CHAPTER II

A SIMPLIFIED POPOV-LIAPUNOV TECHNIQUE FOR THE TRANSIENT STABILITY ANALYSIS OF A SINGLE MACHINE POWER SYSTEM

2.1 INTRODUCTION

In this chapter, we shall like to consider the problem of transient stability of a single machine connected to an infinite bus.* As we have mentioned briefly in Chapter I, this is often taken as the simplest possible model of a power system on the basis of which we can introduce and analyse techniques for the study of the transient stability problems. Our aim here is to point out the possibility of utilizing a simple linear manipulation of the linear and nonlinear parts of the power system model in order to simplify considerably the task of generating an appropriate Liapunov function from the Popov condition for stability. The method essentially helps to shift the pole at the origin to the left hand side as has been first suggested by this author in a slightly different form [37]. This paper has prompted Pai and Varwandkar[38] also to investigate the use of pole shifting techniques in dealing with the problem of generation of Liapunov functions for the single machine power system. The main advantage of the pole shifting

* We shall refer to this model as the Single machine power system.
manipulation is that the method of Walker and McClamor[31] can be utilized straightway for generating the desired Liapunov function for the power system problem and one can thus avoid the rather difficult mathematical manipulations suggested by Willems[15] and Pai et al[19].

In our published work[37] we had also emphasized the possibility of obtaining improved estimates of the region of stability through the Liapunov function which is generated after the pole shifting has been introduced. The results of Pai and Varwandkar[38], however, establish that the improvement depends on the amount of pole shifting and also that with an improper choice there may, in fact, be a deterioration in the estimate of the stability region. Our own experience with this method, as is to be presented in this chapter, indicates that the choice of the machine parameters also has a bearing on the extent of improvement in the estimate of the stability region. We have additionally investigated the use of the pole shifting technique for generating Liapunov functions for single machine as well as multimachine power systems with and without governor actions and the employment of the Liapunov functions so generated not only for computing the regions of stability but also for estimating the quality of the transient response of the system proved stable. In view of this extensive application of the method, we shall make a
systematic presentation in this chapter of the pole shifting technique for application of Kalman's procedure for generation of Liapunov functions for the single machine power system. The discussions here will then serve as a ready reference for the contributions to be presented in the subsequent chapters.

In section 2.2 we give a statement of the problems which are specifically studied in this chapter. In section 2.3, we then generate the Liapunov function for the case where the governor time constant is assumed to be large compared to the transient period. An investigation of the effect of a finite governor time constant is then carried out in section 2.4. The results of both sections 2.3 and 2.4 are illustrated numerically by considering a generator with assumed parameters as given in Willems[15],[20]. Finally in section 2.5 we give a summary of the main results of this chapter.

2.2 PROBLEM STATEMENT

For our study here, we have made the following assumptions to start with:

(i) The voltage behind the transient reactance is assumed constant. In other words, it is assumed that the flux linkage of the machine remains practically constant during the transient period.
Damping is assumed to be directly proportional to the slip velocity and this is primarily due to mechanical friction and an asynchronous torque.

The mechanical input power to the machine is assumed constant during the transient period i.e. the effect of governor time constant is large compared to the transient period and thus neglected.

The effect of saturation is neglected.

Resistances of stator windings of the machine and connecting lines are neglected.

The angular momentum of the synchronous generator is constant.

For mathematical simplicity the effect of saliency is not considered and the nonsalient (round) rotor machine is considered.

Based on these assumptions, the following simplified model can be obtained for describing the behaviour of the machine in the post fault condition[1]-[4];

\[ \ddot{\delta}(t) + \dot{\delta}(t) = P_m - \frac{E V}{X^2} \sin(\delta(t)) \]  

(2.1)

where \( \delta(t) \) is the rotor angle, \( P_m \) is the mechanical power input, \( E \) is the voltage behind the transient reactance of the machine, \( V \) is the voltage of the infinite busbar and \( X \)
is the total reactance of stator windings and transmission lines connecting the machine with the infinite bus, \( M \) and \( d \) are respectively the angular momentum and damping constant of the machine.

Let \( \delta^0 \) be the equilibrium value of \( \delta(t) \). From eq. (2.1) we then see that the mechanical power input can be expressed in terms of this equilibrium value of the rotor angle \( \delta^0 \) as follows

\[
P_m = \frac{E}{X} V \sin \delta^0
\]

We make use of this expression of \( P_m \) in eq. (2.1) and we select the state variables \( x_1(t) \) and \( x_2(t) \) as follows

\[
x_1 = \delta(t) - \delta^0
\]

\[
x_2 = \dot{\delta}(t)
\]

We then obtain the following state variable model for the generator in the post fault condition:

\[
\dot{x}(t) = Ax(t) + bf(y) \quad (2.4)
\]

\[
y(t) = c^T x(t) \quad (2.5)
\]

with the matrix \( A \) and the vectors \( b \) and \( c \) given by

\[
A = \begin{bmatrix}
0 & 1 \\
0 & -d/M
\end{bmatrix}
\]

\[
b = \begin{bmatrix}
0 \\
-1
\end{bmatrix}
\]

\[
c = \begin{bmatrix}
1 \\
0
\end{bmatrix}
\]
The nonlinear function \( f(y) \) is given by the following equation:

\[
f(y) = \frac{EV}{XH} (\sin(y+\delta^o) - \sin\delta^o)
\]  

(2.6)

A plot of this function is shown in Fig.2.1 and the slope \( \frac{df(y)}{dy} \) at the equilibrium point \( y=0 \) (which corresponds to \( \delta^o \)) is easily found to be given by

\[
\frac{df(y)}{dy} \bigg|_{y=0} = \frac{EV}{XH} \cos\delta^o
\]  

(2.7)

Schematic representation of the system is shown in Fig.2.2, where the transfer function \( G(s) \) of the linear subsystem is given by

\[
G(S) = \frac{1}{s(s+d/M)}
\]

Let us now remove assumption(iii) and let us allow the governor time constant to be comparable to the transient period of the post fault state of the generator. The machine dynamics is then modified to the following equations:

\[
\ddot{M}\delta(t) + d\dot{\delta}(t) = P_m(t) - \frac{EV}{X} \sin\delta(t)
\]  

(2.8)

\[
Te \cdot \dot{P}_m(t) + P_m(t) = P_m^o - G \cdot \dot{\delta}(t)
\]

(2.9)

where \( P_m(t) \) is the instantaneous mechanical input power with \( P_m^o \) its steady state value. If we now define \( x_1(t) \) and \( x_2(t) \) as before and introduce the third state variable...
We obtain the state variable model in the form (2.4)-(2.5) but matrix $A$ and vectors $b$ and $c$ given by the following equations:

$$
A = \begin{bmatrix}
0 & 1 & 0 \\
0 & -d/M & -1/M \\
0 & g & -h
\end{bmatrix}
$$

$$
b = \begin{bmatrix}
0 \\
-1 \\
0
\end{bmatrix}
$$

$$
c = \begin{bmatrix}
1 \\
0 \\
0
\end{bmatrix}
$$

where $g(=G/Te)$ and $h(=\frac{1}{Te})$ are parameters of the governor. The nonlinear function $f(y)$ is, however, given by the equation (2.6). The schematic representation is still given by Fig. 2.2 but the transfer function $G(S)$ is now changed to

$$
G(S) = \frac{(s+h)}{s[s^2 + (h+d/M)s + \frac{1}{M}(hd+g)]}
$$

We note that in both cases there is a pole of the transfer function $G(s)$ located at the origin of the $s$-plane. This
Fig. 2.1 Plot of nonlinear function $f(y)$ vs. $y$ corresponding to given $\delta^\circ$.

$\delta^\circ = 15^\circ$

slope $\approx 0.97 \frac{EV}{XM}$

Fig. 2.1 Plot of nonlinear function $f(y)$ vs. $y$ corresponding to given $\delta^\circ$. 
Fig. 2.2 Schematic Representation Of A Single Machine Power System.

\[ G(s) = \frac{1}{s(s + \frac{d}{M})} \]
creates some difficulties in adoption of Kalman's procedure for constructing a Lure'-Postnikov type Liapunov function as is demonstrated by the works of Willems[15] and Pai et al [19].

The following problems are studied with the above machine models;

(i) For the assumed machine models how to generate an appropriate Liapunov function of the Lure'-Postnikov form particularly by adopting Kalman's procedure after the poles of G(s) are shifted to the left hand side by introducing a minor feedback around the linear subsystem.

(ii) To investigate if the region of stability can be improved by a proper choice of the Liapunov function.

(iii) What are the effects of the governor time constant on the region of stability?

2.3 STABILITY ANALYSIS WITH SECOND ORDER MODEL

Let us start with a second order model(2.1)and introduce the proposed method for generation of the desired Liapunov function.

2.3.1 Generation of the Liapunov Function

Let us redraw the schematic diagram of the system
of Fig.2.2 after introducing two minor feedbacks of opposite sign around the linear subsystem as shown in Fig.2.3. This is purely a manipulation on paper and does not involve any real change in the system. Now if the two minor feedback loops are coupled separately with the linear and the nonlinear subsystems of the original diagram (as indicated in Fig.2.3 in the form of dotted boxes), we can obtain the equivalent system of Fig.2.4. This equivalent system is easily checked to be described by the following state variable model:

\[
\begin{align*}
\dot{x} &= A_1 x + b f_1(y) \\
y &= c^T x
\end{align*}
\]

(2.10)

(2.11)

where the matrix \( A_1 \) and the nonlinear function \( f_1(y) \) are given by the following equations:

\[
A_1 = \begin{bmatrix} 0 & 1 \\ -\alpha u & -\ell \end{bmatrix}
\] (2.12)

\[
f_1(y) = \mu [\sin(y+\delta^\circ) - \sin\delta^\circ - \alpha y]
\] (2.13)

where \( \mu \) is a scalar multiplier given by

\[
\mu = \frac{EV}{XM}
\]

It is clear that the effect of the manipulation is to obtain an equivalent machine model for which both the poles are located in the left half of the complex plane. We note, however, that the nonlinear characteristics \( f_1(y) \) may not satisfy sector conditions analogous to those satisfied by the original nonlinear function \( f(y) \) unless
Fig. 2.3 System diagram showing two equal and opposite minor feedbacks of gain $\alpha$.

$$G_1(s) = \frac{G(s)}{1+\alpha G(s)}$$

$$f_1(y) = f(y) - \alpha y$$

Fig. 2.4 Equivalent system representation.
we restrict the minor loop feedback gain $a$ according to the following relation;

$$a < \frac{d}{dy} f(y) \bigg|_{y=0}$$  \hfill (2.14)

Since our problem is essentially to shift the pole at the origin to the left hand side a small value of $a$ satisfying the bound (2.14) serves our purpose.

After having chosen the value of $a$ we can obtain the new sector boundaries $-y'_1$ and $y'_2$ for which $f_1(-y_1) = f_1(y_2) = 0$. For the $f_1(y)$ given by eq. (2.13) we get the following values of $-y'_1$ and $y'_2$.

$$-ay'_1 = \sin(-y'_1 + 6^\circ) - \sin 6^\circ \quad \hfill (2.15)$$

$$ay'_2 = \sin(y'_2 + 6^\circ) - \sin 6^\circ \quad \hfill (2.16)$$

As shown by Walker and McClamorch[31], a sufficient condition for the system under discussion to be asymptotically stable in the finite region of attraction $-y'_1 < y < y'_2$ is that the following condition be satisfied for all real frequencies $\omega > 0$;

$$\Pi(\omega) = \Re[(1+j\omega) G_1(j\omega)] + \frac{1}{k} > 0 \quad \hfill (2.17)$$

where $G_1(j\omega) = -c^T(j\omega I - A_1)^{-1}b$.

Let us assume that this condition is satisfied by the system under study so that we can conclude that the
system is stable in the finite region of attraction. By making use of Kalman-Yakubovich lemma we can then generate a Liapunov function of the following form:

\[ V(x) = x^T P x + q \int_0^y f_1(y) dy \] (2.18)

The Kalman-Yakubovich lemma implies that the real scalar \( q \) and the real symmetric positive definite matrix \( P \) are guaranteed to exist as the solution of the following set of equations:

\[ A_1^T P + PA_1 = -pp^T \] (2.19)

\[ Pb + \frac{1}{2}qA_1^T c + \frac{1}{2}c^T A_1 - m^{1/2}p \] (2.20)

\[ m = qc^T b - \frac{1}{k} \] (2.21)

where \( m \) is a real positive scalar and \( p \) is a real vector of the same dimension as the state \( x \). In order to generate the Liapunov function (2.18), we can proceed by choosing \( q \) and then solving the problem of spectral factorization

\[ \Pi(\omega) = \frac{\delta(j\omega)\delta(-j\omega)}{\det(j\omega I - A_1) \det(-j\omega I - A_1)} \] (2.22)

So far as the question of selecting the scalar \( q \) is concerned the modified polar plot for verifying the inequality (2.17) can be used as a guidance. With \( q \) so chosen and with the spectral factorization corresponding
to eq. (2.22) achieved, we set the following equation towards the choice of the vector \( p \) (with scalar \( m \) determined from eq. (2.21))

\[
-p^T(j\omega I - A_1)^{-1}b = \frac{\phi(j\omega)}{\det(j\omega I - A_1)} - m^{1/2}
\]  

(2.23)

With the vector \( p \) so selected, we make use of the eq. (2.19) in order to obtain the desired matrix \( P \).

2.3.2 Computation of Region of Stability

The Liapunov function generated above can be utilized for computing the region of the \( x_1-x_2 \) plane where the machine remains asymptotically stable. Once this region is computed, we get an idea of the maximum initial perturbation in \( \delta \) and \( \delta \) under which we can expect the generator to regain equilibrium. We can also compute the fault clearance time as will be discussed in the next section.

The procedure for computing the region of stability as given by Walker and McClamorch [31] has been briefly reviewed in section 1.5. We recall that the following steps are involved:

(i) Compute the values of the sector boundaries \(-y_1^\prime\) and \( y_2^\prime\) as given by equations (2.15)-(2.16). Since \( \delta^0 \) is known for a given case we can plot the characteristics of \( f_1(y) \) versus \( y \) for a chosen
value of feedback gain \( a \). Once this characteristics are obtained the values of \( y_1^1 \) and \( y_2^1 \) become known. It may be pointed out that the values of \( y_1^1 \) and \( y_2^1 \) will be less in magnitude compared to the values of \( y_1 \) and \( y_2 \) obtained for the boundaries of the original non-linear function \( f(y) \) where the sector conditions are satisfied (see Fig.2.1). As will be apparent, with \( a \) restricted by (2.14) there is not much difference between boundaries of \( f_1(y) \) and \( f(y) \).

(ii) With \( y_1^1 \) and \( y_2^1 \) known, we compute the values \( V_{m1} \) and \( V_{m2} \) of Liapunov function \( V(x) \) using the following relations:

\[
V_{m1} = V(x) \big|_{y = y_1^1} = \left( y_1^1 \right)^2 \frac{c^2 P^{-1} c}{c^2 P^{-1} c} + q \int_0^{y_1^1} f_1(y) dy
\]

\[
= \frac{(y_1^1)^2}{c^2 P^{-1} c} + q \int_0^{y_1^1} f_1(y) dy
\]

\[
= \frac{(y_1^1)^2}{c^2 P^{-1} c} + \mu q[\cos \delta^\circ - \cos(-y_1^1 + \delta^\circ)] - \sin \delta^\circ y_1^1\frac{1}{2}a(y_1^1)^2
\] (2.24)

and

\[
V_{m2} = V(x) \big|_{y = y_2^1} = \left( y_2^1 \right)^2 \frac{c^2 P^{-1} c}{c^2 P^{-1} c} + q \int_0^{y_2^1} f_1(y) dy
\]

\[
= \frac{(y_2^1)^2}{c^2 P^{-1} c} + q \int_0^{y_2^1} f_1(y) dy
\]

\[
= \frac{(y_2^1)^2}{c^2 P^{-1} c} + \mu q[\cos \delta^\circ - \cos(y_2^1 + \delta^\circ)] - \sin \delta^\circ y_2^1\frac{1}{2}a(y_2^1)^2
\] (2.25)
(iii) Compute $V_{\text{min}}$ using the following definition:

$$V_{\text{min}} = \min \{ V_{m1}, V_{m2} \} \quad (2.26)$$

(iv) Compute the region of stability as those values of $x_1$ and $x_2$ for which the following inequality is satisfied;

$$V(x) < V_{\text{min}} \quad (2.27)$$

For the function $V(x)$ given by eq.(2.18) we obtain the following explicit relation after the integral is evaluated:

$$P_{11} \frac{x_2^2}{2} + 2p_{12} x_1 x_2 + p_{22} x_2^2 + u q (\cos \delta^o - \cos(x_1 + \delta^o)) - x_1 \sin \delta^o - \frac{1}{2} a x_1^2 < V_{\text{min}} \quad (2.28)$$

For a known system all the coefficients of the terms on the left hand side as well as $V_{\text{min}}$ will be known so that the region of stability can be plotted in $x_1$-$x_2$ plane graphically from the relation (2.28).

2.3.3 A Numerical Example

In order to illustrate the techniques of the preceding two subsections let us take the case of a single machine power system as shown in Fig.1.1 having the following parameters (in per units);
The values of matrix A and the vectors b and c are obtained as follows:

\[ A = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ -1 \end{bmatrix}, \quad c = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \]

The nonlinear function \( f(y) \) is checked to be given by

\[ f(y) = \sin(y + 0.412) - 0.40 \]

Let us choose the gain of minor feedbacks as \( a = 0.10 \). This value of \( a \) is selected so that the modified nonlinear function \( f_1(y) \) has the sector boundaries close to those of the original function \( f(y) \). With this choice of \( a \), we get the following state matrix for the modified system

\[ A_1 = \begin{bmatrix} 0 & 1 \\ -0.1 & -1 \end{bmatrix} \]

The modified nonlinear function is then given by

\[ f_1(y) = \sin(y + 0.412) - 0.40 - 0.10y \]

A plot of this function is shown in Fig.2.5 which gives the following values of \( y'_1 \) and \( y'_2 \):

\[ y'_1 = 3.588, \quad y'_2 = 2.08 \]

The actual sector gain of this function is found to be \( k' = 0.8165 \).

The damping constant \( d \) chosen in this example is larger than those encountered in actual power system but was taken to illustrate the results and compare these conveniently with those available in literature.
Fig. 2.5 Sector boundaries from plots of original and modified nonlinearities.

\[ y_1' = 3.588 \quad y_2' = 2.08 \]
\[ y_1 = 3.965 \quad y_2 = 2.3176 \]
Fig. 2.6 Modified polar plot

\[ q = \frac{1}{\text{slope}} = 0.555 \]

\[ q' = \frac{1}{\text{slope}} = -0.727 \]

\[ \left( -\frac{1}{k}, 0 \right) \quad k = 0.8165 \]

\[ \left( -\frac{1}{k}, 0 \right) \quad k = 5 \]

\[ \omega = \infty \]
The Popov condition for stability of the modified system has the following form:

\[ \text{Re}\left[(1+jwq)G(j\omega)\right] + \frac{1}{k} > 0 \]  \hspace{1cm} (2.29)

where \( G(j\omega) = \frac{1}{(0.1-\omega^2)+j\omega} \)

The modified polar plot for this case is shown in Fig.2.6. We note that the slope of the straight line which passes through \((-\frac{1}{k'},0)\) and is tangent to the modified polar plot shown by the solid line has a negative value which implies that \( q \) will also be negative. If we use this negative value of \( q \), obviously the integral term in \( V(x) \) is going to be negative with a consequent reduction in the values of \( V_m1 \) and \( V_m2 \). Generally the value of \( k \) has been taken to \( k' \) by both Willems[15] and Pai et al[19]. This value, however, is too large compared to the actual value of the sector gain and should be modified to a less amount. We have utilized \( k=k'=5 \) in this study and the value of \( q \) computed from the dotted straight line shown in Fig.2.6 is \( q=0.555 \).

Using these values of \( q \) and \( k' \) in (2.29) we get the following expression for \( \Pi(\omega) \):

\[ \Pi(\omega) = \frac{\Phi(j\omega)\Phi(-j\omega)}{(0.1-\omega^2)^2 + \omega^2} \]  \hspace{1cm} (2.30)

This gives the following numerator polynomial for the factored expression of \( \Pi(\omega) \) corresponding to eq.(2.22).

\[ \Phi(j\omega)=0.44721(j\omega)^2 + 0.033983(j\omega)+0.31937 \]  \hspace{1cm} (2.31)
Using eq.(2.23) we then get
\[
p = \begin{bmatrix} -0.27465 & 0.413227 \end{bmatrix}^T \tag{2.32}
\]

Finally using the matrix relation (2.19) we obtain the following expression for \( P \):
\[
P = \begin{bmatrix}
0.536906 & 0.37715 \\
0.37716 & 0.46254
\end{bmatrix}
\]

It is easy to check that this a real symmetric positive definite matrix.

The Liapunov function corresponding to the values of \( q \) and \( P \) above can be put in the following explicit form:
\[
V(x) = 0.536906 x_1^2 + 0.75432 x_1 x_2 + 0.46254 x_2^2 \\
+ 0.555(0.91632 - \cos(x_1 + 0.412) - 0.40 x_1 - 0.05 x_1^2)
\]
\[
\tag{2.33}
\]

Using the values of \( y_1^\prime \) and \( y_2^\prime \) we get \( V_{m1} \) and \( V_{m2} \)
\[
V_{m1} = 4.3242, \quad V_{m2} = 1.4029
\]

Thus, \( V_{min} = V_{m2} = 1.4029 \)

The stability region is then found by satisfying the following inequality:
\[
0.50913 x_1^2 + 0.75432 x_1 x_2 + 0.46254 x_2^2 \\
+ 0.555(0.91632 - \cos(x_1 + 0.412) - 0.40 x_1^3) < 1.4029 \tag{2.34}
\]

A plot of this region is shown in Fig.2.7 as the solid curve. The dotted curve shows the region computed by
following the procedure of Pai et al[19]. The Liapunov function obtained in this case is

\[ V(x) = 0.50 x_1^2 + x_1 x_2 + x_2^2 + 2[0.91632 - \cos(x_1+0.412) - 0.40 x_1] \]  \hspace{1cm} (2.35)

The sector boundaries of the nonlinear function \( f(y) \) as shown in Fig.2.5 are computed as below;

\[ y_1 = 3.965, \quad y_2 = 2.3176 \]

The values of \( V_{m1} \) and \( V_{m2} \) are then easily computed and we obtain \( V_{min} : \frac{V_{m2}}{y_2} = 3.1540 \). Using this value of \( V_{min} \), the inequality for the region of stability comes out as

\[ 0.5x_1^2 + x_1 x_2 + x_2^2 + 2[0.91632 - \cos(x_1+0.412) - 0.40 x_1] < 3.1540 \]  \hspace{1cm} (2.36)

The two curves in Fig.2.7 show that even though the sector boundaries \(-y_1^*\) and \(y_2^*\) obtained after the linear manipulation are contained in the boundaries \(-y_1\) and \(y_2\) of the original nonlinearities, the proposed technique results in an improved region of stability.

We have also computed the fault clearance time for this case by solving numerically the faulted equation [19].

\[ \delta + \delta = 0.40 - 0.20 \sin \delta \]  \hspace{1cm} (2.37)

with the initial conditions \( \delta(0) = 0.314 \) radian, \( \dot{\delta}(0) = 0 \).

The trajectory of the machine under faulted condition is shown also in Fig.2.7. This shows that the fault clearance time.

We note that these initial conditions correspond to the equilibrium values of \( \delta(t) \) and \( \dot{\delta}(t) \) in the pre-fault state of the machine.
Fig. 2.7 Stability regions

- Curve I stability region: $d=1$, $K=2.5$
- Curve II stability region: $d=0.27$, $K=2.5$
- Curve III stability region: $d=1$, $K=5$

Fig. 2.8 Regions of stability for various damping coefficients and sector gains
time is somewhat reduced in our case and is equal to 3.47 normalized seconds.

We have also repeated the above analysis with different values of the damping constants; \( d = \{0.278, 0.5, 0.8, 1.0\} \) and for \( k' = 2.5 \). It turns out that the region of stability for the same value of \( d \) as used earlier is increased slightly with \( k' \) reduced from 5 to 2.5. Also with less damping the region generally gets reduced. Some of these results are shown in Fig.2.8.

2.4 EFFECTS OF THE GOVERNOR TIME CONSTANTS

Let us now consider the machine model given by equations (2.8)-(2.9) which allows for the effects of the governor time constant to be incorporated. We can proceed in the same manner as described in the preceding section in order to generate the Liapunov function and use this function for computing the region of stability and the fault clearance time. As in the previous case we shall first introduce the minor feedbacks with gain \( a \) so as to obtain the following modified system equations:

\[
\begin{align*}
\dot{x} &= A_1 x + b f_1(y) \\
y &= c^T x
\end{align*}
\]  

\( A_1, b \) and \( c \) are given by

\[
A_1 = \begin{bmatrix}
0 & 1 & 0 \\
-a \mu & -d/M & -\mu \\
0 & g & -h
\end{bmatrix}, \quad b = \begin{bmatrix}
0 \\
-1 \\
0
\end{bmatrix}, \quad c = \begin{bmatrix}
1 \\
0 \\
0
\end{bmatrix}
\]

The modified nonlinear function \( f_1(y) \) is also obtained as below:
Fig. 2.9 Modified polar plot for system with governor action
In order to see the effects of the governor action, let us consider the machine with the following parameters (in per units):

\[ M = 1.0, \quad d = 1.0, \quad \alpha = 1.0, \quad P_m = 0.4, \quad \delta_0 = 0.412 \text{ radian} \]

\[ g = 0.50 \text{ and } h = 0.08. \]

Let us choose again the value of \( \alpha = 0.10. \) The transfer function \( G_1(s) \) is then computed as below:

\[
G_1(s) = \frac{s + 0.08}{s^3 + 1.08s^2 + 0.68s + 0.008}
\]

The nonlinear function \( f_1(y) \) is then plotted as before which gives the sector boundaries (see Fig.2.5)

\[ y_1 = 3.588, \quad y'_1 = 2.08\]

The Popov condition for stability has then the following form:

\[
\Pi(\omega) = \text{Re}\left[ (1 + j\omega) G_1(j\omega) \right] + \frac{1}{k'} \geq 0 \quad (2.40)
\]

From the modified polar plot as shown in Fig.2.9, with the choice of \( k' = 5 \) we compute the scalar \( q = 0.3448. \) Using this in the expression for \( \Pi(\omega) \) corresponding to eq.(2.22) we obtain the following equation after spectral factorization:

\[
\phi(\omega) = 0.44721(j\omega)^3 + 0.24566(j\omega)^2 + 0.8433(j\omega) + 0.02555 \quad (2.41)
\]

Using relation(2.23) with \( m \) obtained from eq.(2.21), we then evaluate the vector \( p \) as below

\[ p = [-0.27465 \quad 0.23733 \quad -0.56706]^T \]

Finally from eq.(2.19) we get the following value of the specific matrix \( P: \)
This is found to be a real symmetric positive definite matrix. Thus an appropriate Liapunov function for this third order system can be written explicitly as below:

\[ V(x) = 1.69556 x_1^2 + 0.37716 x_1 x_2 - 2.45073 x_1 x_3 + 0.37716 x_2^2 - 0.27852 x_2 x_3 + 0.27852 x_3^2 - 0.253609 x_1 + 5.17986 x_3 + 0.3448 [0.91632 - \cos(x_1 + 0.412) - 0.4 x_1] \]

(2.42)

Now with \( y_1 \) and \( y_2 \) given above we find

\[ V_{m1} = 4.6348, \quad V_{m2} = 1.49692 \]

Thus, we get \( V_{\min} = V_{m2} = 1.49692 \)

The region of stability is then obtained from the inequality

\[ 1.67832 x_1^2 + 0.75432 x_1 x_2 - 4.90146 x_1 x_3 + 0.27852 x_2^2 - 0.507218 x_2 x_3 + 5.17986 x_3^2 + 0.3448 [0.91632 - \cos(x_1 + 0.412) - 0.4 x_1] < 1.49692 \]

(2.43)

The fault clearance time computed by using this Liapunov function \( V(x) \) for the same initial conditions used earlier is 3.05 normalized seconds. On the other hand, following the procedure of Pai et al [19] the Liapunov function obtained in this case is

\[ V(x) = 4.64 x_1^2 + 1.28 x_1 x_2 - 16 x_1 x_3 + 0.51931 x_2^2 - 0.48276 x_2 x_3 + 16.51724 x_3^2 + 0.3448 [0.91632 - \cos(x_1 + 0.412) - 0.4 x_1] \]

(2.44)
The minimum value of this function corresponding to the nearest critical point $y_2 = 2.3176$ is then computed

$$V_{\text{min}} = V_{m2} = 3.0722$$

The region of stability is thus obtained by the inequality

$$4.64 x_1^2 + 1.28 x_1 x_2 - 16 x_1 x_3 + 0.51931 x_2^2$$
$$- 0.48276 x_2 x_3 + 16.51724 x_3^2 + 2[0.91632$$
$$- \cos(x_1+0.412) - 0.40 x_1] < 3.0722 \quad (2.45)$$

Since inequalities (2.43) and (2.45) involve three variables we can only show the intersections of the three dimensional closed space of asymptotic stability of the machine in any chosen plane by setting the third variable equal to zero[20]. Fig.2.10 shows the intersection of the plane $x_3 = 0$ with the stability space, along with the region obtained by neglecting governor action and the intersection obtained by following the procedure of Pai et al[19]. Fig.(2.11) and Fig.(2.12) show the intersections of the stability space with $x_1 = 0$ and $x_2 = 0$ planes as obtained through our method and that due to Pai et al(regions shown by dotted curves).

We have also computed the intersections of the stability space with the planes $x_1 = 0$, $x_2 = 0$ and $x_3 = 0$ for various values of damping constants. Some of these results are shown by a set of these intersections in Figures 2.13-2.15.
Fig. 2.10 Intersections of stability spaces with plane $x_3 = 0$ and the region obtained without the effects of governor action.

Fig. 2.11 Intersections of the stability spaces with the plane $x_1 = 0$

Fig. 2.12 Intersection of the stability spaces with the plane $x_2 = 0$
Fig. 2.13 Intersections of stability spaces corresponding to various damping coefficients with the plane $x_3=0$

Fig. 2.14 Intersections of stability spaces corresponding to various damping coefficients with the plane $x_2=0$

Fig. 2.15 Intersections of stability spaces corresponding to various damping coefficients with the plane $x_1=0$
2.5 CONCLUSION

We have presented a systematic investigations of the use of Kalman's procedure for constructing Liapunov function for the study of the transient stability of a single machine power system. We have introduced a minor feedback around the linear subsystem in order to shift the pole at the origin of the transfer function of the linear system to the left hand side of the s-plane. We have annulled the effect of this minor feedback by introducing an equal and opposite feedback which is combined with the nonlinear system. This manipulation helps to simplify the mathematical steps involved in the generation of the Liapunov function.

The numerical results presented in this chapter show that compared to the earlier results we can obtain larger regions of stability by taking a finite value of the sector gain k. All the earlier results of Willems [15],[16] and Pai et al [19] are based on the choice k=\infty even though the actual sector gain is k=0.8165 with \alpha=0.1. Our results show that a choice of k=2.5 is perhaps the best.

We have also presented results to indicate the effects of damping and of the governor time constant on the region of stability. So far as damping is concerned, the region of stability increases as the damping increases. However, the effect of a finite time constant of the velocity governor is to allow larger disturbances in the speed \dot{s}(t) with a marginal reduction in the allowable maximum perturbation in the rotor angle \delta(t).