CHAPTER 2

SHOCK RESPONSE OF THREE-LAYER UNDAMPED LAMINATED PLATES SUBJECTED TO HALF-SINE PULSE

2.1 INTRODUCTION

The analysis for the dynamic response of a three-layer unsymmetrical laminated plate (Fig. 2.1) with an elastic core (undamped plate) is carried out taking into consideration the effects of all inertias; i.e., transverse, rotary and longitudinal inertias (generalised theory). Solution is obtained for the transverse displacement response of a simply supported plate subjected to an excitation in the form of a half-sine shock pulse (Fig. 2.2) applied to all edges simultaneously. Further, a similar analysis is done for the case which considers the effect of transverse inertia only (simplified theory). The influences of varying different parameters of the system on the transverse displacement response of the sandwich plate are studied by using both generalised and simplified theories. Also, the significance of including rotary and longitudinal inertias besides transverse inertia is investigated. In addition, the shock response of three-layer undamped plate is compared with that of undamped homogeneous plate using constant size, constant weight and constant static stiffness criteria.
FIG. 2.1 GEOMETRY OF A THREE-LAYER LAMINATED PLATE
(a) PLATE CONFIGURATION;
(b) VARIATION OF DISPLACEMENT $u$;
(c) VARIATION OF DISPLACEMENT $v$. 
FIG. 2.2 HALF-SINE WAVE PULSE ACCELERATION
ACCELERATION = $-v_0 \sin \omega t$, $0 \leq t \leq \tau$
$= 0$, $t \geq \tau$
2.2 ASSUMPTIONS

The assumptions made in the derivation of equations of motion and boundary conditions for a three-layer laminated plate are as follows:

(i) A plane transverse to the middle plane before bending remains plane and perpendicular to the middle plane after bending.

(ii) Transverse displacement at a section remains constant along the plate thickness.

(iii) Longitudinal displacements at a transverse section are assumed to vary as shown in Fig. 2.1.

(iv) There is perfect continuity at the interfaces and no slip occurs there while the plate is bending.

(v) Extension effects in the core and shear deformations in the face layers are neglected.

(vi) All displacements are small as in linear elasticity theory.

2.3 ANALYSIS OF THREE-LAYER PLATE—UNDAMPED—ALL INERTIAS

2.3.1 Equations of Motion and Boundary Conditions

The analysis for the case of a three-layer plate subjected to a general dynamic loading is carried out by taking into account the effects of flexure and membrane energies in the faces, strain energy due to transverse shear in the core and kinetic energy due to transverse, rotary and longitudinal
inertias for the entire plate. The equations of vibratory bending and the boundary conditions thus obtained (details in Appendix A) by the use of variational method are [32] as follows:

Equations of Motion

\[ \gamma_1 \left( \frac{1+\nu_1}{2} \frac{1}{2} v_1^\star + \frac{1-\nu_1}{2} u_1^\star \right) + \gamma_2 \left( \frac{c}{t_2^2} w' - \frac{u_1-u_3}{t_2^2} \right) - \left[ m_1 \ddot{u}_1 + \frac{m_2}{2} \left( \ddot{u}_1 + \frac{\ddot{u}_3}{2} + \frac{t_3-2t_1}{4} \ddot{w} \right) \right] = 0 \] (2.1a)

\[ \gamma_1 \left( \frac{1+\nu_1}{2} \frac{1}{2} u_1^\star + \frac{1-\nu_1}{2} v_1^\star \right) + \gamma_2 \left( \frac{c}{t_2^2} w' - \frac{v_1-v_3}{t_2^2} \right) - \left[ m_1 \ddot{v}_1 + \frac{m_2}{2} \left( \ddot{v}_1 + \frac{\ddot{v}_3}{2} + \frac{t_3-2t_1}{4} \ddot{w} \right) \right] = 0 \] (2.1b)

\[ \gamma_3 \left( \frac{1+\nu_2}{2} \frac{1}{2} v_3^\star + \frac{1-\nu_2}{2} u_3^\star \right) - \gamma_2 \left( \frac{c}{t_2^2} w' - \frac{u_1-u_3}{t_2^2} \right) - \left[ m_3 \ddot{u}_3 + \frac{m_2}{2} \left( \ddot{u}_3 + \frac{\ddot{u}_1}{2} + \frac{2t_3-t_1}{4} \ddot{w} \right) \right] = 0 \] (2.1c)

\[ \gamma_3 \left( \frac{1+\nu_2}{2} \frac{1}{2} u_3^\star + \frac{1-\nu_2}{2} v_3^\star \right) - \gamma_2 \left( \frac{c}{t_2^2} w' - \frac{v_1-v_3}{t_2^2} \right) - \left[ m_3 \ddot{v}_3 + \frac{m_2}{2} \left( \ddot{v}_3 + \frac{\ddot{v}_1}{2} + \frac{2t_3-t_1}{4} \ddot{w} \right) \right] = 0 \] (2.1d)

\[ (D_1+D_2) \nabla^4 w - \gamma_2 \frac{c}{t_2^2} \left\{ \frac{c}{t_2^2} (w''+w^\star\star) + \frac{u_1-u_1'}{t_2^2} + \frac{v_3-v_3^\star}{t_2^2} \right\} 
+ \rho \ddot{w} - f(x,y)g(t) \right] \]
$$+ m_2 \left\{ \frac{t_3-t_1}{12} (\dddot{u}_1 + \dddot{v}_1) + \frac{2t_3-t_1}{12} (\dddot{u}_3 + \dddot{v}_3) \right\}$$

$$+ \left( \frac{e_1^2}{12} + \frac{e_2^2}{12} \right) (\dddot{w} + \dddot{w}^**)\right] = 0$$

(2.1e)

where

$$\nabla^4 w = w^{****} + 2w'''''' + w^{****}$$

$$\gamma_1 = \frac{E_1 t_1}{1-v_1^2}, \quad \gamma_2 = \frac{G_2 t_2}{t_2}, \quad \gamma_3 = \frac{E_3 t_3}{1-v_3^2},$$

$$D_1 = \frac{E_1 t_1}{12(1-v_1^2)}, \quad D_2 = \frac{E_2 t_2}{12(1-v_2^2)}, \quad D_3 = \frac{E_3 t_3}{12(1-v_3^2)},$$

$$c = t_2 + (t_1 + t_3)/2, \quad e_1 = \frac{(t_3 - t_1)}{4}, \quad e_2 = \frac{(t_1 + t_3)}{2},$$

$f(x,y)g(t)$ is a general dynamic loading and other symbols are as explained in the Terminology.

**Boundary Conditions**

Along $x = 0$ and $x = a$,

(i) $u_1^* + v_1 v_1^* = 0$,  
(ii) $v_1^* + u_1^* = 0$,

(iii) $u_3^* + v_3 v_3^* = 0$,  
(iv) $v_3^* + u_3^* = 0$,

(v) $w = 0$,  
(vi) $D_1 (w'' + v_2 w^{**}) + D_2 (w'' + v_2^{**}) = 0$  

(2.2a)

Along $y = 0$ and $y = b$,

(vii) $u_1^* + v_1^* = 0$,  
(viii) $v_1^* + u_1^* = 0$,

(ix) $u_3^* + v_3^* = 0$,  
(x) $v_3^* + v_3 u_3^* = 0$,

(xi) $w = 0$,  
(xii) $D_1 (w^{**} + v_1 w'') + D_2 (w^{**} + v_2 w'') = 0$  

(2.2b)
2.3.2 Solution for Shock Response

As per the discussion in reference [121], when the edge of the plate is stiff enough as is usually the case in practice, the torque corresponding to the shear strains represented by the boundary conditions (ii), (iv), (vii) and (ix) of Eqs. (2.2) can be replaced by statically equivalent couples consisting of shear stresses acting in the core perpendicular to the faces which largely cancel one another. The noncancelling part which corresponds to the rate of change of torque along the edge can be equilibrated by distributed reaction forces along the edges and concentrated reaction forces at the corners which can be produced by rigid supports. Thus, the above referred boundary conditions representing the shear strains in the faces need not be satisfied. Instead, they may be replaced [121] by the more realistic boundary conditions obtained by the requirements of vanishing $u$-and $v$-displacements along the edges parallel to $x$-and $y$-axes, respectively. Thus, along $x = 0$ and $x = a$, $v_1 = 0$, $v_3 = 0$; and along $y = 0$ and $y = b$, $u_1 = 0$, $u_3 = 0$.

These along with all the remaining boundary conditions are satisfied if the following forms of series for the displacement components $w$, $u_1$, $u_3$, $v_1$ and $v_3$ are taken:

$$w(x,y,t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} W_{mn}(t) \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b},$$

(2.3a)
$u_1(x,y,t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} U_{1mn}(t) \cos \frac{\pi m}{a} \sin \frac{\pi n}{b}, \quad (2.3b)$

$u_3(x,y,t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} U_{3mn}(t) \cos \frac{\pi m}{a} \sin \frac{\pi n}{b}, \quad (2.3c)$

$v_1(x,y,t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} V_{1mn}(t) \sin \frac{\pi m}{a} \cos \frac{\pi n}{b}, \quad (2.3d)$

$v_3(x,y,t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} V_{3mn}(t) \sin \frac{\pi m}{a} \cos \frac{\pi n}{b}, \quad (2.3e)$

Excitation applied to the system is taken to be in the form of a half-sine pulse acceleration as shown in Fig. 2.2. This loading can be represented as:

$$f(x,y)g(t) = f(x,y) \sin \omega t, \quad 0 \leq t \leq \tau$$

$$= 0, \quad t > \tau \quad (2.4a)$$

where $\omega$ is the angular frequency and $\tau(=\pi/\omega)$ is the pulse duration.

The load function $f(x,y)$ can be expressed as a double Fourier series:

$$f(x,y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} f_{mn} \sin \frac{\pi m}{a} \sin \frac{\pi n}{b} \quad (2.4b)$$

At a given instant, the load is constant over the whole area bounded by $x = 0$, $x = a$ and $y = 0$, $y = b$ and thus $f(x,y) = f$, where $f = \rho v \omega$ as in the case of a drop test [80].
The expansion coefficient $f_{mn}$ is determined by multiplying both sides of Eq. (2.4b) with $\sin(m\pi x/a) \sin(n\pi y/b)$ and integrating from 0 to $b$ and 0 to $a$ giving:

$$f_{mn} = \frac{16f}{(mnR^2)} \quad \text{for } m,n = 1,3,5,...$$

$$= 0 \quad \text{for } m,n = 2,4,6,... \quad (2.4c)$$

Substituting Eqs. (2.3) and (2.4) in Eqs. (2.1) and non-dimensionalizing the coefficients, we get the following set of differential equations:

$$\left( A_a - A_b q^2 \right) W_{mn} - \left( B_a + A_d q^2 \right) U_{1mn} + \left( A_c - A_e q^2 \right) U_{3mn} - A_f V_{1mn} = 0 \quad (2.5a)$$

$$\left( B_b - B_c q^2 \right) W_{mn} - A_f U_{1mn} - \left( B_d + A_d q^2 \right) V_{1mn} + \left( A_c - A_e q^2 \right) V_{3mn} = 0 \quad (2.5b)$$

$$\left( A_a + A_g q^2 \right) W_{mn} - \left( A_c - A_e q^2 \right) U_{1mn} + \left( B_e + A_h q^2 \right) U_{3mn} + A_k V_{3mn} = 0 \quad (2.5c)$$

$$\left( B_b + A_l q^2 \right) W_{mn} + A_k U_{3mn} - \left( A_c - A_e q^2 \right) V_{1mn} + \left( B_f + A_h q^2 \right) V_{3mn} = 0 \quad (2.5d)$$

$$\left( A_m + A_n q^2 \right) W_{mn} - \left( A_a - A_b q^2 \right) U_{1mn} + \left( A_a + A_g q^2 \right) U_{3mn} - \left( B_b - B_c q^2 \right) V_{1mn} + \left( B_b + A_l q^2 \right) V_{3mn} = B_g \sin \omega t \quad (2.5e)$$
where

\[ A_a = G(1 + \frac{1 + t_{1.3}}{2t_{2.3}}), \quad A_b = \frac{1}{12} \mu_{m_{2.3}}(1 - 2t_{1.3}), \]

\[ A_c = \frac{6}{(t_{2.3} \lambda m)}, \quad A_d = (\mu_{3}/\lambda m)(m_{1.3} + m_{2.3}/3), \]

\[ A_e = \mu_{m_{2.3}}/(6\lambda m), \quad A_f = \frac{E_{1.3}^{+}t_{1.3}^{3} \lambda \varphi n}{2(1 - \nu_{1.3}^{2} \nu_{3})}, \]

\[ A_g = \frac{1}{12} \mu_{m_{2.3}}(2-t_{1.3}), \quad A_h = (\mu_{3}/\lambda m)(1 + m_{2.3}/3), \]

\[ A_k = \lambda \varphi n/[2(1-\nu_{3})], \quad A_l = (n\varphi/m)A_g, \quad A_m = B_5 + B_6 + B_7, \]

\[ A_n = \frac{1}{12} \mu_{3}^{2}\lambda m(1 + \frac{n^2 \varphi^2}{m^2}) [(1 + m_{1.3}^{2}t_{1.3}^{2}) + m_{2.3}(1-t_{1.3}^{2}t_{1.3}^{2})]A_p, \]

\[ A_p = \frac{\mu_{3}}{\lambda m} (1 + m_{1.3} + m_{2.3}), \quad A_q = B_1 + A_c, \quad B_b = (n\varphi/m)A_a, \]

\[ B_c = (n\varphi/m)A_b, \quad B_d = B_2 + A_c, \quad B_e = B_3 + A_c, \quad B_f = B_4 + A_c, \]

\[ B_g = 16n\omega A_p/(\pi^2 mn), \]

\[ B_1 = \frac{E_{1.3}^{+}t_{1.3}^{3} \lambda m}{1 - \nu_{1.3}^{2} \nu_{3}^{2}} [1 + (\frac{1 - \nu_{3}^{2} \nu_{3}^{2}}{2})(n\varphi/m)^2], \]

\[ B_2 = \frac{E_{1.3}^{+}t_{1.3}^{3} \lambda m}{1 - \nu_{1.3}^{2} \nu_{3}^{2}} [(n\varphi/m)^2 + \frac{1 - \nu_{3}^{2} \nu_{3}^{2}}{2}], \]

\[ B_3 = \frac{\lambda m}{1 - \nu_{3}^{2}} [1 + \frac{1 - \nu_{3}^{2}}{2} (n\varphi/m)^2], \]

\[ B_4 = \frac{\lambda m}{1 - \nu_{3}^{2}} [(n\varphi/m)^2 + \frac{1 - \nu_{3}^{2}}{2}], \]

\[ B_5 = \frac{E_{1.3}^{+}t_{1.3}^{3} \lambda m^3}{12(1 - \nu_{1.3}^{2} \nu_{3}^{2})} [1 + (n\varphi/m)^2]^2, \]
If Eqs. (2.5) are reduced to the case of a beam, the three resulting equations can be written in a form so that they are identical to those derived [86] for the case of a three-layer undamped laminated beam.

Elimination of \( U_{1mn}, U_{3mn}, V_{1mn} \) and \( V_{3mn} \) from Eqs. (2.5) gives:

\[
(M_0 + \sum_{i=2,4}^{10} M_i q^i)W_{mn} = P_1 \sin \omega t \quad (2.6)
\]

where the constants \( M_0, M_i \) and \( P_1 \) are worked out during the process of elimination.

**Determination of Transverse Displacement**

The solution for the displacement component \( W_{mn} \) in Eq. (2.6) consists of the sum of complimentary function and particular integral as given below:

\[
W_{mn}(t) = \sum_{i=1}^{10} J_i e^{R_i t} + M_{12} \sin \omega t \quad (2.7)
\]

where \( R_1 \) to \( R_{10} \) are the roots of characteristic tenth order polynomial in \( R \), viz.,
$$M_0 + \sum_{i=2,4}^{10} M_i R^i = 0 , \text{ and}$$

$$M_{12} = \frac{P_1}{M_{11}},$$

$$M_{11} = M_0 - M_2 \omega^2 + M_4 \omega^4 - M_6 \omega^6 + M_8 \omega^8 - M_{10} \omega^{10} .$$

Unknown constants $J_1$ to $J_{10}$ are determined from the following initial conditions based on the assumption that the plate is in a quiescent condition when the shock loading is applied.

At $t = 0$, $W_{mn} = 0$ and $\frac{d^i W_{mn}}{dt^i} = 0$, $i = 1,2,\ldots,9$. (2.8)

Substitution of these initial conditions in Eq. (2.7) yields ten non-homogeneous complex simultaneous algebraic equations which can be written in matrix form as:

$$\begin{bmatrix}
R \\
J_1 \\
J_2 \\
\vdots \\
J_{10}
\end{bmatrix}
= 
\begin{bmatrix}
S_1 \\
S_2 \\
\vdots \\
S_{10}
\end{bmatrix}
$$

where $[R]$ denotes a $(10 \times 10)$ coefficient matrix whose element in $i^{th}$ row and $j^{th}$ column is given by $R_{ij} = S_{i-j} = 0$, $S_7 = S_9 = 0$, $S_2 = -M_{12} \omega$, $S_4 = M_{12} \omega^3$, $S_6 = -M_{12} \omega^5$, $S_8 = M_{12} \omega^7$ and $S_{10} = -M_{12} \omega^9$.

Evaluating $R_1$ to $R_{10}$, $M_{12}$ and $J_1$ to $J_{10}$ as indicated above and placing in Eq. (2.7), we get the value of $W_{mn}(t)$. 
Because only odd modes are relevant in view of Eq. (2.4c), the expression for the displacement component \( w \) (Eq. 2.3a) may be written as:

\[
w(x,y,t) = \sum_{m=1,3}^{\infty} \sum_{n=1,3}^{\infty} \hat{W}_{mn}(t) \sin \frac{mx}{a} \sin \frac{ny}{b} \] \hspace{1cm} (2.9)

Substitution of \( \hat{W}_{mn}(t) \), determined as mentioned above, into Eq. (2.9) yields the expression for the transverse displacement response within the pulse era as:

for \( 0 \leq t \leq \tau \),

\[
w(x,y,t) = \sum_{m=1,3}^{\infty} \sum_{n=1,3}^{\infty} \left[ \sum_{i=1}^{10} J_i e^{i\omega t} + M_{12} \sin \omega t \right] \sin \frac{mx}{a} \sin \frac{ny}{b} \] \hspace{1cm} (2.10a)

Replacing \( t \) by \( (t-\tau) \) in the above equation and adding the resulting equation to it, we get the shock response in the residual vibration era as:

for \( t > \tau \),

\[
w(x,y,t) = \sum_{m=1,3}^{\infty} \sum_{n=1,3}^{\infty} \left[ \sum_{i=1}^{10} J_i e^{i\omega t} (1+e^{-i\omega \tau}) \right] \sin \frac{mx}{a} \sin \frac{ny}{b} \] \hspace{1cm} (2.10b)

2.4 ANALYSIS OF THREE-LAYER PLATE—UNDAMPED—TRANSVERSE INERTIA ONLY

The analysis for the case considering only transverse inertia is similar to the one carried out in Section 2.3 except that rotary and longitudinal inertia terms are neglected.
2.4.1 Solution for Shock Response

Governing Equations

Omitting rotary and longitudinal inertia terms contained in square brackets \([-\ldots-]\) in Eqs. (2.1), we get the following equations of motion including the effect of transverse inertia only.

\[
\begin{align*}
\gamma_1 \left( u_1'' + \frac{1+v_1}{2} v_1'' + \frac{1-v_1}{2} u_1''' \right) + \gamma_2 \left( \frac{c}{t_2^2} w' - \frac{u_1-u_3}{t_2^2} \right) &= 0 \quad (2.11a) \\
\gamma_1 \left( v_1'' + \frac{1+v_1}{2} u_1'' + \frac{1-v_1}{2} v_1' \right) + \gamma_2 \left( \frac{c}{t_2^2} w' - \frac{v_1-v_3}{t_2^2} \right) &= 0 \quad (2.11b) \\
\gamma_3 \left( u_3'' + \frac{1+v_3}{2} v_3'' + \frac{1-v_3}{2} u_3''' \right) - \gamma_2 \left( \frac{c}{t_2^2} w' - \frac{u_1-u_3}{t_2^2} \right) &= 0 \quad (2.11c) \\
\gamma_3 \left( v_3'' + \frac{1+v_3}{2} u_3'' + \frac{1-v_3}{2} v_3' \right) - \gamma_2 \left( \frac{c}{t_2^2} w' - \frac{v_1-v_3}{t_2^2} \right) &= 0 \quad (2.11d) \\
(D_1 + D_3) \nabla^4 w - \gamma_2 \frac{c}{t_2^2} \left\{ \frac{c}{t_2^2} (w'''' + w''') + \frac{u_1-u_3}{t_2^2} + \frac{v_3-v_1}{t_2^2} \right\} \\
+ \rho \ddot{w} - f(x,y)g(t) &= 0 \quad (2.11e)
\end{align*}
\]

The boundary conditions for this case are the same as specified in Eqs. (2.2). Thus, using the series forms for the displacement components \( w, u_1, v_1, u_3 \), and \( v_3 \) given by Eqs. (2.3) and following the procedure of Section 2.3.2, we obtain the differential equations pertaining to the present case.

\[
\begin{align*}
A_a W_{mn} - B_a U_{1mn} + A_c U_{3mn} - A_f V_{1mn} &= 0 \quad (2.12a) \\
B_b W_{mn} - A_f U_{1mn} - B_d V_{1mn} + A_c V_{3mn} &= 0 \quad (2.12b)
\end{align*}
\]
where the terms $A_a$, $B_a$, $A_c$ etc. are the same as already defined in Section 2.3.2.

Elimination of $U_{1mn}$, $U_{3mn}$, $V_{1mn}$ and $V_{3mn}$ from Eqs. (2.12) yields:

$$(M_o + M_1 q^2)W_{mn} = p_1 \sin \omega t$$

(2.13)

where $M_o$, $M_1$ and $p_1$ are determined during the process of elimination.

**Determination of Transverse Displacement**

The solution of Eq. (2.13) can be written as:

$$W_{mn}(t) = K_1 \sin Rt + K_2 \cos Rt + p_1 \sin \omega t/(M_o - M_1 \omega^2)$$

(2.14)

where $R = (M_o/M_1)^{1/2}$, constants $K_1$ and $K_2$ are to be evaluated from the initial conditions.

Assuming the plate to be initially quiescent, the initial conditions are:

At $t = 0$, $W_{mn}(t) = \dot{W}_{mn}(t) = 0$

Using these conditions, we get $K_1 = -\frac{p_1 \omega}{R(M_o - M_1 \omega^2)}$ and $K_2 = 0$. 
Thus knowing $w_{mn}(t)$ completely, the shock response as obtained from Eq. (2.9) is as follows:

for $0 \leq t < \tau$,

$$w(x,y,t) = \sum_{m=1,3}^\infty \sum_{n=1,3}^\infty \frac{P_1}{(M_0 - M_1 \omega^2)} \left[ \sin \omega t - \frac{\omega}{R} \sin Rt \right] \cdot \sin \frac{m \pi x}{a} \sin \frac{n \pi y}{b}$$

(2.15a)

and for $t \geq \tau$,

$$w(x,y,t) = \sum_{m=1,3}^\infty \sum_{n=1,3}^\infty \frac{-P_1 \omega}{R(M_0 - M_1 \omega^2)} \left[ \sin Rt + \sin R(t-\tau) \right] \cdot \sin \frac{m \pi x}{a} \sin \frac{n \pi y}{b}$$

(2.15b)

2.5 RESULTS AND DISCUSSION

2.5.1 Introduction

Investigation of the shock response behaviour of a three-layer simply supported undamped laminated plate subjected to a half-sine pulse loading is done for various geometrical and physical parameters of the system. The results are given for both theories - generalised theory which includes all inertias, viz., transverse, rotary and longitudinal inertias (Section 2.3) and simplified theory which considers only transverse inertia (Section 2.4). It is found that the analysis
which accounts for all inertias is very tedious. Its computations take much more time than that taken by the computations of the case with only transverse inertia. From the comparison of the results obtained by using the two theories, an assessment is made of the situations where rotary and longitudinal inertias may be neglected without any detrimental effect on the prediction of response. Further, the investigation aims at finding out the significance of various parameters of the undamped sandwich plate in limiting the peak response.

Computations are done for the transverse displacement response at the centre of the plate ($w_c$). For both theories, the response series converges rapidly and nine modes ($m=n=5$) of the series have been found to be enough for evaluation of the results.

2.5.2 Effects of Rotary and Longitudinal Inertias

In order to study the influence of inclusion of rotary and longitudinal inertias on the prediction of the shock response of the laminated plate, a comparison is made of the numerical results computed from both the theories (generalised and simplified). The results are obtained for different values of shock pulse duration and a wide range of system parameters of practical interest. In Figs. 2.3 to 2.12, symbol G stands for generalised theory and symbol S stands for simplified theory. With the exceptions mentioned in the respective discussions, the various parameters for these figures are as follows:
\[ t_{1.3} = 1, \quad t_{2.3} = 3, \quad \varphi = 1, \quad \rho_{1.3} = 1, \quad \rho_{2.3} = 0.5, \]
\[ E_{1.3} = 1, \quad \nu_{1.3} = 1, \quad \bar{G} = 5 \times 10^{-3}, \quad t_3 = 0.25 \text{ cm}, \]
\[ a = 0.3 \text{ m}, \quad \rho_3 = 2746 \text{ kg/m}^3, \quad E_3 = 68.65 \text{ GN/m}^2, \]
\[ \nu_3 = 0.34, \quad v = 0.25 \text{ m/s}. \quad (2.16) \]

Figures 2.3 to 2.6 show the time-history of shock response \( w_c \) during the pulse era \((0 \leq t \leq \tau)\) as well as the residual vibration era \((t > \tau)\) for two laminated plates which differ only in the values of their core shear modulus. Taking face layer 3 as the reference layer, modulus ratio \( \bar{G} \) is defined as the ratio of core shear modulus to Young's modulus of the reference layer. The values of core material constant \( \bar{G} (= \frac{G_2}{E_3}) \) for the two plates are \( 3 \times 10^{-4} \) and \( 5 \times 10^{-3} \) and the remaining parameters are as given in Eq. (2.16).

From the response curves of Figs. 2.3 and 2.4 in which the values of pulse duration \((\tau)\) are \( \pi/250 \text{ s} \) and \( \pi/1500 \text{ s} \) respectively, it is observed that the two theories give the same results; whereas, in the case of relatively sharp transients as shown in Figs. 2.5 and 2.6, the generalised theory which includes rotary and longitudinal inertias in addition to transverse inertia predicts a lower displacement response as compared to that predicted by the theory considering transverse inertia only. It is also seen that the difference between the peak responses obtained by the use of the two theories increases as the pulse duration is decreased. Further,
FIG. 2.3 TIME-HISTORY OF SHOCK RESPONSE W/C
FIG. 2.4  TIME-HISTORY OF SHOCK RESPONSE $w_c$
Fig. 2.5 Time-history of shock response $w_c$
FIG.2.6 TIME-HISTORY OF SHOCK RESPONSE $w_c$

- $\tau = T/10000$ s
- G - GENERALISED THEORY
- S - SIMPLIFIED THEORY

- $G = 3 \times 10^{-4}$
- $G = 5 \times 10^{-3}$

TIME (s)
for a given value of $\tau$, the above mentioned difference is larger when the core material is less rigid (less $G$).

Figures 2.7 to 2.12 show the curves of variation of $w_{cp}$ (peak value of $w_c$ found by plotting $w_c$ versus time curves) for different values of shock pulse duration against various geometrical and physical parameters of the undamped three-layer plate. The effect of coefficient of unsymmetry on $w_{cp}$ is illustrated in Fig. 2.7. This coefficient is taken equal to $(t_3 - t_1)/(t_3 + t_1)$. The thicknesses $t_1$ and $t_3$ are varied but their sum is kept constant ($=0.5$ cm in the present data). For this figure, the core thickness is 2.5 cm and remaining parameters are as specified in Eq. (2.16). An increase in coefficient of unsymmetry or a decrease in $\tau$ is found to increase the gap between the results of the two theories. It may be observed that $(t_1 + t_3)$ being a constant ($=0.5$ cm in this case), the above defined unsymmetry coefficient can be rewritten into the form

$$(t_3 + t_1 - 2t_1)/(t_3 + t_1) = (1 - 4t_1) > 0,$$

from which it follows that the results of Fig. 2.7 are limited for $0 < t_1 < 0.25$ cm. It is obvious that in a general case, $0 < t_1 < (t_1 + t_3)/2$, when the coefficient of unsymmetry is varied keeping $(t_1 + t_3)$ a constant.

The curves of $w_{cp}$ versus core density ratio $\rho_{2.3}$ for various values of shock pulse duration $\tau$ are drawn in Fig. 2.8 in which the core density is varied and rest of the data is as listed in Eq. (2.16). Core density ratio $\rho_{2.3}$ is defined as the
FIG. 2.7 VARIATION OF $w_{cp}$ WITH COEFFICIENT OF UNSYMMETRY

$G$ - GENERALISED THEORY
$S$ - SIMPLIFIED THEORY

$\tau = \pi / 7500$
$\tau = \pi / 5000$
$\tau = \pi / 1500$
$\tau = \pi / 500$

$w_{cp} \times 10^2$ (cm)

COEFFICIENT OF UNSYMMETRY

FIG. 2.7 VARIATION OF $w_{cp}$ WITH COEFFICIENT OF UNSYMMETRY
FIG. 2.8 VARIATION OF $w_{cp}$ WITH CORE DENSITY RATIO

G - GENERALISED THEORY
S - SIMPLIFIED THEORY

$\tau = \pi/500$
$\tau = \pi/5000$
$\tau = \pi/7500$
$\tau = \pi/10000$

$w_{cp} \times 10^2 \text{ (cm)}$

CORE DENSITY RATIO ($\rho_{2,3}$)
FIG. 2.9 VARIATION OF $w_{cp}$ WITH FACE THICKNESS RATIO
FIG. 2.10 VARIATION OF W_{cp} WITH CORE THICKNESS RATIO

G - GENERALISED THEORY
S - SIMPLIFIED THEORY

$w_{cp} \times 10^2$ (cm)

$\tau = \pi / 5000$

$\tau = \pi / 15000$

$\tau = \pi / 500$

CORE THICKNESS RATIO ($t_{2.3}$)

FIG. 2.10 VARIATION OF W_{cp} WITH CORE THICKNESS RATIO
FIG. 2.11 VARIATION OF $w_{cp}$ WITH ASPECT RATIO
FIG. 2.12 VARIATION OF \( w_{cp} \) WITH \( \bar{G} \)

G - GENERALISED THEORY
S - SIMPLIFIED THEORY

\( \tau = \pi / 10000 \)
\( \tau = \pi / 5000 \)
\( \tau = \pi / 1500 \)
\( \tau = \pi / 500 \)
ratio of densities of the core and the reference layer. It is noticed that the deviation between the predictions of the two theories increases with an increase in $\rho_{2.3}$ or a decrease in $\tau$.

Figure 2.9 depicts the variation of $w_{cp}$ with face thickness ratio $t_{1.3}$ where the thickness $t_1$ is varied and other parameters are kept as per Eq. (2.16). This figure shows that decrease in $t_{1.3}$ or $\tau$ brings about greater difference between the results obtained from the two theories.

The plots of peak response at the centre of the plate ($w_{cp}$) against core thickness ratio $t_{2.3}$ for different values of $\tau$ are given in Fig. 2.10. In this case, thickness of the core is varied, $a = 0.45$ m and remaining data is as given in Eq. (2.16). The above figure reveals that there is a considerable difference between the predictions of the two theories if $t_{2.3}$ is large and $\tau$ is small. For example, at $\tau = \pi/10 \ 000$ s, simplified theory gives $w_{cp}$ which is nearly 20%, 28% and 49% higher than that given by generalised theory when $t_{2.3}$ is 1.3 and 10 respectively. The deviation between the results of the two theories increases with increase in $t_{2.3}$ or decrease in $\tau$.

Figure 2.11 shows the behaviour of $w_{cp}$ with a change in aspect ratio $\phi$ (length-width ratio). This ratio is varied by changing the dimensions $a$ and $b$, keeping the plate area constant ($0.09 \ m^2$ in the present illustration) and retaining the other data as per Eq. (2.16). The gap between the predictions of the two theories is seen to decrease with increase in aspect ratio or shock pulse duration.
The peak response curves of Fig. 2.12 are plotted to study the variation of $w_{cp}$ with change in $\bar{G}$ for various values of $\tau$. The core shear modulus $G_2$ is varied while rest of the parameters remain as in Eq. (2.16). This figure affirms the inferences already drawn from Figs. 2.5 and 2.6 regarding the increase in the difference between the predictions of the two theories with decrease in modulus ratio ($\bar{G}$) or pulse duration ($\tau$).

Thus, the observations made from Figs. 2.3 to 2.12 as mentioned above, yield the conclusion that the two theories (generalised and simplified) predict almost the same vibration response for a large duration shock pulse, but they deviate in case of relatively sharp transients for which the shock response given by generalised theory tends to be lower than that predicted by simplified theory. This happens because in generalised theory which includes transverse, rotary and longitudinal inertias, a part of the transient input of energy gets apportioned to rotary and longitudinal inertias and the flexural energy of the system is reduced resulting in lesser amplitude of vibration. Further, it is noted that the difference in the predictions of the two theories increases with decrease in pulse duration. Similar observations have been made for elastic homogeneous beams [81,82] and in the case of sandwich beams [86] as well. This confirms the importance of inclusion of longitudinal and rotary inertias in the analysis when the transients are sharp.
2.5.3 Peak Response Characteristics

A study of the time-histories of shock response given in Figs. 2.3 to 2.6 reveals that the peak value of transverse displacement response $w_{cp}$ occurs within the excitation period when the pulse duration is large; whereas, it occurs during the residual vibration era in case of a relatively sharp pulse. Similar conclusion is drawn by Mindlin et al. [80] for the case of an undamped single degree of freedom system subjected to a half-sine shock pulse. It is also seen that the peak response occurs earlier in time when $G$ is more, i.e., the core material is more rigid. Further, the response curve shows one or more undulations within the excitation period if $\tau$ is large and the number of such undulations is more when $G$ is more. As expected for an undamped system, there is no decay of vibration after the pulse era.

The effects of various parameters of the system on the peak displacement response characteristics are depicted in Figs. 2.7 to 2.12. The curves of Fig. 2.7 show that $w_{cp}$ increases with an increase in coefficient of unsymmetry and least response is obtained for a symmetric configuration. This happens because the constraining effect on the core is reduced with an increase in the coefficient of unsymmetry.

Figure 2.8 indicates that $w_{cp}$ increases with increase in core density ratio $\rho_{2.3}$. It is so because an increase in core density causes a higher intensity of shock loading ($\rho v w$).

The peak displacement response $w_{cp}$ is found to decrease with an increase in $t_{1.3}$ and $t_{2.3}$ as observed from Figs. 2.9
and 2.10 respectively. It may be explained due to the fact that an increase in $t_{1.3}$ adds to bending stiffness of the plate and increases the constraining effect on the core; whereas, increase in $t_{2.3}$ results in an increase in stiffness of the system.

It is inferred from Fig. 2.11 that a rectangular configuration with a higher aspect ratio $\phi$ gives a better displacement response. This behaviour is analogous to that observed for homogeneous simply supported plates [122] and is attributed to the increase in plate stiffness because of increase in aspect ratio.

An increase in core shear modulus ratio $\bar{G}$ (Fig. 2.12) decreases $w_{cp}$ rapidly up to a certain value of $\bar{G}$ due to increase in plate stiffness. However, a further increase in $\bar{G}$ becomes less and less effective in reducing the peak response because of the larger shear coupling between the faces.

It is further noticed from Figs. 2.7 to 2.12 that pulse duration $\tau$ has a marked effect on the peak response. In general, $w_{cp}$ rises with sharpness of the pulse when comparison is made on constant impulse basis (i.e., equal area of pulse basis). The area of half-sine pulse ($= \frac{2}{\pi} \rho \nu \omega \tau$) remains constant even when the pulse duration is changed because a decrease in $\tau(=\pi/\omega)$ causes a corresponding increase in amplitude ($\rho \nu \omega$) of the pulse. However, for certain parameters of the three-layer laminated plate, a sharper transient may predict a lower peak displacement response as seen from $\tau = \pi/10 000$ s curve in Figs. 2.9, 2.10.
and 2.12. These observations are similar to those reported in [86] for the case of sandwich beams.

2.6 COMPARISON OF UNDAMPED THREE-LAYER AND UNDAMPED HOMOGENEOUS PLATES

2.6.1 Shock Response Analysis—Undamped Homogeneous Plate—Transverse Inertia Only

Transient response of a simply supported undamped homogeneous plate (a single-layer plate) subjected to an excitation in the form of a half-sine pulse acceleration is analyzed, taking into account the effect of transverse inertia only. The equations of motion for this case can be deduced from the corresponding expressions for the three-layer undamped laminated plate (Section 2.4.1). Putting $t_2 = t_3 = 0$ (thus $\gamma_2 = \gamma_3 = 0$) in Eqs. (2.11), we get the following equations of motion for a single-layer homogeneous plate.

\[ \gamma_1 \left( \frac{1+v}{2} \right) u_1'' + \frac{1-v_1}{2} u_1^* = 0 \] (2.17a)

\[ \gamma_1 \left( \frac{1+v_1}{2} \right) v_1^* + \frac{1-v_1}{2} v_1' = 0 \] (2.17b)

\[ D_1 \nabla^4 w + \rho_1 \tau_1 \ddot{w} - f(x,y)g(t) = 0 \] (2.17c)

Replacing subscript 1 by h in Eq. (2.17c), we get:

\[ D_h \nabla^4 w + \rho_h \tau_h \ddot{w} - f(x,y)g(t) = 0 \] (2.18)

where

\[ D_h = E_h t_h^3/12(1-v_h^2), \] the terms $E_h$, $\rho_h$, $v_h$ and $t_h$ refer to Young's modulus, mass density per unit volume, Poisson's ratio
and thickness of the single-layer (homogeneous) plate.

Equation (2.18) is the equation of motion for the transverse vibrations of an elastic (undamped) homogeneous plate subjected to a general dynamic loading and is the same as derived ab-initio [122].

The displacement component $w$ is taken in the form of a double series:

$$w(x,y,t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} w_{mn}(t) \sin \frac{m \pi x}{a_h} \sin \frac{n \pi y}{b_h}$$  \hspace{1cm} (2.19)

where $a_h$ and $b_h$ are the length and width of homogeneous plate.

It is seen that the above series satisfies the simply supported end conditions, i.e., $w = w'' = w''' = 0$ at the boundary of the plate.

The homogeneous plate is also assumed to be subjected to the loading (half-sine pulse) represented by Eq. (2.4a), which is,

$$f(x,y)g(t) = f(x,y) \sin \omega t, \hspace{1cm} 0 \leq t \leq \tau$$

$$= 0 \hspace{1cm} t > \tau$$  \hspace{1cm} (2.4a)

The load function $f(x,y)$ is taken as a double Fourier series:

$$f(x,y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} f_{mn} \sin \frac{m \pi x}{a_h} \sin \frac{n \pi y}{b_h}$$

where

$$f_{mn} = 16f/(m \pi n^2) \hspace{1cm} \text{for} \hspace{1cm} m,n = 1,3,5,\ldots$$

$$= 0 \hspace{1cm} \text{for} \hspace{1cm} m,n = 2,4,6,\ldots$$
The term \( f = \rho_H \nu \omega \), as explained in Section 2.3.2; \( \rho_H = \rho_n t_h \) is the mass/area of the homogeneous plate.

Substitution of Eqs. (2.19) and (2.4a) in Eq. (2.18) gives the following differential equation:

\[
(M + M_0 q^2) w_{mn} = f_{mn} \sin \omega t
\]

where

\[
M_0 = D_n [(m\pi/a_h)^2 + (n\pi/b_h)^2] \quad \text{and} \quad M_1 = \rho_H.
\]

The above differential equation is similar to Eq. (2.13); thus adopting the procedure of Section 2.4.1 and using the initial conditions \( \dot{w}_{mn} = \ddot{w}_{mn} = 0 \) at \( t = 0 \), we get the following expressions for the transverse displacement response of the homogeneous plate:

for \( 0 \leq t \leq \tau \),

\[
\begin{aligned}
& w(x,y,t) = \sum_{m=1,3}^{\infty} \sum_{n=1,3}^{\infty} \frac{f_{mn}}{M_0 - M_1 \omega^2} \left[ \sin \omega t - \frac{\omega}{R} \sin R t \right] \\
& \times \sin \frac{m\pi x}{a_h} \sin \frac{n\pi y}{b_h}
\end{aligned}
\]

(2.20a)

and for \( t > \tau \),

\[
\begin{aligned}
& w(x,y,t) = \sum_{m=1,3}^{\infty} \sum_{n=1,3}^{\infty} \frac{-f_{mn} \omega}{R(M_0 - M_1 \omega^2)} \left[ \sin R t + \sin R(t-\tau) \right] \\
& \times \sin \frac{m\pi x}{a_h} \sin \frac{n\pi y}{b_h}
\end{aligned}
\]

(2.20b)

where \( R = (M_0/M_1)^{\frac{1}{2}} \)
2.6.2 Comparison Criteria

The criteria for studying the performance of a three-layer laminated plate vis-à-vis a single-layer (homogeneous) plate may be based on constant size, constant weight and constant static stiffness. A symmetrical configuration of the three-layer plate is selected for the purpose of comparison with a reference homogeneous plate. Both the plates are taken to be of the same length and width. The material of homogeneous plate is taken to be same as that of the faces of the sandwich plate, i.e., $E_h = E_3$ and $\rho_h = \rho_3$.

(a) Constant Size Criterion

Size ratio may be defined as the ratio of the thickness of laminated plate to that of the reference homogeneous plate. In case of a symmetrical sandwich ($t_1 = t_3$),

$$\text{size ratio} = \frac{t_3(2+t_2)}{t_h}$$  \hspace{1cm} (2.21a)

For constant size criterion, size ratio will be unity and thus

$$\frac{t_3}{t_h} = \frac{1}{2+t_2}$$ \hspace{1cm} (2.21b)

(b) Constant Weight Criterion

Weight ratio may be defined as the ratio of weight per unit area of the sandwich plate to that of the reference homogeneous plate and is given by

$$\text{weight ratio} = \frac{t_3(2+\rho_2 t_2)}{t_h}$$ \hspace{1cm} (2.22a)
For constant weight criterion, the weight ratio is unity which gives
\[ \frac{t_3}{t_h} = \frac{1}{2 + \rho_{2.3}^t} \] (2.22b)

(c) Constant Static Stiffness Criterion

As in the case of a three-layer beam [60], the static stiffness of a three-layer plate may be defined as the sum of flexural stiffness of the face layers about their own neutral axes and the flexural stiffness due to the displacement of their neutral axes from the neutral axis of the entire plate. The assumption made for the static analysis of conventional sandwich structures [17] is that the normal transverse stiffness of the elastic core is sufficient to enable the face layers to bend about the neutral axis of the entire sandwich. For a symmetrical configuration,

static stiffness of a three-layer plate is

\[ \frac{E_{3h}}{12(1-\nu^2_h)} \left( \frac{3}{2} t_2^2 + \frac{3}{4} t_2^2 t_3 + t_3^2 \right) \]

static stiffness of a homogeneous plate = \[ \frac{E_{3h}}{12(1-\nu^2_h)} \]

Defining the static stiffness ratio as the ratio of static stiffness of sandwich plate to that of reference homogeneous plate, we have

static stiffness ratio = \( \left( \frac{t_3}{t_h} \right)^3 \left[ 2 + 6t_2^2 \right] \)

(2.23a)
The above ratio is unity for constant static stiffness criterion, so that
\[
t_3/t_h = 1/[2 + 6(1+t_2^2)]^{1/3}
\] (2.23b)

2.6.3 Results and Discussion

On the basis of the comparison criteria described in the preceding section, the results for the shock response of undamped three-layer plate and reference homogeneous plate (obtained by the use of Eqs. (2.15) and (2.20) respectively) are compared to find the conditions favourable for achieving a high displacement response effectiveness (capacity to give reduced displacement response) of the laminated plate. Excepting the parameters specified in the concerned discussions, the remaining data used in the computation of the results presented in the current section is taken from the following:

\[\begin{align*}
E_{1.3} &= 1, \quad \rho_{2,3} = 0.05, \quad \rho_{1.3} = 1, \quad \nu_{1.3} = 1, \\
E_3 &= E_h = 68.65 \text{ GN/m}^2, \quad \rho_3 = \rho_h = 2746 \text{ kg/m}^3, \\
a &= b = a_h = b_h = 0.3 \text{ m}, \quad t_h = 1.0 \text{ cm}, \\
\nu_3 &= \nu_h = 0.34, \quad v = 0.25 \text{ m/s}, \quad \tau = \pi/500 \text{ s}
\end{align*}\] (2.24)

A comparison of the time-histories of shock response (\(w_c\)) of the three-layer and the single-layer undamped plates is made in Figs. 2.13a,b,c which are drawn for constant size, constant weight and constant static stiffness criteria, respectively. For
FIG. 2.13(a)

- THREE-LAYER PLATE
- HOMOGENEOUS PLATE

$W_c \times 10^4$ (cm)

TIME (s)

0 0.005 0.01 0.015

FIG. 2.13(b)
FIG. 2.13 TIME-HISTORY OF SHOCK RESPONSE $w_c$ ON:
(a) CONSTANT SIZE CRITERION;
(b) CONSTANT WEIGHT CRITERION;
(c) CONSTANT STATIC STIFFNESS CRITERION.
these figures, \( \bar{G} = 5 \times 10^{-3}, t_{2.3} = 5 \) and the remaining data is as given in Fig. (2.24). In this illustration, it is observed that for all the three criteria, the three-layer laminated plate gives a lower transverse displacement response during the shock pulse duration as well as after it has ceased to act. The results for a higher value of \( \rho_{2.3}(0.2) \) with other parameters as in Figs. 2.13 were also computed and checked. It was found from the time-histories that the dynamic response of the three-layer plate was less than that of the corresponding homogeneous plate for all the three different criteria. In this regard, the results are similar to those obtained in case of \( \rho_{2.3} = 0.05 \) (Figs. 2.13). Further, as expected for an undamped system, the time-histories of the laminated and homogeneous plates show no decay of response during the residual vibration era.

Keeping in view the wide range of parameters involved, a detailed investigation of the displacement response effectiveness of the three-layer undamped plate is further carried out in Figs. 2.14 to 2.16 plotted on constant size, constant weight and constant static stiffness criteria respectively. The curves of \( \omega_{cp} \) versus core thickness ratio \( t_{2.3} \) of the laminated plate for different values of modulus ratio \( \bar{G} \) are drawn in Figs. 2.14c, 2.15c and 2.16c. For the purpose of direct comparison, the peak transverse displacement response of the homogeneous plate is also shown (dotted horizontal line) in these figures. It is seen that for all the three-different comparison criteria, a laminated plate with a relatively thick and rigid core gives a better
FIG. 2.14 VARIATION OF WEIGHT RATIO, STATIC STIFFNESS RATIO AND $w_{cp}$ WITH $t_{2.3}$ ON CONSTANT SIZE CRITERION
FIG. 2.15 VARIATION OF SIZE RATIO, STATIC STIFFNESS RATIO AND \( w_{cp} \) WITH \( t_{2,3} \) ON CONSTANT WEIGHT CRITERION
FIG. 2.16 VARIATION OF SIZE RATIO, WEIGHT RATIO AND $w_{cp}$ WITH $t_{2.3}$ ON CONSTANT STATIC STIFFNESS CRITERION
displacement response as compared to that of a homogeneous plate. However, in case of constant size criterion (Fig. 2.14c) an increase in core thickness ratio gives better displacement response effectiveness only up to a certain value of $t_{2.3}$ after which even the sandwich plates with higher values of $\theta$ tend to give $w_{cp}$ higher than that given by a corresponding homogeneous plate. When the size ratio is unity, an increase in $t_{3.2}$ decreases weight ratio (Fig. 2.14a), but at the same time it also reduces the static stiffness of the sandwich (Fig. 2.14b).

On constant weight basis, the size ratio and stiffness ratio increase with increase in $t_{2.3}$ as observed from Figs. 2.15a,b. For a core thickness ratio of 5, 10 and 20, the size ratio is found to be 3.11, 4.8 and 7.33 (not shown in figure) respectively. Such a high increase in size ratio may inhibit the use of higher values of core thickness ratio under this criterion.

For constant static stiffness criterion (Figs. 2.16a,b), the size ratio increases while the weight ratio decreases as a result of increase in $t_{2.3}$. The increase in size ratio may again limit the value of core thickness ratio to be used on this basis.

Taking a higher value of core density ratio ($\rho_{2.3} = 0.2$) and keeping the other parameters as per Eq. (2.24), figures for the three criteria were drawn (figures not included for want of space) and it was found that the results were similar in
character to those of Figs. 2.14 to 2.16. It is therefore concluded that when a particular ratio is kept as unity, an increase in $t_{2.3}$ brings about a variation in the remaining two ratios which may not be desirable. Thus, for achieving a good displacement response effectiveness for a three-layer undamped laminated plate vis-à-vis a corresponding homogeneous plate, the choice of design criteria, geometrical and physical parameters and the core thickness ratio would have to be optimised.