CHAPTER V

USE OF A DIFFERENT AGGREGATION DECOMPOSITION TECHNIQUE IN DEALING WITH THE PROBLEM OF TRANSIENT STABILITY OF MULTIMACHINE POWER SYSTEMS

5.1 INTRODUCTION

The decomposition technique of the preceding chapter has the disadvantage of yielding a number of subsystems which may be much larger than the number of machines in a multimachine power system. To avoid this difficulty, Jocic et al[55] has preferred to use the nth machine as the reference machine in terms of which the dynamic equations of the remaining machines are rewritten. This approach has the advantage of reducing the number of subsystems to (n-1) and is thus much simpler to deal with particularly when n is large. These authors also make use of the comparison principle in order to obtain conditions for stability of the composite system. However, the choice of the Liapunov function is somewhat different from the choice of Pai and Narayana[54].

The aim of this chapter is to extend the new method of aggregation and decomposition introduced by us in the previous chapter to the multimachine model of Jocic et al[55]. We shall first introduce the system model in section 5.2 in some details and then we shall review the work of Jocic et al in section 5.3 so as to provide a ready comparison of our own results with those of Jocic et al. In section 5.4 we then present the details of application of our approach of aggregation of inter-
action effects and decomposition of the given system into decoupled subsystems. In section 5.5 we illustrate the results by presenting a numerical example based on a second order model for each subsystem. In section 5.6 we then study the effects of governor action by using a third order model for each subsystem. Finally, the main conclusions are summarized in section 5.7.

5.2 PROBLEM STATEMENT

Let us again consider the n-machine power system characterized by the set of equations (4.1) and (4.2) which are reproduced below for ready reference

\[ M_i \dot{\delta}_i(t) + d_i \delta_i(t) = P_{mi} - P_{ei}(t) \]  

(5.1)

where \( P_{ei} \) is given by

\[ P_{ei} = E_i^2 G_{ii} + \sum_{j=1}^{n} E_i E_j Y_{ij} \cos(\delta_i - \delta_j - \theta_{ij}), \quad i=1,2,\ldots,n \]  

(5.2)

We refer to our discussion in sections 3.2 and 4.2 regarding the various assumptions under which the set of equations (5.1) and (5.2) can adequately represent the n-machine power system. Further, the various symbols appearing above have also been explained in the two above mentioned sections. Finally, as in Chapter IV, we shall confine our discussions in this chapter to the case of machines with uniform damping corresponding to \( d_i/M_i = \lambda, \quad i=1,2,\ldots,n \).

In order to decompose the multimachine system into \((n-1)\) interconnected second order subsystems, it is possible to
choose the state variables of the ith subsystem as follows:

\[
\begin{align*}
\dot{x}_{i1} & = \delta_i - \delta_n \\
\dot{x}_{i2} & = (\delta_i - \delta_n) - (\delta_0 - \delta_0)
\end{align*}
\] (5.3) (5.4)

In other words, we use the speed and the angle of the nth machine \( \delta_n \) and \( \delta_n - \delta_0 \) as the reference levels for the state variables of all the subsystems. \( (\delta_i, \delta_i) \) are components of a (locally) stable equilibrium obtained as solutions of the relation

\[
P_{ei}(\delta_i, \delta_i) = P_{mi}
\]

where \( i \) can have the values \( i = 1, 2, \ldots, (n-1) \). If we now make use of equations (5.1) and (5.2) we get the following description of the ith subsystem:

\[
\begin{align*}
\dot{x}_i & = A_i x_i + b_i f_i(y_i) + h_i(x) \\
y_i & = c_i^T x_i
\end{align*}
\] (5.5) (5.6)

where matrix \( A_i \) and vectors \( b_i \) and \( c_i^T \) are given by

\[
A_i = \begin{bmatrix}
-\lambda & 0 \\
1 & 0
\end{bmatrix}
\]

\[
b_i = \begin{bmatrix}
-1 \\
0
\end{bmatrix}
\]

\[
c_i = \begin{bmatrix}
0 \\
1
\end{bmatrix}
\]

The nonlinear functions \( f_i(y_i) \) and \( h_i(x) \) are given by the following equations:

\[
f_i(y_i) = \nu_i \left[ \sin(y_i + (\delta_i - \delta_0)) - \sin(\delta_i - \delta_0) \right] \] (5.7)

where

\[
\nu_i = (M_i^{-1} + K_n^{-1})E_i E_n Y_i \sin \theta_i
\]

\[
h_i(x) = \begin{bmatrix}
h_{i1}(x) \\
0
\end{bmatrix}
\] (5.8)
with

\[ h_{11}(x) = (M_{-1}^{n-1}M_{1}^{n}E_{1}E_{n}Y_{m} \cos \theta_{m} [\cos(\delta_{1} - \delta_{n}) - \cos(\delta_{1} - \delta_{n})] + \sum_{j=1}^{n} (M_{j}^{n}f_{1j}^{1-1}f_{1j}^{1} - f_{1j}^{1}) \] (5.9)

where \( f_{1j} = E_{1}E_{j}Y_{ij}[\cos(\delta_{1} - \delta_{j}) - \cos(\delta_{1} - \delta_{j})] \) (5.10)

We note that the function \( f_{1}(y_{1}) \) satisfies the following sector conditions:

(i) \( f_{1}(0) = 0 \)

(ii) \( 0 \leq y_{1}f_{1}(y_{1}) \leq k_{1}y_{1}^{2}, -y_{11} < y_{1} < y_{12} \) (5.11)

where \( y_{11}, y_{12} \) and \( k_{1} \) are some positive quantities

and \( k_{1} = \frac{d}{dy_{1}} f_{1}(y_{1}) \bigg|_{y_{1}=0} \)

The two boundaries \( y_{11} \) and \( y_{12} \) of the region where the sector conditions (5.11) are satisfied can be obtained from the condition, \( f_{1}(y_{1}) = 0 \). A schematic representation of the system is shown in Fig. 5.1.

We note again that the \( i \)th subsystem model (5.5)-(5.6) resembles the model of the single machine power system except for the presence of the term \( h_{1}(x) \) which accounts for the interaction between the \( i \)th and the other subsystems. Our problem is to apply the techniques of section 4.4, in order to obtain frequency domain stability conditions for the \( (n-1) \) subsystems (5.5)-(5.6) which would then provide a test for stability for the multimachine system.
Fig. 5.1 Schematic representation of the ith subsystem
5.3 REVIEW OF THE WORK OF JOCIC et al [55]

Let us briefly review the work of Jocic et al [55] before introducing our own contributions regarding the solution of the above mentioned problems. These authors also make use of the pole shifting to obtain the following model for the interaction free part of the ith subsystem

\[ \dot{x}_i = A_{il} x_i + b_i f_{il}(y_i) \] (5.12)
\[ y_i = c_i^T x_i \] (5.13)

where the system matrix \( A_{il} \) and the nonlinear function \( f_{il}(y_i) \) are given by the following equations:

\[ A_{il} = \begin{bmatrix} -\lambda & -a_i \nu_i \\ 1 & 0 \end{bmatrix} \] (5.14)
\[ f_{il}(y_i) = \nu_i \left[ \sin(y_i + \delta_i - \delta_o) - \sin(\delta_i - \delta_o) - a_i y_i \right] \] (5.15)

with \( \nu_i = (W_i^{-1} + \kappa_i^{-1})E_i E_n Y_{in} \sin \theta_{in} \) and \( a_i \) is a positive number. Then they employ the method of Kalman for generating a Lure'-Postnikov type Liapunov function for this interaction free subsystem. However, they assume \( k_i = \infty \) while making the spectral factorization and also take an arbitrary value of \( q_i \) satisfying the constraint \( \lambda q_i > 1 \). After generating the Liapunov function, the feedback gain \( a_i \) is set equal to zero which, in effect, reduces equations (5.12)-(5.13) to the original interaction free parts of equations (5.5)-(5.6). This procedure yields the Lure'-Postnikov type of Liapunov function \( V_i(x_i) \)

\[ V_i(x_i) = x_i^T P_i x_i + q_i \int_0^{y_i} f_i(y) dy \] (5.16)

Subsequently, the Liapunov function for analysing the effects of interaction is chosen as follows:

\[ V_i(x_i) = x_i^T P_i x_i + q_i \int_0^{y_i} f_i(y) dy \] (5.16)
It is assumed that one can compute the positive numbers \( n_{i1}, n_{i2}, \) and \( n_{i3} \) and the nonnegative interaction constraints \( z_{ij} \) such that the following bounds are satisfied:
\[
\begin{align*}
 n_{i1} ||x_i|| &\leq v_i(x_i) \leq n_{i2} ||x_i|| \\
v_i(x_i) &= [\text{grad } v_i(x_i)]^T x_i \\
&\leq -n_{i3} ||x_i||, i=1,2,\ldots,(n-1)
\end{align*}
\]
and
\[
[\text{grad } v_i(x_i)]^T h_i(x) \leq \sum_{j=1}^{n-1} z_{ij} ||x_j||, i=1,2,\ldots,(n-1)
\]
Finally, the Liapunov function for the composite system is taken as the vector Liapunov function \( v(x) \) given below
\[
v(x) = [V_1(x_1) \ V_2(x_2) \ldots V_{n-1}(x_{n-1})]^T
\]
The derivative of this Liapunov function is shown to satisfy a vector differential inequality
\[
\dot{V}(x) \leq Wv(x)
\]
where \( W \) is \((n-1)x(n-1)\) constant aggregate matrix having the following elements:
\[
\begin{align*}
w_{ij} &= \begin{cases} 
-n_{i2}^2 & i=j \\
n_{i3} z_{ij} n_{i1}^{-1} & i\neq j
\end{cases} \\
&= (-1)^{i+j} n_{i3} z_{ij} n_{i1}^{-1}, i\neq j
\end{align*}
\]
A sufficient condition for stability of the interconnected system is then obtained in form of the following determinantal inequality:
\[
(-1)^j \begin{vmatrix} w_{11} & w_{12} & \cdots & w_{1j} \\
w_{21} & w_{22} & \cdots & w_{2j} \\
\vdots & \vdots & \ddots & \vdots \\
w_{j1} & w_{j2} & \cdots & w_{jj} \end{vmatrix} > 0, j=1,2,\ldots,(n-1)
\]
The authors also consider the question of constructing the overall...
stability region of the (n-1) subsystems through the method of Weissenberger[60].

5.4 DERIVATION OF FREQUENCY DOMAIN STABILITY CONDITIONS

In this section we shall make use of our own approach to the problem of stability analysis of interconnected system which has been presented in section 4.4. For this purpose we shall start by introducing the pole shifting manipulation which has, in fact, also been done by Jocic et al as mentioned in section 5.3. However, these authors have subsequently allowed the feedback gain $a$ to become zero. On the contrary, we retain a finite $a$ and generate the Liapunov function for the pole shifted system. Another point of difference between our approach and that of Jocic et al lies in the choice of the Liapunov function. As described in section 5.3, these authors choose $V_i(x_i)$ as the Liapunov function for the $i$th subsystem, $V_i(x_i)$ being the Lure'-Postnikov type function generated by Kalman's procedure. As described in section 4.4, we, however, find it convenient to make use of $V_i(x_i)$ directly for our analysis.

5.4.1 Generation of $V_i(x_i)$ for Interaction Free Subsystems

Let us consider first the interaction free part of the $i$th subsystem as given by equations (5.12)-(5.13) which have also been considered by Jocic et al. The modified nonlinear function $f_{i1}(y_i)$ as given by (5.15) is shown in Fig.5.2, where the original nonlinear function $f_i(y_i)$ is also shown as a dotted curve. It is apparent that the boundaries $-y_{i1}$ and $y_{i2}$ of the region of $y_i$ over
Fig. 5.2 Characteristics of modified and original nonlinearities of ith subsystem with their sector boundaries and sector gains
which the function \( f_{i_1}(y_i) \) satisfies the sector conditions

\begin{align*}
\text{i)} & \quad f_{i_1}(0) = 0 \\
\text{ii)} & \quad 0 \leq y_i f_{i_1}(y_i) \leq k_i y_i^2
\end{align*} 

is contained in the region \(-y_{i_{11}} \leq y_i \leq y_{i_{12}}\) of the original non-linear function \( f_i(y_i) \). The sector gain \( k_i \) is also slightly different from the gain \( k_i \) of \( f_i(y_i) \).

For the modified system (5.12) - (5.13) a sufficient condition for the system to be asymptotically stable in the finite region \(-y_{i_{11}} \leq y_i \leq y_{i_{12}}\) is that the following frequency domain inequality be satisfied for all real frequencies \( 0 < \omega < \infty \):

\[
\Pi_i(\omega) = \text{Re} [1 + j\omega q_i] G_{i_1}(j\omega)] + \frac{1}{k_i} > 0 
\]

(5.26)

where \( G_{i_1}(j\omega) \) is the transfer function of the linear part of the system (5.12)-(5.13) and is given by

\[
G_{i_1}(j\omega) \triangleq -c_i^T(j\omega I - A_{i_1})^{-1} b_i
\]

Let us assume that the system under consideration is asymptotically stable so that we can make use of the modified polar plot shown in Fig. 5.3 for choosing an appropriate value of \( q_i \) with selected \( k_i \).

After choosing \( k_i \) and \( q_i \), we employ the spectral factorization to get the relation

\[
\Pi_i(\omega) = \frac{\phi_i(j\omega)\phi_i(-j\omega)}{\det(j\omega I - A_{i_1})\det(-j\omega I - A_{i_1})}
\]

(5.27)

After having found the polynomial \( \phi_i(j\omega) \) we proceed as in section 2.3 so as to obtain the desired matrix \( P_i \) and thus the Lyapunov function \( V_i(x_i) \) in the form

\[
V_i(x_i) = x_i^T P_i x_i + q_i \int_0^y f_{i_1}(y_i) dy_i
\]

(5.28)
Fig. 5.3 Modified polar plot for ith subsystem for choosing an appropriate $q_i$ for selected $k_i$. 

$q_i = \frac{1}{\text{slope}} = 0.228, k_i = 5$

$k_i = 0.8165$

$q_i = \frac{1}{\text{slope}} = -1.11, k_i = 0.8165$
5.4.2 Aggregation of Interaction Effects

After obtaining $V_i(x_i)$ we proceed as in section 4.4.2 in order to aggregate the effects of the interaction terms in the various subsystems. Let us consider for the time being the $i$th subsystem (5.5)-(5.6) of the given multimachine system and let us assume that $V_i(x_i)$ generated above (5.28) is also an appropriate Liapunov function for this subsystem. The derivative of this function $\dot{V}_i(x_i)$ is then computed as below

$$
\dot{V}_i(x_i) = [\text{grad } V_i(x)]^T \dot{x}_i
$$

$$
= x_i^T [A_{i1}^T P_i + P_i A_{i1}] x_i + f_{i1}(y_i) [q_i c_i^T A_{i1}]
$$

$$
+ 2 b_i P_i x_i + c_i b_i q_i f_{i1}^2(y_i)
$$

$$
+ [\text{grad } V_i(x)]^T h_i(x) \quad (5.29)
$$

For the given $V_i(x_i)$ and the given interaction function $h_i(x)$ it is generally possible to obtain the following bound for the last term on the right hand side of the eq.(5.29);

$$
[\text{grad } V_i(x)]^T h_i(x) \leq \sum_{j=1}^{n-1} \beta_{ij} V_j(x_j) \quad (5.30)
$$

Using this inequality and adding the set of terms $[f_{i1}^2(y_i)c_i^T] x_i - f_{i1}^2(y_i)/k_i^2$ and subtracting the equivalent terms $f_{i1}(y_i) [y_i - f_{i1}(y_i)/k_i^2]$ to the right hand side of eq.(5.29) as to include the sector information we get the following inequality:

$$
\dot{V}_i(x_i) \leq x_i^T [A_{i1}^T P_i + P_i A_{i1}] x_i + \sum_{j=1}^{n-1} \beta_{ij} x_j^T P_j x_j
$$

$$
+ f_{i1}(y_i) [q_i c_i^T A_{i1} + 2 b_i P_i x_i + c_i b_i q_i f_{i1}^2(y_i)]
$$

$$
+ \sum_{j=1}^{n-1} \beta_{ij} q_i \int_0^{y_i} f_{i1}(y_i) dy_i + [f_{i1}(y_i)c_i^T] x_i
$$

$$
- f_{i1}^2(y_i)/k_i^2 - f_{i1}(y_i) [y_i - f_{i1}(y_i)/k_i^2] \quad (5.31)
$$
Inequality (5.31), of course, holds for all the (n-1) subsystems.

We note that effects of the interactions in all the (n-1) subsystems are similar in nature in the sense that we are able to bind the interaction effects by \( \sum_{j=1}^{n-1} \beta_{ij} V_j(x_j) \), for \( i=1, 2, \ldots, (n-1) \). This shows that if we choose the Liapunov function \( V(x) \) as the scalar function

\[
V(x) = \sum_{i=1}^{n-1} V_i(x_i)
\]

we should be able to regroup the interaction terms which would permit the establishment of the following inequality:

\[
\dot{V}(x) \leq \sum_{i=1}^{n-1} \dot{W}_i(x_i)
\]

where \( \dot{W}_i(x_i) \) is given by the following expression;

\[
\dot{W}_i(x_i) = x_i^T \left[ (A_{i1} + \sigma_i I)^T P_i + P_i (A_{i1} + \sigma_i I) \right] x_i + 2k_i^1 y_i \left[ b_i^T P_i + \frac{1}{2} c_i^T q_i^1 (A_{i1} + \sigma_i I) + \frac{1}{2} c_i^T \right] x_i
\]

\[
+ \left( c_i^T P_i q_i^1 \frac{1}{k_i^1} \right) k_i^1 y_i^2 - f_{i1}(y_i) [y_i - f_{i1}(y_i)/k_i^1]
\]

where the constant \( \sigma_i \) is defined as below

\[
\sigma_i = \frac{1}{\Delta_i^2} \sum_{j=1}^{n-1} \beta_{ji}
\]

This can be considered as the aggregated interaction coefficient of the ith subsystem.

### 5.4.3 Analysis of Decoupled Subsystems

Let us now introduce (n-1) decoupled subsystems and let the state space model for the ith subsystem given by the following pair of equations:

\[
\dot{x}_i = A_{i1} x_i + b_i f_{i1}(y_i)
\]

\[
y_i = c_i^T x_i, \quad i=1, 2, \ldots, (n-1)
\]
where the matrix $A'_1$ is given by

$$A'_1 = A_1 + \sigma \ I$$

(5.38)

Let us assume that the eigen value of $A'_1$ have negative real parts and the triple $(A'_1, b'_1, c'_1^T)$ is completely controllable and observable.

It then follows that a sufficient condition for the system (5.36)-(5.37) to be asymptotically stable in the finite region $-y_{11} \leq y_1 \leq y_{12}$ is that the following inequality be satisfied for all real frequencies $0 < \omega \leq \omega_0$:

$$\Pi'_1(\omega) = \text{Re}[(1+j\omega q'_1)b'_1] + \frac{1}{k_i^2} > 0$$

(5.39)

Also, from the Kalman-Yakubovich lemma inequality (5.39) is both necessary and sufficient for the existence of Lure'-Postnikov type of Liapunov function $W'_1(x'_1)$ given by

$$W'_1(x'_1) = x'_1 P'_1 x'_1 + q'_1 \int_{y_{11}}^{y_{12}} f'_1(y_i)dy_i$$

(5.40)

The real scalar $q'_1$ and the real symmetric positive definite matrix $P'_1$ must satisfy the following equations:

$$A'_1^T P'_1 + P'_1 A'_1 = -P'_1 P'_1^T$$

(5.41)

$$P'_1 b'_1 + \frac{1}{2} q'_1 A'_1 c'_1 = -m'_1^{1/2} P'_1$$

(5.42)

$$-m'_1^{1/2} = q'_1 c'_1^T b'_1 - \frac{1}{k_i^2}$$

(5.43)

For any given system for which the Popov condition (5.39) is satisfied, we can generate the function $W'_1(x'_1)$ following Kalman's procedure as explained in section 2.3. For this $W'_1(x'_1)$ we get

This, of course, follows from our earlier assumptions in Chapter II that the triplet $(A'_1, b'_1, c'_1^T)$ is completely controllable and completely observable.
Using equations (5.36)-(5.37) we get the following expression for $\dot{W}_i(x_i)$

$$\dot{W}_i(x_i) = x_i^T [A_i^T P_i + P_i A_i] x_i + 2 k_i^1 y_i [b_i^T P_i^1 \gamma_i^1 + c_i b_i q_i^1 f_i^2(y_i)] + \frac{1}{2} c_i^T q_i^1 A_i^1 x_i + c_i^T b_i q_i^1 f_i^2(y_i)$$

(5.45)

In order to incorporate the sector conditions we add the terms $f_{i1}(y_i) c_i^T x_i - f_{i1}^2(y_i)/k_i^1$ and subtract the equivalent terms $f_{i1}(y_i) [y_i - f_{i1}(y_i)/k_i^1]$ to the right hand side of eq.(5.45). This gives the following expression for $\dot{W}_i(x_i)$

$$\dot{W}_i(x_i) = x_i^T [A_i^T P_i + P_i A_i] x_i + 2 k_i^1 y_i [b_i^T P_i^1 \gamma_i^1 + c_i b_i q_i^1 - \frac{1}{2} k_i^1] k_i^1 y_i^2 + f_{i1}(y_i) [y_i - f_{i1}(y_i)/k_i^1]$$

(5.46)

This is a negative definite quantity according to Popov's theorem, if we assume that the inequality (5.39) is satisfied by the given system [8].

5.4.4 Liapunov Function for the Composite System

The results of subsections 5.4.2 and 5.4.3 help us to choose the desired Liapunov function for the n-machine system under consideration. We note first the similarity in the expressions for $W_i(x_i)$ given in eq.(5.34) and $\dot{W}_i(x_i)$ given in eq.(5.46). The two expressions become identical if we replace the matrix $P_i$ in eq.(5.34) by $P_i^1$. This suggests that the choice of Liapunov function $V'(x)$ should be the scalar function

$$V'(x) = \sum_{i=1}^{n-1} W_i'(x_i)$$

(5.47)
where $W_1^i(x_i)$ are the Liapunov functions generated for the decoupled subsystems (5.36)-(5.37). The derivative of this function can be obtained as

$$\dot{V}(x) = \sum_{i=1}^{n-1} W_1^i(x_i)$$

which is negative definite if the Popov inequality (5.39) is satisfied for $i=1,2,...,(n-1)$. Thus the $n$-machine power system will be asymptotically stable if the $(n-1)$ inequalities obtained from (5.39) are satisfied for all real frequencies $\omega$. These conditions are easily verified in the frequency domain through the modified polar plots.

5.5 THE EXAMPLE OF A THREE-MACHINE SYSTEM

Let us now consider the same 3-machine system which we have studied in section 4.5. If we use the third machine as the reference, we get only two subsystems (compared to 3-subsystems in Chapter IV). The state variable models of these two subsystems are given below:

Subsystem 1:

$$
\begin{align*}
\dot{x}_{11} &= [-100 \quad -11] x_{11} + [-1] f_{11}(y_1) + [h_{11}(x)] \\
\dot{x}_{12} &= [1 \quad 0] x_{12} + [0]
\end{align*}
$$

$$y_1 = [0 \quad 1] x_{11}$$

where

$$
\begin{align*}
x_{11} &= \delta_1 - \delta_3 \\
x_{12} &= (\delta_1 - \delta_3) - (\delta_1 - \delta_3)
\end{align*}
$$
The nonlinear functions $f_{11}(y_1)$ and $h_{11}(x)$ are given by

\begin{align}
    f_{11}(y_1) &= 12[\sin(y_1 - 2^\circ) + \sin 2^\circ - 11/12 y_1] \\
    h_{11}(x) &= 0.83 \sin(\frac{1}{2} x_{12}) \sin(\frac{1}{2} x_{12} - 2^\circ) + 0.11 \sin(\frac{1}{2} x_{22}) \\
        &\quad \cdot \sin(\frac{1}{2} x_{22} - 2^\circ) - 0.2 \sin(\frac{1}{2}(x_{12} - x_{22})) \sin(\frac{1}{2}(x_{12} - x_{22} + 18^\circ))
\end{align}

Subsystem 2:

\begin{align}
    \begin{bmatrix}
        \dot{x}_{21} \\
        \dot{x}_{22}
    \end{bmatrix} &=
    \begin{bmatrix}
        -100 & -10 \\
        1 & 0
    \end{bmatrix}
    \begin{bmatrix}
        x_{21} \\
        x_{22}
    \end{bmatrix}
    + \begin{bmatrix}
        -1 \\
        0
    \end{bmatrix} \cdot f_{21}(y_2) \\
    y_2 &= \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix}
        x_{21} \\
        x_{22}
    \end{bmatrix}
\end{align}

where

\begin{align*}
    x_{21} &= \delta_2 - \delta_3 \\
    x_{22} &= (\delta_2 - \delta_3) - (\delta_2 - \delta_3)
\end{align*}

The corresponding nonlinear functions are given by

\begin{align}
    f_{21}(y_2) &= 11[\sin(y_2 - 3^\circ) + \sin 3^\circ - 10/11 y_2] \\
    h_{21}(x) &= 0.382 \sin(\frac{1}{2} x_{22}) \sin(\frac{1}{2} x_{22} - 3^\circ) + 0.12 \sin^2(\frac{1}{2} x_{12}) \\
        &\quad - 0.20 \sin(\frac{1}{2}(x_{12} - x_{22})) \sin(\frac{1}{2}(x_{12} - x_{22} + 18^\circ))
\end{align}

The interaction terms appearing in equations (5.51) and (5.54) are then computed as below

\begin{align*}
    h_{11}(x) &\leq 1.03 |x_{12}| + 0.31 |x_{22}| \\
    h_{21}(x) &\leq 0.32 |x_{12}| + 0.582 |x_{22}|
\end{align*}

The Lyapunov functions for the interaction free subsystems are then computed following Kalman's procedure. While the earlier workers have arbitrarily chosen the values of sector gains $k_1 = k_2 = \infty$, the nonlinear functions in (5.50) and (5.53) are found
to have actual sector gains $k_1^t = k_2^t = 1$ which are used in this analysis. Also if the values of scalar multipliers $q_1$ and $q_2$ corresponding to subsystem 1 and subsystem 2 are determined using the modified polar plots these are found to be relatively large negative constants. In order to ensure the positive definiteness of Liapunov functions $V_1(x_1)$ and $V_2(x_2)$ for the above subsystems the solution for the desired matrices $P_1$ and $P_2$ then come out as matrices with large elements. This, in turn, makes the bounds on the interaction terms $f_{ij}$ also large and both the above subsystems are found to violate the inequality (5.26). In order to avoid this, $q_1$ and $q_2$ have been taken equal to -1. Having chosen the values of $k_i^t$ and $q_i$ the desired matrices $P_i$ are obtained and thus the Liapunov functions $V_i(x_i)$ for the interaction free subsystems are computed as follows:

$$V_1(x_1) = x_1^T \begin{bmatrix} 0.001365 & 0.01135 \\ 0.01135 & 1.40 \end{bmatrix} x_1 + (-1) \int_{12}^{1}[\sin(y_1-2^o) + \sin 2^o] dy_1$$

$$V_2(x_2) = x_2^T \begin{bmatrix} 0.001375 & 0.01250 \\ 0.01250 & 1.515 \end{bmatrix} x_2 + (-1) \int_{11}^{12}[\sin(y_2-3^o) + \sin 3^o] dy_2$$

The bounds on the interaction terms are then computed from the inequality (5.30) and are obtained below:

$$\frac{\partial V_1(x_1)}{\partial x_{11}} h_{11}(x) \leq 0.0145[x_1^T P_1 x_1 + q_1 \int_{0}^{1} f_{11}(y_1) dy_1] + 0.002[x_2^T P_2 x_2 + q_2 \int_{0}^{1} f_{21}(y_2) dy_2]$$
Making use of the defining relation (5.35), the values of aggregated interaction coefficients $\sigma_i$ are then computed

$$\sigma_1 = 0.0085$$  
$$\sigma_2 = 0.006$$

Proceeding with the decomposition-aggregation technique of the preceding subsection 5.4.2, the following expressions for the derivatives of the new Liapunov functions $V_i(x_i)$ of the two subsystems with interaction are obtained:

$$\frac{\partial V_2(x_2)}{\partial x_{21}} h_21(x) \leq 0.0025 \left[ x_1^T P_1 x_1 + q_1 \int_0^1 f_{11}(y_1) dy_1 \right]$$

$$+ 0.01 \left[ x_2^T P_2 x_2 + q_2 \int_0^1 f_{21}(y_2) dy_2 \right]$$

In order to find the values of $P_i$ and $q_i$ which will satisfy the set of equations (5.41)-(5.43) it is noted that the interaction does not alter the nonlinear functions $f_{i1}(y_i), i=1,2$ and accordingly the minimum values of $V_i(x_i)$ and $W_i(x_i)$ should be obtained for the same values of $x_{i1}$ and $x_{i2}$. Using this condition the desired values of scalar multipliers $q_i$ and the real symmetric positive definite matrices $P_i$ are calculated and the following values of the Liapunov functions for the above two subsystems with interaction effects considered are obtained:
\begin{equation}
W_1'(x_1) = x_1 \begin{bmatrix}
0.001705 & 0.014 \\
0.014 & 1.695
\end{bmatrix}
\begin{bmatrix}
x_1 \\
-1.12
\end{bmatrix}
+ \sin^2(y_1-2^\circ) \int_{y_1}^{y_2} [\sin(y_1-2^\circ)] dy_1
\end{equation}

\begin{equation}
W_2'(x_2) = x_2 \begin{bmatrix}
0.001605 & 0.0146 \\
0.0146 & 1.745
\end{bmatrix}
\begin{bmatrix}
x_2 \\
-1.08
\end{bmatrix}
+ \sin^2(y_2-3^\circ) \int_{y_2}^{y_3} [\sin(y_2-3^\circ)] dy_2
\end{equation}

The regions of stability of the two subsystems described by equations (5.49) and (5.52) are then computed on the basis of Liapunov functions \( V_i(x_i) \) and \( W'_i(x_i) \) and shown in Figures 5.4 and 5.5. The dotted curves show the regions of stability without interaction and the continuous curves show the regions of stability with interaction.

As it is obvious that regions obtained in the absence of interaction is somewhat larger.

5.6 EFFECTS OF GOVERNOR TIME CONSTANTS IN THE THREE-MACHINE CASE

In the previous section 5.5, we have neglected the effects of governor time constant which have resulted in a second order model for each of the two subsystems. In this section we shall proceed by including the effects of the governor action. We recall that in Chapter II also we have studied the consequences of the governor action on the region of transient stability of the single machine system by considering the dynamics of governor with single time constant. Since the order of the system becomes three after inclusion of the governor effects, the mathematical steps involved in the generation of the Liapunov
Fig. 5.4
Stability Regions Of Subsystem -1

Fig. 5.5
Stability Regions Of Subsystem -2
function become a little more complicated. However, there is no
basic change in the procedure to be followed. Accordingly we
shall describe below mainly the outcomes of the various steps
without going into details.

The state variable models of the two subsystems
(taking the third machine as the reference one) after inclusion
of the governor time constant can be checked to be given by the
following sets of equations:

\[ \dot{x}_i = A_i x_i + B_i f_i(y_i) + h_i(x) \quad (5.55) \]
\[ y_i = c_i^T x_i \quad , \quad i=1,2 \quad (5.56) \]

Considering the same system parameters as taken in the previous
Chapter IV in section 4.5 with additional governor parameters
\((g=10^3\) and \(a=20\)) the above set of equations for both subsystems
are as follows:

\[ \dot{x}_1 = \begin{bmatrix} -100 & -12a_1 & -1 \\ 1 & 0 & 0 \\ 1000 & 0 & -20 \end{bmatrix} x_1 + \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} f_{11}(y_1) + \begin{bmatrix} h_{11}(x) \\ 0 \\ 0 \end{bmatrix} \quad (5.57) \]
\[ y_1 = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} x_1 \quad (5.58) \]

and

\[ \dot{x}_2 = \begin{bmatrix} -100 & -11a_2 & -1 \\ 1 & 0 & 0 \\ 1000 & 0 & -20 \end{bmatrix} x_2 + \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} f_{21}(y_2) + \begin{bmatrix} h_{21}(x) \\ 0 \\ 0 \end{bmatrix} \quad (5.59) \]
\[ y_2 = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} x_2 \quad (5.60) \]
where the three components of the two state vectors $x_1$ and $x_2$ are as follows:

$$
x_{i1} = \delta_{i3}
$$

$$
x_{i2} = \delta_{i3} - \delta_{i3}
$$

$$
x_{i3} = p_{im}
$$

where $p_{im}$ is the perturbed governor power ($p_{im} = P_{im} - P_{im}(t)$). The nonlinear functions $f_{i1}(y_i)$ and $h_i(x)$ are given by

$$
f_{1i}(y_i) = 12[\sin(y_{i1} - 2\theta) + \sin 2\theta - a_1 y_{i1}]
$$

$$
f_{2i}(y_i) = 11[\sin(y_{i2} - 3\theta) + \sin 3\theta - a_2 y_{i2}]
$$

$$
h_{1i}(x) = 0.83 \sin(\frac{1}{2} x_{12}) \sin(\frac{1}{2} x_{12} - 2\theta) + 0.11 \sin(\frac{1}{2} x_{22}) \sin(\frac{1}{2} x_{22} - 2\theta) + 0.20 \sin(\frac{1}{2} x_{22} - 3\theta) + 0.11 \sin(\frac{1}{2} x_{22} - 2\theta)
$$

$$
-0.20 \sin(\frac{1}{2} x_{12} - 3\theta) \sin(\frac{1}{2} x_{12} - x_{22} + 18\theta)
$$

(5.62)

We note that the nonlinear functions $f_{i1}(y_i)$ as well as the first components of $h_i(x)$ in eq.(5.62) are the same as in the case of second order models. The sector gains $k_i$, the sector boundaries $y_{i1}$ and $y_{i2}$ as well as bounds on $h_i(x)$, therefore, remain unchanged. We take the gains of minor feedbacks $a_1 = 11/12$ and $a_2 = 10/11$.

In order to generate the Liapunov functions $V_i(x_i)$ we use the popov conditions given by

$$
\pi_i(\omega) = \text{Re}[(1 + j\omega a_{i1})G_{i1}(j\omega)] + \frac{1}{k_i} > 0
$$

(5.63)

where $G_{i1}(j\omega)$ are the transfer functions of the linear part of above two subsystems and given by
For the above specified values of $\alpha_i$ these transfer functions of the above subsystems are as follows:

\[
G_{11}(j\omega) = \frac{(20+ju)}{(j\omega)^3 + 120(j\omega)^2 + 3011(j\omega) + 220} \\
G_{21}(j\omega) = \frac{(20+ju)}{(j\omega)^3 + 120(j\omega)^2 + 3010(j\omega) + 200}
\]

Assuming the same values of $\eta_i (\eta_1 = \eta_2 = -1.0)$ as in the preceding section 5.5 and proceeding in the manner described there we get the following values of Liapunov functions $V_i(x_i)$ for the interaction free parts of the above subsystems:

\[
V_1(x_1) = x_1^T \begin{bmatrix}
0.001258 & 0.01088 & -0.00001058 \\
0.01088 & 1.88414 & -0.0000629 \\
-0.00001058 & -0.0000629 & 0.000000529
\end{bmatrix} x_1 \\
\]

\[
+ (-1) \int_0^{y_1} 12\sin(y_1-2^\circ) + \sin 2^\circ \frac{11}{12} y_1^3 \, dy_1 \tag{5.64}
\]

\[
V_2(x_2) = x_2^T \begin{bmatrix}
0.001269 & 0.0119 & -0.000011 \\
0.0119 & 2.0383 & -0.000589 \\
-0.000011 & -0.000589 & 0.00000055
\end{bmatrix} x_2 \\
\]

\[
+ (-1) \int_0^{y_2} 11\sin(y_2-3^\circ) + \sin 3^\circ \frac{10}{11} y_2^2 \, dy_2 \tag{5.65}
\]

Making use of the relation (5.35) (with bounds on interaction terms already known) we compute the aggregated interaction coefficients $\sigma_1 = 0.00553$ $\sigma_2 = 0.00332$

The decoupled subsystems are then modeled by the following pair of equations:
\[ \dot{x}_i = A_i^T x_i + b_i f_i(y_i) \]  
\[ y_i = c_i^T x_i \quad , \quad i=1,2 \]

with subsystem matrices
\[
A_1 = \begin{bmatrix}
-99.9945 & -11 & -1 \\
1 & 0.005535 & 0 \\
1000 & 0 & -19.9945 \\
\end{bmatrix}
\]
\[
A_2 = \begin{bmatrix}
-99.9967 & -10 & -1 \\
1 & 0.003325 & 0 \\
1000 & 0 & -19.9967 \\
\end{bmatrix}
\]

The three-machine system under study is asymptotically stable in the finite regions \(-0.66 < y_1 < 0.765, -0.655 < y_2 < 0.820\) if the following inequalities for all real frequencies \(0 < \omega < \infty\) are satisfied:
\[
\Pi_1(\omega) = \text{Re} \left[ (1-j\omega) G_1(-0.005535+j\omega) + 1 \right] > 0 \tag{5.68}
\]
\[
\Pi_2(\omega) = \text{Re} \left[ (1-j\omega) G_2(-0.003325+j\omega) + 1 \right] > 0 \tag{5.69}
\]

Based on these frequency domain expressions, the following modified Liapunov functions (which include the effects of interactions) are obtained:
\[
W_1(x_1) = x_1^T \begin{bmatrix}
0.001434 & 0.0124 & -0.00001204 \\
0.0124 & 2.132 & -0.000614 \\
-0.00001204 & -0.000614 & 0.000000604 \\
\end{bmatrix} x_1 \]
\[
+ (-1.07) \int_0^{y_1} 12 \left[ \sin(y_1 - 2\theta) + \sin 2\theta \frac{11}{12} y_1 \right] dy_1 \tag{5.70}
\]
\[
W_2(x_2) = x_2^T \begin{bmatrix}
0.001376 & 0.01298 & -0.00001188 \\
0.01298 & 2.209 & -0.000643 \\
-0.00001188 & -0.000643 & 0.000000595 \\
\end{bmatrix} x_2 \]
\[
+ (-1.043) \int_0^{y_2} 11 \left[ \sin(y_2 - 3\theta) + \sin 3\theta \frac{10}{11} y_2 \right] dy_2 \tag{5.71}
\]
The regions of stability in the three dimensional state space of the two subsystems are then obtained from the following inequalities:

\[
0.001434 x_{11}^2 + 2.132 x_{12}^2 + 0.604 \times 10^{-6} x_{13}^2 + 0.0248 x_{11} x_{12} \\
-0.2408 \times 10^{-4} x_{11} x_{13} - 0.001228 x_{12} x_{13} + 12.84 \cos(x_{12} - 0.035) \\
-0.035 x_{12}^2 + 11/24 x_{12}^2 < 13.3789 \quad (5.72)
\]

\[
0.001736 x_{21}^2 + 2.209 x_{22}^2 + 0.595 \times 10^{-6} x_{23}^2 + 0.02596 x_{21} x_{22} \\
-0.2376 \times 10^{-4} x_{21} x_{23} - 0.001286 x_{22} x_{23} + 11.473 \cos(x_{22} - 0.05236) \\
-0.05236 x_{22}^2 + 10/24 x_{22}^2 < 12.1913 \quad (5.73)
\]

These regions of stability of the above subsystems without considering interaction effects are obtained by the following pair of inequalities:

\[
0.001258 x_{11}^2 + 1.884143 x_{12}^2 + 0.529 \times 10^{-6} x_{13}^2 + 0.02176 x_{11} x_{12} \\
-0.0002116 x_{11} x_{13} - 0.001258 x_{12} x_{13} + 12 \cos(x_{12} - 0.035) \\
-0.035 x_{12}^2 + 11/24 x_{12}^2 < 12.3835 \quad (5.74)
\]

\[
0.001269 x_{21}^2 + 2.0383 x_{22}^2 + 0.55 \times 10^{-6} x_{23}^2 + 0.0238 x_{21} x_{22} \\
-0.00022 x_{21} x_{23} - 0.001178 x_{22} x_{23} + 11 \cos(x_{22} - 0.05236) \\
-0.05236 x_{22}^2 + 10/24 x_{22}^2 < 11.6569 \quad (5.75)
\]

In order to see the effects of the governor action we have shown in Figures 5.6 and 5.7 the intersections of the stability space with the planes \(x_{13} = 0\) and \(x_{23} = 0\) and marked as curves I. We have also shown the regions of stability in \(x_{11}, x_{12}\) and \(x_{21}, x_{22}\) planes (as obtained in the previous section) for the above subsystems when the governor actions were not considered.
Fig. 5.6 Stability Regions Of Subsystem-1

Fig. 5.7 Stability Regions Of Subsystem-2
and marked as curves II. The dotted curves shown in these figures are the regions of stability of subsystems without interaction. It is obvious that the governor action helps to improve the regions of stability significantly. The smallest of the regions shown by chain curves in these Figures 5.6 and 5.7 are those obtained by Jocic et al. [55] without interaction effects.

5.7 CONCLUSION

We have used in this chapter the modeling approach of Jocic et al. [55] in order to make use of our own approach regarding the derivation of simplified stability criteria for the transient stability of multimachine power systems. As the results quoted above show, we get larger regions of stability compared to those reported by Jocic et al. Of course, the frequency domain stability criteria derived by us are much simpler to verify than the criteria of Jocic et al.