CHAPTER-3
HUBBLE’S CONSTANT AND REALISTIC VALUE OF THE AGE OF UNIVERSE

3.1 INTRODUCTION

As we know, an open model of universe will expand forever whereas the closed model may contract in future. It is not known exactly that the universe is open or closed. We are also agree with the fact that the observations of total energy & critical energy densities strongly indicated that the geometry of the universe is hyperbolic. In this direction, Frank Steiner et. al [92] have nicely analyzed the CMB anisotropy of the Poincare dodecahedron. There are several interconnecting ways by which the nature of the universe may be determined. The one important way is to measure through critical density, which is defined as

\[
\rho_c = \frac{3H^2}{8\pi G}, \quad H = \frac{\dot{R}}{R}
\]

where \( G \) is the Newton’s Gravitational constant & \( R \) is the scale factor, which is a function of time. If \( t_0 \) denote the present time, then the present value of \( H \), denoted by \( H_0 \) is called the Hubble’s constant, for galaxies which are not too near nor too far. The velocity \( v \) is related to the distance \( D \) by Hubble’s constant as

\[
v = H_0 D
\]

Whereas \( D \) is the proper distance from the galaxy to the observer, measured in mega parsecs (Mpc), in the 3-space defined by given cosmological time. \( H_0 \) is the constant of proportionality (between the distance \( D \) to a galaxy and its velocity \( v \)) and known as Hubble’s constant. It corresponds to the value of \( H \) (often termed the Hubble parameter which is a value that is time dependent) in the Friedmann equations taken at the time of observation denoted by the subscript ‘0’. This value is the same throughout the universe for a given comoving time. The SI unit of \( H_0 \) is \( s^{-1} \) but
it is most frequently quoted in (km/s)/Mpc, There are some uncertainties in the value of the latter. We have decided to find the realistic value of $H_0$ by estimation method. For a value of $H_0$ given by 50km/sec per Mpc, the critical density equals about $5 \times 10^{-30} \text{ g cm}^{-3}$ or about three hydrogen atoms per thousand litres of space. There are several other related ways to determine the universe. One approach is to measure the expansion rate by using deceleration parameter about which there are also uncertainties. Theoretically in a simpler model in suitable units, the deceleration parameter is

$$q = \frac{\rho_a}{2 \rho_c} ; \quad \frac{\rho_a}{\rho_c} = \Omega , \text{ it means } \quad q = \frac{\Omega}{2} .$$

If $\Omega < 1$, the universe will expand forever, the present observed value of $\Omega$ is lying between 0.1 to 2.

Another way to find out $H_0$ if the universe will expand forever is to determine the precise age of the universe and compare it with the “Hubble’s time”. This is the time elapsed since the big bang until now. If the rate of expansion had been the same as at present it is.

![Figure: 3.1: Hubble’s Time](image)

If the tangent at $P$ to the curve $R(t)$ meet t- axis at $M$ at an angle $\alpha$, then

$$\tan \alpha = \frac{PT}{MT} = \dot{R}(t_0) \quad (3.3)$$
Hubble’s constant and realistic value of the age of the universe

\[ MT = \frac{PT}{R(t_0)} \]  

(3.4)

OT denotes the present time ‘\( t_0 \)’, then it is clear that

\[ PT = R(t_0) \]  

(3.5)

From equation \((3.4)\),

\[ MT = \frac{R(t_0)}{R(t_0)} = H_0^{-1} \]  

(3.6)

thus \( MT \), which is in fact Hubble’s time, is the reciprocal of Hubble’s constant.
The reciprocal of \( H_0 \) is called the Hubble’s time, written as \( t_H \). The Hubble’s time is the characteristic timescale of expansion of the Universe. It is related to the Hubble parameter. The present value of the Hubble’s time gives an estimate of the age of the Universe. Hubble’s constant ‘\( H_0 \)’ actually is not a constant, so, Hubble’s time ‘\( t_H \)’ is really only a rough estimate of the age of the universe, which has been presented in this chapter, while that one has to define the age of universe as

\[ t_H = t_H f(\Omega_r, \Omega_b, \Omega_{cdm}, ..., H_0, ...) \]

where \( f(\Omega_r, \Omega_b, \Omega_{cdm}, ..., H_0, ...) \) is a dimensionless function has to be computed from a cosmological model. Here we have just calculated the inverse of \( H_0 \) which is Hubble’s time (estimate age of universe as stated above).

The Hubble’s constant \( H_0 \) has units of inverse time it may be expressed as \( H_0 \sim 2.29 \times 10^{-18} \text{s}^{-1} \). Therefore we can define Hubble’s time \( t_H \) as:

\[ t_H = \frac{1}{H_0} \]  

(3.2)

If the value of \( H_0 \) were to stay constant, a naive interpretation of the Hubble’s time is that, it is the time taken for the universe to increase in size by a factor of \( e \). However, over long periods of time the dynamics are complicated by relativity. The current value of the Hubble’s constant, is hotly debated, with two opposing camps of
researchers generally getting values near the high and low ends of 50 and 100\textit{kms$^{-1}$}/\textit{Mpc} [93-102]. Since we know that the Hubble’s time is the inverse of the Hubble’s constant, also called the Hubble age, or the Hubble period, provides an estimate for the age of the universe by presuming that the universe has always expanded at the same rate as it is expanding today.

### 3.2 ASSUMPTIONS

For the construction of the model we assume the density of the universe at the present epoch as $3\times 10^{-31} \text{ g/cm}^3$. One of the important inference from the Hubble’s law (3.2) is that in the past, the galaxies were much closer together than they are today. Therefore the density of the universe has been very large in the past. Microwave background radiation or Microwave background fluctuation is the afterglow radiation left over from the hot Big-bang. Its temperature is extremely uniform all over the sky. However tinny temperature variations or fluctuations can offer great insight into the origin, evolution and content of the universe. It means, we can say that microwave background suggest a dense radiation which dominated the early phase of universe. As a consequence of this result, it is required here to determine the age of the universe.

For the purpose of our submission regarding the age of the universe, we have investigated the values of Hubble’s constant and Hubble’s time which is being discussed in the next section as per our paper [103]. There have been lots of measurements for the value of the Hubble’s constant $H_0$. Some of these including recent results are listed below:
### 3.3 DIFFERENT VALUES OF $H_0$

From the research findings, various observed/calculated values of $H_0$ have been listed in the following table:

**Table-3.1: Different Values of $H_0$**

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>Method Used</th>
<th>Values of $H_0$ in (Km/sec) / Mpc Max</th>
<th>Min</th>
<th>Ref.</th>
<th>Year</th>
<th>Selected Value of Hubble’s time (In billion Years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>M-101 group velocity and distance</td>
<td>64.2</td>
<td>46.8</td>
<td>[104-106]</td>
<td>1974</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Type-I Supernova</td>
<td>71</td>
<td>41</td>
<td>[108-112]</td>
<td>1979</td>
<td>23.9</td>
</tr>
<tr>
<td>6</td>
<td>Type-I Supernova</td>
<td>57</td>
<td>43</td>
<td>[108-112]</td>
<td>1982</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Infrared Tully-Fisher relations</td>
<td>92</td>
<td>72</td>
<td>[108-112]</td>
<td>1983</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Cepheids in Virgo (M100)</td>
<td>97</td>
<td>63</td>
<td>[108-112]</td>
<td>1984</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Surface brightness fluctuations</td>
<td>100</td>
<td>80</td>
<td>[108-112]</td>
<td>1993</td>
<td>9.79</td>
</tr>
<tr>
<td>10</td>
<td>Type-I Supernova &amp; Cepheids</td>
<td>63</td>
<td>47</td>
<td>[108-112]</td>
<td>1994</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>Gravitational lensing</td>
<td>66</td>
<td>66</td>
<td>[113-117]</td>
<td>1999</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>Cepheid distances to Galaxy</td>
<td>62</td>
<td>62</td>
<td>[118-121]</td>
<td>1999</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>HST</td>
<td>61</td>
<td>61</td>
<td>[126]</td>
<td>2000</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>Geometric distance measures</td>
<td>71</td>
<td>71</td>
<td>[122-125]</td>
<td>2000</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>HST</td>
<td>80</td>
<td>64</td>
<td>[127]</td>
<td>2001</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>Chandra X-ray observatory</td>
<td>77</td>
<td>77</td>
<td>[128]</td>
<td>2006</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>WMAP</td>
<td>72.4</td>
<td>69.2</td>
<td>[129]</td>
<td>2008</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>HST</td>
<td>77.8</td>
<td>70.6</td>
<td>[129]</td>
<td>2009</td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>Average of all above values</td>
<td>77.7</td>
<td>77.7</td>
<td></td>
<td>12.61</td>
<td></td>
</tr>
</tbody>
</table>
3.3.1 OUR ASSUMPTIONS IN THIS CASE

If we consider Hubble’s time as low, average and high for different values of Hubble constant $H_0$ then we can write:

\[ t_{\text{hl}} \text{ (for } H_0 = 100 \text{ (Km/s)/Mpc) } = 9.79 \text{ billion years} \]
\[ t_{\text{AH}} \text{ (for } H_0 = 77.7 \text{ (Km/s)/Mpc) } = 12.61 \text{ billion years} \]
\[ t_{\text{bl}} \text{ (for } H_0 = 41 \text{ (Km/s)/Mpc) } = 23.9 \text{ billion years} \]

where

$t_{\text{hl}}$ : Lowest value of Hubble’s Time
$t_{\text{AH}}$ : Average value of Hubble’s Time
$t_{\text{bl}}$ : Highest value of Hubble’s Time

3.4 RESULT AND DISCUSSIONS

Since here we have three time estimates so by using the formula given below based on the time estimation, we may get the most realistic value of Hubble’s time (Age of the Universe) as: If we denote the most realistic value of Hubble’s time as $t_{\text{rh}}$ then we can express the formula as:

\[ t_{\text{rh}} = \frac{t_{\text{hl}} + 4t_{\text{AH}} + t_{\text{bl}}}{6} \] \hspace{1cm} (3.3)

Putting the value from section (3.3.1), we have

\[ t_{\text{rh}} = \frac{9.79 + 50.44 + 23.9}{6} \text{ billion years} \]
\[ = \frac{84.13}{6} \text{ billion years} \]
\[ = 14.02 \text{ billion years} \] \hspace{1cm} (3.4)
If we compare this value in the table starting from Hubble period then it looks as:

### Table-3.2: Comparison of Hubble’s Time

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>Value of Hubble’s time (Age of Universe)</th>
<th>Scientists</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Less than 2 billion Years</td>
<td>Hubble</td>
<td>[17]</td>
</tr>
<tr>
<td>2</td>
<td>8 billion Years</td>
<td>Freedman</td>
<td>[130]</td>
</tr>
<tr>
<td>3</td>
<td>9 billion Years</td>
<td>Schmidt</td>
<td>[130]</td>
</tr>
<tr>
<td>4</td>
<td>10 billion Years</td>
<td>Kundic</td>
<td>[131]</td>
</tr>
<tr>
<td>5</td>
<td>11 billion Years</td>
<td>Falco</td>
<td>[132]</td>
</tr>
<tr>
<td>6</td>
<td>12 billion Years</td>
<td>Tammann</td>
<td>[131]</td>
</tr>
<tr>
<td>7</td>
<td>More than 13 billion Years</td>
<td>Liddle</td>
<td>[130]</td>
</tr>
<tr>
<td>8</td>
<td>14 billion Years*</td>
<td>Proposed in this thesis</td>
<td></td>
</tr>
</tbody>
</table>

* Proposed here in this thesis as the realistic age of the universe.

On the basis of the above calculations we have found the realistic value of Hubble’s time (age of the universe).

\[ t_{(\text{H})} = 14.02 \text{ billion years} \approx 14 \text{ billion years} \]

This is an estimation for the age of the universe. Further detail is also mentioned in chapter-7 (section 7.6).

### 3.5 CONCLUDING REMARKS

One of the natural query may arise in my mind that why is measuring \( H_0 \) still important? & we are agree upto some extent with the reply that the history has taught us that agreement does not always mean that the answer is right! So Independent confirmation is required in future. Independent measurements of \( H_0 \) may be evolved for correct prediction of age of universe and we hope to continue our efforts in this direction in future too along with the methodology to determine the Hubble’s constant.
Chapter-3 Hubble’s constant and realistic value of the age of the universe

The Hubble time is the amount of time one predicts by assuming the expanding universe at its current rate under following possibilities:

- If the universe used to be expanding much faster than the Hubble’s time overestimates the age of the universe.
- If the universe is expanding faster today than it used to be, the Hubble’s time underestimates the true age of the universe.

So it may be concluded that the realistic value is always required for calculation purposes in future and we may assume the realistic value of Hubble’s time as

\[ t_{\text{HH}} = 14.02 \text{ billion years.} \]