Chapter 7

Bucket Capacity and Force Calculations

In this chapter calculations of the backhoe excavator bucket capacity, bucket crowd force or breakout force, arm curl or digging force according to standards of SAE (Society of Automotive Engineers) are presented. In addition a generalized breakout and digging force model from the proposed backhoe mathematical kinematic model (chapter 3) is presented and can be used as a boundary condition (time varying or dynamic) to carry out the dynamic finite element analysis of the proposed backhoe excavator. Moreover; static force analysis, considering the maximum breakout force condition, is done for the different parts of the backhoe excavator as a boundary condition for static FEA (chapter 8), the static model presented in chapter 4 is according to resistive force condition which is lesser than breakout force, so breakout force has been taken as the boundary condition for static FEA. In this chapter: The bucket capacity calculations are carried out in section 7.1; digging forces according to SAE standard is presented in section 7.2; generalized breakout and digging force model is presented in section 7.3; static force analysis for static FEA is presented in section 7.4 and the section 7.5 presents the comparison of proposed backhoe excavator attachment with the other standard models available in the market for all technical specifications.

7.1 Bucket capacity calculations

Bucket capacity is a measure of the maximum volume of the material that can be accommodated inside the bucket of the backhoe excavator. Bucket capacity can be either measured in struck capacity or heaped capacity as described below:

![Fig. 7.1 Bucket struck and heaped capacities](image)

(a) Struck Capacity

(b) Heaped Capacity
Struck capacity is defined as: The volume capacity of the bucket after it has been struck at the strike plane. The strike plane passes through the top back edge of the bucket and the cutting edge as shown in Fig. 7.1 (a). This struck capacity can directly be measured from the 3D model of the backhoe bucket excavator.

On the other hand the calculation of the heaped capacity is done by following the standards. Globally two standards used to determine the heaped capacity, are: (i) SAE J296: “Mini excavator and backhoe bucket volumetric rating”, an American standard (Mehta Gaurav K., 2006), (Komatsu, 2006) (ii) CECE (Committee of European Construction Equipment) a European standard (Mehta Gaurav K., 2006), (Komatsu, 2006).

Heaped capacity is defined as: The sum of the struck capacity plus the volume of excess material heaped on the bucket at a 1:1 angle of repose (according to SAE) or at a 1:2 angle of repose (according to CECE), as shown in the Fig. 7.1 (b). This in no way implies that the hoe must carry the bucket oriented in this attitude, or that all material will naturally have a 1:1 or 1:2 angle of repose.

As can be seen from the Fig. 7.1 the heaped capacity \( V_h \) can be given as:

\[
V_h = V_s + V_e \quad \ldots (7.1)
\]

Where, \( V_s \) is the struck capacity, and \( V_e \) is the excess material capacity heaped either at 1:1 or at 1:2 angle of repose as shown in Fig. 7.1 (b).

Firstly, from Fig. 7.2 struck capacity \( V_s \) equation will be presented, then by using two methodologies SAE and CECE, two equations of excess material volume or capacity \( V_e \) will be presented from Fig. 7.2. Finally bucket heaped capacity can be found from equation (7.1).

- Struck capacity \( V_s \) can be given from Fig. 7.2 as:
  \[
  V_s = \text{p} \cdot \text{Area} \left( \frac{W_r + W_c}{2} \right) \quad \ldots (7.2)
  \]

- Excess material capacity \( V_e \) for angle of repose 1:1 according to SAE J296 standard as shown in Fig. 7.2 (a).
  \[
  V_e = \left( \frac{L_B W_f^2}{4} - \frac{W_f^3}{12} \right) \quad \ldots (7.3)
  \]

- Excess material capacity \( V_e \) for angle of repose 1:2 according to CECE section VI as shown in Fig. 7.2 (b).
  \[
  V_e = \left( \frac{L_B W_f^2}{8} - \frac{W_f^3}{24} \right) \quad \ldots (7.4)
  \]
The description of the terms used in Fig. 7.2 is as follows:

- \( L_B \): Bucket opening, measured from cutting edge to end of bucket base rear plate.
- \( W_c \): Cutting width, measured over the teeth or side cutters (note that the 3D model of bucket proposed in this thesis is only for light duty construction work, so side cutters are not attached in our model).
- \( W_B \): Bucket width, measured over sides of bucket at the lower lip without teeth of side cutters attached (so this will also not to be the important
parameter for the proposed 3D model of bucket as it does not contain any side cutters).

- \( W_f \): Inside width front, measured at cutting edge or side protectors.
- \( W_r \): Inside width rear, measured at narrowest part in the back of the bucket.
- \( P_{\text{Area}} \): Side profile area of bucket, bounded by the inside contour and the strike plane of the bucket.

Fig. 7.3 shows the important parameters to calculate the bucket capacity for the proposed 3D model of bucket. The calculation done is based on SAE standard as this standard is globally acceptable and used.

![Fig. 7.3 Parameters of the proposed 3D bucket model to calculate the bucket capacity](image)

As can be seen from the left side of the Fig. 7.3 \( P_{\text{Area}} \) is the area bounded by struck plane (blue line) and side protector (red curve), and it is 66836 mm\(^2\).

By using equations (7.1), (7.2) and (7.3) the bucket capacity for the proposed 3D backhoe bucket model comes out to be 0.02781 m\(^3\) = 0.028 m\(^3\).

### 7.2 Backhoe digging forces according to standard SAE J1179

Bucket penetration into a material is achieved by the bucket curling force (\( F_B \)) and arm crowd force (\( F_S \)). The rating of these digging forces is set by SAE J1179 standard “Surface Vehicle Standards - Hydraulic Excavator and Backhoe Digging Forces” (SAE International, 1990). These rated digging forces are the forces that can be exerted at the outermost cutting point (that is the tip of the bucket teeth). These forces can be calculated by applying working relief hydraulic pressure to the cylinders providing the digging force.
Fig. 7.4 Determination of digging forces by following the standard SAE J1179

Fig. 7.4 shows the measurement of bucket curling force $F_B$, arm crowd force $F_S$, the other terms in the figure $d_A$, $d_B$, $d_C$, $d_D$, $d_D^1$, $d_E$, and $d_F$ shows the distances as shown in Fig. 7.4.

According to SAE J1179: Maximum radial tooth force due to bucket cylinder (bucket curling force) $F_B$ is the digging force generated by the bucket cylinder and tangent to the arc of radius $d_D^1$. The bucket shall be positioned to obtain maximum output moment from the bucket cylinder and connecting linkages. $F_B$ becomes maximum when distance $d_A$ reaches maximum, because rest of the distances in the equation (7.5) are constant.

$$F_B = \frac{\text{Bucket cylinder force} \left( \frac{d_A \times d_C}{d_B} \right)}{d_D}$$  

.... (7.5)

Where,

Bucket cylinder force = (Working pressure) $\times$ (End area of bucket cylinder)

If the end diameter of the bucket cylinder = $D_B$ (mm) and the working pressure is $p$ (MPa) and other distances are in mm then the equation (7.5) can be written as:

$$F_B = \frac{p \times \left( \frac{\pi}{4} \right) D_B^2 \left( \frac{d_A \times d_C}{d_B} \right)}{d_D}$$  

.... (7.6)
Equation (7.6) determines the value of the bucket curl or breakout force in N. Now let us determine the maximum radial tooth force due to arm cylinder $F_S$. Maximum tooth force due to arm cylinder is the digging force generated by the arm cylinder and tangent to arc of radius $d_F$. The arm shall be positioned to obtain the maximum output moment from the arm cylinder and the bucket positioned as described in the case of maximum bucket curl force (Max. bucket tangential force). While calculating maximum force $F_S$ occurs, when the axis in the arm cylinder working direction is at a right angle to the line connecting the arm cylinder pin and the boom nose pin as shown in Fig. 7.4.

$$F_S = \frac{p \times (\pi / 4) D_A^2 \times d_E}{d_F}$$

….. (7.7)

Where, $d_F = \text{bucket tip radius (d_D)} + \text{arm link length}$ and $D_A = \text{end diameter of the arm cylinder}$.

When the assembly of proposed model is placed in the position as shown in Fig. 7.4 it holds the values of the parameters as: $d_A = 257 \text{ mm}$, $d_B = 220 \text{ mm}$, $d_C = 181 \text{ mm}$, $d_D = 547 \text{ mm}$, $d_E = 285 \text{ mm}$, and $d_F = (547 + 723) = 1270 \text{ mm}$. The working pressure $p = 157 \text{ bar or 15.7 MPa}$, $D_A = D_B = 40 \text{ mm}$. So by using equations (7.6) and (7.7) the bucket curl or breakout force $F_B = 7626.25 \text{ N} = 7.626 \text{ KN}$, and arm crowd force or digging force $F_S = 4427.419 \text{ N} = 4.427 \text{ KN}$.

The combination of the backhoe excavator’s arm crowd force $F_S$ and bucket curling force $F_B$ give this machine configuration more effective bucket penetration force per mm of bucket cutting edge than is available with other machine types such as wheel and track loaders.

As a result of high penetration force, a backhoe excavator bucket is comparatively easy to load. Also, the higher unit breakout forces allow the backhoe excavator’s economic application range to be extended further into the tougher soils (coral, shale, limestone) before blasting or ripping is required.

7.3 Generalized breakout and digging force model

The SAE J1179 standard provides the bucket curling force $F_B$, and arm crowd force $F_S$, only for the position of the maximum breakout force condition as stated in standards, which is helpful for static analysis, but for autonomous application it is important to know the digging forces generated during the entire digging operation. Hydraulic cylinders

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apply force to boom, arm and the bucket to actuate the mechanism. Depending on the mechanism position, working pressure and the amount of excavation force changes. Therefore, for autonomous application it is necessary to know the breakout and digging forces for all position of the backhoe mechanism. This can be achieved by developing the generalized breakout and digging force model. In practice, boom cylinders are used for adjusting the bucket position not for digging. Arm cylinder and bucket cylinder are used for excavating. Thus, calculation of digging force must be carried out separately when arm cylinder or bucket cylinder is the active cylinder.

7.3.1 Force calculation when the arm cylinder is active

Force created by the arm cylinder $A_7A_8$ (length of the arm cylinder) is $F_{A_7A_8}$ can be found by using its end cylinder diameter and working pressure as described in the previous section.

$$F_{A_7A_8} = p \times \left(\frac{\pi}{4}\right)D_A^2 \quad \ldots (7.8)$$

![Fig. 7.5 Backhoe geometrical parameter assignment](image)

As can be seen from the Fig. 7.5 the digging force from the arm cylinder $F_{Arm}$ (acting on the teeth of the bucket in the tangential direction of $A_2A_4$ radius) will be the moment created by the arm cylinder $M_{Arm}$ divided by the distance $A_2A_4$. This leads to;
\[ F_{\text{Arm}} = \frac{M_{\text{Arm}}}{A_2A_4} \]  \hspace{1cm} (7.9)

This is because while excavating, firstly the X_3- and the X_4- axes are made collinear (i.e. A_2, A_3, and A_4 points are collinear points), then the bucket is made to contact with the ground and curled inwards. While arm cylinder is only active the bucket is made curling inward from the point A_2 to point A_4.

Now, moment created on arm M_{\text{Arm}} will be the product of the force created by the arm cylinder F_{A_7A_8} and the perpendicular distance to the cylinder, so M_{\text{Arm}} can be given by:

\[ M_{\text{Arm}} = (A_2A_8) \sin(\angle A_7A_8A_2) F_{A_7A_8} \]  \hspace{1cm} (7.10)

Now in equation (7.10), the distance A_2A_8 is fixed from the geometry of the backhoe excavator (Fig. 7.5), and the arm cylinder force F_{A_7A_8} can be determined from equation (7.8), now the only unknown remained is the \( \angle A_7A_8A_2 = \Gamma_1 \). From the cosine rule applied to the triangle \( \Delta A_7A_8A_2 \) the angle \( \Gamma_1 \) can be given by:

\[ \Gamma_1 = \tan^{-1} \left[ \frac{4(A_7A_8)^2(A_2A_8)^2 - [(A_7A_8)^2 + (A_2A_8)^2 - (A_2A_7)^2]^{1/2}}{(A_7A_8)^2 + (A_2A_8)^2 - (A_2A_7)^2} \right] \]  \hspace{1cm} (7.11)

The length of the piston rod in the arm cylinder A_7A_8 can be determined either from the sensors (in case of autonomous backhoe operations) or from the joint angle \( \theta_3 \) from chapter 3, equation (3.22) as:

\[ (A_7A_8)^2 = (A_2A_7)^2 + (A_2A_8)^2 - 2(A_2A_7)(A_2A_8)\cos(3\pi - \delta_1 - \delta_2 - \theta_3) \]  \hspace{1cm} (7.12)

Where, \( \angle A_1A_2A_7 = \delta_1 \), and \( \angle A_8A_2A_3 = \delta_2 \) are constant for the geometry of boom and arm respectively (Fig. 7.5).

By using equations (7.11) and (7.12), the moment M_{\text{Arm}} can be determined from equation (7.10).

Now, let us determine the length A_2A_4. If the cosine rule is applied to the \( \Delta A_2A_3A_4 \) one yields:

\[ (A_2A_4)^2 = (A_2A_3)^2 + (A_3A_4)^2 - 2(A_2A_3)(A_3A_4)\cos(\theta_4 - \pi) \]  \hspace{1cm} (7.13)

In equation (7.13), all terms are known except the joint 4 angle \( \theta_4 \). This can be determined from chapter 3 of equation (3.24) if the length of the piston rod in the bucket cylinder A_9A_{10} is known;
\[
\zeta_1 = 2\pi - \epsilon_1 - \tan^{-1}\left\{\frac{4(A_9A_{12})^2(A_{10}A_{12})^2 - [(A_9A_{12})^2 + (A_{10}A_{12})^2 - (A_9A_{10})^2]^2}{[(A_9A_{12})^2 + (A_{10}A_{12})^2 - (A_9A_{10})^2]^2}\right\}^{1/2}
\]

(7.14)

Where, \(\epsilon_1\) = the major \(\angle A_9A_{12}A_3\) and it is constant and thus known for us. From equation (7.14) \(\zeta_1\) can be determined. Now by putting this value of \(\zeta_1\) into the following equation will give the value of \(\zeta_2\) (rest of the terms are known).

\[
(A_3A_{12})^2 + (A_{10}A_{12})^2 - 2(A_3A_{12})(A_{10}A_{12})\cos(\zeta_1) = (A_3A_{11})^2 + (A_{10}A_{11})^2 - 2(A_3A_{11})(A_{10}A_{11})\cos(\zeta_2)
\]

(7.15)

By putting these two values of \(\zeta_1\) and \(\zeta_2\), and \(\angle A_{12}A_{13}\angle A_{11} = \eta_1\) (fixed from the geometry), and \(\angle A_{14}A_{13}A_{11} = \eta_2\) (fixed from the geometry) into the following equation will determine the joint 4 angle \(\theta_4\).

\[
\theta_4 = \zeta_1 + \zeta_2 + \pi - \eta_1 - \eta_2 + \zeta_3
\]

(7.16)

So by using equations (7.14), (7.15), and (7.16) the distance \(A_2A_4\) can be determined from equation (7.13). And by using equation (7.13) and (7.10), the digging force when the arm cylinder is active \(F_{\text{Arm}}\) can be determined by equation (7.9).

### 7.3.2 Force calculation when the bucket cylinder is active

Force created by the bucket cylinder \(A_9A_{10}\) (length of the arm cylinder) is \(F_{A_9A_{10}}\) can be found by using its end cylinder diameter and working pressure as described in the previous section.

\[
F_{A_9A_{10}} = p \times \left(\frac{\pi}{4}\right)D_B^2
\]

(7.17)

As can be seen from the Fig. 7.5 the breakout force from the bucket cylinder \(F_{\text{Bucket}}\) (acting on the teeth of the bucket in the tangential direction of \(A_3A_4\) radius) will be the moment created by the bucket cylinder \(M_{\text{Bucket}}\) divided by the distance \(A_3A_4\). This leads to;

\[
F_{\text{Bucket}} = \frac{M_{\text{Bucket}}}{A_3A_4}
\]

(7.18)

In equation (7.18) the length \(A_3A_4\) is fixed from the geometry of the bucket and thus known to us.

Here, only the bucket cylinder is active and the bucket is made curling inward from the point \(A_3\) to point \(A_4\) for the excavation operation to be carried out by bucket cylinder.
Now, moment created on bucket $M_{\text{Bucket}}$ will be the product of the force created by the bucket cylinder $F_{A_9A_{10}}$ and the perpendicular distance to the cylinder, so $M_{\text{Bucket}}$ can be given by:

$$M_{\text{Bucket}} = (A_{10}A_{12}) \sin(\angle A_9A_{10}A_{12}) F_{A_9A_{10}} \quad \ldots (7.19)$$

Now in equation (7.19), the distance $A_{10}A_{12}$ is fixed from the geometry of the backhoe excavator (Fig. 7.5), and the bucket cylinder force $F_{A_9A_{10}}$ can be determined from equation (7.17), now the only unknown remained in the equation is the $\angle A_9A_{10}A_{12} = \Gamma_2$. From the cosine rule applied to the triangle $\triangle A_9A_{10}A_{12}$ the angle $\Gamma_2$ can be given by;

$$\Gamma_2 = \tan^{-1} \left[ \frac{4(A_9A_{10})^2(A_{10}A_{12})^2 - [(A_9A_{10})^2 + (A_{10}A_{12})^2 - (A_9A_{12})^2]^2}{(A_9A_{10})^2 + (A_{10}A_{12})^2 - (A_9A_{12})^2} \right]^{1/2} \ldots (7.20)$$

From the equation (7.20), the angle $\Gamma_2$ can be determined either from the sensors (in case of autonomous backhoe operations) or from the joint 4 angle $\theta_4$. If the joint 4 angle $\theta_4$ is known then by following the reverse procedure of the end of the section 7.3.1 from equations (7.16), (7.15) and (7.14) the length of the bucket actuator $A_9A_{10}$ can be determined.

Thus by using the equations (7.19) and (7.20) the breakout force or bucket digging force can be determined in the generalized form from equation (7.18).

In this section both the breakout force of bucket cylinder $F_{\text{Bucket}}$ and the digging force of the arm cylinder $F_{\text{Arm}}$ have been determined in the generalized form. These two forces are the function of the respective joint angles, and these joint angles are the function of time while excavating the earth. So equation (7.9) and (7.18) provides the generalized digging and the breakout forces as a function of time (dynamic), and thus can be used as a boundary condition for the dynamic FEA of the backhoe excavator, but the dynamic FEA of the backhoe excavator is not the part of the research reported in this thesis.

This generalized digging force model has been coded in MATLAB (section D.3 of appendix D) and results of the same are discussed in chapter 10.

### 7.4 Static force analysis

In this section calculation for the static force analysis of the backhoe excavator for the condition in which the mechanism produces the maximum breakout force has been
explained. Unlike the previous section’s flexibility (section 7.3) where the force analysis could be done for any of the position and orientation (collectively known as the configuration) of the mechanism from the available breakout and digging forces, in static analysis one configuration of the mechanism has to be decided first for which the analysis is to be carried out. From all the configurations, the maximum breakout force condition is the most critical one as it produces the highest breakout force, and thus for this condition the force analysis is done, and will be used as a boundary condition for static FEA.

Fig. 7.6 Maximum breakout force configuration

Fig. 7.6 shows the configuration in which the mechanism is producing the maximum breakout force. The free body diagram of bucket, arm, and boom, with directions and magnitudes of the forces are explained in this section.

7.4.1 Bucket static force analysis
Fig. 7.7 shows the free body diagram of the bucket. As can be seen the reaction force on the bucket teeth at point A₄ due to the breakout force 7.626 KN acts at the angle 38.23° for configuration of the maximum breakout force condition.
Static forces on joints can be calculated by considering the summation of forces must be equal to zero ($\Sigma F = 0$) and summation of moments equal to zero ($\Sigma M = 0$) for equilibrium condition of the bucket, arm and boom respectively. All the forces in the Fig. 7.7, Fig. 7.8, Fig. 7.9, and Fig. 7.10 are in Kilo Newton (KN).

Firstly the reaction force acting on the bucket teeth (at point $A_4$) is resolved in the horizontal (X) and the vertical (Y) directions using following equations, as 5.933 KN, and 4.716 KN respectively.

\[ F_{4H} = F_B \cdot \cos(\rho) \]  \hspace{1cm} \text{.... (7.21)}
\[ F_{4V} = F_B \cdot \sin(\rho) \]  \hspace{1cm} \text{.... (7.22)}

Where, $\rho$ is the angle between the breakout force of bucket and the ground level as horizontal reference surface of 38.23° as shown in Fig. 7.7. Now considering the bucket in equilibrium $\Sigma M = 0$, taking moment about the bucket hinge point $A_3$ leads to;

\[ F_4 \cdot l_4 - F_{gb} \cdot l_{gb} = F_{11} \cdot l_{11} \]  \hspace{1cm} \text{.... (7.23)}

Where, $F_4$ is the force acting at bucket tool tip when the bucket approaches to the earth in the maximum breakout force condition as shown in Fig. 7.6 and Fig. 7.7, which is equivalent to the bucket breakout force $F_B$. $l_4$ is the distance of the tool tip of the bucket from the bucket hinge point (547 mm), $l_{gb}$ is the distance between the C.G. of the bucket to the bucket hinge point (220 mm), $l_{11}$ is the distance of the bucket hinge point to the
idler link hinge point on bucket (181 mm), $F_{gb}$ is the gravitational force acting on bucket (0.235 KN) and $F_{11}$ is the force acting on hinge point of the idler link on bucket which can be found by using equation (7.23) and acting at an angle $\beta_{11}$ of 64º as shown in Fig. 7.7. The force $F_{11}$ can be resolved in horizontal (X) and the vertical (Y) directions by using the following equations (7.24) and (7.25).

$$F_{11H} = F_{11} \cdot \cos(\beta_{11}) \quad .... (7.24)$$

$$F_{11V} = F_{11} \cdot \sin(\beta_{11}) \quad .... (7.25)$$

<table>
<thead>
<tr>
<th>Joint of the bucket</th>
<th>Forces (KN)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Horizontal (X) component</td>
</tr>
<tr>
<td>A₁₁</td>
<td>-9.977</td>
</tr>
<tr>
<td>A₄</td>
<td>-5.933</td>
</tr>
<tr>
<td>A₃</td>
<td>15.97</td>
</tr>
</tbody>
</table>

The negative sign shows the force acting in the leftward direction for horizontal component of the force and downward direction for vertical component of the force. The forces on each of the joints of the bucket are shown in table 7.1.

### 7.4.2 Arm static force analysis

In Fig. 7.8(a) shows the important dimensions and angles for the moments and the resolution of forces respectively. Fig. 7.8(b) shows the static forces acting at the different points on the arm. The Force ($F_{12}$) is the force acting on the intermediate link (A₁₀A₁₂) from the idler link (A₁₁A₁₀) at an angle $\beta_{10}$ of 70.5º as shown in Fig. 7.8(a).

$$F_{12} = F_{11} \cdot \cos(\beta_{10}) \quad .... (7.26)$$
Fig. 7.8 Free body diagram of arm (a) Dimensions (b) Resolved forces
The force \( F_9 \) is acting on arm through the bucket cylinder, at an angle \( \beta_{10a} \) of 10.13° as shown in Fig 7.8(a).

\[
F_9 = F_{11} \cdot \cos(\beta_{10a})
\]  
\( \text{.... (7.27)} \)

The force \( F_{12} \) can be resolved in horizontal (X) and the vertical (Y) directions by using the equations (7.28) and (7.29). Here, \( \beta_{12} \) is the angle made by intermediate link with horizontal reference of 46.50° as shown in Fig. 7.8(a).

\[
F_{12H} = F_{12} \cdot \cos(\beta_{12})
\]  
\( \text{.... (7.28)} \)

\[
F_{12V} = F_{12} \cdot \cos(\beta_{12})
\]  
\( \text{.... (7.29)} \)

The force \( F_9 \) can be resolved in horizontal (X) and the vertical (Y) directions by using the following equations (7.30) and (7.31). Here, \( \beta_9 \) is the angle made by force on arm through bucket cylinder with horizontal reference of 53.70° as shown in Fig. 7.8(a).

\[
F_{9H} = F_9 \cdot \cos(\beta_9)
\]  
\( \text{.... (7.30)} \)

\[
F_{9V} = F_9 \cdot \cos(\beta_9)
\]  
\( \text{.... (7.31)} \)

Considering the arm in equilibrium \( \Sigma M = 0 \) and taking moment about the arm to boom hinge point \( (A_2) \) leads to;

\[
F_8 \cdot l_8 = (F_{3V} \cdot l_{3H}) + (F_{ga} \cdot l_{ga}) - (F_{3H} \cdot l_{3V}) - (F_{12} \cdot l_{12}) - (F_9 \cdot l_9) \quad \text{.... (7.32)}
\]

Where, \( F_8 \) is the force acting at arm cylinder front end hinge point \( (A_8) \) which can be determined using the equation (7.32). Here, \( l_8 \) is the distance between the arm hinge point \( (A_2) \) and arm cylinder front end hinge point \( (A_8) \) in maximum breakout force condition of 285 mm as shown in Fig. 7.6, \( F_{3V} \) is the vertical force component acts on bucket hinge point \( (A_3) \) of 15.74 KN as shown in Fig. 7.8(b), \( l_{3H} \) is the horizontal distance between the bucket hinge point \( (A_3) \) and arm hinge point \( (A_2) \) of 466 mm as shown in Fig. 7.8(a), \( F_{ga} \) is the gravitational force on arm of 0.289 as shown in Fig. 7.8(b), \( l_{ga} \) is the distance between the C.G. of arm and arm hinge point \( (A_2) \) of 194 mm as shown in Fig. 7.8(a), \( F_{3H} \) is the horizontal force component acts on bucket hinge point \( (A_3) \) of 15.97 KN as shown in Fig. 7.8(b), \( l_{3V} \) is the vertical distance between the bucket hinge point \( (A_3) \) and arm hinge point \( (A_2) \) of 551 mm as shown in Fig. 7.8(a), \( F_{12} \) is the force acting on intermediate link due to idler link of 7.784 KN as shown in Fig. 7.8(b), \( l_{12} \) is the distance between arm hinge point \( (A_2) \) and intermediate link hinge point on arm \( (A_{12}) \) of 591 mm as shown in Fig. 7.6, \( F_9 \) is the force acting on arm through bucket cylinder of 22.405 KN as shown in Fig. 7.8(b), and \( l_9 \) is the distance between arm hinge point \( (A_2) \) and the bucket cylinder end hinge point \( (A_9) \) of 294 mm as shown in Fig. 7.6.
Considering \( \Sigma F = 0 \), force on the arm to boom hinge point \( A_2 \) can be found out as shown in Fig. 7.8(b). The forces on each of the joints of the arm are shown in table 7.2. The minus sign in table 7.2 shows the direction of the forces.

### Table 7.2 Static forces on the arm joints

| Joint of the bucket | Forces (KN) | | |
|---------------------|-------------|-------------------|
|                     | Horizontal (X) component | Vertical (Y) component |
| \( A_3 \)          | -15.97      | -15.74            |
| \( A_{12} \)       | -5.358      | 5.646             |
| \( A_9 \)          | 13.264      | 18.057            |
| \( A_8 \)          | -44.196     | 0                 |
| \( A_2 \)          | 52.26       | -7.949            |

### 7.4.3 Boom static force analysis

Fig. 7.9 shows the free body diagram of the boom, in which Fig. 7.9 (a) shows the important dimensions and angles for the moments and the resolution of forces respectively. The Fig. 7.9 (b) shows the static forces acting at the different points on the boom. The force \( F_7 \) is the force acts by arm at point \( A_7 \) through arm cylinder which is same as the force \( F_8 \) but direction is opposite.

The force \( F_7 \) can be resolved in horizontal (X) and the vertical (Y) directions by using the following equations (7.33) and (7.34). Here, \( \beta_7 \) is the angle made by force on boom through arm cylinder with horizontal reference at point \( A_7 \) of 0° as shown in Fig. 7.9(b).

\[
F_{7H} = F_7 \cdot \cos(\beta_7) \quad \text{.... (7.33)}
\]

\[
F_{7V} = F_7 \cdot \cos(\beta_7) \quad \text{.... (7.34)}
\]
Considering the boom in equilibrium $\Sigma M = 0$ and taking moment about the arm to boom hinge point ($A_1$) leads to:

$$F_5 \cdot l_5 = (F_{2H} \cdot l_{2V}) + (F_{gbo} \cdot l_{gbo}) - (F_{2V} \cdot l_{2H}) - (F_7 \cdot l_7) \hspace{1cm} \ldots (7.35)$$

Where, force $F_5$ is acting at point $A_5$ through boom cylinder which is acting at angle $\beta_5$ at point $A_5$ of 45.58° as shown in Fig. 7.9(b). $l_5$ is the distance between boom hinge
point and boom cylinder end hinge point on swing link of 218 mm as shown in Fig. 7.6. 

\( F_{2H} \) and \( F_{2V} \) are the horizontal and vertical components of the force acting at point \( A_2 \) of 52.26 KN and 7.963 KN respectively as shown in Fig. 7.9(a). \( l_{2H} \) and \( l_{2V} \) are the horizontal and vertical distances of point \( A_2 \) from boom hinge point \( A_1 \) of 1301 mm and 348 mm respectively as shown in Fig. 7.9(a). \( F_{gbo} \) is the gravitational force acts on boom of 0.432 KN as shown in Fig. 7.9(b), and \( l_{gbo} \) is the horizontal distance between C.G. of boom and boom hinge point \( A_1 \) of 524 mm as shown in Fig. 7.9(a). \( l_7 \) is the vertical distance between arm cylinder end hinge point \( A_7 \) and boom hinge point \( A_1 \) of 633 mm as shown in Fig. 7.6. The force \( F_5 \) can be resolved in horizontal (X) and the vertical (Y) directions by using the following equations (7.36) and (7.37).

\[
F_{5H} = F_5 \cdot \cos(\beta_5) \quad \ldots (7.36)
\]

\[
F_{5V} = F_5 \cdot \cos(\beta_5) \quad \ldots (7.37)
\]

Considering \( \Sigma F = 0 \), force on the bucket hinge point \( A_1 \) can be found out as shown in Fig. 7.9(b). The forces on each of the joints of the boom are shown in table 7.3. The minus sign in table 7.3 shows the direction of the forces.

<table>
<thead>
<tr>
<th>Joint of the boom</th>
<th>Forces (KN)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Horizontal (X) component</td>
</tr>
<tr>
<td>( A_2 )</td>
<td>-52.26</td>
</tr>
<tr>
<td>( A_7 )</td>
<td>44.196</td>
</tr>
<tr>
<td>( A_5 )</td>
<td>-69.032</td>
</tr>
<tr>
<td>( A_1 )</td>
<td>-77.033</td>
</tr>
</tbody>
</table>

### Table 7.3 Static forces on the boom joints

#### 7.4.4 Swing link static force analysis

Fig. 7.10 shows the free body diagram of the swing link, it shows the resolved forces in horizontal and vertical directions at each joint of the swing link. The force \( F_6 \) is acting at point \( A_6 \) of boom cylinder end hinge point through the boom cylinder which is equal to the force \( F_5 \) but opposite in direction. \( F_{01} \) and \( F_{02} \) are the forces acts on swing cylinder front end hinge points of \( A_{01} \) and \( A_{02} \) respectively through swing cylinders 1 and 2 of 30.827 KN. These forces can be finding out by using the equation (3.38).
\[ F_{01} = F_{02} = p \cdot \left( \frac{\pi}{4} \right) D_S^2 \] .... (7.38)

Where, \( D_S \) is the swing cylinder end diameter of 50 mm. and \( p \) is the working pressure of the hydraulic circuit of 15.7 MPa. The forces on each of the joints of the swing link are shown in table 7.4. The minus sign in table 7.4 shows the direction of the forces.

**Table 7.4 Static forces on the swing link joints**

<table>
<thead>
<tr>
<th>Joint of the swing link</th>
<th>Forces (KN)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Horizontal (X) component</td>
<td>Vertical (Y) component</td>
</tr>
<tr>
<td>( A_1 )</td>
<td>77.033</td>
<td>-78.407</td>
</tr>
<tr>
<td>( A_6 )</td>
<td>69.032</td>
<td>-70.444</td>
</tr>
<tr>
<td>( A_{01} )</td>
<td>-30.827</td>
<td>0</td>
</tr>
<tr>
<td>( A_{02} )</td>
<td>-30.827</td>
<td>0</td>
</tr>
</tbody>
</table>

**7.5 Comparison of the backhoe excavator models for digging force and bucket capacity**

The standard backhoe models of Komatsu PC 09-1 (Komatsu America Corporation, 2010) and Hitachi Zaxis 8-1 (Hitachi Construction Machinery Co. Ltd., 2010) are smallest model. The comparison of the bucket specifications of the predefined backhoe model, Komatsu PC 09-1, Hitachi Zaxis 8-1 and developed backhoe model are made in table 5.10 of chapter 5. Table 7.5 shows the % reduction in required digging force and
increase in bucket capacity, with the comparison of compact standard backhoe excavator models of Komatsu PC 09-1, and Hitachi Zaxis 8-1 respectively.

| Table 7.5 % reduction in required digging force and increase in bucket capacity |
|---------------------------------|-----------------|-----------------|
| Model                           | Pre-defined     | Komatsu PC 09-1 | Hitachi Zaxis 8-1 |
| Reduction in digging force      | 23.74           | 27.69           | 25.96            |
| Increase in bucket capacity     | 7.14            | 10.71           | 21.43            |

The required digging force is reduced by 27.69 % compared to Komatsu PC 09-1 and 25.96 % compared to Hitachi Zaxis 8-1 model. The bucket capacity also improved by 10.71 % and 21.43 % compared to Komatsu PC 09-1 and Hitachi Zaxis 8-1 models respectively. Reduction in required digging force shows that the power required for operating the link mechanism gets reduced and ultimately the fuel consumption will be reduced.

The breakout force of the proposed model can be increased by increasing the end cylinder diameter of the bucket cylinder and the arm cylinder only. But for the purpose for which the proposed backhoe excavator attachment is designed, this breakout force can be accepted, because as we have seen in chapter 6 that the maximum resistive force offered by the ground for the proposed tool dimensions is 3890.7 N, and the breakout force calculated is 7626 N which is higher than the force required to cut the soil (3890.7 N), thus this calculated breakout force is accepted for the job to be performed (light duty construction work) by the proposed backhoe excavator.

7.6 Pin design

There are three main pins used in the backhoe excavators, which are applied between bucket and arm (pin joint of bucket and arm) called as bucket pin, between arm and boom (pin joint of arm and boom) called as arm pin, and between boom and swing link (pin joint of boom and swing link) called boom pin. During excavation process, the resistive forces offered by the terrain and the digging forces generated by the actuators which must be greater than the resistive forces and are responsible to dig the terrain. In the proposed backhoe model pins are selected based on market survey and reverse engineering
therefore, it is very important to check the pins used in proposed backhoe model for stressed developed during the excavation task. The designed pins are checked for bearing, shear strength and bending strength.

7.6.1 Bucket pin design
The bucket pin is shown in Fig. 7.11 and 3D model of bucket pin is shown in Fig. 5.10 (2) of chapter 5. The soil properties are given in table 6.1 in section 6.5 of chapter 6. There are total two forces acting on the bucket pin ($F_{pb,d}$) during dumping cycle as follows:

1. Soil weight inside of bucket ($F_m$).
2. Bucket self-weight ($F_{sw}$).

Let,

The density of sandy loam soil, $\gamma' = 14912$ N/m$^3$
The bucket heaped capacity, $V_h = 0.028$ m$^3$
Mass of bucket, $m_b = 23.143$ kg

Now the soil weight inside of bucket is,

$$F_m = \gamma' \cdot V_h$$  

\[\text{(7.39)}\]

Therefore, the total force acting on bucket pin during dumping cycle is,

$$F_{pb,d} = F_m + F_{sw}$$  

\[\text{(7.40)}\]

$$F_{pb,d} = 644.56 \text{ N (Downward direction)}$$

But, the forces acting at the bucket hinge joint ($A_3$) are resolved in horizontal and vertical components as explained in section 7.4.1 of bucket static force analysis for maximum breakout force configuration. The resultant force of these forces is acting at the bucket pin which is higher than that of the force acting on bucket pin for dumping cycle. Therefore, the maximum force is considered for checking the design of bucket pin. The bucket pin is checked for bearing, shear and bending failure.
Let us first check the bucket pin for bearing failure. The total length of the bucket pin is 190 mm. But for the calculation, the effective length of the bucket pin (the portion of bucket pin which is in contact with the arm) is considered.

Let,

- Diameter of bucket pin, \( d_b = 30 \) mm
- Shear stress of bucket pin material (IS 2062), \( \tau = 42 \) MPa
- Bearing pressure of bucket pin material (IS 2062), \( p_b = 40 \) MPa
- Total length of the bucket pin, \( l_{pb} = 190 \) mm
- Effective length of the bucket pin, \( l_{pbe} = 173 \) mm
- Horizontal component of force acting at bucket hinge point \( A_3 \), \( F_{3H} = 15970 \) N
- Vertical component of force acting at bucket hinge point \( A_3 \), \( F_{3V} = 15970 \) N
- \( y_{max} = d_b / 2 = 15 \) mm

The resultant force acting on the bucket pin is,

\[
F_{pb} = F_3 = \sqrt{(F_{3H})^2 + (F_{3V})^2}
\]  
\[
F_{pb} = 22422.95 \text{ N}
\]  

Also, the bearing force acting on the bucket pin is,

\[
F_{pb} = d_b \cdot l_{pbe} \cdot p_b
\]  
\[
\therefore p_b = 4.32 \text{ MPa} < [p_b] = 40 \text{ MPa}
\]

Here, the bearing pressure induced in the bucket pint is very less compared to the allowable bearing pressure of bucket pin material IS 2062 as 40 MPa. So, the bucket pin is safe in bearing.

Considering bucket pin is in double shear and the shear area of the bucket pin is,

\[
A_{pb} = 2 \cdot \frac{\pi}{4} \cdot (d_b)^2
\]  
\[
\text{Now, shear force acting on the bucket pin is,}
F_{pbfs} = 2 \cdot \frac{\pi}{4} \cdot (d_b)^2 \cdot \tau_{\text{shear}}
\]

Equating the equations (7.41) and (7.44), we get the shear stresses developed in the bucket pin is,

\[
F_{pb} = F_{pbfs}
\]
\[
F_{pb} = 2 \cdot \frac{\pi}{4} \cdot (d_b)^2 \cdot \tau_{\text{shear}}
\]
\[
\tau_{\text{shear}} = 15.86 \text{ MPa} < [\tau] = 42 \text{ MPa}
\]

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Here, the shear stresses developed in the bucket pin is very less compared to allowable shear stress of bucket pin material IS 2062 as 42 MPa. So, the bucket pin is safe in shear.

Consider bucket pin as simply supported beam with uniformly distributed loading condition as shown in Fig. 7.12 and now let us check bucket pin for bending failure.

Now, the uniformly distributed load acting on bucket pin is,

\[ w_b = \frac{F_{pb}}{l_{pbe}} \quad \ldots (7.45) \]

\[ w_b = 129.61 \text{ N/mm} \]

Maximum bending moment occurs at the bucket pin is,

\[ M_b = \frac{w_b l_{pbe}^2}{8} \quad \ldots (7.46) \]

\[ M_b = 4.85 \times 10^5 \text{ N-mm} \]

Now, the moment of inertia of the bucket pin is,

\[ I_b = \frac{\pi}{64} (d_b)^4 \quad \ldots (7.47) \]

\[ I_b = 3.98 \times 10^4 \text{ mm}^4 \]

The maximum bending stress developed in the bucket pin is,

\[ \frac{\sigma_{\text{max}}}{y_{\text{max}}} = \frac{M_b}{I_b} \quad \ldots (7.48) \]

\[ \sigma_{\text{max}} = \sigma_b = 182.79.02 \text{ MPa} < [\sigma_b] = 205 \text{ MPa} \]

The maximum bending stress developed in the bucket pin is within the stress limit of the bucket pin material, therefore the design of bucket pin is safe in bending.
7.6.2 Arm pin design

The arm pin is shown Fig. 7.13 and 3D model of arm pin is shown in Fig. 5.10 (5) of chapter 5. There are total three forces acting on the arm pin \( F_{pa} \) during dumping cycle as follows:

1. Soil weight inside of bucket \( F_m \).
2. Bucket self-weight \( F_{sw} \).
3. Arm self-weight \( F_{asw} \).

Let,

\[
\text{Mass of arm, } m_a = 30.938 \text{ kg}
\]

Therefore, the total force acting on arm pin is,

\[
F_{pa} = F_m + F_{sw} + F_{asw}
\]

\[
= 948.1 \text{ N (Downward direction)}
\]

![Fig. 7.13 Arm pin](image)

But, the forces acting at the arm hinge joint \( A_2 \) are resolved in horizontal and vertical components as explained in section 7.4.2 of arm static force analysis for maximum breakout force configuration. The resultant force of these forces is acting at the arm pin which is higher than that of the force acting on arm pin for dumping cycle. Therefore, the maximum force is considered for checking the design of arm pin. The arm pin is checked for bearing, shear and bending failure.

Let us first check the arm pin for bearing failure. The total length of the arm pin is 185 mm. But for the calculation, the effective length of the arm pin (the portion of arm pin which is in contact with the arm) is considered.

Let,

- Diameter of arm pin, \( d_a = 50 \text{ mm} \)
- Shear stress of arm pin material (IS 2062), \( [\tau] = 42 \text{ MPa} \)
- Bearing pressure of arm pin material (IS 2062), \( [p_b] = 40 \text{ MPa} \)
- Total length of the arm pin, \( l_{pa} = 185 \text{ mm} \)
Effective length of the arm pin, \( l_{pae} = 169 \text{ mm} \)

Horizontal component of force acting at arm hinge point \( A_2 \), \( F_{2H} = 52260 \text{ N} \)

Vertical component of force acting at arm hinge point \( A_2 \), \( F_{2V} = 79630 \text{ N} \)

\( y_{max} = d_a/2 = 25 \text{ mm} \)

The resultant force acting on the arm pin is,

\[
F_{pa} = F_2 = \sqrt{(F_{2H})^2 + (F_{2V})^2}
\]

\[F_{pa} = 95247.281 \text{ N}\]

Also, the bearing force acting on the arm pin is,

\[
F_{pa} = d_b \cdot l_{pae} \cdot p_b
\]

\[
\therefore p_b = 11.272 \text{ MPa} < [p_b] = 40 \text{ MPa}
\]

Here, the bearing pressure induced in the arm pin is very less compared to the allowable bearing pressure of arm pin material IS 2062 as 40 MPa. So, the arm pin is safe in bearing.

Considering pin in double shear, the shear area of the arm pin is,

\[
A_{pa} = 2 \cdot \frac{\pi}{4} \cdot (d_a)^2
\]

\[A_{pa} = \] (7.52)

Now, shear force acting on the arm pin is,

\[
F_{pqfs} = 2 \cdot \frac{\pi}{4} \cdot (d_a)^2 \cdot \tau_{shear}
\]

\[F_{pqfs} = \] (7.53)

Equating the equations \((7.50)\) and \((7.53)\), we get shear stress developed in the arm pin is,

\[
F_{pa} = F_{pqfs}
\]

\[
F_{pa} = 2 \cdot \frac{\pi}{4} \cdot (d_a)^2 \cdot \tau_{shear}
\]

\[F_{pa} \]

\[
\tau_{shear} = 24.255 \text{ MPa} < [\tau] = 42 \text{ MPa}
\]

Here, the shear stresses developed in the arm pin is very less compared to allowable shear stress of arm pin material IS 2062 as 42 MPa. So, the arm pin is safe in shear.

Consider arm pin as simply supported beam with uniformly distributed loading condition as shown in Fig. 7.12 and check arm pin for bending failure.

Now, the uniformly distributed load on arm pin is,

\[
w_a = \frac{F_{pa}}{l_{pae}}
\]

\[w_a = 563.6 \text{ N/mm}\]

Maximum bending moment occurs at the arm pin is,
\[ M_a = \frac{w_{al_{pa}e}}{8} \]  
\[ M_a = 20.12 \times 10^5 \text{ N} \cdot \text{mm} \]  

The moment of inertia of the arm pin is,

\[ I_a = \frac{\pi}{64} (d_a)^4 \]  
\[ I_a = 3.07 \times 10^5 \text{ mm}^4 \]  

The maximum bending stress developed in the arm pin is,

\[ \frac{\sigma_{\text{max}}}{\nu_{\text{max}}} = \frac{M_a}{I_a} \]  
\[ \sigma_{\text{max}} = \sigma_b = 163.844 \text{ MPa} < [\sigma_b] = 205 \text{ MPa} \]  

The maximum bending stress developed in the arm pin is within the stress limit of the arm pin material, therefore the design of an arm pin is safe in bending (Bansal R. K., 2007).

### 7.6.3 Boom pin design

The boom pin is shown in Fig. 7.14 and 3D model of arm pin is shown in Fig. 5.10 (11) of chapter 5. There are total five forces acting on the boom pin \( F_{\text{phmd}} \) during dumping cycle as follows:

1. Boom force by boom cylinder \( F_{\text{bm}} \),
2. Soil weight inside of bucket \( F_m \),
3. Bucket self-weight \( F_{\text{sw}} \),
4. Arm assembly self-weight \( F_{\text{asw}} \),
5. Boom assembly self-weight \( F_{\text{bsw}} \).

![Fig. 7.14 Boom pin](image-url)
Let,

- Boom cylinder pressure, \( p = 15.7 \) MPa
- Diameter of boom cylinder piston rod, \( d_{rbm} = 30 \) mm
  
- Distance between the centers of boom pin to boom cylinder pin attached on boom cylinder mounting lug of swing link, \( A_1A_6 = 0.218 \) m (Ref. Fig. 7.5 and Fig. 7.6).
- Angle made between the center line of boom cylinder and line joining of boom pin center to boom cylinder pin attached with boom cylinder mounting lug of boom, \( \emptyset = \angle A_1A_5A_6 = 18^\circ \) (for maximum breakout force configuration).
- Distance of boom pin to bucket tip, \( h = 2.21 \) m
- Mass of boom, \( m_{bm} = 51.605 \) kg

So, the force generated by boom cylinder is,

\[
F_{boom} = p \cdot \frac{\pi}{4} \cdot (d_{rbm})^2 
\]  
\[
F_{boom} = 11097.68 \text{ N (Downward direction)}
\]

Now, moment about boom pin is,

\[
M_{bmd} = A_1A_6 \cdot \sin(\emptyset) \cdot F_{boom}
\]  
\[
M_{bmd} = 747.60 \text{ N} \cdot \text{m}
\]

When boom cylinder is in active condition then force exerted at boom pin is,

\[
F_{bmd} = M_{bm} / h 
\]  
\[
F_{bmd} = 338.28 \text{ N}
\]

Now, the total force acting on boom pin during dumping cycle is,

\[
F_{pbmd} = F_{bm} + F_m + F_{sw} + F_{asw} + F_{bsw}
\]  
\[
F_{pbmd} = 1792.6 \text{ N}
\]

But, the forces acting at the boom hinge joint (\( A_1 \)) are resolved in horizontal and vertical components as explained in section 7.4.3 of boom static force analysis for maximum breakout force configuration. The resultant force of these forces is acting at the boom pin which is higher than that of the force acting on boom pin for dumping cycle. Therefore, the maximum force is considered for checking the design of boom pin. The boom pin is checked for bearing, shear and bending failure.

Let us first check the boom pin for bearing failure. The total length of the boom pin is 251 mm. But for the calculation, the effective length of the boom pin (the portion of boom pin which is in contact with the boom) is considered.
Let,

- Diameter of boom pin, \( d_{bm} = 60 \text{ mm} \)
- Shear stress of boom pin material (IS 2062), \( \tau = 42 \text{ MPa} \)
- Bearing pressure of boom pin material (IS 2062), \( p_b = 40 \text{ MPa} \)
- Total length of the boom pin, \( l_{pbm} = 251 \text{ mm} \)
- Effective length of the boom pin, \( l_{pbme} = 236 \text{ mm} \)
- \( y_{max} = d_{bm}/2 = 30 \text{ mm} \)

Horizontal component of force acting at boom hinge point \( A_1 \), \( F_{1H} = 77033 \text{ N} \)

Vertical component of force acting at boom hinge point \( A_1 \), \( F_{1V} = 78407 \text{ N} \)

The resultant force acting on the boom pin is,

\[
F_{pbm} = F_1 = \sqrt{(F_{1H})^2 + (F_{1V})^2} \quad \text{.... (7.62)}
\]

\[
F_{pbm} = 109916.972 \text{ N}
\]

Also, the bearing force acting on the boom pin is,

\[
F_{pbm} = d_{bm} \cdot l_{pbme} \cdot p_b \quad \text{.... (7.63)}
\]

\[
\therefore p_b = 7.8 \text{ MPa} < [p_b] = 40 \text{ MPa}
\]

Here, the bearing pressure induced in the boom pin is very less compared to the allowable bearing pressure of boom pin material IS 2062 as 40 MPa. So, the boom pin is safe in bearing.

Considering pin in double shear, the shear area of the boom pin is,

\[
A_{pbm} = 2 \cdot \frac{\pi}{4} \cdot (d_{bm})^2 \quad \text{.... (7.64)}
\]

Now, shear force acting on the boom pin is,

\[
F_{pbmfs} = 2 \cdot \frac{\pi}{4} \cdot (d_{bm})^2 \cdot \tau_{shear} \quad \text{.... (7.65)}
\]

Now, equating the equations (7.62) and (7.65), we get shear stress developed in the boom pin is,

\[
F_{pbm} = F_{pbmfs}
\]

\[
F_{pbm} = 2 \cdot \frac{\pi}{4} \cdot (d_{bm})^2 \cdot \tau_{shear}
\]

\[
\tau_{shear} = 19.44 \text{ MPa} < [\tau] = 42 \text{ MPa}
\]

Here, the shear stresses developed in the boom pin is very less compared to allowable shear stress of boom pin material IS 2062 as 42 MPa. So, the boom pin is safe
in shear. Consider boom pin as simply supported beam with uniformly distributed loading condition as shown in Fig. 7.12 and check boom pin for bending failure.

Now, the uniformly distributed load on boom pin is,

\[ w_{bm} = \frac{F_{pbm}}{l_{pbme}} \quad \text{.... (7.66)} \]

\[ w_{bm} = 465.75 \text{ N/mm} \]

Maximum bending moment occurs at the boom pin is,

\[ M_{mbm} = \frac{w_{bm} l_{pbme}^2}{8} \quad \text{.... (7.67)} \]

\[ M_{mbm} = 32.43 \times 10^5 \text{ N} \cdot \text{mm} \]

The moment of inertia of the boom pin is,

\[ I_{bm} = \frac{\pi}{64} \cdot (d_{bm})^4 \quad \text{.... (7.68)} \]

\[ I_{bm} = 6.362 \times 10^5 \text{ mm}^4 \]

For maximum bending stress developed in the boom pin is,

\[ \sigma_{max} = \frac{M_{mbm}}{I_{bm}} \quad \text{.... (7.69)} \]

\[ \sigma_{max} = \sigma_b = 152.924 \text{ MPa} < [\sigma_b] = 205 \text{ MPa} \]

The maximum bending stress developed in the boom pin is within the stress limit of the boom pin material, therefore the design of boom pin is safe for bending (Bansal R. K., 2007).

### 7.6.4 Swing link pin design

The swing pin is shown in Fig. 7.15 and 3D model of arm pin is shown in Fig. 5.10 (13) of chapter 5. The swing cylinders are not active during the digging task up to the completion of digging cycle. Also it is considering that, at a time any one swing cylinder is operated to move the backhoe attachment for dumping purpose.

![7.15 Swing link pin](image-url)
During the dumping cycle, the maximum force offered by swing cylinder is faced by swing link pin. The forces acting at the swing link hinge joint \((A_0)\) are resolved as explained in section 7.4.4 of swing link static force analysis for maximum breakout force configuration.

Let us first check the swing link pin for bearing failure. The total length of the boom pin is 175 mm. But for the calculation, the effective length of the swing link pin (the portion of swing link pin which is in contact with the swing link and fixed link) is considered.

There are total six forces acting on the swing link pin \((F_{psl})\) when swing cylinder is active as follows:

1. Soil weight inside of bucket \((F_m)\),
2. Bucket self-weight \((F_{sw})\),
3. Arm assembly self-weight \((F_{asw})\),
4. Boom assembly self-weight \((F_{bsw})\),
5. Swing link self-weight \((F_{slsw})\),
6. Force due to swing cylinder \((F_{sc})\).

Let,

- Diameter of swing link pin, \(d_{sl} = 60\ mm\)
- Shear stress of swing link pin material (IS 2062), \([\tau] = 42\ MPa\)
- Bearing pressure of swing link pin material (IS 2062), \([p_b] = 40\ MPa\)
- Total length of the swing link pin, \(l_{psl} = 175\ mm\)
- Effective length of the swing link pin, \(l_{psle} = 160\ mm\)
- Mass of swing link, \(m_{sl} = 177.41\ kg\)
- \(y_{max} = d_{sl}/2 = 30\ mm\)
- \(F_{sc(\text{offset})} = \) Force developed by the swing cylinder on swing cylinder pin
  \[= 30827\ N\]
- \(L_{V(\text{offset})} = \) Vertical offset distance between line of action of swing cylinder force and center line of swing link in vertical plane = 0.115 m
- \(L_{H(\text{offset})} = \) Horizontal offset distance between line of action of swing cylinder force and center line of swing link in horizontal plane
  \[= 0.11556\ m\]
7.16 Swing link in home position (a) Front View (b) Top view

7.17 Swing link position for maximum bending moment
As shown in Fig. 7.16 the line of action of force produced by the swing cylinder is 115 mm offset from the center line of swing link pin in vertical plane. The Fig. 7.17 shows the offset distance of 115.56 mm between the swing link and the swing cylinder pin for maximum bending moment position in horizontal plane.

Now, the bending moment generated in horizontal plane is,

\[ M_H = L_H(\text{offset}) \times F_{sc}(\text{offset}) \quad \text{.... (7.70)} \]

\[ M_H = 3562.37 \text{ N} \cdot \text{m} \]

Similarly, the bending moment generated in vertical plane is,

\[ M_V = L_V(\text{offset}) \times F_{sc}(\text{offset}) \quad \text{.... (7.71)} \]

\[ M_V = 3545.11 \text{ N} \cdot \text{m} \]

Now, the resultant of the bending moment is,

\[ M = \sqrt{M_H^2 + M_V^2} \quad \text{.... (7.72)} \]

\[ M = 5025.76 \text{ N} \cdot \text{m} \]

The distance of upper swing link pin is higher than the lower swing link pin from the center of swing cylinder pin in vertical plane. Therefore, the force generated due to swing cylinder on lower swing link pin is higher than the force generated on upper swing link pin.

The force developed on the lower swing link due to the resultant moment is,

\[ F_{sc} = \frac{M}{L_V(\text{offset})} \quad \text{.... (7.73)} \]

\[ F_{sc} = 43702.261 \text{ N} \]

Now, the total force acting on the swing link pin is,

\[ F_{psl} = F_m + F_{sw} + F_{asw} + F_{bsw} + F_{dsw} + F_{sc} \quad \text{.... (7.74)} \]

\[ F_{psl} = 46896.97 \text{ N} \]

Now, let us check the swing link pin for bearing. The bearing force acting on the swing link pin is,

\[ F_{psl} = d_{sl} \cdot l_{psl} \cdot p_b \quad \text{.... (7.75)} \]

\[ \therefore p_b = 5.09 \text{ MPa} < [p_b] = 40 \text{ MPa} \]

Here, the bearing pressure induced in the swing link pin is very less compared to the allowable bearing pressure of swing link pin material IS 2062 as 40 MPa. So, the swing link pin is safe in bearing.
Now, check the swing link for shear. Considering pin in double shear, the shear area of the swing link pin,

\[ A_{psl} = 2 \cdot \frac{\pi}{4} \cdot (d_{sl})^2 \]  

\[ \ldots (7.76) \]

Now, shear force acting on the swing link pin,

\[ F_{pslfs} = 2 \cdot \frac{\pi}{4} \cdot (d_{sl})^2 \cdot \tau_{shear} \]  

\[ \ldots (7.77) \]

Equating the equations (7.74) and (7.77), we get shear stress developed in the swing link pin when swing cylinder is active,

\[ F_{psl} = F_{pslfs} \]

\[ F_{psl} = 2 \cdot \frac{\pi}{4} \cdot (d_{sl})^2 \cdot \tau_{shear} \]

\[ \tau_{shear} = 8.29 \text{ MPa} < [\tau] = 42 \text{ MPa} \]

Here, the shear stresses developed in the swing link pin is very less compared to allowable shear stress of swing link pin material IS 2062 as 42 MPa. So, the swing link pin is safe in shear.

Consider swing link pin as simply supported beam with uniformly distributed loading condition as shown in Fig. 7.12 and check swing link pin for bending failure.

Now, the uniformly distributed load acting on swing link pin,

\[ w_{sl} = \frac{F_{psl}}{l_{psle}} \]  

\[ \ldots (7.78) \]

\[ w_{sl} = 293.11 \text{ N/mm} \]

So, the maximum bending moment occurs at the swing link pin,

\[ M_{msl} = \frac{w_{sl}l_{psle}^2}{8} \]  

\[ \ldots (7.79) \]

\[ M_{msl} = 9.38 \times 10^5 \text{ N} \cdot \text{mm} \]

So, using equation (7.69) the moment of inertia of swing link pin,

\[ I_{sl} = \frac{\pi}{64} \cdot (d_{sl})^4 \]  

\[ \ldots (7.80) \]

\[ I_{sl} = 6.362 \times 10^5 \text{ mm}^4 \]

Now, using equation (7.70) the maximum bending stress developed in the swing link,

\[ \sigma_{max} = \frac{M_{msl}}{I_{sl}} \]  

\[ \ldots (7.81) \]

\[ \sigma_{max} = \sigma_b = 44.23 \text{ MPa} < [\sigma_b] = 205 \text{ MPa} \]

The maximum bending stress developed in the swing link pin is within the stress limit of the swing link pin material, therefore the design of swing link pin is safe in bending (Bansal R. K., 2007).
7.7 Summary

The capacity of the bucket has been calculated according to the standard SAE J296 and comes out to be 0.028 m$^3$ as given in section 7.1. This bucket specification is the most superior when compared to all the other standard mini hydraulic excavator models available in the market as can be seen from section 7.5. The breakout force calculation is done by following the standard SAE J1179 and comes out to be 7626 N as given in section 7.2. This bucket specification can also be the most superior if the end cylinder diameter of the bucket cylinder and the arm end cylinder are increased.

A generalized breakout force (when the bucket cylinder is active), and the digging force (when the arm cylinder is active) are presented in section 7.3 as a function of time and can be used as a boundary condition for the dynamic FEA of the backhoe excavator. The developed generalized breakout and digging force models can be utilized for autonomous application of backhoe excavator.

Section 7.4 describes the necessary static force analysis considering the maximum breakout force configuration and will be used as a boundary condition for static FEA of the backhoe parts in the next chapter 8.

Section 7.5 covers the comparison of the backhoe excavator models for digging force and bucket capacity. These results shows overall improvement in the proposed backhoe excavator model by reducing required digging force and by increasing the bucket capacity compared to standard backhoe excavator models.

Section 7.6 includes the design of bucket, arm, boom and swing link pins and checked them for shear and bending strength. The results show that all the pins are safe in bearing, shear and bending.