CHAPTER 4

TCM ON BANDLIMITED ISI CHANNELS

4.1 INTRODUCTION

In a band-limited digital communication system, the effect of each symbol transmitted over time-dispersive channel extends beyond the symbol interval [31]. Consequently, overlapping of received symbols occur which results in a linear distortion called Inter-symbol Interference (ISI), which turns out to be the primary obstacle in high speed data transmission over bandlimited channels. In addition to linear ISI distortion, the signals transmitted over a bandlimited channel are subjected to other impairments such as nonlinear distortion, frequency offset, phase jitter, impulse noise, and thermal noise. But for mathematical tractability the channel model that is normally adopted for a bandlimited channel is a linear time-invariant filter that introduces the ISI together with additive white Gaussian noise. Coded modulation schemes such as TCM can effectively enhance noise immunity of a band-limited communication channel. But to mitigate the effect of channel ISI powerful equalization techniques are essential.

In a practical system, it is assumed that ISI affects a finite number of symbols [10]. Consequently, the cascade of a TCM encoder and the ISI channel can be viewed as a combined finite-state machine (FSM), and hence as a combined ISI-Code trellis whose states are given by the product of TCM encoder states and the ISI states. The receiver performs Maximum-Likelihood Sequence Estimation of the received data sequence using Viterbi Algorithm that searches for a minimum cost path in the ISI-Code trellis.

In this chapter, we consider the study of basic equalizer structures to combat the effect of ISI and decoding of TCM signals in the presence of ISI and AWGN. Optimum combined ISI-code receiver structure and various sub-optimum reduced complexity ISI-code receiver structures employing ML sequence estimation have been considered for the study and simulation results are presented.
4.2 BANDLIMITED ISI CHANNEL AND DISCRETE TIME MODEL

Typical baseband digital transmission system subjected to ISI is shown in Fig. 4.1.

And the optimum receiver for a bandlimited ISI channel corrupted by AWGN is shown in Fig. 4.2. It transmits the data sequence \(\{a(n)\}\) at a rate of one symbol for every symbol interval \(T\) over a baseband channel whose impulse response is \(g(t)\). The channel is assumed to be linear and time invariant, delivers to the receiver a distorted time-smeared version of the transmitted signal. The symbols \(\{a(n)\}\) and the channel impulse response \(g(t)\) are complex valued [67]. The channel output is corrupted by the complex additive white Gaussian noise \(w(t)\). Thus the received waveform is given by

\[
r(t) = \sum_{k} a(n)g(t-kT) + w(t) \tag{4.1}
\]

It was shown by Forney [31] that the samples of a whitened matched filter form sufficient statistics for the detection of the transmitted sequence \(\{a(n)\}\). Thus the cascade of the linear channel representing the modulator, the transmitter filter, actual channel and the receiver filter consisting of a whitened matched filter and a symbol rate sampler,
which can be modeled as a discrete-time white noise channel as shown in Fig. 4.1. Referring to this model the received signal at the time instant \( nT \) is given by

\[
\mathbf{r}(n) = \sum_{i=0}^{L} \mathbf{g}_i \mathbf{a}(n - i) + \mathbf{w}(n) \tag{4.2}
\]

where \( \{g_i\} \) are the tap gains that correspond to the sampled channel impulse response, and \( \{w(n)\} \) are samples of complex-valued Gaussian noise with zero mean and variance \( 2\sigma^2 \). The noise and the data sequences are assumed to be uncorrelated, the number of taps are \((L+1)\), where \( L \) represents the channel memory length.

If the signal constellation used for the transmission of \( \{a(i)\} \) has an alphabet size of \( M \) symbols then the discrete-time channel can be represented by either \( M^L \)-state finite state machine [20], and the system state at any time instant \( n \) is defined by \( L \) symbols, given by

\[
s_n = (a(n-1), a(n-2), \ldots, a(n-L)) \tag{4.3}
\]

where \( s_n \) may assume one of the possible values represented by

\[
\{a(n-i)\}, \text{ for } 1 \leq i \leq L \tag{4.4}
\]

### 4.3 BASIC EQUALIZER STRUCTURES

To combat the effect of ISI, a variety of receiver structures have been proposed in the literature [37,56]. The general form remains the same in almost all approaches which take the form of matched filter followed by a suitable equalizer algorithm. While the matched filter reduces the errors due to AWGN, the equalizer minimizes the error due to ISI. The nature of the equalizer may vary from simple linear transversal filter through nonlinear decision-feedback equalizer structures to the more sophisticated MLSE algorithms like the Viterbi algorithm.

An optimum linear equalizer consists of an infinite length transversal filter which minimizes the error due to ISI, whereas in a practical system the length is limited to a finite value \( L \). Linear Equalizer yields good performance on channels such as telephone lines, where the spectral characteristics of the channels are well behaved and do not exhibit spectral nulls. However, on channels with severe amplitude distortion it enhances and correlates noise. This basic limitation of the linear equalizer to cope with severe ISI has motivated a considerable amount of research into nonlinear equalizers.
DFE consists of a feed-forward filter and feedback filter. Both have taps spaced at the symbol interval. The input to the feed-forward section is the received signal sequence \( \{ r_n \} \). In this respect the feed-forward filter is identical to the linear transversal equalizer described above. The feedback filter has as its input the sequence of decisions on previously detected symbols. Functionally the feedback filter is used to remove that part of the inter-symbol interference from the present estimate caused by previously detected symbols, mitigate the effect of ISI due to precursor as well as post cursor symbol. DFE is superior over the linear equalizer when the effect of decision errors on performance is neglected. On severely distorted channels the DFE provides an improved performance as compared to an LE. If the past decisions are assumed correct, then the ISI caused by them can be subtracted from the received signal in arriving at a correct decision about the present symbol provided the channel response is known exactly. However, there is still significant degradation due to the residual Intersymbol Interference, especially on channels with severe distortion, DFE suffers from severe error propagation due to the feedback filter section. An incorrect decision fed into the feedback filter results in error bursts, as if an impulsive noise has been injected into the decoder. And in practice since the channel response is not known a priori it is not possible to design an ideal matched filter.

Forney [31] has shown that the VA indeed is a Maximum-Likelihood Sequence Estimation technique, for TCM. As explained in the previous chapter, although MLSE is an optimum solution, its computational complexity and storage requirements grow exponentially with memory length \( L \) thereby limits its practical use.

4.4 DECODING OF TCM IN THE PRESENCE OF ISI AND AWGN

The two primary impediments to reliable high-speed transmission of digital data are the ISI and AWGN. TCM scheme in combination with an optimum MLSE equalization technique may be employed to realize reliable digital transmission at rates close to channel capacity [28]. Chevillant and Eleftheriou [21], and, Eyuboglu and Qureshi [30] have independently proposed a new integrated approach to the TCM receiver design, wherein the equalization and TCM decoding are combined into a single entity. Based on this approach, we consider various combined equalization and TCM decoding schemes for linear ISI channels corrupted by AWGN. Various reduced complexity suboptimum decoding structures have been considered for the study.
4.4.1 Finite State Machine Model

Combined discrete-time finite state machine model of a data transmission system employing Trellis Coded Modulation shown in Fig. 4.2. It consists of a TCM encoder followed by a linear minimum phase matched filter with $L$ coefficients \{1, $g_1$, $g_2$, \ldots $g_L$,\} followed by MLSD. The transmitted data symbol $a_n$ is given by

$$a_n = g_1(X_n, \sigma_n)$$  \hspace{1cm} (4.5)

where $X_n$ is the information transmitted at time $n$, and $\sigma_n$ is the encoder state. And the state transition of the encoder is given by

$$\sigma_{n+1} = g_2(X_n, \sigma_n)$$  \hspace{1cm} (4.6)

The output samples of the discrete time model is

$$q_n = a_n + u_n$$  \hspace{1cm} (4.7)

where

$$u_n = \sum_{i=1}^{L} g_i a_{n-i}$$  \hspace{1cm} (4.8)

describes the ISI at the output of the matched filter. Combined ISI-Code trellis states of the finite-state machine model is given by

$$\mu_n = (a_{n-L}, a_{n-L+1}, \ldots, a_{n-1}; \sigma_n)$$  \hspace{1cm} (4.9)

where the symbol sequence $(a_{n-L}, a_{n-L+1}, \ldots, a_{n-1})$ correspond to a path which takes the TCM encoder from a previous state $\sigma_{n-1}$ to the present state $\sigma_n$ in compliance with the TCM coding rule. Correspondingly state transition of the FSM can be written as

$$\mu_n : a_n \rightarrow \mu_{n+1}$$  \hspace{1cm} (4.10)

Equivalently the combined state can be expressed in terms of the transmitted symbol sequence as

$$\mu_n = (y_{n-L}, y_{n-L+1}, \ldots, y_{n-1}; \sigma_n),$$  \hspace{1cm} (4.11)

and in terms of the information sequence as

$$\mu_n = (\sigma_{n-L}; x_{n-L}, x_{n-L+1}, \ldots, x_{n-1})$$  \hspace{1cm} (4.12)

Associated with each encoder state are $2^m$ ISI states. Hence, in the case of an $N_c$-state TCM encoder and a signal constellation with $M=2^m$ points the combined trellis has $N_c$ states with $M/2$ transitions emerging from each state.
4.5 COMBINED EQUALIZATION AND DECODING

The optimum decoder for trellis encoded data signals in the presence of AWGN determines the coded sequence \( \{ \hat{a}_n \} \) which is closest in Euclidean distance to the noisy received sequence \( \{ z_n \} \). The noisy received sequence \( \{ z_n \} \) given by

\[
\{ z_n \} = \{ a_n \} + \{ w_n \}
\]

where \( w_n \) represents the whitened noise. This is accomplished by a soft decision Viterbi decoder which operates on the combined ISI and code trellis called super trellis and recursively minimizes the metric

\[
M_n(\ldots a_n) = M_{n-1}(\ldots a_{n-1}) + |z_n - a_n|^2
\]

over all coded sequences \( \{ a_n \} \).

\[\text{Fig. 4.3 Combined Discrete Finite State Machine Model of a Communication system}\]

In the presence of ISI and whitened noise the optimum decoder determines the sequence \( \{ \hat{a}_n \} \) which is closest to the received sequence given by

\[
\{ z_n \} = \{ a_n \} + u_n + w_n
\]

and minimizes the metric accordingly, which takes into account ISI due to past symbols \( \{ a_{n-1} \} \), given by

\[
M_n(\ldots a_n) = M_{n-1}(\ldots a_{n-1}) + |z_n - \sum_{i=1}^f g_{n,i} a_{n-i} - a_n|^2
\]
The state complexity of the optimum combined ISI-code receiver can be reduced by truncating the effective channel memory length to $J$, and using a built-in decision feedback mechanism to cancel out the residual ISI terms which are not represented by the truncated combined state trellis. This reduced complexity receiver has the state complexity $N_r(M/2)^J$, where $N_r$ is the number of states of Trellis-Code generator. Resulting receiver structure which operates on this reduced state combined ISI-code trellis is sub-optimum.

The performance degradation due to $(L-J)$ ISI terms not represented by the truncated combined state $\mu_n^J$ is compensated by incorporating an ISI-cancellation mechanism into the branch metric computation. Each truncated combined state $\mu_n^J$ gives information on $J$ past symbols $\{a_{n-i}\}$, for $1 \leq i \leq J$ associated with that state. Associated with state $\mu_n^J$ there will be a unique survivor path with a history of path symbol estimates and a survivor path metric defined by

$$M_n(a_n) = M_{n-1}(a_{n-1}) + \left| z_n - \sum_{i=1}^{J} g_{n+i} \hat{a}_{n+i} - \sum_{i=1}^{J} g_i a_{n+i} - a_n \right|^2$$

---(4.17)

The second term of the branch metric computation corresponds to the cancellation of ISI due to L-J residual terms. The above expression suggest a family of reduced state truncated combined ISI code receiver structures with state complexity ranging from $N_r$ to $N_r(M/2)^J$.

In the following we consider some of the combined MLSE-ISI structures.

### 4.5.1 The 32-State combined ISI-Code Trellis for 4-State 16-QAM TCM Scheme

For 4-state 16-QAM TCM scheme, the TCM encoder uses 16-QAM signal constellation where $M = 2^{m+1} = 2^{3+1} = 16$ specifies the constellation size. The number of bits transmitted per symbol interval is $m = 3$ and the encoder has $N_s = 4$ number of states. Each encoder state is associated with $(M/2)^J = 2^{m-1} = 2^{3-1} = 8$ states, therefore the number of states in the combined ISI-code trellis becomes $(M/2)^J N_r = 32$. The encoder receives 3-bits per symbol interval, and we have the following:

Input $X_n = (x_n^1, x_n^2, x_n^3)$

---(4.18)

Present State of the encoder $\mu_n = (\sigma_n; a(n-1))$

---(4.19)

Next state of the encoder $\mu_{n+1} = (\sigma_{n+1}; a(n))$

---(4.20)
where, Encoder present state \( \sigma_n = (x_{n-2}^i, x_{n-1}^i) \) ---(4.21) 
and Encoder next state \( \sigma_{n+1} = (x_{n-1}^i, x_n^i) \) ---(4.22) 
Symbol transmitted is \( a(n) \), and, the trellis code corresponding to the symbol \( a(n) \) is \((y_n^3, y_n^2, y_n^1, y_n^0)\) 
\[ y_n^3 = x_n^3 \]
\[ y_n^2 = x_n^2 \]
where, 
\[ y_n^1 = x_n^1 \oplus x_{n-2}^1 \quad \text{---(4.23)} \]
\[ y_n^0 = x_n^1 \]
Therefore in terms of input \( X_n \) present-state and next-state of the encoder is given by 
\[ \mu_n = (x_{n-3}^i, x_{n-2}^i, x_{n-1}^i, x_n^2, x_n^3) \] ---(4.24) 
and, \[ \mu_{n+1} = (x_{n-2}^i, x_{n-1}^i, x_n^1, x_n^2, x_n^3) \] ---(4.25)

4.5.2 The 64-State combined ISI-Code Trellis for 8-State 16-QAM TCM Scheme

The 8-state 16-QAM encoder/modulator receives 5-bit input \( X_n \) per symbol duration and the encoded bits are mapped into a signal point selected from the 64-QAM signal constellation. The encoder has a rate-\( \frac{1}{2} \) convolutional encoder as an integral part of it, having 3 memory elements, which defines eight states of the encoder. Each code state has \((M/2)^l = 2^m = 2^{3*1} = 8\) ISI states and the number of states in the ISI code trellis is \((M/2)^l N_s = 64\). The present and next states are given by (4.6) and (4.12). From each state of the combined code trellis there will be \(2^m = 8\) distinct transitions and each state provides information about previous symbol \( a(n-1) \) as depicted by 4.6.

With respect to 64-state ISI-code trellis structure of the scheme, we have the following:

The signal constellation size \( M \)
No of bit transmitted per symbol interval=\( m=3 \)
Input \( X_n = (x_n^1, x_n^2, x_n^3) \)
Number of ISI-code trellis states= \( N_s(2^m)^l = 8(2^3)^l = 64 \) ---(4.26)
Encoder state \( \sigma_{n+1} = (x_{n-2}^i, x_{n-1}^i, x_n^1) \) ---(4.27)
Next state of the super trellis = \( \mu_{n+1} = (\sigma_n; x_n^1, x_n^2, x_n^3) \) ---(4.28)
Fig. 4.4 The 32-State combined ISI-Code Trellis for 4-State 16-QAM TCM transmission over an ISI channel of memory length $L=1$
Trellis code corresponding to the symbol $a(n)$ is $(y_n^3, y_n^2, y_n^1, y_n^0)$, where

\begin{align*}
y_n^3 &= x_n^3 \\
y_n^2 &= x_n^2 \\
y_n^1 &= x_n^1 \oplus x_{n-2}^1 \\
y_n^0 &= x_n^1 \\
\end{align*}

---(4.29)

### 4.5.3 The 128-State combined ISI-Code Trellis for 4-State 64-QAM TCM Scheme

The TCM encoder considered is the one shown in Fig. 2.1. With respect to 64-state ISI-code trellis structure of the scheme, we have the following:

- **Signal constellation size** $M = 2^{m+1} = 64$
- **No of bit transmitted per symbol interval** $m = 5$
- **Input** $X_n = (x_n^1, x_n^2, x_n^3, x_n^4, x_n^5)$
- **Number of ISI-code trellis states** $N_s(2^m)^L = 4(2^5)^1 = 128$ ---(4.30)
- **Next state** $\mu_{n+1} = (\sigma_{n+1}; a(n))$ ---(4.31)
- **Encoder state** $\sigma_{n+1} = (x_{n-1}^1, x_n^1)$ ---(4.32)

Treillis code corresponding to the symbol $a(n)$ is $(y_n^3, y_n^2, y_n^1, y_n^0, y_n^1)$. From each state there will be 8 distinct transitions. The state complexity of the combined ISI-Code trellis structure increases with an increase in constellation size $M$ and memory length $L$, and the complexity grows exponentially with $L$. For the 4-state 16-QAM TCM scheme and ISI channel memory length $L = 3$, the number of states in the ISI-code trellis increases to 2048 from 32 as depicted in example 1. Similarly for a 4-state 64-QAM TCM with $L = 3$, the combined ISI-Code trellis increases to $2^{17}$. These examples illustrates that although combined ISI-Code trellis structure is optimum, it becomes unrealistic even for moderate ISI memory length $L = 2$.

Decoding is accomplished by the implementation of (4.17) through the Viterbi algorithm. It operates on the combined and ISI-Code trellis and the resulting receiver structure is optimum.

### 4.6 RESULTS AND DISCUSSIONS

In this section the error performance evaluation of various combined MLSE receiver structures are considered which are designed for the decoding of 16-QAM and 64-QAM TCM schemes. The error performances are evaluated through simulation and the reference system employed for the error performance evaluation is the uncoded.
M-QAM system having the same data rate and bandwidth as that of coded systems considered.

Combined MLSE receivers use the combined ISI-Code trellis for the decoding of Trellis Coded QAM signals corrupted by channel ISI and AWGN. To simulate the communication system corrupted by ISI and AWGN, we have considered the equivalent discrete time finite state machine model of the communication system corrupted by ISI and AWGN, shown in Fig. 4.3. It is assumed that the channel is time in-variant and its discrete time impulse response characteristics are known at the receiver. For the assumed impulse response \( \{g_i\} \) of the ISI channel, We have considered four different communication channels in our study, whose impulse response samples are given by \( \{g_i\} \) chosen arbitrarily in such a way that for each channel \( \sum_{k=0}^{L}|g_k|^2 = 1 \), where \( L \) represents memory length of the ISI channel. Table 4.1 show various communication channels with their impulse responses considered for simulation.

For the error performance evaluation of 16-QAM and 64-QAM TCM schemes, the two references considered are uncoded 8-QAM and uncoded 32-QAM schemes. In the trellis structure of the uncoded QAM schemes, the number of combined ISI-code trellis states is same as ISI trellis states. The VA operates only on the ISI-trellis structure.

For illustration, we consider 8-QAM uncoded scheme transmission over an ISI channel of memory length \( L=1 \). The information is transmitted at the rate of 3-bits per baud interval which defines the signal points of the 8-QAM constellation. Since there is no specific sequence pattern for the data symbols generation, the trellis state-state structure of the uncoded scheme has only one state and there are eight parallel transitions. Combined ISI-code trellis for the uncoded 8-QAM scheme contain eight ISI trellis states and from each state of the ISI trellis there will be 8-distinct transitions that corresponds to the possible binary input sequence. Decoding is accomplished by MLSE through the implementation of Viterbi algorithm. The error performance of this uncoded MLSE structure has been considered as the reference to compute the coding gain achieved with the use of various combined ISI-Code trellis structures designed for the decoding of 16-QAM TCM schemes over an ISI channel of memory length \( L=1 \). Similarly in the 32-QAM uncoded scheme, 5-bits binary inputs are transmitted per baud interval which defines the signal points of the 32-QAM constellation. The 32-QAM scheme is used as
reference scheme to evaluate the performance of various receiver structures designed to decode 64-QAM TCM systems.

The error performance of 32 state combined MLSE structure designed for 4-State 16-QAM TCM scheme transmission over bandlimited ISI channel of memory length $L=1$, as discussed in section 4.5.1 is shown in Fig. 4.5. In the legend of the Fig. 4.5(a)-(d) the error performance characteristics obtained for TCM schemes are marked as ‘Combined MLSE’ and those obtained for uncoded QAM schemes as reference characteristics are marked as ‘Uncoded MLSE’. The label ‘Coded ISI-free’ represents the error performance characteristic obtained for TCM schemes under ISI-free condition. The Fig. 4.5(a) depicts the error performance characteristic obtained for transmission over the channel CH11. Improved performance noted is 2.5 dB over uncoded 8-QAM scheme MLSE performance, and performance degradation of 2.0 dB is noted over optimum MLSE performance. Similarly the error performance characteristics obtained for transmission over the channels CH12, CH13 and CH14 are shown in Fig. 5.4(b)-(d). It is noted that the channel CH14 yields the best performance with a performance degradation of about 1.0 dB over optimum MLSE at an error rate $p_e=10^{-4}$. This is followed by the performances of channels CH13, CH12 and CH11 in order.

Figure 4.6 shows the error performance characteristics obtained for 128-state combined MLSE structure designed for 4-state 64-QAM TCM scheme transmission on bandlimited ISI channels with the ISI length $L=1$, as discussed in section 4.5.3. Simulation was run for $10^5$ symbols transmission over the channels CH12 and CH14. The Fig. 4.6(a) shows the error performance characteristic for transmission on channel CH14. It is noted that the gain improvement over the uncoded 32 QAM scheme receiver performance is about 2.8 dB at an error rate of $10^{-4}$. From the Fig. 4.6 (b) it is noted that MLSE for 4-state 64-QAM scheme, transmission over the communication channel channel CH12 provides a coding gain of about 3.2 dB with a degradation in performance is of 2.2 dB over Coded ISI-free performance. Details of the results are given in the Table 4.3.

From the error performance characteristics of the Fig. 4.5-4.6 we find that combined MLSE provides a performance close to the ISI-free performance with some amount of performance degradation about 0.4 dB-2.0 dB depending on the impulse response of the channel. It is noted that MLSE structure for 4-state 16-QAM TCM scheme with the channel memory length $L=1$ consists of 32-states.
Table 4.1
Equivalent Discrete Time Impulse Response of Different ISI Channels

<table>
<thead>
<tr>
<th>ISI Length</th>
<th>Channel Label</th>
<th>Channel Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>g0</td>
</tr>
<tr>
<td>L=1</td>
<td>CH11</td>
<td>.707</td>
</tr>
<tr>
<td></td>
<td>CH12</td>
<td>.7746</td>
</tr>
<tr>
<td></td>
<td>CH13</td>
<td>.8366</td>
</tr>
<tr>
<td></td>
<td>CH14</td>
<td>.8944</td>
</tr>
<tr>
<td>L=2</td>
<td>CH21</td>
<td>.707</td>
</tr>
<tr>
<td></td>
<td>CH22</td>
<td>.7746</td>
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<td>CH33</td>
<td>.8366</td>
</tr>
<tr>
<td></td>
<td>CH34</td>
<td>.8944</td>
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</table>
Table 4.2

Different Combined ISI-Code MLSE Structures and Corresponding Reference Systems

<table>
<thead>
<tr>
<th>ISI Length</th>
<th>Coded System</th>
<th>Uncoded Reference</th>
<th>Reference Section</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4 State 16_QAM</td>
<td>32 State Combined ISI Code Trellis</td>
<td>8 QAM</td>
</tr>
<tr>
<td>1</td>
<td>8 State 16_QAM</td>
<td>64 State Combined ISI Code Trellis</td>
<td>8 QAM</td>
</tr>
<tr>
<td>1</td>
<td>4 State 64_QAM</td>
<td>128 State Combined ISI Code Trellis</td>
<td>32 QAM</td>
</tr>
</tbody>
</table>
Fig. 4.5  Error performance of 32-state MLSE receiver for 4-State 16-QAM TCM scheme transmission over ISI channel, L=1
The MLSE structure for 4-state 64-QAM contain 128-states. As the number of symbols transmitted increases the computational complexity of the optimum MLSE receivers grows exponentially. Also as discussed earlier, the design procedure says that as the memory length increases it becomes a practically unrealizable system. Hence Reduced complexity suboptimum receivers are the alternate approach.