APPENDIX F
DETAIL DERIVATION OF RESULTS FOR EXAMPLE IN SUBSECTION 6.2.3

The Popov's frequency domain condition for given \( \sigma=0.0025 \) with \( q=0.3585 \) and \( k'=5 \) is given by

\[
P(\sigma, \omega) = \text{Re} \left( \frac{(0.0775+j \omega)}{(0.0063068-1.0725 \omega^2)+j \omega(0.67462-\omega^2)} \right)^
\]

for all \( \omega \geq 0 \) \hspace{1cm} (F.1)

From solution of the spectral factorization problem in order to obtain \( \phi(j \omega) \) we have

\[
\phi(j \omega) = 0.4472(j \omega)^3 + 0.17557(j \omega)^2 + 0.83814(j \omega) + 0.022287
\]

Solutions of analog relation of eq.(2.23) gives the value of a real 3x1 vector \( p_0 \)

\[
p_0 = [-0.2600713 \quad 0.3040636 \quad -0.59835]^T \hspace{1cm} (F.3)
\]

With this vector \( p_0 \) and the system matrix \( A_k \), known use is made of the matrix relation (6.7) to obtain \( P_0 \)

\[
P_0 = \begin{bmatrix}
1.83179 & 0.38398 & -2.68221 \\
0.38398 & 0.2955 & -0.270894 \\
-2.68221 & -0.270894 & 5.805223
\end{bmatrix} \hspace{1cm} (F.4)
\]

The Liapunov function \( V(x, \sigma) \) corresponding to above values of the scalar \( q \) and the matrix \( P_0 \) is thus given by

\[
V(x, \sigma) = 1.81518 x_1^2 + 0.2955 x_2^2 + 5.805223 x_3^2 + 0.76976 x_1 x_2 \\
-5.3645 x_1 x_3 - 0.5418 x_2 x_3 + 0.3585[0.91632 - \cos(x_1 + 0.412)] \\
-0.40x_1 \hspace{1cm} (F.5)
\]

The minimum value of this function corresponding to \( y_2 = 2.08 \) is then computed as

\[
V_{\text{min}} = 1.54285 \hspace{1cm} (F.6)
\]

The region of exponential stability is thus obtained from the inequality
The above steps in derivation of results are repeated for the case with $\sigma = 0.005$. Region of exponential stability for $\sigma = 0.005$ is obtained by the inequality

$$1.99325 x_1^2 + 0.31525 x_2^2 + 6.4882 x_3^2 + 0.78534 x_1 x_2 - 6.01418 x_1 x_3 - 0.56878 x_2 x_3 + 0.3125 [0.91632 - \cos(x_1 + 0.412) - 0.40 x_1] < 1.5976$$

(F.8)

The above analysis is also repeated for the system with various damping coefficients and also for various choice of sector gains with specified degree $\sigma = 0.0025$. A few results i.e. with $d = 0.5$, $k' = 3.5$ and $d = 1$, $k' = 3.5$ are presented here for deriving some conclusions.

Region of exponential stability with $d = 0.5$, $k' = 3.5$ with specified $\sigma = 0.0025$ is obtained by the inequality

$$1.5045 x_1^2 + 0.45062 x_2^2 + 5.8396 x_3^2 + 0.72996 x_1 x_2 - 5.1354 x_1 x_3 - 0.5952 x_2 x_3 + 0.762 [0.91632 - \cos(x_1 + 0.412) - 0.40 x_1] < 1.7041$$

(F.9)

Region of exponential stability with $d = 1$, $k' = 3.5$ and given degree $\sigma = 0.0025$ is computed from the inequality

$$1.804 x_1^2 + 0.2988 x_2^2 + 5.682 x_3^2 + 0.7299 x_1 x_2 - 5.2768 x_1 x_3 - 0.5829 x_2 x_3 + 0.173 [0.91632 - \cos(x_1 + 0.412) - 0.40 x_1] < 1.8525$$

(F.10)

These regions of exponential stability are shown in Figures 6.5-6.7.