CHAPTER-10
CONCLUSIONS AND SUGGESTIONS FOR FUTURE ASPECTS

10.1 INTRODUCTION

This chapter presents the salient and significant contributions of the thesis along with few suggestions for future aspects. The main results achieved in different chapters are listed and discussed here in brief.

10.2 HOMOGENEOUS AND ISOTROPIC COSMOLOGICAL MODELS WITH VARIABLE G AND Λ

In the previous chapters we have discussed the construction of homogeneous and isotropic cosmological models under the following assumptions:
Assumption 1: \(8\pi G \rho = C_1 t^{-2n}, \Lambda c^2 = -C_2 t^{-2n}\)
Assumption 2: \(8\pi G \rho = C_1 R^{-2n}, \Lambda c^2 = -C_2 R^{-2n}\)

According to the assumptions, flat model of the universe is possible for different positive values of \(n\) and open model & closed models are possible only for \(n = 2\). We have already seen in chapter 4 & 6 for determination of scale factor as a function of time with the above assumptions. In these assumptions we have determined the constants \(C_1\) & \(C_2\) from the observation and estimated data.

10.3 HUBBLE’S CONSTANT \((H_0)\) AND HUBBLE’S TIME \((t_H)\)

Since everyday where we require the value of Hubble’s constant and during investigations it has been noticed that the different authors have used different values of \(H_0\) & \(t_H\) in the range between 10 billion years to 24 billion years, so it felt necessity to find the realistic value of Hubble’s time and accordingly it is estimated as
14.02 billion years. In using Hubble’s law to determine distances, only the velocity due to expansion of the universe can be used, because gravitationally interacting galaxies move relative to each other independent of the expansion of the universe.

The mathematical description of the Hubble’s law for a uniformly expanding universe is a fairly elementary theorem of geometry in 3-dimensional Cartesian, Newtonian coordinate space, which considered as metric space, is entirely homogeneous & isotropic, because its properties do not vary with location or direction.

10.4 VALIDITY OF FUNDAMENTAL CONSTANTS

One of the most valued principles of science is that there ought to be a single, immutable set of laws governing the universe and that these laws should remain the same everywhere and at all times. A striking common feature of almost all cosmological models throughout cosmological time history from 500BC to 2000AD (pl. look Sec-1.1 of Chapter-1, page 2-4) have been noticed, so the interpretation regarding variability of physical constants $G$, $c$, $h$, $a$, $e$ & cosmological constant $\Lambda$ have been discussed in Chapter-2. The main feature which we wish to discuss here is that one common view of constants is their asymptotic states. For example, the speed of light ‘$c$’ is (in Special theory of Relativity) the maximum velocity of a massive particle moving in flat space-time. Similarly if we talk about gravitational constant ‘$G$’, then we may say that it define the limiting potential for a mass that does not form a black-hole in curved space-time.

So for any physical theory we know that there should be one such constant. This view is acceptable in practice, but unsatisfactory in principle, mainly because it does not address the question of the constant origin. Another important conclusion I wish to present here that in our opinion ‘these constants’ are not fundamental but simply ways of relating quantities of different dimensional types. They are simply conversion constants that make the equations of physics dimensionally homogeneous.
10.5 VARIATION OF $\rho$, $G$ AND $\Lambda$

The variation of $\rho$ is derived from the conservation equation (4.16) with the help of equation (4.19)

$$\frac{\dot{\rho}}{\rho} = -3(1 + \beta) \frac{\dot{R}}{R} \tag{10.1}$$

For the determination of the variation of $G$ & $\Lambda$ from the equation

$$8\pi G \rho + \dot{\Lambda} c^2 = 0 \tag{10.2}$$

we assume the dependence of $G\rho$ & $\Lambda$ as from equations (6.3) & (6.4)

$$8\pi G \rho = -\alpha_1 \Lambda c^2 \tag{10.3}$$

where $\alpha_1$ is a positive constant, by assuming the condition written below

\[
\begin{align*}
G > 0 & \text{ & } \dot{G} < 0 \\
\Lambda < 0 & \text{ & } \dot{\Lambda} > 0
\end{align*}
\tag{10.4}
\]

In equations (10.4), the limitation which we have taken here is $\alpha_1 \neq 1$, otherwise the equation (10.2) gives

$$8\pi G \dot{\rho} = 0$$

$$\Rightarrow \quad \dot{\rho} = 0 \quad \Rightarrow \quad \rho = \text{constant}$$

which may not be true, because the universe is expanding so our assumption $\alpha_1 \neq 1$ is correct.

Differentiating equation (10.3), we get

$$8\pi G \dot{\rho} + 8\pi G \dot{\rho} = -\alpha_1 \dot{\Lambda} c^2 \tag{10.5}$$

By using equations (10.2) in equation (10.5), we get

$$8\pi G \dot{\rho} = (1 - \alpha_1) \dot{\Lambda} c^2 \tag{10.6}$$

Dividing equation (10.6) by equation (10.3), we get

$$\frac{\dot{\Lambda}}{\dot{\Lambda}} = -\frac{\alpha_1}{1 - \alpha_1} \frac{\dot{\rho}}{\rho} \tag{10.7}$$
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Summary of cosmological models

By using equation (10.1) in equation (10.7), we get

$$\frac{\dot{\Lambda}}{\Lambda} = \frac{3\alpha_1 (1 + \beta)}{(1 - \alpha_1)} \frac{\dot{R}}{R} \quad (10.8)$$

From equations (10.2) & (10.3), we have

$$\frac{\dot{G}}{G} = \frac{1}{\alpha_1} \frac{\dot{\Lambda}}{\Lambda} \quad (10.9)$$

By using equation (10.8) in equation (10.9), we get

$$\frac{\dot{G}}{G} = \frac{3(1 + \beta)}{\alpha_1 (1 - \alpha_1)} \frac{\dot{R}}{R} \quad (10.10)$$

So from equations (10.1), (10.8) & (10.10), we get

$$\frac{\dot{\rho}}{\rho} + \frac{\dot{G}}{G} = \frac{\dot{\Lambda}}{\Lambda} \quad (10.11)$$

Integrating the equations (10.1), (10.8) & (10.10) under the conditions

at $t = t_1$; $R = R_1$, $G = G_1$, $\rho = \rho_1$ & $\Lambda = \Lambda_1$

we get

$$\rho = \rho_1 \left( \frac{R}{R_1} \right)^{-3(1 + \beta)} \quad (10.12)$$

$$G = G_1 \left( \frac{R}{R_1} \right)^{3(1 + \beta)} \quad (10.13)$$

$$\Lambda = \Lambda_1 \left( \frac{R}{R_1} \right)^{3\alpha_1 (1 + \beta)} \quad (10.14)$$

The constants $R_1, G_1, \rho_1$ & $\Lambda_1$ are to be decided via observational and estimated data. During the expansion of the universe $\rho$ & $G$ are positive and decreasing whereas $\Lambda$ is negative and increasing.
10.6 VALIDITY OF THE MODELS

According to f-gravity theory a model of the universe should satisfy the following initial conditions

\[
\begin{align*}
R &= R_i = 10^{13} \text{ cm} \\
\rho &= \rho_i = 10^{17} \text{ g.cm}^{-3} \\
G &= G_i = 6.7 \times 10^{31} \text{ c.g.s.units} \\
\Lambda &= \Lambda_i = -10^{28} \text{ cm}^{-2} \\
t &= t_i = 10^{-23} \text{ sec}
\end{align*}
\]

and with present day observation data

\[
\begin{align*}
G &= G_0 = 6.7 \times 10^{-8} \text{ c.g.s.units} \\
\rho &= \rho_0 = 3 \times 10^{-31} \text{ g.cm}^{-3} \\
t &= t_0 = 6.3 \times 10^{17} \text{ sec} \\
R &= R_0 = 10^{28} \text{ cm} \\
\Lambda &= \Lambda_0 = -10^{-37} \text{ cm}^{-2}
\end{align*}
\]

with the help of equations (10.15), (10.16) & (10.12), we have

\[
\frac{\rho_i}{\rho_0} = \left( \frac{R_i}{R_0} \right)^{(3(1+\beta))} \\
\Rightarrow \beta = \frac{1}{15}
\]

and it is clearly \( 0 < \beta = \frac{1}{15} < \frac{1}{3} \)

then by using equation (10.13), (10.15), (10.16) & (10.17), we get

\[
\frac{G_i}{G_0} = \left( \frac{R_i}{R_0} \right)^{(3(1+\beta))} \\
\Rightarrow \alpha_i = \frac{87}{39}
\]

then from equation (10.3), we have
\[ \Lambda_i = -\frac{8 \pi G_i \rho_i}{\alpha_i c^2} \]  

(10.19)

Putting all the values, we get

\[ \Lambda_i = -8.4 \times 10^{28} \text{ cm}^{-2} \]  

(10.20)

Again from equation (10.3), we have

\[ \Lambda_0 = -\frac{8 \pi G_0 \rho_0}{\alpha_1 c^2} \]  

(10.21)

Putting all the values, we get

\[ \Lambda_0 = -2.5 \times 10^{-58} \text{ cm}^{-2} \]  

(10.22)

Therefore in the cosmological model, \( \Lambda \) is negative and increasing from \(-8.4 \times 10^{28} \text{ cm}^{-2}\) to \(-2.5 \times 10^{-58} \text{ cm}^{-2}\) as \( R \) increases from \(10^{13} \text{ cm}\) to \(10^{28} \text{ cm}\).

On the basis of this result discussed here and in previous chapters, we can say that the exact solution is possible for flat models only, whereas for closed and open models exact solution are not possible, which represents the realistic model of the universe. Therefore suitable approximate calculations have been proposed.

**10.7 APPROXIMATE CALCULATIONS**

As discussed in previous sections we have seen that only flat model may be treated as a realistic model of the universe, provided we assume missing mass in the universe. The possibilities of open models and closed models have been briefly discussed in chapter-4 and chapter-6 under the assumptions

\[ 8 \pi G \rho = C_1 t^{-2n} \& \Lambda c^2 = -C_2 t^{-2n}, \]

and

\[ 8 \pi G \rho = C_1 R^{-2n} \& \Lambda c^2 = -C_2 R^{-2n} \]

In chapter-6, under section 6.6 & 6.7, we have discussed the cosmological models in the radiation dominated era and in the matter dominated era, but in between these two, there must be an era when the matter and radiation are comparable. In this case \( \beta \) is neither equal to 1/3 nor equal to 0, but a value in between 0 & 1/3. The discussion of
such models are significant in the sense that we have $G$ and $\Lambda$ as variables and the variations of $G$ and $\Lambda$ may be observed only on the time scale comparable to the age of the universe depending upon the estimated data. In chapter-6, under section-6.8, we have also calculated the values of $C_1$ & $C_2$ as

$$C_1 = 1.7 \times 10^{125} \text{ c.g.s units} \quad \text{(approx.)}$$

$$C_2 = 6.8 \times 10^{124} \text{ c.g.s. units} \quad \text{(approx.)}$$

$$n = 2.9, \quad \Lambda_0 = -7.6 \times 10^{-58} \text{ cm}^{-2}$$

We have already verified and validated conclusions in concluding remarks in each & every chapter of the thesis.

### 10.8 FUTURE ASPECT

As stated in the thesis the cosmology is the study of the origins and the development of the universe. It is connected with the large scale, both with respect to the distance, and with respect to the past & future for the universe.

Most important problem in the cosmology is associated with the understanding how galaxies and cluster of galaxies formed and determining the nature of the mass of the universe. Following are the suggestions (but not limited to) regarding future aspects:

- In General theory of Relativity, the cosmological constant ‘$\Lambda$’ may be regarded as a measure of the energy density of the vacuum and can in principle lead to the avoid of big-bang singularity that is characteristic of other Friedman-Robertson-Walker models. However the simplest property of the vacuum that follows from the usual form of Einstein’s equations can be made more realistic if that theory is extended, which in general lead to the variable cosmological constant ‘$\Lambda$’. It requires more attention of cosmologists.

- Higher dimension cosmological models in the presence of bulk viscous fluid source may provide some interesting results for the spatially flat homogeneous & isotropic models.
To discuss the geometry of the universe more accurately, research in the area of Cosmic Topology is required and needed in future.

The nature of both Dark Mater and Dark Energy components remain unknown and in future we may hope that the large hadrons colloidal (LHC) or any other proposal may be able to provide hint on the nature of Dark Matter.

Under the assumptions that universe is dominated by radiations, cosmologists may discuss here at Big Bang nucleosynthesis (BBN) time.

With the same assumptions, the chaotic behavior of mix master universe may be discussed along with analysis of BKL time.

The accuracy in determining the planetary orbital motions will further improve with data from future space-missions.

More research regarding nature of baryonic matter is required & there is possibility to discuss the dual dark matter/dark energy properties in term of dark fluid. This need to be further investigated with chaotic behavior of universe.