6.1 INTRODUCTION

MHD is the science of motion of electrically conducting fluid in presence of magnetic field. There are numerous examples of application of MHD principles, including MHD generators, MHD pumps and MHD flow meters etc. The dynamo and motor is a classical example of MHD principle. MHD principles also find its application in medicine and biology. The principle of MHD is also used in stabilizing a flow against the transition from laminar to turbulent flow. Convection problems of electrically conducting fluid in presence of transverse magnetic field have got much importance because of its wide application in Geophysics, Astrophysics, Plasma Physics and Missile Technology etc. The present form of MHD is due to the pioneer contribution of several authors like Alfven (1942), Cowling (1957), Crammer and Pai (1978), Shercliff (1965) and Ferraro and Pulmption (1966). Model studies on MHD free and forced convection with heat and mass transfer problems have been carried out by many of the authors due to their application in many branches of science and technology. Some of them are Ahmed (2010), Elbashbeshy (2003) and Singh and Singh (2000).

The mass flux caused due to temperature gradient is known as the thermal diffusion or Soret effect or Thermophoresis or Thermomigration or Thermodiffusion or Luding-Soret effect. The experimental investigation of the thermal diffusion effect on mass transfer related problems was first performed by Charles Soret in 1879. Thereafter this effect is termed as Soret effect in the honour of his name. In general, the Soret effect is of smaller order in magnitude than the effect described in Fick’s law and is often ignored in mass transfer process. Though this effect is quite small, but the devices may be arranged to produce very sharp temperature gradient so that the separation of components in mixtures are affected. Eckert and Drake (1972) have emphasized that the Soret effect assumes significance in cases concerning isotope separation and in mixtures between gases with very light molecular
weight $H_2, \text{He}$. It is noted that in mass transfer processes involving very low species concentration levels, the effect of thermal diffusion may be ignored.

Based on Eckert and Drake’s work (1972) many other investigators have carried out model studies on the Soret effect in different heat and mass transfer problems. Some of them are Dursunkaya and Worek (1992), Kafoussias and Williams (1995), Sattar and Alam (1994), Hurle and Jakeman (1971), Postelnicu (2004), Raju et al. (2008), Ahmed (2010) and Ahmed et al. (2010).

The main objective of the present investigation is to study the Soret effect as well as the MHD effects on the unsteady mixed convective mass transfer flow past an infinite vertical porous plate with variable suction, where the plate temperature oscillates with the same frequency as that of variable suction velocity. It is assumed that the plate is embedded in a uniform porous medium and moves with a constant velocity in the direction of the flow in presence of a transverse magnetic field.

### 6.2 MATHEMATICAL FORMULATION OF THE PROBLEM

We now consider an unsteady MHD conducting flow of a viscous incompressible electrically conducting fluid past an infinite vertical porous plate with variable suction under the influence of a uniform transverse magnetic field. Our investigation is restricted to the following assumptions:

- The polarization effects are assumed to be negligible and hence the electric field is also negligible.
- The variations of all fluid properties other than the variations of density except in so far as they give rise to a body force are ignored completely.
- All the physical variables are functions of $y'$ and $t'$ only as the plate are infinite.
It is assumed that the variation of expansion co-efficient is negligibly small and the pressure and influence of the pressure on the density are negligible.

We introduce a co-ordinate system \((x', y', z')\) with \(X\) - axis vertically upwards along the plate, \(Y\)- axis perpendicular to the plate and directed into the fluid region and \(Z\)- axis along the width of the plate as shown in figure 6.1. Let the components of velocity along with \(X\) and \(Y\) axes should be \(u'\) and \(v'\). Let these velocity components are chosen in the upward direction along the plate and normal to the plate respectively.

![Figure 6.1 Flow Configuration](image)

Under the assumptions just stated above, the equations that describe the physical situation are given by :

\[
\frac{\partial v'}{\partial y'} = 0 \tag{6.2.1}
\]
\[
\frac{\partial u'}{\partial t'} + v' \frac{\partial u'}{\partial y'} = -\frac{1}{\rho} \frac{\partial p'}{\partial x'} + \nu \frac{\partial^2 u'}{\partial y'^2} + g\beta \ (T' - T_w') + g\beta' \ C' - C_w' - \nu \frac{u'}{K_I} - \frac{\sigma B_0^2 u'}{\rho} \quad (6.2.2)
\]

\[
\frac{\partial T'}{\partial t'} + v' \frac{\partial T'}{\partial y'} = \frac{K_I}{\rho c_p} \frac{\partial^2 T'}{\partial y'^2} \quad (6.2.3)
\]

\[
\frac{\partial C'}{\partial t'} + v' \frac{\partial C'}{\partial y'} = D_m \frac{\partial^2 C'}{\partial y'^2} + D_t \frac{\partial^2 T'}{\partial y'^2} \quad (6.2.4)
\]

All the physical quantities involved in the above equations are defined in the nomenclature.

The boundary conditions are:

\[
\begin{align*}
& u' = 0, T' = T_w' + (1 + e^{i\theta}) \ (T_0' - T_w'), C' - C_w' \text{ at } y' = 0 \quad (6.2.5) \\
& u' \rightarrow U' = U_0 + e^{i\theta}, T' \rightarrow T_0', C' \rightarrow C_w' \text{ as } y' \rightarrow \infty \quad (6.2.6)
\end{align*}
\]

The equation (6.2.1) yields that the suction velocity at the plate is either a constant or a function of time and we take the suction velocity normal to the plate in the form

\[
v' = -V_0 \ (1 + e^{i\theta}) \quad (6.2.7)
\]

where \( A \) is a real positive constant, \( \varepsilon \) is a small value less than unity, \( V_0 \) is a scale of suction velocity which is non-zero positive constant. The negative sign indicates that the suction is towards plate.

Outside the boundary layer, equation (2) gives

\[
\frac{1}{\rho} \frac{\partial p'}{\partial x'} = \frac{dU'}{dt'} + \frac{\nu}{K_I} U' + \frac{\sigma B_0^2 U'}{\rho} \quad (6.2.8)
\]
In order to write the governing equations and the boundary conditions in non-dimensional form, the following non-dimensional quantities are introduced:

\[ u = \frac{u'}{V_0}, v = \frac{v'}{V_0}, y = \frac{Y'}{\nu}, U = \frac{U'}{U_0}, \lambda = \frac{U_u}{V_0}, t = \frac{tV_0^2}{U}, \]

\[ \theta = \frac{T' - T_v}{T_w - T_v}, \phi = \frac{C' - C_v}{C'_w - C_v}, K_i = \frac{K_iV_0^2}{\nu^3}, \text{Pr} = \frac{\nu}{\alpha}, \]

\[ \text{Sc} = \frac{\nu}{D_M}, M = \frac{\nu B_{\text{nd}}^3}{\rho V_0^2}, \text{Gr} = \frac{\nu \beta g V_0}{V_0^3}, \text{Sr} = \frac{D_1 T_w - T_v}{\nu C'_w - C_v}, \]

\[ \text{Gm} = \frac{\nu B^2 g C'_w - C_v}{V_0^3}, \alpha = \frac{4\nu g^2}{V_0^2}, \]

In view of the equations (6.2.7) – (6.2.9), the equations (6.2.2) – (6.2.4) reduce to the following non-dimensional form:

\[ \frac{1}{4} \frac{\partial u}{\partial t} - 1 + \varepsilon A e^{i\phi} \frac{\partial u}{\partial y} = \frac{\lambda}{4} \frac{\partial^2 u}{\partial y^2} + \text{Gr} \theta + \text{Gm} \phi + N \lambda U - u \]

\[ \frac{1}{4} \frac{\partial \theta}{\partial t} - 1 + \nu A e^{i\phi} \frac{\partial \theta}{\partial y} = \frac{1}{\text{Pr}} \frac{\partial^2 \theta}{\partial y^2} \]

\[ \frac{1}{4} \frac{\partial \phi}{\partial t} - 1 + \nu A e^{i\phi} \frac{\partial \phi}{\partial y} = \frac{1}{\text{Sc}} \frac{\partial^2 \phi}{\partial y^2} - \text{Sr} \frac{\partial^2 \alpha}{\partial y^2} \]

The corresponding boundary conditions are:

\[ u = 0, \theta = 1 + \alpha e^{i\phi}, \phi = l \quad \text{at} \quad y = 0 \]

\[ u \to U = 1 + \varepsilon e^{i\phi}, \theta \to 0, \phi \to 0 \quad \text{as} \quad y \to \infty \]
6.3 METHOD OF SOLUTION

Equations (6.2.10) – (6.2.12) are coupled non-linear partial differential and these cannot be solved in closed-form. However, these equations can be reduced to a set of ordinary differential equations, which can be solved analytically. This can be done by representing the velocity, temperature and concentration of the fluid in the neighbourhood of the plate as

\[ u = u_0 + \varepsilon^2 u_1 + \varepsilon^2 + \ldots \]

\[ \theta = \theta_0 + \varepsilon^2 \theta_1 + \varepsilon^2 + \ldots \]

\[ \phi = \phi_0 + \varepsilon^2 \phi_1 + \varepsilon^2 + \ldots \]  

(6.3.1)

\[ U = 1 + \varepsilon^2 \]

Substituting (6.3.1) in equations (6.2.10) – (6.2.12), equating the harmonic terms and neglecting the higher order terms of \( \varepsilon^2 \), we obtain

\[ u_0'' + Lu_0' - Nu_0 = -N\lambda - Gr\theta_0 - Gm\phi_0 \]  

(6.3.2)

\[ u_1'' + Lu_1' - \left(N + \frac{i\alpha}{4}\right)u_1' - \lambda\left(N + \frac{i\alpha}{4}\right)\theta_1 - Gr\theta_0 - Gm\phi_1 \]  

(6.3.3)

\[ \theta_0'' + Pr L\theta_0' = 0 \]  

(6.3.4)

\[ \theta_1'' + Pr L\theta_1' - \frac{i\omega}{4} Pr \theta_1 = 0 \]  

(6.3.5)

\[ \phi_0'' + LSc \phi_0' = -ScSr \theta_0'' \]  

(6.3.6)

\[ \phi_1'' + LSc \phi_1' - \frac{i\omega}{4} \phi_1' = -ScSr \theta_1'' \]  

(6.3.7)

where primes denotes ordinary differentiation with respect to \( y \).
The corresponding boundary conditions can be written as

\[ u_0 = 0, u_1 = 0, \theta_0 = 1, \theta_1 = 1, \phi_0 = 1, \phi_1 = 0 \quad \text{at} \quad y = 0 \] (6.3.8)

\[ u_0 \to 1, u_1 \to 1, \theta_0 \to 0, \theta_1 \to 0, \phi_0 \to 0, \phi_1 \to 0 \quad \text{as} \quad y \to \infty \] (6.3.9)

Solving equations (6.3.2) – (6.3.7) under the boundary conditions (6.3.8)-6.3.9), we obtain the velocity, temperature and concentration distribution in the boundary layer as:

\[ u(y, t) = \lambda + m_1 e^{m_{0y}} - m_{10} e^{PrLy} - m_s e^{-LScy} + \varepsilon e^{i\omega t} (\lambda + m_{17} e^{-m_{12}y} - m_{16} e^{-m_{15}y} + m_{15} e^{-m_{12}y}) \] (6.3.10)

\[ \theta(y, t) = e^{-PrLy} + \varepsilon e^{i\omega t} e^{-m_{12}y} \] (6.3.11)

\[ C(y, t) = m_3 e^{-LScy} + m_2 e^{-PrLy} + \varepsilon e^{i\omega t} m_4 (e^{-m_{12}y} - e^{-m_{15}y}) \] (6.3.12)

### 6.4 COEFFICIENT OF SKIN FRICTION

The non dimensional form of Coefficient of skin friction at the plate is given by:

\[ \text{Cf} = \frac{\partial u}{\partial y} \bigg|_{y=0} = \left[ (-m_1 m_6 + m_{10} Pr L + m_s LS) + \varepsilon e^{i\omega t} (-m_{12} m_{17} + m_{16} m_1 - m_{15} m_4) \right] \] (6.4.1)

### 6.5 NUSSELT NUMBER

The non dimensional form of the rate of heat transfer in terms of Nusselt number at the plate is given by:

\[ \text{Nu} = -\frac{\partial \theta}{\partial y} \bigg|_{y=0} = \left[ -Pr L + (-m_1 \varepsilon e^{i\omega t}) \right] \] (6.5.1)
6.6 SHERWOOD NUMBER

The non-dimensional form of the rate of mass transfer in terms of Sherwood number at the plate is given by:

\[ Sh = -\left(\frac{\partial \phi}{\partial y}\right)_{y=0} = \left(-m_1 L Sc - m_2 Pr L\right) + \epsilon e^{y(\text{at})} (m_4 m_5 - m_1 m_3) \]  (6.6.1)

6.7 RESULTS AND DISCUSSION

In order to get the physical insight into the problem, we have carried out numerical calculations for non-dimensional velocity field, temperature field, concentration field, coefficient of skin friction \( C_f \) at the plate, the rate of heat transfer in terms of Nusselt number \( Nu \) and the rate of mass transfer in terms of Sherwood number \( Sh \) have been carried out by assigning some specific arbitrary values to the different parameters involved in the problem, viz, Hartmann number \( M \), Soret number \( S_r \), Thermal Grashof number \( Gr \), Solutal Grashof number \( G_m \), Prandtl number \( Pr \), Schmidt number \( Sc \) and velocity ratio parameter \( \lambda \). The effects of these values are demonstrated through different graphs and the results are interpreted physically.

The figures 6.2-6.8 exhibit the variation of the velocity field \( u \) against \( y \) under the influence of Hartmann number \( M \), Soret number \( S_r \), Schmidt number \( Sc \), Prandtl number \( Pr \), Thermal Grashof number \( Gr \), Solutal Grashof number \( G_m \) and velocity ratio parameter \( \lambda \). Figure 6.2 indicates that the fluid velocity decreases with the increase in magnetic intensity indicating the fact that the fluid motion is decelerated under the action of transverse magnetic field.

Figure 6.3 and 6.4 illustrate the effect of Soret number \( S_r \) and Schmidt number \( Sc \) on fluid velocity. It is simulated from these two figures that an increase in Soret number \( S_r \) and Schmidt number \( Sc \) lead to an increase in fluid velocity. i.e the fluid motion is accelerated.
under the action of thermal diffusion effect and low mass diffusivity. The influence of Prandtl number Pr on velocity profile is demonstrated in figure 6.5. It is observed from this figure that the fluid velocity is enhanced for higher values of Prandtl number.

For the case of different values of thermal Grashof number Gr and Solutal Grashof number Gm, the velocity profiles in the boundary layer are shown in figure 6.6 and 6.7. It is found from the above figures that the velocity field falls under the effects of thermal Grashof number as well as solutal Grashof number which establish the fact that the fluid motion is retarded due to thermal buoyancy force and concentrated buoyancy effects represented by solutal Grashof number.

Figure 6.8 demonstrates the effect of velocity ratio against y on the velocity field. It is inferred from the figure that under the action of velocity ratio, the fluid motion is accelerated. We recall that the velocity ratio parameter increases means that the free stream velocity increases i.e. the boundary layer velocity is enhanced due to increase in the free stream velocity.

Figures 6.9-6.10 show the variation of Co efficient of Skin-friction Cf against magnetic field under the influence of Soret number Sr and velocity ratio parameter \( \lambda \). In both the figures it is observed that an increase in thermal-diffusion effect and velocity ratio parameter tends the viscous drag to rise and to fall significantly.

It is simulated from the figure 6.11 that the Nusselt number (co-efficient of rate of heat transfer) from the plate to the fluid increases due to the effect of Prandtl number. Here it is also observed that the co-efficient of rate of heat transfer is not so affected under the imposition of the transverse magnetic field. Figure 6.12 presents the variation of the rate of mass transfer from the plate to the fluid. It is inferred from this figure that the Sherwood
number decreases under thermal-diffusion effect. i.e. the mass flux from the plate to the fluid is reduced under the influence of diffusion-thermo effect.

6.8 CONCLUSIONS

Our investigation of the problem setup leads to the following conclusions:

1. The fluid motion is accelerated due to increase in Schmdit number, Soret number and decelerated under the action of tranverse magnetic field.

2. The velocity ratio parameter accelerates the fluid motion and the viscous drag.

3. Thermal Diffusion effect raised the magnitude of the viscous drag on the plate and reduced the mass flux from the plate to the fluid.

4. The co-efficient of rate of heat transfer in terms of Nusselt number from the plate to the fluid increases due to increase in Prandtl number.

6.9 NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Dimension</th>
<th>M.K.S Unit</th>
</tr>
</thead>
<tbody>
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<td>$\bar{B}$</td>
<td>Magnetic induction vector</td>
<td>-</td>
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<tr>
<td>$B_0$</td>
<td>Strength of the applied magnetic field</td>
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<td>$y$</td>
<td>$y$ – component of $\bar{B}$</td>
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<tr>
<td>$C_f$</td>
<td>Coefficient of Skin friction</td>
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<tr>
<td>$C'$</td>
<td>Concentration</td>
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<td>$\text{Mol m}^{-3}$</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
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<tr>
<td>--------</td>
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<td>------------------------------------------</td>
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<tr>
<td>(C_p)</td>
<td>Specific heat at constant pressure</td>
<td>[L \cdot T^{-1} \cdot \Theta^{-1}]</td>
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<tr>
<td>(C'_w)</td>
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<td>Mol m^{-3}</td>
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<tr>
<td>(C'_{\infty})</td>
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<td>Mol m^{-3}</td>
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<td>(D_t)</td>
<td>Thermal diffusion ratio</td>
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<td>Kg m^{-3} s^{-1} K^{-1}</td>
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<tr>
<td>(g)</td>
<td>Acceleration due to gravity</td>
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</tr>
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</tr>
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<tr>
<td>Subscript $\infty$</td>
<td>Refers to the values of the physical quantities away from the plate</td>
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</tr>
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</table>
APPENDIX

\[ L = P + iQ, \quad P = 1 + \cos \omega t, \quad Q = eA \sin \omega t, \]
\[ m_1 = \frac{\Pr L + \sqrt{\Pr^2 L^2 + i\omega \Pr}}{2}, \quad m_2 = \frac{-\Sc \Sr \Pr^2 L^2}{\Pr^2 L^2 + \Pr L \Sc}, \quad m_3 = 1 - m_2, \]
\[ m_4 = \frac{\Sc \Sr \m_2^2}{\m_2^2 + \Sc \m_1}, \quad \m_3 = \frac{-\Sc \Sr}{m_2^2 \Sc \m_1}, \quad N = \frac{\rho \beta^2}{\nu^2} + \frac{1}{K_1} \]
\[ m_6 = \frac{L + \sqrt{L^2 + 4N}}{2}, \quad m_7 = \frac{\Gr}{\Pr^2 L^2 + \Pr L^2 - N}, \quad \m_8 = \frac{\Gm \m_1}{\Sc^2 L^2 + \Sc L^2 - N} \]
\[ m_9 = \frac{\Gm \m_2}{\Pr^2 L^2 + \Pr L^2 - N}, \quad \m_{10} = m_7 + m_8, \quad \m_{11} = -(\lambda - m_{10} - m_9), \quad \lambda = \frac{U_0}{V_0} \]
\[ m_{12} = \frac{L + \sqrt{L^2 + 4(\frac{i\omega}{4} + N)}}{2}, \quad m_{13} = \frac{\Gr}{m_7 - \m_7 - \left(\frac{i\omega}{4} + N\right)}, \]
\[ m_{14} = \frac{\Gm \m_5}{\m_7 - \m_7 - \left(\frac{i\omega}{4} + N\right)} \]
\[ m_{15} = \frac{\Gm \m_6}{\m_7 - \m_7 - \left(\frac{i\omega}{4} + N\right)}, \quad \m_{16} = m_{13} + m_{14}, \]
\[ m_{17} = -(\lambda - m_{16} + m_{13}) \]

\[ m_1 = X_2 + iY_2, \quad X_2 = \frac{\Pr P + X_1}{2}, \quad Y_2 = \frac{\Pr Q + Y_1}{2}, \]
\[ X_1 = \sqrt{\frac{R + \sqrt{R^2 + S^2}}{2}}, \quad Y_1 = \sqrt{\frac{\sqrt{R^2 + S^2} - R}{2}}, \quad R = \Pr^2 P^2 - \Pr^2 Q^2, \]
\[ S = 2 \Pr^2 P Q + \omega \Pr \]

\[ m_4 = X_4 + iY_4, \quad X_4 = \frac{\Sc P + X_3}{2}, \quad Y_4 = \frac{\Sc Q + Y_3}{2}, \]
\[ X_3 = \sqrt{\frac{\Lambda_i + \sqrt{\Lambda_i^2 + B_i^2}}{2}}, \quad Y_3 = \sqrt{\frac{\sqrt{\Lambda_i^2 + B_i^2} - \Lambda_i}{2}}, \quad A_i = P^2 \Sc^2 - Q^2 \Sc^2, \quad B_i = 2 \Pr \Sc^2 + \omega \]

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\[ m_5 = X_5 + iY_5, X_5 = \frac{X_5X_6 + Y_5Y_6}{X_6^2 + Y_6^2}, Y_5 = \frac{Y_5X_6 - X_5Y_6}{X_6^2 + Y_6^2}, \]

\[ X_5 = -\text{ScSr}(X_2^2 - Y_2^2), Y_5 = -2\text{ScSr}X_2Y_2, \]

\[ X_6 = X_5^2 - Y_5^2 + \text{Sc}(PX_2 - QY_2), Y_6 = 2X_2Y_2 + \text{Sc}(QX_2 + PY_2) - \frac{\omega}{4}, \]

\[ m_6 = X_6 + iY_6, X_6 = \frac{P + X_4}{2}, Y_6 = \frac{Q + Y_4}{2}, \]

\[ X_8 = \sqrt{A_2 + \frac{A_2^2 + B_2^2}{2}}, Y_8 = \frac{\sqrt{A_2^2 + B_2^2} - A_4}{2}, \]

\[ A_2 = P^2 - Q^2 + 4N, B_2 = 2PQ, \]

\[ m_7 = X_{11} + iY_{11}, X_{11} = \frac{\text{Gr}X_{10}}{X_{10}^2 + Y_{10}^2}, Y_{11} = \frac{-\text{Gr}Y_{10}}{X_{10}^2 + Y_{10}^2}, \]

\[ X_{10} = \text{Pr}^2(P^2 - Q^2) + \text{Pr}(P^2 - Q^2) - N, Y_{10} = 2\text{Pr}^2PQ + 2\text{Pr}PQ, \]

\[ m_8 = X_{14} + iY_{14}, X_{14} = \frac{X_{13}(\text{Gm Pr} + \text{Gm Sc} + \text{Gm Sc Sr Pr})}{X_{13}^2 + Y_{13}^2}, \]

\[ Y_{14} = \frac{-Y_{13}(\text{Gm Pr} + \text{Gm Sc} + \text{Gm Sc Sr Pr})}{X_{13}^2 + Y_{13}^2}, \]

\[ X_{13} = \text{Pr}X_{12} + \text{Sc}X_{12}, Y_{13} = \text{Pr}Y_{12} + \text{Sc}Y_{12}, \]

\[ X_{12} = \text{Sc}^2(P^2 - Q^2) + \text{Sc}(P^2 - Q^2) - N, Y_{12} = 2\text{Sc}^2PQ + 2\text{ScPQ}, \]

\[ m_9 = X_{16} + iY_{16}, X_{16} = \frac{-\text{Gm Sc Sr Pr}X_{15}}{X_{15}^2 + Y_{15}^2}, \]

\[ Y_{16} = \frac{\text{Gr Sc Sr Pr}Y_{15}}{X_{15}^2 + Y_{15}^2}, X_{15} = \text{Pr}X_{10} + \text{Sc}X_{10}, Y_{15} = \text{Pr}Y_{10} + \text{Sc}Y_{10}, \]

\[ m_{10} = X_{17} + iY_{17}, X_{17} = X_{11} + X_{14}, Y_{17} = Y_{11} + Y_{14}, \]

\[ m_{11} = X_{18} + iY_{18}, X_{18} = X_{17} + X_{16} - \lambda, Y_{18} = Y_{17} + Y_{16}, \]

\[ m_{12} = X_{20} + iY_{20}, X_{20} = \frac{P + X_{19}}{2}, Y_{20} = \frac{Q + Y_{19}}{2}, \]

\[ X_{19} = \sqrt{A_1 + \frac{A_1^2 + B_1^2}{2}}, Y_{19} = \frac{\sqrt{A_1^2 + B_1^2} - A_4}{2}, \]

\[ A_3 = P^2 - Q^2 + 4N, B_3 = 2PQ + \omega. \]
\[ m_{13} = X_{22} + iY_{22}, \quad X_{22} = \frac{GrX_{21}}{X_{21}^2 + Y_{21}^2}, \quad Y_{22} = \frac{-GrY_{21}}{X_{21}^2 + Y_{21}^2}, \]

\[ X_{21} = X_2^2 - Y_2^2 + PX_2 - QY_2 - N, \quad Y_{21} = 2X_2Y_2 + QX_2 + PY_2 - \frac{\omega}{4}, \]

\[ m_{14} = X_{23} + iY_{23}, \quad X_{23} = \frac{-GmX_7X_{21} - GmY_7Y_{21}}{X_{21}^2 + Y_{21}^2}, \quad Y_{23} = \frac{GmX_7Y_{21} - GmY_7X_{21}}{X_{21}^2 + Y_{21}^2}, \]

\[ m_{15} = X_{25} + iY_{25}, \quad X_{25} = \frac{-GmX_7X_{24} + GmY_7Y_{24}}{X_{24}^2 + Y_{24}^2}, \quad Y_{25} = \frac{GmY_7X_{24} + GmX_7Y_{24}}{X_{24}^2 + Y_{24}^2}, \]

\[ X_{24} = X_4^2 - Y_4^2 + PX_4 - QY_4 - N, \quad Y_{24} = 2X_4Y_4 + QX_4 + PY_4 - \frac{\omega}{4}, \]

\[ m_{16} = X_{26} + iY_{26}, \quad X_{26} = X_{22} + X_{23} + X_{24}, \quad Y_{26} = Y_{22} + Y_{23}, \]

\[ m_{17} = X_{27} + iY_{27}, \quad X_{27} = X_{26} + X_{25} - \lambda, \quad Y_{27} = Y_{26} - Y_{25}, \]

Figure 6.2: Velocity versus \( y \) for Pr = 0.71, Gr=5, Gm=5, K_t=5, Sc=0.60, t = 1, \( \lambda=1, \omega=1, \varepsilon=0.002, A=0.5, \text{Sr}=1 \)

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Figure 6.3: Velocity versus $y$ for $Pr = .71$, $Gr=5$, $Gm=5$, $K_i=5$, $Sc=.60$, $t = 1$, $\lambda=1$, $\omega=1$, $\varepsilon=.002$, $A=.5$, $M=1$

Figure 6.4: Velocity versus $y$ for $Pr = .71$, $Gr=5$, $Gm=5$, $K_i=5$, $M=1$, $t = 1$, $\lambda=1$, $\omega=1$, $\varepsilon=.002$, $A=.5$, $Sr=1$
Figure 6.5: Velocity versus $y$ for $Sc = .60$, $Gr=5$, $Gm=5$, $K_i=5$, $M=1$, $t = 1$, $\lambda=1$, $\omega=1$, $\varepsilon=.002$, $A=.5$, $Sr=1$.

Figure 6.6: Velocity versus $y$ for $Sc = .60$, $Pr=.71$, $Gm=5$, $K_i=5$, $M=1$, $t = 1$, $\lambda=1$, $\omega=1$, $\varepsilon=.002$, $A=.5$, $Sr=1$.
Figure 6.7: Velocity versus y for $Sc = .60$, $Pr=.71$, $Gr=5$, $K=5$, $M=1$, $t = 1$, $\lambda=1$, $\omega=1$, $\varepsilon=.002$, $A=.5$, $Sr=1$

Figure 6.8: Velocity versus y for $Sc = .60$, $Pr=.71$, $Gr=5$, $K=5$, $M=1$, $t = 1$, $Gm=1$, $\omega=1$, $\varepsilon=.002$, $A=.5$, $Sr=1$
Figure 6.9: Skin friction versus $M$ for $Sc = .60$, $Pr=.71$, $Gr=5$, $\kappa_i=5$, $\lambda=1$, $t = 1$, $Gm=1$, $\omega=1$, $\varepsilon=.002$, $A=.5$

Figure 6.10: Skin friction versus $M$ for $Sc = .60$, $Pr=.71$, $Gr=5$, $\kappa_i=5$, $Sr=1$, $t = 1$, $Gm=1$, $\omega=1$, $\varepsilon=.002$, $A=.5$
Figure 6.11: Nusselt number versus M for $Sc = .60$, $Gr=5$, $K_i=5$, $Sr=1$, $t = 1$, $Gm=1$, $\omega=1$, $\varepsilon=.002$, $A=.5$, $\lambda=1$

Figure 6.12: Sherwood number versus M for $Sc = .60$, $Gr=5$, $K_i=5$, $Pr=.71$, $t = 1$, $Gm=1$, $\omega=1$, $\varepsilon=.002$, $A=.5$, $\lambda=1$