
<tr>
<td>15</td>
<td>0.736010</td>
</tr>
</tbody>
</table>

Table 8.1. Availability vs. Time

![Graph showing Availability vs. Time](image-url)

Fig. 8.5. Availability vs. Time
(b) Model-X

For the availability analysis of considered system put the values of different parameters as 
\[ \lambda_A = 0.04, \lambda_B = 0.04, \lambda_C = 0.045, \lambda_D = 0.09, \lambda_E = 0.21, \lambda_F = 0.21, \lambda_h = 0.11 \quad \text{and} \quad \mu_A = \mu_B = \mu_C = \mu_D = \mu_E = \mu_F = \mu_h = \mu_{AC} = \mu_{BC} = \mu_{CD} = \mu_{CE} = \mu_{CF} = 1 \]
then using inverse Laplace transform, the availability of the system is obtained as:

\[
P_{up}(t) = 0.4107799801 e^{(-1.703033898t)} - 0.00257837510 e^{(-0.7380277981t)}
+ 0.5917983951 e^{(-0.00393830385t)}
\]

(8.41)

Now changing \( t \) in the equation (8.41), the following Table 8.2 and Fig. 8.6 for availability for the sugar mill is obtained as:

<table>
<thead>
<tr>
<th>Time (t)</th>
<th>Availability ( P_{up}(t) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.00000</td>
</tr>
<tr>
<td>25</td>
<td>0.53630</td>
</tr>
<tr>
<td>50</td>
<td>0.48602</td>
</tr>
<tr>
<td>75</td>
<td>0.44044</td>
</tr>
<tr>
<td>100</td>
<td>0.39914</td>
</tr>
<tr>
<td>150</td>
<td>0.32780</td>
</tr>
<tr>
<td>200</td>
<td>0.26921</td>
</tr>
<tr>
<td>250</td>
<td>0.22109</td>
</tr>
</tbody>
</table>

Table 8.2. Availability vs. Time
8.5.2 Reliability Analysis

(a) Model-IX

For computing the reliability of the MPP setting the values of different parameters as
\[ \lambda_G = 0.003, \quad \lambda_M = 0.011, \quad \lambda_D = 0.111[114] \quad \text{and} \quad \mu_G = \mu_M = \mu_D = \mu_{MD} = \mu_{GMD} = \mu_{GM} = \mu_{GD} = 0 \]
in equation (8.14), then taking the inverse Laplace transform, we get the reliability of MPP as:

\[
R(t) = 5 \ e^{(-0.139t)} + 4 \ e^{(-0.1375t)} \ \sinh(0.0015t) + 4 \ e^{(-0.1335t)} \ \sinh(0.0055t) + 4 \ e^{(-0.125t)}
- 4 \ e^{(-0.128t)} - 4 \ e^{(-0.136t)}
\]

(8.42)

Now varying the time unit \( t \), one gets Table 8.3 and corresponding Fig. 8.7 for reliability of the MPP as:
Time \((t)\) | Reliability \(R(t)\)  
---|---
0 | 1.00000 
1 | 0.894823 
2 | 0.800507 
3 | 0.715957 
4 | 0.640185 
5 | 0.572301 
6 | 0.511500 
7 | 0.427060 
8 | 0.408328 
9 | 0.364716 
10 | 0.325698 
11 | 0.290797 
12 | 0.259587 
13 | 0.231684 
14 | 0.206743 
15 | 0.184455 

Table 8.3. Reliability vs. Time

Fig. 8.7. Reliability vs. Time
(b) Model-X

For the reliability analysis, put all repairs equal to zero and various parameters as \( \lambda_A = 0.04, \lambda_B = 0.04, \lambda_C = 0.045, \lambda_D = 0.09, \lambda_E = 0.21, \lambda_F = 0.21, \lambda_g = 0.11 \) in equation (8.3) and using inverse Laplace transform, the reliability of the sugar mill is given as:

\[
R(t) = e^{(-0.745t)} + 2e^{(-0.7225t)} \sinh(0.0225 t) \tag{8.43}
\]

Now varying time unit \( t \) in the equation (8.43), one gets Table 8.4 and corresponding Fig. 8.8.

<table>
<thead>
<tr>
<th>Time ((t))</th>
<th>Reliability (R(t))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.00000</td>
</tr>
<tr>
<td>1</td>
<td>0.49658</td>
</tr>
<tr>
<td>2</td>
<td>0.24659</td>
</tr>
<tr>
<td>3</td>
<td>0.12245</td>
</tr>
<tr>
<td>4</td>
<td>0.06081</td>
</tr>
<tr>
<td>5</td>
<td>0.03019</td>
</tr>
<tr>
<td>6</td>
<td>0.01499</td>
</tr>
<tr>
<td>7</td>
<td>0.00744</td>
</tr>
<tr>
<td>8</td>
<td>0.00369</td>
</tr>
<tr>
<td>9</td>
<td>0.00183</td>
</tr>
<tr>
<td>10</td>
<td>0.00091</td>
</tr>
<tr>
<td>11</td>
<td>0.00045</td>
</tr>
<tr>
<td>12</td>
<td>0.00022</td>
</tr>
<tr>
<td>13</td>
<td>0.00011</td>
</tr>
<tr>
<td>14</td>
<td>0.00005</td>
</tr>
<tr>
<td>15</td>
<td>0.00002</td>
</tr>
</tbody>
</table>

Table 8.4. Reliability vs. Time
8.5.3 MTTF Analysis

(a) Model-IX

For computing the MTTF of the MPP, taking all repairs equal to zero in equation (8.14) and \( s \) tends to zero; one can obtained the MTTF of the MPP as:

\[
\text{MTTF} = \frac{1}{(2\lambda_G + 2\lambda_M + \lambda_D)} + \frac{2\lambda_G}{(2\lambda_G + 2\lambda_M + \lambda_D)(\lambda_G + 2\lambda_M + \lambda_D)} + \frac{2\lambda_M}{(2\lambda_G + 2\lambda_M + \lambda_D)(2\lambda_G + \lambda_M + \lambda_D)} + \frac{4\lambda_G\lambda_M}{(\lambda_G + \lambda_M + \lambda_D)(2\lambda_G + \lambda_M + \lambda_D)} + \frac{4\lambda_G\lambda_M}{(\lambda_G + \lambda_M + \lambda_D)(\lambda_G + 2\lambda_M + \lambda_D)} \tag{8.44}
\]

Setting \( \lambda_G = 0.003, \lambda_M = 0.011, \lambda_D = 0.111 \) [114] and varying failure rates from 0.01 to 0.10 one by one in equation (8.44), one gets the MTTF of the MPP tabulated as in Table 8.5 and Fig. 8.9.
## Table 8.5. MTTF vs. Failure rates

<table>
<thead>
<tr>
<th>Variations in $\lambda_G$, $\lambda_M$, $\lambda_D$</th>
<th>MTTF</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\lambda_G$</td>
</tr>
<tr>
<td>0.01</td>
<td>8.768456</td>
</tr>
<tr>
<td>0.02</td>
<td>8.531786</td>
</tr>
<tr>
<td>0.03</td>
<td>8.238186</td>
</tr>
<tr>
<td>0.04</td>
<td>7.924509</td>
</tr>
<tr>
<td>0.05</td>
<td>7.609688</td>
</tr>
<tr>
<td>0.06</td>
<td>7.303434</td>
</tr>
<tr>
<td>0.07</td>
<td>7.010532</td>
</tr>
<tr>
<td>0.08</td>
<td>6.733079</td>
</tr>
<tr>
<td>0.09</td>
<td>6.471686</td>
</tr>
<tr>
<td>0.10</td>
<td>6.22614</td>
</tr>
</tbody>
</table>

Fig. 8.9. MTTF vs. Failure rates
(b) Model-X

Taking all repairs equal to zero in equation (8.38) and $s$ tends to zero, one gets the MTTF of the system, and then setting $\lambda_A = 0.04, \lambda_B = 0.04, \lambda_C = 0.045, \lambda_D = 0.09, \lambda_E = 0.21, \lambda_F = 0.21, \lambda_h = 0.11$ and varying failure rates from 0.01 to 0.09 one by one in MTTF expression, one gets the MTTF of the sugar mill as tabulated in Table 8.6 and Fig. 8.10.

<table>
<thead>
<tr>
<th>Variations in $\lambda_A, \lambda_B,$ $\lambda_C, \lambda_D,$ $\lambda_E, \lambda_F,$ $\lambda_h$</th>
<th>MTTF</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_A$</td>
<td>$\lambda_B$</td>
</tr>
<tr>
<td>0.01</td>
<td>1.49253</td>
</tr>
<tr>
<td>0.02</td>
<td>1.47058</td>
</tr>
<tr>
<td>0.03</td>
<td>1.44927</td>
</tr>
<tr>
<td>0.04</td>
<td>1.42857</td>
</tr>
<tr>
<td>0.05</td>
<td>1.40845</td>
</tr>
<tr>
<td>0.06</td>
<td>1.38888</td>
</tr>
<tr>
<td>0.07</td>
<td>1.36986</td>
</tr>
<tr>
<td>0.08</td>
<td>1.35135</td>
</tr>
<tr>
<td>0.09</td>
<td>1.33333</td>
</tr>
<tr>
<td>0.10</td>
<td>1.31578</td>
</tr>
</tbody>
</table>

Table 8.6. MTTF vs. Failure rates
8.5.4 Sensitivity Analysis

1) Sensitivity with Respect to Reliability

(a) Model-IX

The sensitivity analysis of reliability is carried out by differentiating the reliability expression with respect to various failure rates, and then setting $\lambda_G = 0.003, \lambda_M = 0.011, \lambda_D = 0.111[114]$. One gets the values of $\frac{\partial R(t)}{\partial \lambda_G}, \frac{\partial R(t)}{\partial \lambda_M}, \frac{\partial R(t)}{\partial \lambda_D}$. Now, taking time unit $t$ from 0 to 10 units of time in these partial derivatives, Table 8.7 and corresponding Fig. 8.11 for MPP is obtained as:
Table 8.7. Sensitivity of Reliability vs. Time

<table>
<thead>
<tr>
<th>Time (t)</th>
<th>$\frac{\partial R(t)}{\partial \lambda_G}$</th>
<th>$\frac{\partial R(t)}{\partial \lambda_M}$</th>
<th>$\frac{\partial R(t)}{\partial \lambda_D}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>-0.005344</td>
<td>-0.019366</td>
<td>-0.894823</td>
</tr>
<tr>
<td>2</td>
<td>-0.019040</td>
<td>-0.068191</td>
<td>-1.601014</td>
</tr>
<tr>
<td>3</td>
<td>-0.038146</td>
<td>-0.135061</td>
<td>-2.147872</td>
</tr>
<tr>
<td>4</td>
<td>-0.060370</td>
<td>-0.211361</td>
<td>-2.560742</td>
</tr>
<tr>
<td>5</td>
<td>-0.083954</td>
<td>-0.290708</td>
<td>-2.861506</td>
</tr>
<tr>
<td>6</td>
<td>-0.107576</td>
<td>-0.368494</td>
<td>-3.069005</td>
</tr>
<tr>
<td>7</td>
<td>-0.130267</td>
<td>-0.441499</td>
<td>-3.199423</td>
</tr>
<tr>
<td>8</td>
<td>-0.151342</td>
<td>-0.507595</td>
<td>-3.266625</td>
</tr>
<tr>
<td>9</td>
<td>-0.170343</td>
<td>-0.565488</td>
<td>-3.282451</td>
</tr>
<tr>
<td>10</td>
<td>-0.186990</td>
<td>-0.614520</td>
<td>-3.256983</td>
</tr>
</tbody>
</table>

(b) Model-X

The sensitivity analysis of reliability of the considered system is carried out by differentiating the reliability expression with respect to various failure rates, then setting
$\lambda_A = 0.04, \lambda_B = 0.04, \lambda_C = 0.045, \lambda_D = 0.09, \lambda_E = 0.21, \lambda_F = 0.21, \lambda_h = 0.11$, the values of \( \frac{\partial R(t)}{\partial \lambda_A}, \frac{\partial R(t)}{\partial \lambda_B}, \frac{\partial R(t)}{\partial \lambda_C}, \frac{\partial R(t)}{\partial \lambda_D}, \frac{\partial R(t)}{\partial \lambda_E}, \frac{\partial R(t)}{\partial \lambda_F}, \frac{\partial R(t)}{\partial \lambda_h} \) is obtained. Now, taking $t = 0$ to 10 unit of time in these partial derivatives, one can obtained the Table 8.8 and corresponding Fig. 8.12.

<table>
<thead>
<tr>
<th>Time ($t$)</th>
<th>$\frac{\partial R(t)}{\partial \lambda_A}$</th>
<th>$\frac{\partial R(t)}{\partial \lambda_B}$</th>
<th>$\frac{\partial R(t)}{\partial \lambda_C}$</th>
<th>$\frac{\partial R(t)}{\partial \lambda_D}$</th>
<th>$\frac{\partial R(t)}{\partial \lambda_E}$</th>
<th>$\frac{\partial R(t)}{\partial \lambda_F}$</th>
<th>$\frac{\partial R(t)}{\partial \lambda_h}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>-0.49658</td>
<td>-0.49658</td>
<td>-0.01084</td>
<td>-0.49658</td>
<td>-0.49658</td>
<td>-0.49658</td>
<td>-0.49658</td>
</tr>
<tr>
<td>2</td>
<td>-0.49319</td>
<td>-0.49319</td>
<td>-0.02090</td>
<td>-0.49319</td>
<td>-0.49319</td>
<td>-0.49319</td>
<td>-0.49319</td>
</tr>
<tr>
<td>3</td>
<td>-0.36736</td>
<td>-0.36736</td>
<td>-0.02267</td>
<td>-0.36736</td>
<td>-0.36736</td>
<td>-0.36736</td>
<td>-0.36736</td>
</tr>
<tr>
<td>4</td>
<td>-0.24324</td>
<td>-0.24324</td>
<td>-0.01943</td>
<td>-0.24324</td>
<td>-0.24324</td>
<td>-0.24324</td>
<td>-0.24324</td>
</tr>
<tr>
<td>5</td>
<td>-0.15098</td>
<td>-0.15098</td>
<td>-0.01464</td>
<td>-0.15098</td>
<td>-0.15098</td>
<td>-0.15098</td>
<td>-0.15098</td>
</tr>
<tr>
<td>6</td>
<td>-0.08997</td>
<td>-0.08997</td>
<td>-0.01016</td>
<td>-0.08997</td>
<td>-0.08997</td>
<td>-0.08997</td>
<td>-0.08997</td>
</tr>
<tr>
<td>7</td>
<td>-0.05212</td>
<td>-0.05212</td>
<td>-0.00667</td>
<td>-0.05212</td>
<td>-0.05212</td>
<td>-0.05212</td>
<td>-0.05212</td>
</tr>
<tr>
<td>8</td>
<td>-0.02958</td>
<td>-0.02958</td>
<td>-0.00420</td>
<td>-0.02958</td>
<td>-0.02958</td>
<td>-0.02958</td>
<td>-0.02958</td>
</tr>
<tr>
<td>9</td>
<td>-0.01652</td>
<td>-0.01652</td>
<td>-0.00256</td>
<td>-0.01652</td>
<td>-0.01652</td>
<td>-0.01652</td>
<td>-0.01652</td>
</tr>
<tr>
<td>10</td>
<td>-0.00911</td>
<td>-0.00911</td>
<td>-0.00152</td>
<td>-0.00911</td>
<td>-0.00911</td>
<td>-0.00911</td>
<td>-0.00911</td>
</tr>
</tbody>
</table>

Table 8.8. Sensitivity of Reliability vs. Time

![Graph showing the sensitivity of reliability vs. time](image)

Fig. 8.12. Sensitivity of Reliability vs. Time
2) Sensitivity of MTTF

(a) Model-IX

For computing the sensitivity analysis of the MPP with respect to MTTF, differentiating equation (8.44) with respect to various failure rates, then putting the values of various failure rates as $\lambda_G = 0.003$, $\lambda_M = 0.011$, $\lambda_D = 0.111$ [114, 95], the values of

$$\frac{\partial(\text{MTTF})}{\partial \lambda_G}, \frac{\partial(\text{MTTF})}{\partial \lambda_M}, \frac{\partial(\text{MTTF})}{\partial \lambda_D}$$

is obtained. Now varying the failure rates one by one respectively as 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09, 0.10 in these partial derivatives, one can obtain the Table 8.9 and corresponding Fig. 8.13 as:

<table>
<thead>
<tr>
<th>Variations in $\lambda_G$, $\lambda_M$ &amp; $\lambda_D$</th>
<th>$\frac{\partial(\text{MTTF})}{\partial \lambda_G}$</th>
<th>$\frac{\partial(\text{MTTF})}{\partial \lambda_M}$</th>
<th>$\frac{\partial(\text{MTTF})}{\partial \lambda_D}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>-18.828004</td>
<td>-19.937357</td>
<td>-3260.827933</td>
</tr>
<tr>
<td>0.02</td>
<td>-27.345142</td>
<td>-28.684763</td>
<td>-1445.660690</td>
</tr>
<tr>
<td>0.03</td>
<td>-30.788850</td>
<td>-32.067973</td>
<td>-796.836785</td>
</tr>
<tr>
<td>0.04</td>
<td>-31.644330</td>
<td>-32.776529</td>
<td>-499.058284</td>
</tr>
<tr>
<td>0.05</td>
<td>-31.164750</td>
<td>-32.137029</td>
<td>-339.838967</td>
</tr>
<tr>
<td>0.06</td>
<td>-30.009906</td>
<td>-30.834707</td>
<td>-245.450799</td>
</tr>
<tr>
<td>0.07</td>
<td>-28.537386</td>
<td>-29.234359</td>
<td>-185.174359</td>
</tr>
<tr>
<td>0.08</td>
<td>-26.944078</td>
<td>-27.533202</td>
<td>-144.459072</td>
</tr>
<tr>
<td>0.09</td>
<td>-25.338557</td>
<td>-25.837752</td>
<td>-115.724449</td>
</tr>
<tr>
<td>0.10</td>
<td>-23.779663</td>
<td>-24.204187</td>
<td>-94.719457</td>
</tr>
</tbody>
</table>

Table 8.9. Sensitivity of MTTF vs. Failure rates
Fig. 8.13. Sensitivity of MTTF vs. Failure rates

(b) Model-X

By differentiating MTTF expression with respect to failure rates and putting the values of various failure rates as $\lambda_A = 0.04, \lambda_B = 0.04, \lambda_C = 0.045, \lambda_D = 0.09$, $\lambda_E = 0.21, \lambda_F = 0.21, \lambda_h = 0.11$, the values of $\frac{\partial \text{MTTF}}{\partial \lambda_A}, \frac{\partial \text{MTTF}}{\partial \lambda_B}, \frac{\partial \text{MTTF}}{\partial \lambda_C}, \frac{\partial \text{MTTF}}{\partial \lambda_D}$, $\frac{\partial \text{MTTF}}{\partial \lambda_E}, \frac{\partial \text{MTTF}}{\partial \lambda_F}, \frac{\partial \text{MTTF}}{\partial \lambda_h}$ is obtained. Varying failure rates one by one respectively as 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09 in these partial derivatives, one can obtained Table 8.10 and corresponding Fig. 8.14.
Variations in failure rates

<table>
<thead>
<tr>
<th>Variations in failure rates</th>
<th>$\frac{\partial (MTTF)}{\partial \lambda_A}$</th>
<th>$\frac{\partial (MTTF)}{\partial \lambda_B}$</th>
<th>$\frac{\partial (MTTF)}{\partial \lambda_C}$</th>
<th>$\frac{\partial (MTTF)}{\partial \lambda_D}$</th>
<th>$\frac{\partial (MTTF)}{\partial \lambda_E}$</th>
<th>$\frac{\partial (MTTF)}{\partial \lambda_F}$</th>
<th>$\frac{\partial (MTTF)}{\partial \lambda_h}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>-2.22766</td>
<td>-2.22766</td>
<td>-0.02833</td>
<td>-2.60145</td>
<td>-4.00000</td>
<td>-4.00000</td>
<td>-2.77777</td>
</tr>
<tr>
<td>0.02</td>
<td>-2.16262</td>
<td>-2.16262</td>
<td>-0.05511</td>
<td>-2.51952</td>
<td>-3.84467</td>
<td>-3.84467</td>
<td>-2.68744</td>
</tr>
<tr>
<td>0.03</td>
<td>-2.10039</td>
<td>-2.10039</td>
<td>-0.08042</td>
<td>-2.44140</td>
<td>-3.69822</td>
<td>-3.69822</td>
<td>-2.60145</td>
</tr>
<tr>
<td>0.04</td>
<td>-2.04081</td>
<td>-2.04081</td>
<td>-0.10435</td>
<td>-2.36686</td>
<td>-3.55998</td>
<td>-3.55998</td>
<td>-2.51952</td>
</tr>
<tr>
<td>0.05</td>
<td>-1.98373</td>
<td>-1.98373</td>
<td>-0.12698</td>
<td>-2.29568</td>
<td>-3.42935</td>
<td>-3.42935</td>
<td>-2.44140</td>
</tr>
<tr>
<td>0.06</td>
<td>-1.92901</td>
<td>-1.92901</td>
<td>-0.14839</td>
<td>-2.22766</td>
<td>-3.30578</td>
<td>-3.30578</td>
<td>-2.36686</td>
</tr>
<tr>
<td>0.07</td>
<td>-1.87652</td>
<td>-1.87652</td>
<td>-0.16866</td>
<td>-2.16262</td>
<td>-3.18877</td>
<td>-3.18877</td>
<td>-2.29568</td>
</tr>
<tr>
<td>0.08</td>
<td>-1.82615</td>
<td>-1.82615</td>
<td>-0.18784</td>
<td>-2.10039</td>
<td>-3.07787</td>
<td>-3.07787</td>
<td>-2.22766</td>
</tr>
<tr>
<td>0.09</td>
<td>-1.77777</td>
<td>-1.77777</td>
<td>-0.20601</td>
<td>-2.04081</td>
<td>-2.97265</td>
<td>-2.97265</td>
<td>-2.16262</td>
</tr>
</tbody>
</table>

Table 8.10. Sensitivity of MTTF vs. Failure rates

![Fig. 8.14. Sensitivity of MTTF vs. Failure rates](image-url)
8.5.5 Expected Profit

(a) Model-IX

Using equation (8.40) in equation (3.155) of section 3.5.5, the profit function is obtained as:

\[
E_p(t) = \left\{ K_1 [-0.1112012153 e^{-1.1176920110} \cos(0.01482285241t) - 0.1307049142 e^{-1.1176920208} \sin(0.01482285241t) - 64.32839971 e^{(-0.01414023771)} + 0.443660375 \times 10^{-6} e^{(-2.152746028)} + 0.01290028474 e^{(-1.130558144)} - 0.01581024641 e^{(-1.123157408)} + 64.41088996] - tK_2 \right\}
\]

(8.45)

Setting revenue \( K_1 = 1 \) and varying service cost as 0.1, 0.2, 0.3, 0.4, 0.5 and 0.6 respectively, then varying time scale \( t \) in equation (8.45), Table 8.1 and correspondingly Fig. 8.15 for expected profit of MPP is obtained as:

<table>
<thead>
<tr>
<th>Time ( t )</th>
<th>Expected Profit ( E_p(t) )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( K_2 = 0.1 )</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0.858386</td>
</tr>
<tr>
<td>2</td>
<td>1.667683</td>
</tr>
<tr>
<td>3</td>
<td>2.452236</td>
</tr>
<tr>
<td>4</td>
<td>3.220346</td>
</tr>
<tr>
<td>5</td>
<td>3.974915</td>
</tr>
<tr>
<td>6</td>
<td>4.717029</td>
</tr>
<tr>
<td>7</td>
<td>5.447163</td>
</tr>
<tr>
<td>8</td>
<td>6.165586</td>
</tr>
<tr>
<td>9</td>
<td>6.872497</td>
</tr>
<tr>
<td>10</td>
<td>7.568067</td>
</tr>
</tbody>
</table>

Table 8.11. Expected Profit vs. Time
Using equation (8.41) in equation (3.155) of section 3.5.5, the profit function for sugar mill is given as:

\[ E_p(t) = \{K_1[-0.2412048172 e^{(-1.70303898 t)} + 0.003493601605 e^{(-0.738027798 t)}] - 150.2673393 e^{(0.0039380354 t)} + 150.5050505] - K_2 t \} \]  

(8.46)

Setting \( K_1 = 1 \) and \( K_2 = 0.1, 0.2, 0.3, 0.4, 0.5 \) and 0.6 respectively then varying \( t \) in (8.46), Table 8.12 and corresponding Fig. 8.16 is obtained as:
Table 8.12. Expected profit vs. Time

<table>
<thead>
<tr>
<th>Time (t)</th>
<th>Expected Profit $E_p(t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$K_2 = 0.1$</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0.68608</td>
</tr>
<tr>
<td>2</td>
<td>1.20945</td>
</tr>
<tr>
<td>3</td>
<td>1.70158</td>
</tr>
<tr>
<td>4</td>
<td>2.18627</td>
</tr>
<tr>
<td>5</td>
<td>2.66779</td>
</tr>
<tr>
<td>6</td>
<td>3.14691</td>
</tr>
<tr>
<td>7</td>
<td>3.62373</td>
</tr>
<tr>
<td>8</td>
<td>4.09830</td>
</tr>
<tr>
<td>9</td>
<td>4.57061</td>
</tr>
<tr>
<td>10</td>
<td>5.04067</td>
</tr>
</tbody>
</table>

Fig. 8.16. Expected profit vs. Time
8.6 Results Discussion

(a) Model-IX

In this model, the authors have tried to find the various reliability characteristics of the marine power plant. The findings are:

The graph between availability and time shown in Fig. 8.5, it reveals that availability of marine power plant decrease smoothly as time passes. The behavior of reliability of MPP with respect to the time scale \( t \) is shown in Fig. 8.7. It can be seen from the figure that the reliability of MPP decreases faster than availability as time passes. This reflects the importance of repair policy. Fig. 8.9 shows the graph between MTTF of marine power plant and variation in failure rates. From the graph, it is observed that the MTTF on marine power plant decreases with respect to all types of failure rates and it is highest with respect to failure rates of the DSB i.e. MTTF of MPP is much effected by the failure rate of DSB. Fig. 8.11 shows the sensitivity analysis of reliability of MPP with respect to different failure rates. This shows the reliability of MPP is utmost sensitive with respect to the failure rate of the DSB. This shows that in order to make the system reliability less sensitive one have to control the failure rate of DSB. The sensitivity of MTTF for marine power plant is shown in Fig. 8.13. It reflects that the MTTF is equally sensitive with respect to the failure rate of generator and MSB. Also, it is very much sensitive with respect to the failure rate of the DSB. From this, one can say that we have to focus more on the failure rate of DSB to enhance reliability of MPP. Keeping the revenue per unit time fixed at 1 and varying service cost as 0.1, 0.2, 0.3, 0.4, 0.5 and 0.6, Fig. 8.15 is obtained. It is very clear from the graph that the profit decreases as the service cost increases with the passage of time unit. So, in order to optimize the profit function, one has to control service cost as well as various failure rates of marine power plant.

(b) Model-X

- From Fig. 8.6, we observe that the availability of the system first decreases rapidly and then swiftly as time passes.
- From Fig. 8.8, the reliability of the system decreases smoothly as time passes.
From Fig. 8.10, we observe that the MTTF of the system is decreasing with respect to all type of failure rates except failure rate of bagasse carrying system. MTTF with respect to failure rate of bagasse carrying system is approximately constant.

Fig. 8.12 shows the sensitivity analysis of reliability with respect to different failure rates. We can see that the sensitivity of reliability is approximately constant with respect to failure rate of bagasse carrying system and for remaining failure rates of the system it first decreases and then increases.

Fig. 8.14 shows the sensitivity of MTTF. We can see that the MTTF is equally sensitive with respect to the failure rate of cutting and crushing system, evaporation and crystallization and it is approximately constant with respect to failure rate of bagasse carrying system.

Keeping the revenue per unit time fixed as 1 and varying service cost as 0.1, 0.2, 0.3, 0.4, and 0.5, one can obtain Fig. 8.16. It is very clear from the graph that the profit decreases as the service cost increases with passage of time unit and in order to maximize the profit one has to control its failure rates, which is possible by providing sufficient repair facility and necessary amendment in the operations.

8.7 Conclusion

(a) Model-IX

This model investigated the various performance measures of a MPP. MPP was also investigated by Kumar et al. [114] with the aid fuzzy theory with interval valued vague sets for finding its reliability. Compared to the work done by Kumar et al. [114], this model investigated MPP with the help of Markov process and supplementary variable technique and found some more reliability characteristics for the same which was not founded by [114]. It is found that the MPP is highly sensitive with respect to failure rate of DSB. Also, it is observed that, MPP is equally sensitive with respect to failure rates of generator and MSB. The difference between the graph of availability and reliability shows the importance of repair facility. Hence, we can say that a good maintenance is the major requirement for the successful functioning of any system.
(b) Model-X

In this model, the feeding, evaporation and crystallization system of a sugar mill is investigated. Compared to the work done by Kumar et al. [61] for calculating availability analysis of feeding system of a sugar mill, here, author investigated not only the feeding system, but also evaporation and crystallization system of a sugar mill. On the basis of the above calculation, one can say that the failure rate of bagasse carrying system has not much impact on the production and MTTF is sensitive with respect to the failure rate of sub parts of feeding system (cutting and crusher system). Also most surprised thing is that reliability is equally sensitive with respect to all type of failure rates, accepts bagasse carrying system. So to make our system more reliable, we have to consider these points and try to reduce these failure rates.