Milling is one of the most commonly used machining processes in manufacture industry. In the aeronautical sector it represents 70% of overall machining operations. Surface quality of parts and productivity are critical issues in milling processes. Besides, objects like airplane, vehicle, ship, golf club head have complicated profiles involving free form surfaces that need to be manufacture precisely. Machining of these surfaces using conventional ball-end or face-end milling cutters may not lead to the best efficiency of machining and the quality of the machined surface. Machining these free-form surfaces with a custom-engineered (special) form milling (CEFM) cutter is a desirable alternative to obtain the defined shape of the component with improved surface accuracy. That is why in metal cutting industry, the shape of the CEFM cutter is one of the key factors to affect the machining accuracy and dynamic stability.
A CEFM cutter is a peripheral cutter whose cutting edges are shaped so as to generate a special profile on the surfaces machined. For a single pass, the exact contour of the cutting edge of a form mill is reproduced on the surface of the work piece. This CEFM cutter can be used to machine various hard and soft metals, leathers, woods, etc. to reproduce the desired surface profile. In the case of an insert type form milling cutter, the cutting teeth of the solid type cutter are replaced with the inserts. Here, the body of the cutter is made of one piece of material (low cost steel) and the insert teeth of a different material (normally made of materials like carbide or ceramic). The inserts are generally mechanically locked to the cutter body with the help of wedges and clamps or brazed to it. The major advantage of insert based cutters is that body of the cutter need not be replaced when the insert wears out. There exists a variety of customized form milling cutters designed by the manufacturers on the case to case basis and no standardization in the definition of the CEFM cutter explicitly exist. This chapter deals with brazed insert-based custom-engineered form milling cutter and develops the three-dimensional surface based geometric model of the generic form milling cutter. The proposed definition of the CEFM cutter is generic in nature. Besides, a new CEFM cutter is developed for metal cutting industries.

3.1 Surface Modeling of CEFM Cutter

The geometry of CEFM cutter projected on two-dimensional orthographic planes is shown in Figure 3.1. $D_1, D_2, d, D_r$ and $D_i$ are diameter of hub1, diameter of hub2, bore diameter, root circle diameter of insert seat and outer circle diameter of insert seat respectively. $R$ is the radius of fillet and $W$ is the width of the insert seat / insert. Keyway in the hub1 has the specification $a, b$ as the width and depth of the keyway respectively. In terms of surface modeling paradigm, the geometry of the CEFM cutter can be taken as to be made up of two sets of surface patches, namely,

(i) Surface patches constituting insert body

(ii) Surface patches constituting cutter body

All the insert teeth of the form milling cutter are similar in geometry. In this section, for the purpose of modeling, a unified insert tooth is considered and modeled in detail. Later, this insert is placed in the proper position and orientation on the periphery of the cutter body.
as many times as the number of inserts to complete the model. Based on the criticality of function performed, the surface patches forming the insert tooth and the cutter body can be grouped into two categories:

- Critical surfaces or functional surfaces
- Non-critical surfaces or transitional surfaces meant for completing the geometry

The geometry of a single insert body consists of a NURBS sweep surface and planar surface patches formed by transforming suitable unbounded two-dimensional planes with their centre initially coinciding with the local coordinate system $C_2$. The cutter body consists of insert seats and core cutter body. An insert seat geometry depends on the type of insert profile and is defined by a NURBS sweep surface and planar surface patches. The main body of the cutter consists of surface patches that are either planar or cylindrical in geometry.

### 3.1.1 Shape Design of a Unified Insert Tooth

A unified insert tooth is designed to consist of eight functional surfaces and one chamfered surface. Table 3.1 lists the surface patches of the insert of the CEFM cutter. These surface patches are labeled as $\Sigma_1$ to $\Sigma_8$ and $\sigma_{8,1}$ (chamfered surface) respectively and shown with the

![Figure 3.1: Two-Dimensional Projected Geometry of a CEFM Cutter](image-url)
help of Figure 3.2. To model the insert surfaces, the insert is placed in a local right-hand Cartesian frame of reference $C_2 \{O_2 : X_2, Y_2, Z_2\}$ with $X_2$-axis along the rake face, $Y_2$-axis along the end face and $Z_2$-axis along the intersection of these two surfaces. All the above surface patches except the surface patch $\Sigma_4$ are planar surfaces and are defined by transforming suitable unbounded two-dimensional planes with their origins initially coinciding with the origin in $C_2$. Surface $\Sigma_4$ is developed by linearly sweeping a curve, modeled as a non-uniform rational B-spline (NURBS); called NURBS sweep surface here. NURBS sweep surface formulations enable predictable shape controls of the cutting edge and flank surface by changing only a few simple parameters. These geometric representations have been used with success to represent free form surfaces [10, 96, 145, 155]. Also, flank surface of the tooth insert can be modified by sweeping the NURBS profile along user-defined trajectories.

A vertex of the surface $\Sigma_i$ is given by $p_i(u,v) = [u, v, 1]$. An infinite XY plane is given by $p(u,v) = [u, v 0 1]$, where $-\infty \leq (u, v) \leq \infty$. Similarly, YZ and ZX infinite planes can be defined by $p(v,w) = [0, v, w, 1]$ and $p(u,w) = [u, 0, w, 1]$ respectively $(-\infty \leq (u, v, w) \leq \infty)$.

### 3.1.1.1 Rake Face

The rake face ($\Sigma_1$) is formed by the $Z_2X_2$ plane kept at origin $O_2$. The surface $\Sigma_1$ is mathematically defined as,

$$p_1(u_1, w_1) = [u_1, 0, w_1]$$

### Table 3.1: Surface Patches of a Unified Insert Tooth of the CEFM Cutter

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Surface Patch Name</th>
<th>Symbol</th>
<th>Surface Patch Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Sigma_1$</td>
<td>Rake Face</td>
<td>$\Sigma_7$</td>
<td>Back of Tooth</td>
</tr>
<tr>
<td>$\Sigma_2, \Sigma_3$</td>
<td>Peripheral Lands</td>
<td>$\Sigma_8$</td>
<td>End Face</td>
</tr>
<tr>
<td>$\Sigma_4$</td>
<td>Major Flank</td>
<td>$\sigma_{8,1}$</td>
<td>Chamfered Surface</td>
</tr>
<tr>
<td>$\Sigma_5, \Sigma_6$</td>
<td>Face Land (Right and Left )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 3.2: Modeling of an Insert in Local Coordinate System $C_2$

### 3.1.1.2 Peripheral Land

The peripheral land ($\Sigma_2$ & $\Sigma_3$) is formed by rotating $Z_2X_2$ plane by an angle $-\gamma_2 = -\gamma_3$ about $Z_2$ axis $[R_{z,\gamma_2}]$, followed by translation by an amount $d_{21} = d_{31} (= l_1 \cos \gamma_2 + l_2)$ along the $X_2$ axis $[T_{x,d_{21}}]$. Here, $l_1$ and $l_2$ are the lengths of the peripheral land and back of tooth of the insert respectively. Position vector of any point on the transformed surface patch $\Sigma_2$ & $\Sigma_3$ can be found as,

$$\mathbf{p}_2(u_2, w_2) = p(u_2, w_2) \cdot \begin{bmatrix} \gamma_2 \end{bmatrix} \cdot \begin{bmatrix} T_{x,d_{21}} \end{bmatrix}$$

$$= \begin{bmatrix} u_2 & 0 & w_2 & 1 \end{bmatrix} \begin{bmatrix} \cos(-\gamma_2) & \sin(-\gamma_2) & 0 & 0 \\ -\sin(-\gamma_2) & \cos(-\gamma_2) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ d_{21} & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} u_2 \cos \gamma_2 + d_{21} & -u_2 \sin \gamma_2 & w_2 & 1 \\ u_2 \cos \gamma_2 + l_1 \cos \gamma_2 + l_2 & -u_2 \sin \gamma_2 & w_2 & 1 \end{bmatrix}$$

### 3.1.1.3 Face Land

Right and left face lands ($\Sigma_5$ & $\Sigma_6$) are formed by translating the $X_2Y_2$ plane by an amount $d_{53} (= W/2)$ along the $Z_2$ axis $[T_{z,d_{53}}]$ and taking its reflection about the $Z_2 = 0$ plane. Here $W$ is the width of the insert. The surfaces are defined as,
\[ \mathbf{p}_5(u_5, v_5) = [u_5 \quad v_5 \quad d_{53} \quad 1] \]  
(3.3)

\[ \mathbf{p}_6(u_6, v_6) = [u_6 \quad v_6 \quad -d_{53} \quad 1] \]  
(3.4)

### 3.1.1.4 Back of Tooth

The back of tooth (\( \Sigma_7 \)) is formed by translating the \( Z_2X_2 \) plane by an amount \( d_{72} = l_i \sin \gamma_2 \) along the \( Y_2 \) axis \([T_{y,d_{72}}] \). The surface \( \Sigma_7 \) can be mathematically written as,

\[ \mathbf{p}_7(u_7, w_7) = [u_7 \quad d_{72} \quad w_7 \quad 1] \]  
(3.5)

### 3.1.1.5 End Face

The end face (\( \Sigma_8 \)) is formed by the \( Y_2Z_2 \) plane kept at origin \( O_2 \). The surface \( \Sigma_8 \) is given by,

\[ \mathbf{p}_8(v_8, w_8) = [0 \quad v_8 \quad w_8 \quad 1] \]  
(3.6)

### 3.1.1.6 Chamfered Surface

The chamfered surface (\( \sigma_{8,1} \)) is the only transitional surface on the unified insert body and is modeled as a linear sweep surface. A straight \( u \) parametric edge of unit width lying on \( X_2Y_2 \) plane, inclined at 45° (for 45° chamfer) along \( X_2 \) axis is given by \([0.707(1-u) \quad 0.707u \quad 0 \quad 1] \), with \( 0 \leq u \leq 1 \). This edge is swept along \( Z_2 \) axis in terms of parameter \( v \) and forms \( \sigma_{8,1} \), where

\[ \mathbf{p}_{8,1}(u,v) = \begin{bmatrix} 0.707(1-2u) & 0.707u & \frac{W}{2}(1-2v) & 1 \end{bmatrix} \text{ for } 0 \leq u, v \leq 1 \]  
(3.7)

### 3.1.1.7 Major Flank

The major flank (\( \Sigma_4 \)) is formed by sweeping a NURBS curve \( V_2V_4 \) lying on rake face (\( \Sigma_1 \)) along the peripheral land (\( \Sigma_2 \)) in the direction \( V_1-V_7 \) (as shown in Figures 3.2 and 3.3). Here, \( V_2 \) and \( V_4 \) are the vertices of the major cutting edge of the insert. Vertex \( V_1 \) is the intersection point of surface patches \( \Sigma_1, \Sigma_2 \& \Sigma_4 \) and vertex \( V_7 \) is the intersection point of surface patches \( \Sigma_2, \Sigma_5 \& \Sigma_7 \). The major cutting edge \( V_2V_4 \) is developed using a NURBS faired curve approach [26] as shown with the help of Figures 3.3 and 3.4.
Figure 3.3: Projected View of the Control Polygon and the Major Cutting Edge on Rake Face of Insert

Figure 3.4: NURBS Faired Curve

In the NURBS faired curve approach, the input data points act as control points and the curve approximates the control points. Such NURBS curves are known to represent the freeform objects closely. This method has excellent model coverage and a high degree of freedom to develop the major cutting edge of the insert. An instantaneous point \( p(t) \) in a parametric domain is represented by NURBS [109] as follows,

\[
p(t) = \sum_{i=0}^{n} D_i R_{i,k}(t)
\]

where \( R_{i,k}(t) = \frac{h_i N_{i,k}(t)}{\sum_{i=0}^{n} h_i N_{i,k}(t)} \), for \( 0 \leq t \leq t_{\max} \).
Here, \( D_j(V_2, B_1, B_2, B_3, V_4) \) are the position vectors of the \( n+1 \) vertices of the control polygon, \( t \) is the parameter, \( R_{ik}(t) \) is the rational basis function of the NURBS curve, \( N_{ik}(t) \) is the normalized basis function of the NURBS curve, \( k \) is the order of the NURBS curve and \( h_i \) is the weight corresponding to \( D_i \). In this work, for modeling the sample cutter, the NURBS curve is plotted with \( k = 4 \) and \( n+1 = 5 \) and is mathematically defined as

\[
p_4(t, s) = \left[ p(t) \right][T_s]
\]

where \( [T_s] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s \cos \gamma_2 & s \sin \gamma_2 & 0 & 1 \end{bmatrix} \), for \( 0 \leq s \leq 1 \).

For \( 0 \leq t \leq 1 \),

\[
p(t) = \frac{V_t h_0(1 - 3t + 3t^2 - t^3) + B_t h_1(3t - 4.5t^2 + 1.75t^3) + B_t h_2(1.5t^2 - t^3) + B_t h_3(0.25t^3)}{h_0(1 - 3t + 3t^2 - t^3) + h_1(3t - 4.5t^2 + 1.75t^3) + h_2(1.5t^2 - t^3) + h_3(0.25t^3)}
\]

For \( 1 \leq t \leq 2 \),

\[
p(t) = \frac{\begin{cases} B_t h_1(2 - 3t + 1.5t^2 - 0.25t^3) + B_t h_2(-2 + 5t - 3t^2 + 0.5t^3) \\ + B_t h_3(2 - 6t + 6t^2 - 1.75t^3) + V_t h_4(-1 + 3t - 3t^2 + t^3) \end{cases}}{h_1(2 - 3t + 1.5t^2 - 0.25t^3) + h_2(-2 + 5t - 3t^2 + 0.5t^3) + h_3(2 - 6t + 6t^2 - 1.75t^3) + h_4(-1 + 3t - 3t^2 + t^3)}
\]

where \( V_2, B_1, B_2, B_3 \) and \( V_4 \) are the control points and \( h_0 - h_4 \) are the corresponding weights of the NURBS curve \( V_2 V_4 \).

### 3.1.2 Shape Design of the Cutter Body

The body of CEFM cutter broadly consists of two segments. They are

(i) Insert seat

(ii) Core cutter body
Figure 3.5 shows the schematic two-dimensional projected orthographic view of the body of the CEFM cutter with its sectional view comprising of insert seat and the core cutter body. The surface patches comprising insert seat are geometrically modeled in a local coordinate system $C_1$, while those of core cutter body are modeled placing them in the global coordinate system $C_0$. The surface patches that make up a single insert seat of the CEFM cutter are nine in number, labeled $\Sigma_1$ to $\Sigma_9$ as mentioned in Table 3.2, while there are twelve surface patches that form the core cutter body ($\Sigma_{50}$ to $\Sigma_{61}$) of the cutter.

![Figure 3.5: Two-Dimensional Projected Geometry and the Sectional View of the Cutter Body of a CEFM Cutter](image)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Surface Patch Name</th>
<th>Symbol</th>
<th>Surface Patch Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Sigma_1$</td>
<td>Face</td>
<td>$\Sigma_6$</td>
<td>Fillet</td>
</tr>
<tr>
<td>$\Sigma_2, \Sigma_3$</td>
<td>Land</td>
<td>$\Sigma_7$</td>
<td>End Face</td>
</tr>
<tr>
<td>$\Sigma_4$</td>
<td>Flank</td>
<td>$\Sigma_8, \Sigma_9$</td>
<td>Right &amp; Left Face Land</td>
</tr>
</tbody>
</table>
3.1.2.1 Surface Geometry of Insert Seat

An insert seat is a cavity created in the cutter body to place the insert. Surface patches $\Sigma_1$ to $\Sigma_9$ form the insert seat that houses the insert of the CEFM cutter. Figure 3.6 shows the geometric model of the insert seat. Surfaces $\Sigma_1 - \Sigma_3$ and $\Sigma_5 - \Sigma_7$ are modeled as linear sweep surfaces, surfaces $\Sigma_8 - \Sigma_9$ are planar surfaces, while surface $\Sigma_4$ is modeled by linearly sweeping a NURBS curve. For a CEFM cutter, the composite section curve profile $(V_0 - V_6)$ in $X_1Y_1$ plane, which is swept to generate the insert seat surfaces, is shown in Figure 3.7. Segments $V_0V_1V_2$ of the composite curve is a circular arc of radius $R$ with center of the arc at vertex $c_1$, while segments $V_2V_3$, $V_3V_4$, $V_4V_5$ and $V_5V_6$ are straight lines in two-dimensional projected plane. The sectional geometry of the insert seat is mathematically evolved and presented below.

Sectional Geometry of an Insert Seat

The composite profile of the sectional geometry of the insert seat is described with the help of five cross-sectional segments. Out of these five, four segments correspond to the four land widths, namely, face, land, back of insert seat and end face, and are straight lines in two-dimensional projected plane. The fifth segment is a circular arc and corresponds to the fillet of an insert seat. The circular arc is meant to provide smooth surfaces, primarily for easy and convenient chip disposal.

To model the cross-sectional profile in two-dimensional plane, the input parameters are (i) length of the face ($l_1$), (ii) angles obtained to form face ($\gamma_1$), and land ($\gamma_2$) about $Z_1$ axis, (iii) radius of fillet (R), (iv) outer diameter of insert seat ($D_1$) and root circle diameter of insert seat ($D_R$) and (v) number of flutes ($N$).

When the insert seat is placed in a local coordinate system $C_1 \{O_1 : X_1, Y_1, Z_1\}$, the position vectors of end vertices of different sections of the composite profile curve and centre points of the two circular arcs ($c_1, c_2$) are satisfied by the following relations:

$$V_0 = \left[ c_{1x} - R \sin \gamma_1, c_{1y} - R \cos \gamma_1, 0 \right]$$

$$V_1 = \left[ V_{1x}, V_{1y}, 0 \right]$$
Figure 3.6: Surface Patches Modeling the Insert Seat

Figure 3.7: Composite Sectional Curve
\[
V_{1x} = \frac{D_R \sin^2 \gamma_{li} [2D_i \cot^2 \gamma_{li} - 4l_i \cot \gamma_{li} \cos ec \gamma_{li} + \sqrt{2} \sqrt{-(D_i^2 + 8l_i^2 - 2(D_R + 2R)^2 - 8D_i l_i \cos \gamma_{li} + D_i^2 \cos 2\gamma_{li}) \cos ec^2 \gamma_{li}}]}{4(D_R + 2R)}
\]

\[
V_{1y} = \frac{D_R \cos^2 \gamma_{li} [\sqrt{-(D_i^2 + 4D_i l_i \sec \gamma_{li} + (D_R - 2l_i + 2R)(D_R + 2(l_i + R)) \sec^2 \gamma_{li}}] - (D_i - 2l_i \sec \gamma_{li}) \tan \gamma_{li}]}{2(D_R + 2R)}
\]

\[
V_2 = \begin{bmatrix} (c_{1x} + R \sin \gamma_{li}) & (c_{1y} + R \cos \gamma_{li}) & 0 & 1 \end{bmatrix}
\]

\[
V_3 = \begin{bmatrix} \frac{D_i}{2} - l_i \cos \gamma_{li} & (l_i \sin \gamma_{li}) & 0 & 1 \end{bmatrix}
\]

\[
V_4 = \begin{bmatrix} \frac{D_i}{2} & 0 & 0 & 1 \end{bmatrix}
\]

\[
V_5 = \begin{bmatrix} V_{5x} & V_{5y} & 0 & 1 \end{bmatrix}
\]

\[
V_{5x} = \frac{c_{1x} \sin \gamma_{li} + c_{1y} \cos \gamma_{li} - R - \frac{D_i}{2} \cot \gamma_{li} \cos (\gamma_{li} - \psi)}{\sin (\gamma_{li} - \psi) - \cos (\gamma_{li} - \psi) \cot \gamma_{li}}
\]

\[
V_{5y} = \frac{c_{1x} \sin \gamma_{li} + c_{1y} \cos \gamma_{li} - R - \frac{D_i}{2} \sin (\gamma_{li} - \psi)}{\cos (\gamma_{li} - \psi) - \sin (\gamma_{li} - \psi) \tan \gamma_{li}}
\]

\[
V_6 = \begin{bmatrix} V_{6x} & V_{6y} & 0 & 1 \end{bmatrix}
\]

\[
V_{6x} = (c_{1x} - R \sin \gamma_{li}) \cos \psi - (c_{1y} - R \cos \gamma_{li}) \sin \psi
\]

\[
V_{6y} = (c_{1x} - R \sin \gamma_{li}) \sin \psi + (c_{1y} - R \cos \gamma_{li}) \cos \psi
\]

\[
c_1 = \begin{bmatrix} c_{1x} & c_{2y} & 0 & 1 \end{bmatrix}
\]

\[
c_{1x} = \frac{\sin^2 \gamma_{li} [2D_i \cot^2 \gamma_{li} - 4l_i \cot \gamma_{li} \cos ec \gamma_{li}]}{4} + \sqrt{2} \sqrt{-(D_i^2 + 8l_i^2 - 2(D_R + 2R)^2 - 8D_i l_i \cos \gamma_{li} + D_i^2 \cos 2\gamma_{li}) \cos ec^2 \gamma_{li}}
\]

\[
c_{1y} = \frac{\cos^2 \gamma_{li} \sqrt{-[D_i^2 + 4D_i l_i \sec \gamma_{li} + (D_R - 2l_i + 2R)(D_R + 2(l_i + R)) \sec^2 \gamma_{li}]} - (D_i - 2l_i \sec \gamma_{li}) \tan \gamma_{li}]}{2}
\]
\[ c_2 = \begin{bmatrix} (c_{1x} \cos \psi - c_{1y} \sin \psi) & (c_{1x} \sin \psi + c_{1y} \cos \psi) & 0 & 1 \end{bmatrix} \]

where \( \psi = \frac{2\pi}{N} \).

**Parametric Representation of Section Curve**

As discussed earlier, the cross-sectional profile of an insert seat consists of one parametric circular arc and four parametric linear edges, labeled as, \( p_1(s) \) to \( p_4(s) \). Curve \( p_1(s) \) is a circular arc of radius \( R \), while \( p_2(s), p_3(s), p_4(s) \) and \( p_5(s) \) are straight lines between vertices \( V_2V_3, V_3V_4, V_4V_5 \) and \( V_5V_6 \) respectively in two-dimensional space. The equation of \( p_2(s) \) is evolved as,

\[ p_2(s) = V_2 + s(V_3 - V_2), \text{ where } 0 \leq s \leq 1. \]

Substituting the relations for the \( x, y \) and \( z \) coordinates of \( V_2 \) and \( V_3 \), the homogeneous equation of \( p_2(s) \) becomes

\[ p_2(s) = \begin{bmatrix} (c_{1x} + R \sin \gamma_{li})(1-s) + s\left(\frac{D_1}{2} - l_3 \cos \gamma_{li}\right) & (c_{1x} + R \cos \gamma_{li})(1-s) + s l_3 \sin \gamma_{li} & 0 & 1 \end{bmatrix} \]

where \( 0 \leq s \leq 1 \).

Similarly, other curve segments are given parametrically by

\[ p_3(s) = \begin{bmatrix} \left(\frac{D_2}{2} - l_3 \cos \gamma_{li} + s l_3 \cos \gamma_{li}\right) & l_3 \sin \gamma_{li}(1-s) & 0 & 1 \end{bmatrix} \]

\[ p_4(s) = \begin{bmatrix} \left(\frac{D_2}{2} + \frac{s(c_{1x} \sin \gamma_{li} + c_{1y} \cos \gamma_{li} - R - \frac{D_1}{2} \sin(\gamma_{li} - \psi))}{\sin(\gamma_{li} - \psi) - \cos(\gamma_{li} - \psi) \cot \gamma_{li}}\right) & \left\{\frac{s(c_{1x} \sin \gamma_{li} + c_{1y} \cos \gamma_{li} - R - \frac{D_1}{2} \sin(\gamma_{li} - \psi))}{\cos(\gamma_{li} - \psi) - \sin(\gamma_{li} - \psi) \tan \gamma_{li}}\right\} & 0 & 1 \end{bmatrix}^T \]
Chapter 3. Custom-Engineered Form Milling (CEFM) Cutter

The parametric equation of the circular arc \( p_t(s) \) is defined as

\[
p_t(s) = \begin{bmatrix}
(1-s)(c_{ix} \sin \gamma_i + c_{iy} \cos \gamma_i - R - \frac{D}{2} \cot \gamma_i \cos(\gamma_i - \psi)) \\
\sin(\gamma_i - \psi) - \cos(\gamma_i - \psi) \cot \gamma_i \\
(1-s)(c_{ix} \sin \gamma_i + c_{iy} \cos \gamma_i - R - \frac{D}{2} \sin(\gamma_i - \psi)) \\
\cos(\gamma_i - \psi) - \sin(\gamma_i - \psi) \tan \gamma_i \\
0 \\
1
\end{bmatrix}^T
\]

where \( s_1 \leq s \leq s_2 \), with \( s_1 = \frac{\pi}{2} + \gamma_{ii} \) and \( s_2 = \frac{3\pi}{2} + \gamma_{ii} \).

**Insert Seat Surface Patches**

There are in all nine surface patches of the form milling insert seat as shown in Figure 3.6 and labeled in Table 3.2. The right and left end surface patches are the surfaces on the ends of the cutter insert seat, perpendicular to the axis of the cutter rotation. During cutting, the right end and the left end surface patches rotates in clockwise and counterclockwise direction respectively about the cutter axis while viewed in the direction of that surface patch.

Let \( t \) be the parameter for linear sweep along \( Z_2 \) axis and \( [T_s] \) the transformation matrix meant for sweeping operation. Then, the surface patches \( \Sigma_1 - \Sigma_3 \) and \( \Sigma_5 - \Sigma_7 \) are sweep surfaces and are parametrically formed as

\[
p(s,t) = p(s) [T_s]
\]

where \( s \) is the parameter of the section curve profile in \( X_2 Y_2 \) plane and \( p(s) \) is the equation for curve segments defined earlier. The transformation matrix \( [T_s] \) for linear (parallel) sweep is given by the matrix,

\[
[T_s] = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & \frac{W}{2} (1 - 2t) & 1
\end{bmatrix}, \text{ with } 0 \leq t \leq 1
\]
In the above equation, \( W \) is the width of the insert seat / insert. At \( t = 0 \), right end surface patch of the cutter is obtained while at \( t = 1 \), one gets left end surface patch of the cutter. The sweeping operation \( p_1(s)[T_s] \) models fillet (\( \Sigma_6 \)). End face (\( \Sigma_7 \)) is obtained by using relation \( p_2(s)[T_s] \), while face (\( \Sigma_4 \)), back of insert seat (\( \Sigma_5 \)) are modeled by performing the operations \( p_3(s)[T_s] \) and \( p_5(s)[T_s] \) respectively. The operation \( p_4(s)[T_s] \) leads to the surface patch \( \Sigma_2' \) from which the land (\( \Sigma_2 \& \Sigma_3 \)) will be obtained if the Boolean intersection between \( \Sigma_2' \) and \( \Sigma_4 \) is performed.

The two end surfaces i.e. \( \Sigma_8 \) and \( \Sigma_9 \) are modeled as bounded circular \((X_2Y_2)\) plane, positioned at \( z = + \frac{W}{2} \) and \( z = - \frac{W}{2} \) respectively. They may be defined as

\[
p_8(u_8, v_8) = \begin{bmatrix} u_8 \cos v_8 & u_8 \sin v_8 & W/2 & 1 \end{bmatrix}
\]

and

\[
p_9(u_9, v_9) = \begin{bmatrix} u_9 \cos v_9 & u_9 \sin v_9 & -W/2 & 1 \end{bmatrix}
\]

where \( \frac{D_2}{2} \leq u_8, u_9 \leq \frac{D_1}{2} \), \( -\delta \leq v_8, v_9 \leq (-\delta + \psi) \) and \( \delta = \tan^{-1} \left( \frac{c_{1y} - R \cos \gamma_{k2}}{c_{1x} - R \sin \gamma_{k2}} \right) \).

Here, \( D_2 \) and \( D_1 \) are the diameter of hub2 and the outer circle diameter of insert seat respectively.

Flank (\( \Sigma_4 \)) of the insert seat is formed in a way similar to that of major flank of the insert. To form the flank surface of the insert seat, the NURBS curve \( V_{r7}, V_{l7} \) lying on face \( \Sigma_1 \) of the insert seat is swept along the land \( \Sigma_2 \) in the direction of the vertex \( V_{r4} \) to \( V_{r5} \) (as shown in Figures 3.6 and 3.8). \( V_{r7} \) and \( V_{l7} \) are the vertices on the face of the insert seat corresponding to the major cutting edge of insert. Vertices \( V_{r4} \) and \( V_{r5} \) are projection of vertex \( V_4 \) and \( V_5 \) at right face land \( \Sigma_8 \) of the insert seat. The geometric model of flank surface \( \Sigma_4 \) is parametrically defined as

\[
p_4(t, s) = [p(t)]\{T_s\}
\]
where \([T_s] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s.c_x & s.c_y & s.c_z & 1 \end{bmatrix}\), for \(0 \leq s \leq l_s\).

\[
\begin{bmatrix} c_x & c_y & c_z & 1 \end{bmatrix}
\]

are the direction cosines of the edge \(V_4V_5\), where

\[
c_x = \frac{V_{5x} - \left(\frac{D_s}{2}\right)}{\sqrt{(V_{5x} - \left(\frac{D_s}{2}\right))^2 + (V_{5y})^2}}, \quad c_y = \frac{V_{5y}}{\sqrt{(V_{5x} - \left(\frac{D_s}{2}\right))^2 + (V_{5y})^2}}\]

and \(c_z = 0\).

For \(0 \leq t \leq 1\),

\[
p(t) = \frac{V_{R7}h_0(1-3t + 3t^2 - t^3) + V_{R8}h_1(3t - 4.5t^2 + 1.75t^3) + V_{L8}h_3(0.25t^3)}{h_0(1-3t + 3t^2 - t^3) + h_1(3t - 4.5t^2 + 1.75t^3) + h_3(0.25t^3)}
\]

For \(1 \leq t \leq 2\),

\[
p(t) = \frac{\left\{V_{R8}h_1(2-3t + 1.5t^2 - 0.25t^3) + V_{L8}h_3(2-6t + 6t^2 - 1.75t^3)\right\}}{h_1(2-3t + 1.5t^2 - 0.25t^3) + h_3(2-6t + 6t^2 - 1.75t^3)}
\]

\[
+ V_{L7}h_4(-1+3t - 3t^2 + t^3)
\]

where \(V_{R7}, V_{R8}, V_9, V_{L8}\) and \(V_{L7}\) are the control points and \(h_0 - h_4\) are the corresponding weights of the NURBS curve \(V_{R7}V_{L7}\).

The generic design of the cutter can be used to control the shape design of major flank and flank surface of the insert and insert seat respectively while varying the weights of the NURBS curve \(V_2V_4\) and \(V_{R7}V_{L7}\) respectively. Figure 3.9 shows another case, when the order of the NURBS curve in Eqns. (3.8) and (3.12) is changed to 3, while the number of control points is still 5 with weights \([1 \ 1 \ 0 \ 1 \ 1]\) respectively. Similarly, with this definition different profiles of the cutters can be generated by varying weights.
3.1.2.2 Surface Geometry of Core Cutter Body

The surface patches that join all the insert seats and completes the body of CEFM cutter are identified as surface patches of core cutter body. They are twelve in number, labeled as \( \Sigma_{50}, \ldots, \Sigma_{61} \). Besides, there are two transitional (chamfered) surfaces, labeled as \( \sigma_{50,52} \) & \( \sigma_{51,52} \) and all these surfaces patches are shown explicitly in Figure 3.10. These surface patches are either planar or cylindrical in geometry. Cylindrical surface patches are modeled as surface of revolution. The mathematical modeling of the geometry of the cutter body is presented herewith:
Figure 3.10: Surface Patches Modeling the Core Cutter Body

**Surface Patch** $\Sigma_{50}$

Surface patch $\Sigma_{50}$ is formed by displacing $X_0Y_0$ plane along $Z_0$ axis by a distance $d_{503}$ ($= W/2$). The mathematical definition of $\Sigma_{50}$ can be given by

$$p_{50} = \begin{bmatrix} u_{50} & v_{50} & \frac{W}{2} & 1 \end{bmatrix}, \text{ for } -\infty \leq u_{50}, v_{50} \leq \infty. \quad (3.13)$$

For specific / bounded surface, $\Sigma_{50}$ may be parametrically defined by

$$p_{50} = \begin{bmatrix} u_{50} \cos v_{50} & u_{50} \sin v_{50} & \frac{W}{2} & 1 \end{bmatrix}, \text{ for } \frac{d}{2} \leq u_{50} \leq \frac{D_1}{2} \text{ and } 0 \leq v_{50} \leq 2\pi. \quad (3.14)$$

**Surface Patch** $\Sigma_{51}$

Surface patch $\Sigma_{51}$ is the reflection of surface patch $\Sigma_{50}$ about $X_0Y_0$ plane and is given by,

$$p_{51} = \begin{bmatrix} u_{51} \cos v_{51} & u_{51} \sin v_{51} & -\frac{W}{2} & 1 \end{bmatrix}, \text{ for } \frac{d}{2} \leq u_{51} \leq \frac{D_1}{2} \text{ and } 0 \leq v_{51} \leq 2\pi. \quad (3.15)$$
Surface Patch $\Sigma_{52}$

Surface patch $\Sigma_{52}$ is the surface forming the bore / mounting hole of the cutter. It is in contact with cutter arbor. Surface $\Sigma_{52}$ is formed as a surface of revolution, when the straight edge, lying in ZX plane is rotated about $Z_0$ axis by an angle $v_{s2}$. Therefore,

$$
p_{s2} = \begin{bmatrix}
\frac{d}{2} 
\cos v_{s2} \\
\frac{d}{2} \sin v_{s2} \\
-\frac{W}{2} (1 - 2w_{s2}) \\
1
\end{bmatrix},
$$

(3.16)

where $0 \leq w_{s2} \leq 1$ and $\sin^{-1}(a/d) \leq v_{s2} \leq 2\pi - \sin^{-1}(a/d)$.

Surface Patch $\Sigma_{53}$, $\Sigma_{54}$ and $\Sigma_{55}$

Surface Patches $\Sigma_{53}$, $\Sigma_{54}$ and $\Sigma_{55}$ comprise the keyway surfaces. The keyway plane, surface patch $\Sigma_{53}$ is formed by translating a $Z_0X_0$ plane along $Y_0$ axis by a distance $y = (c+b)$ and is expressed as

$$
p_{53} = \begin{bmatrix}
-a 
\left(1 - 2u_{53}\right) 
(c+b) \\
\frac{d}{2} 
\frac{d}{2} \\
-\frac{W}{2} (1 - 2w_{53}) \\
1
\end{bmatrix},
$$

(3.17)

for $0 \leq u_{53}, w_{53} \leq 1$, $c = \sqrt{(d^2-a^2)/2}$ and $a$ and $b$ are the width and depth of keyway respectively.

The side walls of keyway, surface patch $\Sigma_{54}$ and $\Sigma_{55}$ are formed by positioning $Y_0Z_0$ plane at $x = -\frac{a}{2}$ and $x = \frac{a}{2}$ respectively. Thus, the equations of $\Sigma_{54}$ and $\Sigma_{55}$ are

$$
p_{54} = \begin{bmatrix}
-a 
\left(1 - 2u_{54}\right) 
(c+bv_{s4}) \\
\frac{d}{2} 
\frac{d}{2} \\
-\frac{W}{2} (1 - 2w_{54}) \\
1
\end{bmatrix}
$$

and

$$
p_{55} = \begin{bmatrix}
\frac{a}{2} 
\left(c + bv_{55}\right) \\
\frac{d}{2} 
\frac{d}{2} \\
-\frac{W}{2} (1 - 2w_{55}) \\
1
\end{bmatrix},
$$

(3.18)

where $0 \leq u_{54}, w_{54}, u_{55}, w_{55} \leq 1$. (3.19)
Surface Patch $\Sigma_{56}$

Surface patch $\Sigma_{56}$ is formed by positioning $X_0Y_0$ plane at $z = \frac{W_b}{2}$, where $W_b$ is the width of the cutter body. It can be parametrically defined as

$$p_{56} = \begin{bmatrix} u_{56} & v_{56} & \frac{W_b}{2} & 1 \end{bmatrix}, \text{ for } -\infty \leq u_{56}, v_{56} \leq \infty. \quad (3.20)$$

It can also be defined as a bounded circular surface defined by

$$p_{56} = \begin{bmatrix} u_{56} \cos v_{56} & u_{56} \sin v_{56} & \frac{W_b}{2} & 1 \end{bmatrix}, \text{ for } \frac{D_1}{2} \leq u_{56} \leq \frac{D_2}{2} \text{ and } 0 \leq v_{56} \leq 2\pi. \quad (3.21)$$

Surface Patch $\Sigma_{57}$

Surface patch $\Sigma_{57}$ is formed by taking reflection of $\Sigma_{56}$ about $X_0Y_0$ plane. This gives the vector equation of the plane $\Sigma_{57}$ as

$$p_{57} = \begin{bmatrix} u_{57} \cos v_{57} & u_{57} \sin v_{57} & -\frac{W_b}{2} & 1 \end{bmatrix}, \text{ where } \frac{D_1}{2} \leq u_{57} \leq \frac{D_2}{2} \text{ and } 0 \leq v_{57} \leq 2\pi. \quad (3.22)$$

Surface Patch $\Sigma_{58}$

Surface patch $\Sigma_{58}$ is formed as a surface of revolution, when a straight edge lying in $Z_0X_0$ plane and given by $\begin{bmatrix} \frac{d}{2} & 0 & w_{58} & 1 \end{bmatrix}$ is rotated about $Z_0$ axis by an angle $v_{58}$. Therefore,

$$p_{58} = \begin{bmatrix} \frac{d}{2} \cos v_{58} & \frac{d}{2} \sin v_{58} & w_{58} & 1 \end{bmatrix}, \text{ with } 0 \leq v_{58} \leq 2\pi \text{ and } \frac{W_b}{2} \leq w_{58} \leq \frac{W}{2}. \quad (3.23)$$

Surface Patch $\Sigma_{59}$

Surface patch $\Sigma_{59}$ is the mirror plane of $\Sigma_{58}$ about $X_0Y_0$ plane and is given by,

$$p_{59} = \begin{bmatrix} \frac{d}{2} \cos v_{59} & \frac{d}{2} \sin v_{59} & -w_{59} & 1 \end{bmatrix}, \text{ with } 0 \leq v_{59} \leq 2\pi \text{ and } \frac{W_b}{2} \leq w_{59} \leq \frac{W}{2}. \quad (3.24)$$
Surface Patch $\Sigma_{60}$

Surface patch $\Sigma_{60}$ is formed as a surface of revolution, when a straight edge lying in $Z_0X_0$ plane and given by $\begin{bmatrix} \frac{d_2}{2} & 0 & w_{60} & 1 \end{bmatrix}$ is rotated about $Z_0$ axis by an angle $v_{60}$. This helps to define $\Sigma_{60}$ by,

$$p_{60} = \begin{bmatrix} \frac{d_2}{2} \cos v_{60} & \frac{d_2}{2} \sin v_{60} & w_{60} & 1 \end{bmatrix} \text{ for } 0 \leq v_{60} \leq 2\pi \text{ and } \frac{W_b}{2} \leq w_{60} \leq \frac{W}{2}. \quad (3.25)$$

Surface Patch $\Sigma_{61}$

Surface patch $\Sigma_{61}$ is the reflection of $\Sigma_{60}$ about $X_0Y_0$ plane and given by,

$$p_{61} = \begin{bmatrix} \frac{d_2}{2} \cos v_{61} & \frac{d_2}{2} \sin v_{61} & -w_{61} & 1 \end{bmatrix} \text{ for } 0 \leq v_{61} \leq 2\pi \text{ and } \frac{W_b}{2} \leq w_{61} \leq \frac{W}{2}. \quad (3.26)$$

Chamfer $\sigma_{50,52}$ and $\sigma_{51,52}$

A transitional surface $\sigma_{i,j}$ is formed by blending two surface patches $\Sigma_i$ and $\Sigma_j$ to form a smooth localized transition between neighboring surfaces at their edge of intersection. The body of the CEFM cutter has two transitional surfaces in the form of chamfers. They are $\sigma_{50,52}$ and $\sigma_{51,52}$ and one of them is shown in Figure 3.10. Chamfers $\sigma_{50,52}$ and $\sigma_{51,52}$ are modeled as surfaces of revolution. A straight edge of unit width on $Y_0Z_0$ plane and inclined at 45° (for 45° chamfer) is revolved about $Z_0$ axis to form $\sigma_{50,52}$. The coordinates of the ends of this straight edge are $(0, \frac{d}{2}, (\frac{W}{2} - 0.707))$ and $(0, (\frac{d}{2} + 0.707), \frac{W}{2})$. This gives the parametric equation of the straight line in terms of parameter $u$ as

$$0 \leq u \leq 1. \quad (3.27)$$

$$p_{50,52}(u, \theta) = \left[ -\left( \frac{d}{2} + 0.707u \right) \sin \theta \quad \left( \frac{d}{2} + 0.707u \right) \cos \theta \quad \left( \frac{W}{2} - 0.707(1-u) \right) \quad 1 \right]. \quad (3.27)$$
where \(0 \leq u \leq 1\) and \(0 \leq \theta \leq 2\pi\) with the chamfer \(\sigma_{50, 52}\) non existent when \(\theta \in (\frac{\pi}{4} - \sin^{-1}(\frac{d}{2}), \frac{\pi}{4} + \sin^{-1}(\frac{d}{2}))\) due to formation of keyway.

Chamfer \(\sigma_{51, 52}\) is the reflection of chamfer \(\sigma_{50, 52}\) about \(X_0 Y_0\) plane and thus can be represented as

\[
p_{51, 52}(u, \theta) = \left\{ \begin{array}{c} \left( -\frac{d}{2} + 0.707u \right) \sin \theta \\ \left( \frac{d}{2} + 0.707u \right) \cos \theta \\ \frac{W}{2} + 0.707(1-u) \end{array} \right\} \right]. \quad (3.28)
\]

### 3.2 Customized 3D Cutter (C\(^3\)C) Design Interface

For the convenience of modeling complex profiled cutting tools, a cutter design interface called ‘Customized 3D Cutter (C\(^3\)C) Design’ has been developed in this work and illustrated here. This design tool is supported by a universal and modular code as well as by a user friendly graphical interface, for pre- and post-processing user interactions. The interface converts the proposed three-dimensional model defined with the help of parametric equations into an intermediate neutral CAD format. Developing an interface is advantageous in comparison to using the APIs of existing CAD packages, as it does not limit the convenience of the proposed modeling paradigm, primarily the conversion of free-form parametric surfaces. Besides, with this cutter design interface, the cutter model can be directly modeled in any commercial CAD environment for validation and down-stream technological applications.

The C\(^3\)C design interface was developed in Microsoft Visual C++ using Dot Net 3.5 technology for Windows XP / Win 2003 operating systems. Graphical User Interface (GUI) of the C\(^3\)C design has four modules; (a) variable management, (b) entity plotter, (c) assembler and (d) IGES generator. The data-entry windows of all the four modules of C\(^3\)C design graphical interface are presented in Figures 3.11-3.14. The users enter the 3D geometric parameters proposed in the work through variable management section of the C\(^3\)C modeler. Various surface geometries (surface geometry of insert, insert seat and core cutter body) of the CEFM cutter are developed in entity plotter section and are saved separately in dot txt format. In the assembler section, different part files of the cutter are assembled. Finally, the IGES generator section generates the IGES [164] file of the complete surface model of the CEFM cutter. The users can change the geometric parameters of the CEFM
cutter in the variable management section of the C³C design tool. This cutter design modeler also provides the ability to model similar cutters by simply changing the values of input parameters. The models output format in IGES provides realistic surface model which can be easily managed for further investigations, or as an input to other commercial CAD, CAM, or FEA environments. Besides, this C³C design interface can also be embedded to any existent commercial CAD package.

Figure 3.11: Data-Entry Window of the C³C Design Interface: Variable Management Module
Figure 3.12: Data-Entry Window of the C³C Design Interface: Entity Plotter Module

Figure 3.13: Data-Entry Window of the C³C Design Interface: Assembler Module
Chapter 3. Custom-Engineered Form Milling (CEFM) Cutter

3.3 Implementation in Commercial CAD Environment

Based on the above proposed generic definition (as described in Section 3.1), the geometric parameters needed to completely describe a CEFM cutter are shown in Tables 3.3(a) and 3.3(b). The geometric parameters of the CEFM cutter are entered in a custom-GUI of the C³C design interface, and the output CEFM cutter model was generated in the IGES 5.3 format. This IGES file is then rendered in a CAD modeling environment (here CATIA V5) as shown in Figure 3.15. The displayed cutter validates the geometric modeling methodology proposed in the work of the CEFM cutter. Besides, the surface model of the CEFM cutter is converted into a solid model in CATIA modeling environment which can be saved in a variety of file formats (CAT PART, IGES, STL, STEP, etc).
Table 3.3 (a): Geometric Parameters (Dimensional) and Input Data for a CEFM Cutter

<table>
<thead>
<tr>
<th>Dimensional Parameters</th>
<th>Value (mm)</th>
<th>Special Milling Cutter 1</th>
<th>Special Milling Cutter 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outer diameter of insert seat ((D_i)) (mm)</td>
<td>92.7</td>
<td>92.7</td>
<td></td>
</tr>
<tr>
<td>Root circle diameter of insert seat ((D_R))</td>
<td>73.0</td>
<td>73.9</td>
<td></td>
</tr>
<tr>
<td>Outer diameter of hub2 ((D_2))</td>
<td>69.0</td>
<td>69.0</td>
<td></td>
</tr>
<tr>
<td>Outer diameter of hub1 ((D_1))</td>
<td>34.0</td>
<td>34.0</td>
<td></td>
</tr>
<tr>
<td>Bore diameter ((d))</td>
<td>20.0</td>
<td>20.0</td>
<td></td>
</tr>
<tr>
<td>Fillet radius ((R))</td>
<td>3.4</td>
<td>3.65</td>
<td></td>
</tr>
<tr>
<td>Length of peripheral land ((l_1))</td>
<td>5.5</td>
<td>5.5</td>
<td></td>
</tr>
<tr>
<td>Length of back of tooth ((l_2))</td>
<td>6.5</td>
<td>6.5</td>
<td></td>
</tr>
<tr>
<td>Length of face of insert seat ((l_3))</td>
<td>5.0</td>
<td>5.0</td>
<td></td>
</tr>
<tr>
<td>Width of the insert seat / insert ((W))</td>
<td>10.0</td>
<td>16.0</td>
<td></td>
</tr>
<tr>
<td>Width of the cutter body ((W_b))</td>
<td>9.0</td>
<td>15.0</td>
<td></td>
</tr>
<tr>
<td>Width’s of peripheral land ((\Sigma_2 &amp; \Sigma_3))</td>
<td>1.0</td>
<td>4.0</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.3 (b): Geometric Parameters (Angular) and Input Data for a CEFM Cutter

<table>
<thead>
<tr>
<th>Rotational Angles</th>
<th>Special Milling Cutter 1</th>
<th>Special Milling Cutter 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\gamma_1)</td>
<td>-30</td>
<td>-71</td>
</tr>
<tr>
<td>(\gamma_{1i})</td>
<td>-55</td>
<td>-14</td>
</tr>
<tr>
<td>(\gamma_{2i})</td>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>
3.4 Model Verification through Reverse Engineering

In this chapter, the accuracy of the proposed model is verified through Reverse Engineering (RE) technique. For this purpose, the digitized point cloud data of the existing CEFM cutter (used in leather cutting industries) is captured with the help of a 3D laser scanner (as shown in Figure 3.16 (a)), available at the Institute. Utilizing these point cloud data, surfaces are formed to generate the surface model of the cutter in the surface modeling environment of CATIA (Figure 3.16 (b)). A surface-based model of the same tool is developed with the proposed modeling approach. In order to verify the accuracy of our modeling methodology, surface-surface comparisons are carried out using the surface model developed by the proposed methodology to the surface model generated using point cloud data of the same physical cutter. The deviation plot between these two surface models is shown in Figure 3.17. The scanning accuracy of the scanner used is ±0.05mm. The deviation between the corresponding surfaces is around 0.135 mm, which is negligible and hence the modeling paradigm is validated. Similar exercise can be carried out for a worn or used tool to locate the worn areas on a cutting tool.
Chapter 3. Custom-Engineered Form Milling (CEFM) Cutter

Figure 3.16: (a) Digitized Point Cloud Data of actual Cutter; (b) Surface Model generated with the Digitized Data of actual Cutter

Figure 3.17: Deviation Plot for Surface Comparison of the CEFM Cutter Surfaces
3.5 Design Improvement and Redesign of the Cutter Model

The 3D model of a cutting tool offers a lot of utility and can provide a variety of down-stream applications. This section presents one such application of redesigning the cutter based on the feedback of the finite element analysis of the CEFM cutter. Besides, two special shaped milling cutters are re-designed for leather and metal industries.

Finite element based engineering analysis (FEA) is performed on the 3D model of the CEFM cutter created using the proposed methodology to optimize its design. This exercise helps in obtaining the optimum values of different angles of the cutter in terms of the least stresses on the cutter teeth to obtain the best milling practice. The CEFM cutter generated in CATIA V5 as described in Section 3.2.2 is exported for Structural Transient Dynamic Analysis in ANSYS. ANSYS [80] is an implicit finite element (FE) program which uses the Newmark time integration method for the solution of the transient dynamic equilibrium equation. The cutting insert was analyzed under transient dynamic load conditions while varying the cutter radial rake angle of the insert and insert seat. The nodal solution results of contour plots of stress distribution, deformation and elemental total strain intensity are presented for the insert and its body under a variety of cutter radial rake angles. The CEFM cutter model is modified and optimized based on the feedbacks from the analysis. This is an iterative process and the result is an optimized CAD model of a CEFM cutter.

3.5.1 Special Shaped Milling Cutter Designed for Leather Industries

The CEFM cutter shown in Figure 3.15 (special shaped milling cutter 1 based on the data presented in Table 3.3) designed for generating shapes on objects made of leather. The cutter is re-designed as per the feedback from the finite element analysis. Material used here for the insert and the cutter body are cemented carbide (WC) and gray cast iron respectively, whose mechanical and thermal properties [24, 42] are given in Table 3.4 (a). The insert material has a hardness of 1700HV. The material used for the work piece is ecor sole leather. Its mechanical properties [165] are shown in Table 3.4 (b). The cutter is meshed using solid brick 8node45 element. This 3-D brick element is defined by eight nodes having three degrees of freedom at each node, translations in the nodal x, y, and z directions. The element also has plasticity, creep, swelling, stress stiffening, large deflection, and large strain capabilities. As the geometry of the CEFM cutter is not much complex, so, linear hexahedral (LH) element is
preferred for mesh generation of the cutter. Hexahedral elements are highly favored and these elements are characterized by high robustness, better mesh quality and require a lower element count to fill a physical domain [148]. Because of their better mesh quality, hexahedral elements have less likelihood to be inverted compared to tetrahedral elements (inversion results in failure of FE analysis [51]). Using LH and quadratic tetrahedral (QT) elements did not evidence significant differences, fact referred by Cifuentes & Kalbag [29] for FE analysis of various structural problems. Various studies reveals that LH, quadratic hexahedral (QH) and QT models all provided acceptable results, even with relatively coarse meshes [12]. LH elements results in fewer degrees of freedom when compared to the QT elements. Thus, they require less CPU time and disk storage. For adaptive meshing in ANSYS, using solid 45 elements, with reduced integration gives good results in solutions that are very accurate [147]. The meshed model shown in Figure 3.18 is built as a general orthogonal cutting model. The tool is modeled with a fine mesh at the insert, with less refinement at insert seat, while the core cutter body portion is modeled coarsely. During the cutting tests, the cutting conditions are spindle speed, $N = 150$ rpm, feedrate, $f = 240$ mm/min for a depth of cut 6 mm and width of cut, $w = 10$ mm. For our simulation, load has been applied on a single insert in radial and tangential directions. The direction of load application on the cutter is shown in Figure 3.19.

The transient dynamic analysis is carried out for a total time domain of 0.026 sec (for cutter angle rotation of 24°), with a time step size of 0.0066 sec. A number of iterations were performed for limiting the optimum range of radial rake angles. Based on the minimum nodal stress intensity, minimum elemental total strain and deformation developed in the cutter model, the optimal range of radial rake angles is 50° to 60°. Figure 3.20 shows the results of nodal stress intensity distribution, elemental total strain intensity distribution and deformation of the cutter at tool radial rake angle of 52°. During transient dynamic load conditions, Table 3.5 shows the maximum values of nodal stress intensity, elemental total strain intensity and deformation of the cutter model with radial rake angles varying from 51° to 57°. These results shows that the radial rake angle has a complex influence on the magnitude of stress intensity, deformation and total strain intensity components. It is clear from the simulated results, for the radial rake angles (51° to 57° - Table 3.5), that the optimum radial rake angle is 52° and 54°, where it minimizes the effective stress to reach a maximum value of 275 Pa and 288.75 Pa
Table 3.4 (a): Mechanical and Thermal Properties of Tool Insert (P20 Cemented Carbide) and Cutter Body (Grey Cast Iron)

<table>
<thead>
<tr>
<th>Material</th>
<th>Cemented Carbide</th>
<th>Gray Cast Iron</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density (kg/m$^3$)</td>
<td>12100</td>
<td>7710</td>
</tr>
<tr>
<td>Young’s Modulus, E (GPa)</td>
<td>558</td>
<td>103</td>
</tr>
<tr>
<td>Poisson’s ratio, $\nu$</td>
<td>0.22</td>
<td>0.27</td>
</tr>
<tr>
<td>Yield stress, $s_y$ (MPa)</td>
<td>850</td>
<td>207</td>
</tr>
<tr>
<td>Thermal conductivity (W/m°C)</td>
<td>46</td>
<td>35</td>
</tr>
</tbody>
</table>

Table 3.4 (b): Mechanical Properties of Work piece (Ecor Sole Leather Sheets)

<table>
<thead>
<tr>
<th>Material</th>
<th>Ecor Sole Leather</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density (kg/m$^3$)</td>
<td>1000</td>
</tr>
<tr>
<td>Shear Modulus, E (MPa)</td>
<td>3</td>
</tr>
<tr>
<td>Coefficient of friction, $\mu$</td>
<td>0.6</td>
</tr>
<tr>
<td>Bending Stiffness, (MPa)</td>
<td>100</td>
</tr>
<tr>
<td>Ultimate tensile stress, $s_t$ (MPa)</td>
<td>15</td>
</tr>
</tbody>
</table>

Figure 3.18: Meshing of Special Milling Cutter designed for Leather Industries

Figure 3.19: Forces Acting on the Special Milling Cutter
respectively. The existing special shaped milling cutter used in leather sole industries has a radial rake angle of 55º. Therefore, to reach the goal of minimizing the stress zones so as to extend tool life, it is recommended that for the special shaped milling cutter designed for sole leather industries, the radial rake angle of 52º should be adopted and hence the cutter is re-designed accordingly.

Figure 3.20: During Transient Dynamic Load Conditions, for Cutter Model of Radial Rake Angle 52º (a) Nodal Stress Intensity Distribution (b) Elemental Total Strain Intensity Distribution (c) Deformation
Table 3.5: Maximum values of Nodal Stress Intensity, Total Strain Intensity and Deformation of Special Milling Cutter designed for cutting Leather Sole through FEA

<table>
<thead>
<tr>
<th>Radial Rake Angle (Degrees)</th>
<th>Maximum Nodal Stress Intensity (Pa)</th>
<th>Maximum Total Strain Intensity</th>
<th>Maximum Deformation (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>51</td>
<td>293.78</td>
<td>0.290e-08</td>
<td>0.681e-08</td>
</tr>
<tr>
<td>52</td>
<td>275.0</td>
<td>0.250e-08</td>
<td>0.659e-08</td>
</tr>
<tr>
<td>53</td>
<td>299.35</td>
<td>0.323e-08</td>
<td>0.746e-08</td>
</tr>
<tr>
<td>53.5</td>
<td>301.25</td>
<td>0.334e-08</td>
<td>0.771e-08</td>
</tr>
<tr>
<td>54</td>
<td>288.75</td>
<td>0.297e-08</td>
<td>0.715e-08</td>
</tr>
<tr>
<td>55</td>
<td>426.63</td>
<td>0.378e-08</td>
<td>0.880e-08</td>
</tr>
<tr>
<td>56</td>
<td>323.95</td>
<td>0.343e-08</td>
<td>0.890e-08</td>
</tr>
<tr>
<td>57</td>
<td>482.237</td>
<td>0.550e-08</td>
<td>0.110e-07</td>
</tr>
</tbody>
</table>

3.5.2 Special Shaped Milling Cutter Designed for Metal Cutting Industries

Figure 3.21 shows the CEFM cutter (special shaped milling cutter 2 - Table 3.3) designed for form generation in cutting of mild steel. Material used here for the insert is high speed steel (HSS), the cutter body is of gray cast iron and the material used for the work piece is mild steel AISI 1020. The mechanical and thermal properties [42, 168] of the above mentioned materials are given in Tables 3.6 and 3.4 (a). The special milling cutter is meshed using solid brick 8node45 element and is shown in Figure 3.22. For this cutting operation, the cutting conditions are spindle speed, \( N = 80 \) rpm, feedrate, \( f = 65 \) mm/min, i.e. feed per tooth, \( f = 0.081 \) mm for a depth of cut 6 mm and width of cut, \( w = 10 \) mm. For the simulation of above cutting, a single insert will be in contact with the work piece. Therefore, load has been applied on a single insert in radial and tangential directions.

The transient dynamic analysis is carried out for a total time domain of 0.0625 sec (for cutter angle rotation of 30°), with a time step size of 0.0125 sec. A number of iterations were performed within the prescribed limiting range of radial rake angles from 10° to 15° [75]. Figure 3.23 shows the results of stresses, strains and displacements of the cutter at tool radial rake angle of 14.5°. For the dynamic conditions, Table 3.7 shows the maximum values of nodal stress intensity distribution, deformation and elemental total strain intensity distribution of the cutter models at radial rake angles of 10° to 15°. The results tabulated in Table 3.7 shows that the optimum radial rake angle is 14.5°, where it minimizes the effective stress to reach a maximum value of 10404 Pa.
So, as per the feedback of the transient dynamic analysis results, the special shaped milling cutter for the above two cases are re-designed and their geometry is optimized. The accuracy of the simulated physical phenomena is reasonable and within acceptable limits. The cost parameter could not be compared due to lack of any similar work published in the literature.

Table 3.6: Mechanical and Thermal Properties of Tool Insert (HSS) and Work piece (Mild Steel AISI 1020)

<table>
<thead>
<tr>
<th>Materials</th>
<th>High Speed Steel</th>
<th>Mild Steel AISI 1020</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density (kg/m³)</td>
<td>7980</td>
<td>7870</td>
</tr>
<tr>
<td>Young’s Modulus, E (GPa)</td>
<td>210</td>
<td>200</td>
</tr>
<tr>
<td>Poisson’s ratio, n</td>
<td>0.30</td>
<td>0.29</td>
</tr>
<tr>
<td>Tensile strength (MPa)</td>
<td>970</td>
<td>394.7</td>
</tr>
<tr>
<td>Thermal conductivity (W/m°C)</td>
<td>20.9</td>
<td>51.9</td>
</tr>
<tr>
<td>Yield stress, $s_y$ (MPa)</td>
<td>-</td>
<td>294.8</td>
</tr>
</tbody>
</table>
Designing a customized cutting tool includes many aspects. To obtain the optimum geometry of a special tool, modeling and analysis of the cutting profiles of the tool based on cutting forces during machining plays an important role. In the most of the cases, the geometry of the cutting tools are developed in a manner that it is tough to define a unified representation scheme for the family of the cutter. Hence, in this section different design activities, such as

Figure 3.23: During Transient Dynamic Load Conditions, for Cutter Model of Radial Rake Angle 14.5° (a) Nodal Stress Intensity Distribution (b) Elemental Total Strain Intensity Distribution (c) Deformation

3.6 Chapter Summary
Table 3.7: Maximum values of Nodal Stress Intensity, Total Strain Intensity and Deformation of Special Milling Cutter designed for cutting Mild Steel Jobs

<table>
<thead>
<tr>
<th>Radial Rake Angle (Degrees)</th>
<th>Maximum Nodal Stress Intensity (Pa)</th>
<th>Maximum Total Strain Intensity</th>
<th>Maximum Deformation (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>10595</td>
<td>0.656e-07</td>
<td>0.869e-07</td>
</tr>
<tr>
<td>11</td>
<td>10571</td>
<td>0.654e-07</td>
<td>0.877e-07</td>
</tr>
<tr>
<td>12</td>
<td>10534</td>
<td>0.652e-07</td>
<td>0.884e-07</td>
</tr>
<tr>
<td>13</td>
<td>10511</td>
<td>0.651e-07</td>
<td>0.892e-07</td>
</tr>
<tr>
<td>13.5</td>
<td>10527</td>
<td>0.652e-07</td>
<td>0.895e-07</td>
</tr>
<tr>
<td>14</td>
<td>10412</td>
<td>0.645e-07</td>
<td>0.898e-07</td>
</tr>
<tr>
<td>14.5</td>
<td>10404</td>
<td>0.644e-07</td>
<td>0.902e-07</td>
</tr>
<tr>
<td>15</td>
<td>10481</td>
<td>0.649e-07</td>
<td>0.108e-06</td>
</tr>
</tbody>
</table>

surface modeling, finite element analysis and design optimization have been integrated and used to accurately model a designated CEFM cutter. A shape design methodology to model and improve the geometry of a generic CEFM cutter is illustrated. Two different types of special shaped milling cutters are generated from the same definition and shown here, to illustrate the generic definition of the CEFM cutters. These cutters are redesigned, based on the feedbacks obtained while performing the finite element analysis for the optimum rake angle of the insert.

Besides, a dedicated customized cutter design interface, named Customized 3D Cutter (C³C) design is developed. The interface developed here is not intended to be the final word on the modeling paradigm, but rather it may be evolved and generalized to act as independent cutting tool design software or a module of any of the existing commercial CAD packages. It can be used not only to model complex tools but complex mechanical components also. This approach illustrates an advanced modeling paradigm that can be used to accurately model a special shaped milling cutter and thus, opens up avenues to define conveniently not only various customized cutters but also components in the domain of bioCAD, custom implants in orthopedics etc.