Chapter 7

DYNAMIC STABILITY OF THIN SHELLS FILLED WITH LIQUID

7.1 Introduction

Thin cylindrical shells are used extensively in several branches of engineering especially in civil, off-shore, nuclear, petrochemical, mechanical, marine and aerospace engineering. These cylindrical shells are utilized as a containment vessels or tanks for the storage of liquids. This is mainly due to the important role played by these shells as efficient load carrying members, particularly axial and lateral loads. Under dynamic loadings like seismic excitations, these thin-walled cylindrical shells experience axial compressive loads and exhibit highly nonlinear behaviour and lose stability there by failing at load levels very much less than the material’s ultimate strength. The inertial coupling between fluid motion (sloshing) and shell wall motions may affect significantly the dynamic behaviour and stability of fluid filled shells. When a thin cylindrical storage tank is subjected to vertical excitations, axisymmetric dynamic liquid loads act on the shell wall resulting in large amplitude vibrations with circumferential wave numbers equal to or larger than one. The frequencies of these vibrations will be half the frequency of the excitation force and this type of vibration can be explained as parametric resonance. Hence, in order to utilize the thin cylindrical shells effectively without failing under dynamic loads, it is important to study the dynamic behaviour of shells under dynamic excitations.

In 1960, Bublik and Merkulov [136] analyzed the dynamic stability of a simply supported cylindrical tank under axial excitations theoretically and showed
that the problem is governed by Hill’s equation. Kana and Craig [137, 138] considered a cantilever cylindrical shell completely filled with liquid and analyzed the stability of the shell theoretically and experimentally. Vijayaraghavan and Evan-Iwanowski [139] investigated the parametric instability of thin cylindrical shells subjected to in-plane longitudinal inertia loading arising from sinusoidal base excitation analytically and experimentally. Shkenev [140] and Pavlovskii and Filin [141, 142] analyzed dynamic stability of an elastic shell filled with ideal liquid theoretically. Tani [143] studied the dynamic stability of truncated conical shells under periodic axial load theoretically. Yamaki and Nagai [144] investigated dynamic stability of cylindrical shells under periodic shearing forces theoretically. Haroun [145] and Veletsos [146] studied the axisymmetric response of a cylindrical tank subjected to vertical excitation theoretically by making few simplified assumptions. Chiba and Tani [147 – 152] studied the dynamic stability of liquid filled cylindrical shells under horizontal and vertical excitations experimentally and theoretically. Chiba and Tani carried out experimental studies with polyester test cylinders and their studies were at high frequency range (200-900 Hz) whereas the normal seismic loadings are of frequency range 1-30 Hz. The studies on investigation of dynamic buckling in the seismic frequency range were carried out by Uras and Liu [153 – 159]. Uras and Liu investigated the dynamic stability of liquid-filled shells with fluid-structure interaction theoretically through Galerkin finite element discretization procedure under the seismic loadings. Kochupillai and Ganesan [160, 161] studied the parametric instability in flexible pipes conveying fluid under time-periodic flow fluctuations of fluid numerically using finite element method. Goncalves and Silva [162 – 164] analyzed dynamic instability of circular
cylindrical shells under static and harmonic axial loadings theoretically using Poincare maps and Lyapunov exponents. A detailed review of linear and non-linear shell vibrations, including fluid-shell interaction, can be found in a book by Amabili [165].

In all the references cited so far, the dynamic stability studies in fluid-filled cylindrical shells were carried out either theoretically or experimentally. Theoretical studies are possible for simple geometries with simple boundary conditions. If the geometry of the shell or the boundary conditions are complex, theoretical solution goes complex and may be even impossible for some situations. It is possible to apply numerical methods like finite element method for such cases. In the present chapter the dynamic stability of fluid-filled cylindrical shells is investigated numerically using finite element method. Both the thin shell and fluid are discretized using finite element method. Hsu’s stability criteria are applied to study the dynamic stability.

7.2 Governing equations

The liquid-shell system under consideration is shown in Figure 7.1; it is a ground-supported circular thin-walled cylindrical shell of radius $R$, height $L$ and thickness $h$, with the wall connected to rigid base. The tank is partly filled with an inviscid, compressible liquid of mass density $\rho_f$ to a height $H$. $E$, $v$, $\rho_s$ are the structures Young’s modulus, poisons ratio and density respectively.

7.2.1 Fluid field equations

The linearized governing equation for the inviscid, compressible, irrotational fluid domain in terms of pressure variable is the wave equation given as follows in the frequency domain
7.2.1.1 Moving wall boundary condition (S)

\[
\frac{1}{\rho_f} \frac{\partial^2 p}{\partial x_i^2} + \frac{1}{\rho_f c^2} \frac{\partial^2 p}{\partial t^2} = 0 \quad \text{in} \; V_f
\]  

(7.1)

where \( V_f \) is the fluid volume, \( c \) is the acoustic wave propagation velocity in fluid, \( p \) is the dynamic pressure field. To the volume equation of fluid various boundary conditions are associated, as shown in Figure 7.1.

7.2.1.2 Free surface in gravitational field (\( \Sigma \))

In the gravitational field \( g \), the dynamic pressure on the free surface is related to the normal displacement \( Z \) of fluid, by the equation

\[
p = \rho_f g z
\]  

(7.3)
\[
\frac{1}{\rho_f} n_i \frac{\partial p}{\partial x_i} - \omega^2 z = -\mathbf{f}_i(t) \cdot n_i \quad (7.4)
\]

Fixed pressure on the free surface

\[
p = p_s \quad (7.5)
\]

### 7.2.2 Structural filed equations

Let \( u \) be the structure displacement, \( \varepsilon(u) \) be the strain and \( \sigma(u) \) be the stress fields of the thin-walled structure. The structure is described using an elastic shell model. The structure is assumed to have an elastic, linear, isotropic behaviour. The kinetic and potential energies of the structure are given by,

\[
T(t) = \frac{1}{2} \int_{V_m} \rho_s \left( \frac{\partial u}{\partial t} \right)^2 \, dv \quad (7.6)
\]

\[
U(t) = \frac{1}{2} \int_{V_m} \sigma(u) \varepsilon(u) \, dv \quad (7.7)
\]

The governing equation for structure in frequency domain is given by,

\[
\rho_s \omega^2 u_i + \frac{\partial \sigma_{ij}(u)}{\partial x_{ij}} + F_i = 0 \text{ in } V_m \quad (7.8)
\]

with boundary condition:

\[
\sigma_{ij}(u) n_j = -pn_j \text{ on } S \quad (7.9)
\]

\( F_i \) corresponds to volume forces of the structure.

### 7.2.3 Fluid-structure coupling

Coupling between the fluid and structure is done by the boundary conditions Eq. (7.2) and Eq. (7.9). Equation (7.2) expresses the continuity of the normal displacement component of the structure. On \( S \), the structure acts on the fluid through an imposed displacement in the normal direction at the fluid boundary.
Equation (7.9) expresses the continuity of the normal component of the stress tensor at the fluid-structure interface. On $S$, the fluid acts on the structure through imposed pressure that creates a structure loading in the normal direction at the structure boundary.

7.3 Numerical treatment of the coupled problem

7.3.1 Variational formulation of the coupled problem

To obtain the numerical approximation of the coupled problem, finite element method is employed. A start of employing finite element method is to use a variational formulation approach. The variational formulation of the structure for any virtual displacement field $\delta u$ satisfying the required boundary conditions is written as:

$$
\int_{V_s} \sigma_{ij}(u) e_{ij}(u) \, dv - \omega^2 \int_{V_s} \rho_s u \, \delta u \, dv - \int_{S} F \delta u \, dv - \int_{S} p n \delta u \, ds = 0 \tag{7.10}
$$

The variational formulation of the fluid for any virtual pressure field $\delta p$ is written as:

$$
\int_{V_f} \frac{1}{\rho_f} \frac{\partial}{\partial x_i} \frac{\partial \delta p}{\partial x_i} \, dv - \omega^2 \int_{V_f} \frac{1}{\rho_f c_s^2} p \delta \, dv - \omega^2 \int_{S_s} u \, n \, \delta p \, ds - \int_{\Sigma} z \delta \, p ds + \int_{S+\Sigma} \bar{\nu}(t) \cdot n \delta p \, ds = 0 \tag{7.11}
$$

The sloshing of the free surface is governed by Eq. (7.3), the corresponding variational formulation with any virtual normal displacement $\delta z$ can be written as:

$$
\int_{\Sigma} \rho_g g z \delta z \, ds - \int_{\Sigma} p \delta z = 0 \tag{7.12}
$$

7.3.2 Finite element discretization

To the above variational formulations equations, setting a suitable shape functions for each variable and spatially discretizing using finite elements gives
mass, stiffness, load and fluid-structure interaction matrices [72, 80] for the structure and fluid as follows:

Structure:

\[
\int_{V_s} \rho u \delta u d\nu \rightarrow \delta U^T M U \tag{7.13}
\]

\[
\int_{V_s} \sigma_{ij} (u) \varepsilon_{ij} (u) d\nu \rightarrow \delta U^T K U \tag{7.14}
\]

\[
\int_{V_s} F_i \delta u d\nu \rightarrow F_i \tag{7.15}
\]

Fluid:

\[
\int_{V_f} \frac{1}{\rho_f c_f^2} p \delta p d\nu \rightarrow \delta p^T M_f p \tag{7.16}
\]

\[
\int_{V_f} \frac{1}{\rho_f} \frac{\partial p}{\partial x_i} \frac{\partial \delta p}{\partial x_i} d\nu \rightarrow \delta p^T K_f p \tag{7.17}
\]

\[
\int_{s+\Sigma} \mathbf{\bar{f}}_i(t) n_i \delta p ds \rightarrow F_f \tag{7.18}
\]

Free surface of fluid:

\[
\int_{z} \rho_f g z \delta z ds \rightarrow \delta z^T K_z z \tag{7.19}
\]

Fluid-structure interaction:

\[
\int_{S} p n \delta u ds \rightarrow \delta U^T R_{hi} p \tag{7.20}
\]

\[
\int_{S} u n \delta p ds \rightarrow \delta p^T R_{hi} U \tag{7.21}
\]

\[
\int_{z} z \delta p ds \rightarrow \delta p^T R_{zi} z \tag{7.22}
\]

\[
\int_{z} p \delta z \rightarrow \delta z^T R_{zi} p \tag{7.23}
\]
In the above equations the suffix \( s \) stand for structure and \( f \) stand for fluid. Finite element discretization leads to the following coupled equations:

\[
-\omega^2 M_s U + K_s U - R_s p = F_s \\
-\omega^2 M_f p + K_f p - \omega^2 R_f U - \omega^2 R_z z = F_f
\]

(7.24)

(7.25)

\[
K_z z - R_z^T p = 0
\]

(7.26)

The above dynamic equations of the coupled liquid-elastic shell system can be combined to obtain complete fluid-structure dynamic interaction matrix equation as follows:

\[
\begin{bmatrix}
K_s & -R_s & 0 \\
0 & K_f & 0 \\
0 & -R_z^T & K_z
\end{bmatrix}
\begin{bmatrix}
U \\
p \\
z
\end{bmatrix}
- \omega^2
\begin{bmatrix}
M_s & 0 & 0 \\
R_f^T & M_f & R_z \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
U \\
p \\
z
\end{bmatrix}
= \begin{bmatrix}
F_s \\
F_f \\
0
\end{bmatrix}
\]

(7.27)

The above matrix eq. (7.27) is non-symmetric and extraction of eigenvalues and eigenvectors become extremely difficult. Although the eigenvalues and eigenvectors can be computed using available non-symmetric algorithms [166], they require lots of computational time. From that point of view, the non-symmetric matrix equations are converted to symmetric form. Several methods have been proposed to convert the non-symmetric coupled problem to symmetric coupled problem. In some formulations [167], the matrices are forced to take symmetrical forms by including some additional unknowns.

A new variable \( \pi \) is introduced, such as:

\[
\pi = -\frac{p}{\omega^2}
\]

(7.28)

Thus the coupled problem is formulated with the unknowns as \((u, p, z, \pi)\), where the structure has displacement, \( u \) as unknown and the fluid has pressure, \( p \), normal free
surface displacement, \( z \) and \( \pi \) as the unknowns. Equation (7.28) can be written as follows, using the fluid stiffness matrix, \( K_f \):

\[
K_f p + \omega^2 K_f \pi = 0
\]  
(7.29)

Combining Eq. (7.29) with Eq. (7.27) gives the following system of coupled equations:

\[
\begin{bmatrix}
K_s & 0 & 0 & 0 & M_s & 0 & -R_f & 0 & U \\
0 & K_f & 0 & 0 & -R_f & -K_f & 0 & p \\
0 & 0 & K_s & 0 & 0 & 0 & -R_f & \pi \\
0 & 0 & 0 & K_s & 0 & 0 & 0 & z
\end{bmatrix}
- \omega^2
\begin{bmatrix}
M_s & 0 & -R_f & 0 & U \\
0 & 0 & -K_f & 0 & p \\
-R_f^T & -K_f & -M_f & R_z & \pi \\
0 & 0 & -R_f^T & 0 & z
\end{bmatrix}
= 
\begin{bmatrix}
F_m \\
0 \\
F_f \\
0
\end{bmatrix}
\]  
(7.30)

The above matrix equation is in symmetric form and could be easily solved. The system of equation is in the general form of a usual structural system equation \( K-\omega^2 M=F \), the usual numerical methods used for dynamic analysis of structural systems can be applied without any change to the coupled fluid-structure system of equations.

From the above coupled fluid-structure equations, when the equation of motion for the structure with fluid-structure interaction is considered the mass matrix gets an additional term called added mass matrix denoted by \( M_{\text{Add}} \). A portion of the liquid vibrates with the tank; this portion called as impulsive mass and is characterized by the added mass. The fluid structure interaction in the frequency domain is described by the following eigenvalue problem:

\[
-\omega^2 \left[ M_s + M_{\text{Add}} \right] U + K_s U = 0
\]  
(7.31)

where

\[
M_{\text{Add}} = R_f M_f^{-1} R_f^T
\]  
(7.32)
The added mass matrix $M_{add}$ is positive definite. Due to the added mass, the eigenfrequencies of the structure with fluid are lower than the eigenfrequencies of the structure without fluid.

**7.4 Governing equation for dynamic stability analysis of liquid-filled shells**

Thus the governing equation for dynamic stability of liquid-filled shells under vertical seismic excitations can be written as

$$M \ddot{U} + KU + K_G(t)U = 0$$  \hspace{1cm} (7.33)

where $M$, $K$, $K_G$ is the mass matrix (including added mass of the fluid), the stiffness matrix and the time dependent geometrical stiffness matrix. $U$ is the generalized displacement vector. The formulations of these matrices are shown in Chapter 6. Eq. (7.33) can be transformed to the following equation using natural vibration mode shapes as per the mode superposition technique

$$\ddot{u} + \left( K^{(0)} + K(t) \right)u = 0$$ \hspace{1cm} (7.34)

By applying the transformation, the total mass matrix is normalized to the identity matrix and stiffness matrix is reduced to a diagonal matrix $K^{(0)}$ of natural frequencies $\omega_{in}$ where $i$ for axial mode number and $n$ stand for circumferential mode number and $u$ is the generalized displacement in transformed coordinates. The time dependent geometric stiffness matrix $K(t)$ varies periodically with time and can be expanded as Fourier series as follows

$$K(t) = \sum_{s=1,2,\ldots}^{S} \left( D^{(s)} \cos(s\omega t) + E^{(s)} \sin(s\omega t) \right)$$ \hspace{1cm} (7.35)

By adding a diagonal damping matrix $C$, whose components are given by

$$c_{in} = 2\xi\omega_{in}$$ \hspace{1cm} (7.36)
where $\xi$ is the damping ratio and the coupling of each mode is assumed to be negligible, the governing equation of motion for dynamic stability analysis takes the form

$$
\ddot{u} + C\dot{u} + K^{(0)}u + \sum_{s=1}^{S} \left( D^{(s)} \cos(st\omega) + E^{(s)} \sin(st\omega) \right) u = 0 \quad (7.37)
$$

In seismic analysis, the response of the fluid-structure system is dominated by only a few modes, with this assumption Eq. (7.37) can be simplified to

$$
\ddot{u} + C\dot{u} + K^{(0)}u + \varepsilon Du \cos\omega t = 0 \quad (7.38)
$$

where $\omega$ and $\varepsilon$ represent a typical dominant frequency and the normalized amplitude of the seismic excitation respectively. The above equation in the component form can be written as follows

$$
\ddot{u}_j + 2\xi\omega_j\omega \dot{u}_j + \omega^2 u_j + \varepsilon \sum_j d_{ij} \cos\omega t = 0 \quad (7.39)
$$

Eq. (7.39) is a set of coupled Mathieu equations. The stability of Eq. (7.39) can be sought from the methods discussed in Chapter 1.

### 7.5 Dynamic stability analysis

For a given dimensions and physical properties of the shell and liquid, Eq. (7.39) shows the solution growing indefinitely with time under certain combinations of $\varepsilon$ and $\omega$. The dynamic stability Eq. (7.39) is obtained using Hsu’s stability criteria. According to Hsu’s results, the instability boundaries are given by the following equations [148]

$$
\frac{\omega}{\omega_j} = 1 \pm \overline{\theta}_j \quad (7.40)
$$

$$
\omega_j = \omega_m + \omega_n, \quad \overline{\theta}_j = \sqrt{\frac{\varepsilon^2 d_{ij} d_{ijn}}{16\omega_m^2 \omega_n^2 - \varepsilon^2}} \quad (7.41)
$$
In the above stability conditions, $\omega_{ij}$ and $\bar{\theta}_{ij}$ correspond to the central frequency and the relative width parameter of the instability region respectively. The instability regions obtained using Eq. (7.40 – 7.41) are the combination resonance instability regions of sum type. In addition, Eq. (7.39) has parametric instability regions when the excitation frequency $\omega$ is almost twice the natural frequency $\omega_{\text{na}}$. The boundaries of this instability region is obtained by putting $i=j$ in Eq. (7.40) and Eq. (7.41).

### 7.6 Numerical results and discussion

The dynamic stability of fluid filled shells is studied for two different cases; a tall tank and a broad tank. The geometrical data for the tall and broad storage tank used in this chapter are given in Table 7.1. Both the tanks are assumed to be filled with water to 75% of height. The dimensions of the tank are same as the dimensions of tanks taken in Chapter 5. Density of water is taken as 1000 kg/m$^3$. Before analyzing the dynamic stability of shells, it is needed to know the natural frequencies of the fluid-structure system. A free-vibration analysis is carried out in the next section.

#### 7.6.1 Free vibration analysis of fluid-structure system

The natural frequencies and mode shapes of the fluid-structure system can be obtained by solving the eigenvalue problem given in Eq. (7.30). The complete analysis of the dynamic stability of fluid-structure system is carried out in CAST3M [168], an object oriented finite element software package. The validity of CAST3M for fluid-structure interaction problems can be referred in [169-171]. The vibrational modes of a circular cylindrical shell filled with fluid can be classified as
Table 7.1: Geometric and material data of the cylindrical shells

<table>
<thead>
<tr>
<th>Shell data</th>
<th>Tall shell</th>
<th>Broad shell</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$ Radius</td>
<td>7.32 m</td>
<td>18.130 m</td>
</tr>
<tr>
<td>$L$ Length</td>
<td>21.96 m</td>
<td>12.20 m</td>
</tr>
<tr>
<td>$t$ Thickness</td>
<td>0.0254 m</td>
<td>0.0254 m</td>
</tr>
<tr>
<td>$E$ Young’s modulus</td>
<td>206.7 GPa</td>
<td>206.7 GPa</td>
</tr>
<tr>
<td>$\nu$ Poisson ratio</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>$\rho$ Shell mass density</td>
<td>$7.84 \times 10^3$ Kg/m$^3$</td>
<td>$7.84 \times 10^3$ Kg/m$^3$</td>
</tr>
</tbody>
</table>

Vertical nodal patterns

Circumferential nodal patterns

Figure 7.2: Fluid filled circular cylindrical shell vibrational modes

the cos $\theta$-type modes for which there is a single cosine wave of deflection in the circumferential direction, and as the cos $n\theta$-type modes for which the deflection of the shell involves a number of circumferential waves higher than 1. These circumferential cos $n\theta$-type modes can be further denoted as beam-type modes because the shell behaves like a vertical cantilever beam across the length. Figure 7.2 shows the vertical nodal patterns and circumferential modes for a circular cylindrical shell filled with fluid. Table 7.2 and Table 7.3 shows the frequencies obtained for tall and broad shell respectively filled with 75% of fluid using
CAST3M. Eleven circumferential modes and respective first four beam bending modes for tall and broad tanks are listed in the tables.

Table 7.2: Natural frequencies of 75% water filled tall tank in Hz

<table>
<thead>
<tr>
<th>N</th>
<th>i = 1</th>
<th></th>
<th>i = 2</th>
<th></th>
<th>i = 3</th>
<th></th>
<th>i = 4</th>
</tr>
</thead>
<tbody>
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<td>0</td>
<td>8.3900</td>
<td></td>
<td>21.7788</td>
<td></td>
<td>30.6000</td>
<td></td>
<td>37.0870</td>
</tr>
<tr>
<td>1</td>
<td>7.5664</td>
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<td>19.0684</td>
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<td>27.2553</td>
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</tr>
<tr>
<td>2</td>
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<td></td>
<td>13.3654</td>
<td></td>
<td>23.3592</td>
<td></td>
<td>30.6960</td>
</tr>
<tr>
<td>3</td>
<td>2.3316</td>
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<td>9.2367</td>
<td></td>
<td>18.3713</td>
<td></td>
<td>26.1492</td>
</tr>
<tr>
<td>4</td>
<td>1.8390</td>
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<td>6.8814</td>
<td></td>
<td>14.3715</td>
<td></td>
<td>22.1290</td>
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<tr>
<td>5</td>
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<td></td>
<td>5.6799</td>
<td></td>
<td>11.7428</td>
<td></td>
<td>18.9999</td>
</tr>
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<td>8</td>
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<td></td>
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<td>9</td>
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<td></td>
<td>11.4886</td>
<td></td>
<td>12.2500</td>
<td></td>
<td>13.2498</td>
</tr>
</tbody>
</table>

Table 7.3: Natural frequencies of 75% water filled broad tank in Hz

<table>
<thead>
<tr>
<th>N</th>
<th>i = 1</th>
<th></th>
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</tr>
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<td>13.6227</td>
<td></td>
<td>18.8837</td>
</tr>
</tbody>
</table>

7.6.2 Dynamic stability of tall and broad fluid filled shells

In the present section dynamic stability analysis is carried out for the fluid-filled shells using Hsu’s conditions as given in Eq. (7.40). Figure 7.2 and Figure 7.3 shows the dynamic stability chart of tall and broad shell respectively. The simple
resonance regions are shown in blue colored lines and combination resonance regions are shown in red colored lines. From the stability charts it can be observed that, if the peak ground acceleration (PGA) of vertical base excitation exceeds approximately 0.2g, the tanks undergo parametric instability and below this excitation the tanks are stable. From the figures we can infer that, the instability regions are more dense and broad for tall tank compared to broad tank. Thus tall tank is more prone to dynamic instability under vertical excitation.

7.7 Summary

The dynamic stability of bottom clamped cylindrical shells filled with fluid under vertical base excitation is obtained using Hsu’s stability criteria. The governing Mathieu-Hill equation is obtained by employing finite element formulation. Two tanks of different aspect ratio are taken and analysis is carried out in CAST3M.

![Figure 7.3: Dynamic stability chart for tall tank](image-url)
Figure 7.4: Dynamic stability chart for broad tank