Chapter 7  Industrial System Performance Assessment under Multi Failures with Standby Mode

Model-VIII

7.1 Introduction

In the present chapter, we have considered a complex industrial system which consist $n$-units, connected in parallel configuration, with a standby unit. The designed type industrial system can be easily found in a paper plant system, sugar industry based system, hydro power plant etc. The considered system can work in three different modes, namely good, degraded and failed state. There are four types of failure that we have considered, namely, unit failure, catastrophic failure, human error and standby unit failure. When all the parallel units of the system fail, standby unit will automatically start. The system goes to complete failed state due to human error, catastrophic failure, all units’ failure with failure of standby unit. The system can repair after a unit failure (partially failed) as well as the major failure (completely failed).

7.2 Assumptions and Nomenclature

Including the assumptions mentioned in section 3.2, there are some more assumptions associated with the model. These are:

(i) The change over time from active to standby unit is negligible.

Including the nomenclatures mentioned in abbreviations, there are some more nomenclatures associated with the model. These are:

In the considered system $P_0(t), P_1(t), P_2(t), P_3(t), P_{n-1}(t), P_c(x,t), P_h(x,t), P_r(t), P(x,t)$ are the probabilities that at time $t$, when the system is working with full capacity, with one failed unit, with two failed units, with three failed units, with $(n-1)$ failed units, failed due to catastrophic failure, failed due to human failure, working with standby unit, failed due to failure of standby unit with failure of all $n$ units respectively. $P_c(x,t), P_h(x,t), P(x,t)$ are the probability that the system is in complete failed state and the system is running under repair and elapsed repair time is $x$. $\lambda, \lambda_c, \lambda_h, \alpha$ are the failure rate of parallel unit, catastrophic failure, human error, human error,
complete system failure/standby unit failure respectively. \( \phi, \phi_h, \phi_C, \delta \) are the repair rates of parallel unit, human failure, catastrophic failure and standby unit respectively.

### 7.3 System Configuration

The considered industrial system consist \( n \) units in parallel configuration with a standby unit, as shown in Fig. 7.1. There are four types of failure that we considered throughout the task; these are unit failure, catastrophic failure, human failure and the failure of standby unit. The corresponding state transition diagram is shown in Fig. 7.2.

![System Configuration Diagram]

*Fig. 7.1. System Configuration*
7.4 Mathematical Formulation and Solution

By the probability considerations and continuity arguments, the following set of differential equations governing the present mathematical model is obtained:

\[
\frac{\partial}{\partial t} + n\lambda + \lambda_h + \lambda_c + \alpha P_0(t) = \delta P_s(t) + \phi P_1(t) + \int_0^\infty \phi_h P_n(x,t)dx + \int_0^\infty \delta P(x,t)dx + \int_0^\infty \phi_c P_C(x,t)dx
\]

(7.1)

\[
\frac{\partial}{\partial t} + \phi + (n-1)\lambda + \lambda_c + \lambda_h P_1(t) = n\lambda P_0(t) + \phi P_2(t)
\]

(7.2)

\[
\frac{\partial}{\partial t} + \phi + (n-2)\lambda + \lambda_c + \lambda_h P_2(t) = (n-1)\lambda P_1(t) + \phi P_3(t)
\]

(7.3)

\[
\frac{\partial}{\partial t} + \phi + (n-3)\lambda + \lambda_c + \lambda_h P_3(t) = (n-2)\lambda P_2(t) + \phi P_{n-1}(t)
\]

(7.4)
\[
\left( \frac{\partial}{\partial t} + \phi + \lambda + \lambda_c + \lambda_h \right) P_{n-1}(t) = (n - 3) \lambda P_3(t) \tag{7.5}
\]

\[
\left( \frac{\partial}{\partial t} + \alpha + \delta + \lambda_c + \lambda_h \right) P_s(t) = \alpha P_0(t) + \lambda P_{n-1}(t) \tag{7.6}
\]

\[
\left( \frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \phi \right) P_n(x,t) = 0 \tag{7.7}
\]

\[
\left( \frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \phi_c \right) P_C(x,t) = 0 \tag{7.8}
\]

\[
\left( \frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \delta \right) P(x,t) = 0 \tag{7.9}
\]

\[
P(O, t) = \alpha P_s(t) \tag{7.10}
\]

\[
P_n(O, t) = \lambda_n \left[ P_s(t) + P_0(t) + P_1(t) + P_2(t) + P_3(t) + P_{n-1}(t) \right] \tag{7.11}
\]

\[
P_C(O, t) = \lambda_c \left[ P_s(t) + P_0(t) + P_1(t) + P_2(t) + P_3(t) + P_{n-1}(t) \right] \tag{7.12}
\]

Initial condition

\[
P_0(0) = 1 \text{ and all other state probabilities are zero at } t = 0. \tag{7.13}
\]

Taking Laplace transformation from equation (7.1) to (7.12):

\[
(s + n\lambda + \lambda_h + \lambda_c + \alpha) \tilde{P}_0(s) = 1 + \delta \tilde{P}_s(s) + \phi \tilde{P}_1(s) + \int_0^\infty \phi_h \tilde{P}_h(x, s) dx
\]

\[
+ \int_0^\infty \delta \tilde{P}(x, s) dx + \int_0^\infty \phi_c \tilde{P}_C(x, s) dx \tag{7.14}
\]
\[ (s + \phi + (n-1)\lambda + \lambda_c + \lambda_h)\overline{P}_1(s) = n\lambda \overline{P}_0(s) + \phi \overline{P}_2(s) \quad (7.15) \]
\[ (s + \phi + (n-2)\lambda + \lambda_c + \lambda_h)\overline{P}_2(s) = (n-1)\lambda \overline{P}_1(s) + \phi \overline{P}_3(s) \quad (7.16) \]
\[ (s + \phi + (n-3)\lambda + \lambda_c + \lambda_h)\overline{P}_3(s) = (n-2)\lambda \overline{P}_2(s) + \phi \overline{P}_{n-1}(s) \quad (7.17) \]
\[ (s + \phi + \lambda + \lambda_c + \lambda_h)\overline{P}_{n-1}(s) = (n-3)\lambda \overline{P}_3(s) \quad (7.18) \]
\[ (s + \alpha + \delta + \lambda_c + \lambda_h)\overline{P}_{1}(s) = \alpha \overline{P}_0(s) + \lambda \overline{P}_{n-1}(s) \quad (7.19) \]
\[ \left( \frac{\partial}{\partial x} + s + \phi_h \right) \overline{P}_h(x,s) = 0 \quad (7.20) \]
\[ \left( \frac{\partial}{\partial x} + s + \phi_c \right) \overline{P}_c(x,s) = 0 \quad (7.21) \]
\[ \left( \frac{\partial}{\partial x} + s + \delta \right) \overline{P}(x,s) = 0 \quad (7.22) \]
\[ \overline{P}(0,s) = \alpha \overline{P}_s(s) \quad (7.23) \]
\[ \overline{P}_h(0,s) = \lambda_h[\overline{P}_s(s) + \overline{P}_0(s) + \overline{P}_1(s) + \overline{P}_2(s) + \overline{P}_3(s) + \overline{P}_{n-1}(s)] \quad (7.24) \]
\[ \overline{P}_c(0,s) = \lambda_c[\overline{P}_s(s) + \overline{P}_0(s) + \overline{P}_1(s) + \overline{P}_2(s) + \overline{P}_3(s) + \overline{P}_{n-1}(s)] \quad (7.25) \]

Solving equations (7.14) to (7.22) with the help equations (7.23-7.25) and initial condition, one can obtain:

\[ \overline{P}_0(s) = \frac{1}{(H_8 - H_5 - H_6 - H_7)} \quad (7.26) \]

\[ \overline{P}_1(s) = \frac{\overline{P}_0(s)n\lambda}{s + (n-1)\lambda + \lambda_c + \lambda_h + \phi - \frac{\phi(n-1)\lambda}{H_1}} \quad (7.27) \]

\[ \overline{P}_2(s) = \frac{\overline{P}_0(s)n(n-1)\lambda^2}{H_2} \quad (7.28) \]

\[ \overline{P}_3(s) = \overline{P}_0(s)H_3 \quad (7.29) \]

\[ \overline{P}_{n-1}(s) = \overline{P}_0(s)H_4 \quad (7.30) \]
\[ \overline{P}_s(s) = \frac{(\alpha + \lambda H_4) \overline{P}_0(s)}{(s + \alpha + \lambda_c + \lambda_h + \delta)} \]  

(7.31)

Where

\[ H_1 = \left( \frac{s + (n-2)\lambda + \lambda_h + \lambda_c + \phi}{(s + n-2)\lambda + \phi} \right) \]

\[ H_2 = H_1 \left[ s + (n-1)\lambda + \lambda_c + \lambda_h + \phi \right] - \phi(n-1)\lambda \]

\[ H_3 = \frac{n(n-1)(n-2)\lambda^3}{H_2 \left[ s + (n-1)\lambda + \lambda_c + \lambda_h + \phi \right] - \frac{(n-3)\phi \lambda}{s + \lambda + \lambda_c + \lambda_h + \phi}} \]

\[ H_4 = \frac{H_3 \lambda(n-3)}{(s + \lambda + \lambda_c + \lambda_h + \phi)} \]

\[ H_5 = \frac{n\lambdaH_1 \left[ \phi + \frac{\lambda_h \phi_h}{s + \phi_h} + \frac{\lambda_c \phi_c}{s + \phi_c} \right]}{H_1} \]

\[ H_6 = (\alpha + \lambda H_4) \left[ \delta + \frac{\lambda_h \phi_h}{s + \phi_h} + \frac{\lambda_c \phi_c}{s + \phi_c} + \frac{\alpha \delta}{(s + \delta)} \right] \]

\[ H_7 = \left[ \frac{\lambda_h \phi_h}{s + \phi_h} + \frac{\lambda_c \phi_c}{s + \phi_c} \right] \left[ 1 + \frac{n(n-1)\lambda^2}{H_2} + H_3 + H_4 \right] \]

\[ H_8 = (s + n\lambda + \lambda_c + \lambda_h + \alpha) \]

From the state transition diagram (Fig. 7.2), the working (good and degraded i.e. up state) and failed (i.e. down state) state probability of the system is given as:

\[ \overline{P}_{up}(s) = \overline{P}_0(s) + \overline{P}_s(s) + \overline{P}_1(s) + \overline{P}_2(s) + \overline{P}_3(s) + \overline{P}_{n-1}(s) \]  

(7.32)

\[ \overline{P}_{down}(s) = \overline{P}_h(x, s) + \overline{P}(x, s) + \overline{P}_c(x, s) \]  

(7.33)
7.5 Numerical Computations

7.5.1 Availability Analysis

Taking number of unit in the system as 500 i.e. \( n = 500 \) and the values of different parameters as \( \lambda = 0.025 \), \( \lambda_c = 0.001 \), \( \lambda_h = 0.03 \), \( \alpha = 0.015 \) and \( \phi = \phi_c = \phi_h = \delta = 1 \) in equation (7.32) then taking inverse Laplace transform, we get the availability of the system. Now varying \( t \) from 0 to 10 unit of time, the following Table 7.1 and Fig. 7.3 for availability of the considered system is obtained as:

<table>
<thead>
<tr>
<th>Time (( t ))</th>
<th>Availability ( P_{up}(t) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.000000</td>
</tr>
<tr>
<td>1</td>
<td>0.980599</td>
</tr>
<tr>
<td>2</td>
<td>0.973593</td>
</tr>
<tr>
<td>3</td>
<td>0.971061</td>
</tr>
<tr>
<td>4</td>
<td>0.970147</td>
</tr>
<tr>
<td>5</td>
<td>0.969818</td>
</tr>
<tr>
<td>6</td>
<td>0.969699</td>
</tr>
<tr>
<td>7</td>
<td>0.969657</td>
</tr>
<tr>
<td>8</td>
<td>0.969641</td>
</tr>
<tr>
<td>9</td>
<td>0.969636</td>
</tr>
<tr>
<td>10</td>
<td>0.969634</td>
</tr>
</tbody>
</table>

Table 7.1. Availability vs. Time
7.5.2 Reliability Analysis

For reliability of the system, substituting all repairs as zero in equation (7.32) and $\lambda = 0.025$, $\lambda_c = 0.001$, $\lambda_h = 0.03$, $\alpha = 0.015$ then taking inverse Laplace transform, the reliability of the system is given as:

$$R(t) = \begin{cases} 292.7164134 e^{(-12.546t)} + 625 e^{(-12.526t)} \sinh(0.02t) \\ -115.7495976 e^{(-12.481t)} - 219.0183527 e^{(-12.506t)} + 42.05155976 \\ e^{(-12456t)} - 1.510280805 e^{(-0.056t)} + 2.510257987 e^{(-0.046t)} \end{cases}$$

(7.34)

Now varying time unit $t$ from 0 to 10 in equation (7.34), Table 7.2 and Fig. 7.4 for reliability of the system is obtained as:
Table 7.2. Reliability vs. Time

<table>
<thead>
<tr>
<th>Time ($t$)</th>
<th>Reliability $R(t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.000000</td>
</tr>
<tr>
<td>1</td>
<td>0.969372</td>
</tr>
<tr>
<td>2</td>
<td>0.939361</td>
</tr>
<tr>
<td>3</td>
<td>0.909960</td>
</tr>
<tr>
<td>4</td>
<td>0.881183</td>
</tr>
<tr>
<td>5</td>
<td>0.853038</td>
</tr>
<tr>
<td>6</td>
<td>0.825534</td>
</tr>
<tr>
<td>7</td>
<td>0.798676</td>
</tr>
<tr>
<td>8</td>
<td>0.772467</td>
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<tr>
<td>9</td>
<td>0.746908</td>
</tr>
<tr>
<td>10</td>
<td>0.721998</td>
</tr>
</tbody>
</table>

Fig. 7.4. Reliability vs. Time
7.5.3 MTTF Analysis

The MTTF of the considered system can be obtained by taking all repairs equal to zero for exponential distribution in equation (7.32) and taking $s$ tends to zero. The MTTF for the considered system is obtained, now setting $\lambda = 0.025$, $\lambda_c = 0.001$, $\lambda_h = 0.03$, $\alpha = 0.015$ and varying $\lambda, \lambda_c, \lambda_h, \alpha$ one by one from 0.01 to 0.10 in the MTTF expression, one can get the MTTF as tabulated in Table 7.3 and corresponding Fig. 7.5.

<table>
<thead>
<tr>
<th>Variations in $\lambda$, $\lambda_c$, $\lambda_h$, $\alpha$</th>
<th>MTTF</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>$\lambda_c$</td>
</tr>
<tr>
<td>0.01</td>
<td>29.731480</td>
</tr>
<tr>
<td>0.02</td>
<td>28.174278</td>
</tr>
<tr>
<td>0.03</td>
<td>27.122119</td>
</tr>
<tr>
<td>0.04</td>
<td>26.365150</td>
</tr>
<tr>
<td>0.05</td>
<td>25.794668</td>
</tr>
<tr>
<td>0.06</td>
<td>25.349379</td>
</tr>
<tr>
<td>0.07</td>
<td>24.992170</td>
</tr>
<tr>
<td>0.08</td>
<td>24.699268</td>
</tr>
<tr>
<td>0.09</td>
<td>24.454746</td>
</tr>
<tr>
<td>0.10</td>
<td>24.247534</td>
</tr>
</tbody>
</table>

Table 7.3. MTTF vs. Failure rates
7.5.4 Sensitivity Analysis

a) Sensitivity of Reliability

We carry out the sensitivity analysis of reliability by differentiating the reliability expression with respect to failure rates $\lambda$, $\lambda_c$, $\lambda_h$ and $\alpha$ respectively and then putting $\lambda = 0.025$, $\lambda_c = 0.001$, $\lambda_h = 0.03$, $\alpha = 0.015$. The values of $\frac{\partial R(t)}{\partial \lambda}$, $\frac{\partial R(t)}{\partial \lambda_c}$, $\frac{\partial R(t)}{\partial \lambda_h}$, $\frac{\partial R(t)}{\partial \alpha}$ is obtained. Now, taking $t=0$ to 10 units of time in these partial derivatives of reliability, one can obtained Table 7.4 and corresponding Fig. 7.6 for sensitivity of reliability as:
b) Sensitivity of MTTF

By differentiating MTTF expression obtained in section 7.5.3, with respect to failure rates and then putting the values of $\lambda = 0.025$, $\lambda_c = 0.001$, $\lambda_h = 0.03$, $\alpha = 0.015$, the values
of $\frac{\partial (\text{MTTF})}{\partial \lambda}$, $\frac{\partial (\text{MTTF})}{\partial \lambda_c}$, $\frac{\partial (\text{MTTF})}{\partial \lambda_h}$, $\frac{\partial (\text{MTTF})}{\partial \alpha}$ is obtained. Varying the failure rates one by one respectively as 0.01 to 0.09 in these partial derivatives of MTTF, one can obtained Table 7.5 and Fig. 7.7 for sensitivity of MTTF as:

<table>
<thead>
<tr>
<th>Variations in $\lambda$, $\lambda_c$, $\lambda_h$, $\alpha$</th>
<th>$\frac{\partial (\text{MTTF})}{\partial \lambda}$</th>
<th>$\frac{\partial (\text{MTTF})}{\partial \lambda_c}$</th>
<th>$\frac{\partial (\text{MTTF})}{\partial \lambda_h}$</th>
<th>$\frac{\partial (\text{MTTF})}{\partial \alpha}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>-192.477648</td>
<td>-472.374439</td>
<td>-2548.059262</td>
<td>-263.561602</td>
</tr>
<tr>
<td>0.02</td>
<td>-125.680758</td>
<td>-325.649537</td>
<td>-1223.184168</td>
<td>-170.843426</td>
</tr>
<tr>
<td>0.03</td>
<td>-88.059865</td>
<td>-237.232681</td>
<td>-704.727656</td>
<td>-119.850275</td>
</tr>
<tr>
<td>0.04</td>
<td>-65.065249</td>
<td>-180.096418</td>
<td>-453.752154</td>
<td>-88.840124</td>
</tr>
<tr>
<td>0.05</td>
<td>-50.017850</td>
<td>-141.158008</td>
<td>-314.800730</td>
<td>-68.587561</td>
</tr>
<tr>
<td>0.06</td>
<td>-39.641820</td>
<td>-113.488873</td>
<td>-230.399072</td>
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</tr>
<tr>
<td>0.07</td>
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<td>-93.153596</td>
<td>-175.531560</td>
<td>-44.618827</td>
</tr>
<tr>
<td>0.08</td>
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<td>-77.787880</td>
<td>-137.965970</td>
<td>-37.184298</td>
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<tr>
<td>0.09</td>
<td>-22.432654</td>
<td>-65.904445</td>
<td>-111.173601</td>
<td>-31.515144</td>
</tr>
</tbody>
</table>

Table 7.5. Sensitivity of MTTF vs. Failure rates
7.6 Results Discussion

In this chapter, the availability, reliability, MTTF and sensitivity analysis of a complex industrial system is investigated. For numerically examining the behavior of system, put numerical values of various parameters of the system for various reliability measures.

- On the basis of calculation, it can be easily seen from Fig. 7.3 that the availability of the system decreases swiftly when the time increases and then attains a uniform value after a specific time.
- Fig. 7.4 represents the variation of reliability of the system. It shows that the reliability of the system decreases more stately with respect to availability as the increment in time.
- From Fig. 7.5, MTTF of the system decreases with respect to all type of failures. We can see that MTTF declines rapidly with respect to human error. The MTTF is highest with respect to human error and attains the lowest value with respect to catastrophic failure.
- The sensitivity of the system reliability is shown in Fig. 7.6. It reveals that sensitivity of reliability decreases as time passes. It is clear from the graph that system reliability is
more sensitive with respect to human error and catastrophic failure. So, we can say that the system can be made less sensitive with respect to reliability by controlling human error and catastrophic failure.

- Moreover, Fig. 7.7 shows the sensitivity of MTTF with respect to failure rates of the system. This shows that it increases with the increment in failure rates. Critical observation of graph point out that MTTF of the system is highly sensitive with respect to human error.

7.7 Conclusion

This model investigated an industrial system which consist $n$ unit with a standby unit with multistate failure. Similar model was also discussed [28, 34] in the history of reliability but they did not consider catastrophic failure and human error simultaneously in the system which can exist in every system. In result, it is concluded that MTTF of the system is highly sensitive with respect to human error and system reliability is more sensitive with respect to human error and catastrophic failure both compare to other system parameter.