Chapter 5

DYNAMIC STABILITY OF THIN SHELLS

5.1 Introduction

A shell is defined as a curved surface which develops membrane and bending stresses under external loadings. Thin-shell structures find wide applications in many branches of engineering. Examples include aircraft, spacecraft, cooling towers, nuclear reactors, steel silos and tanks for bulk solid and liquid storage, pressure vessels, pipelines and offshore platforms. Because of the slenderness of these structures, buckling is often the controlling failure mode. It is therefore essential that their stability behavior must be properly understood for safety and reliability. A detailed review on static buckling of thin shells can be found in [85]. The formulation of shell governing equations based on different theories can be found in [86].

If thin walled structures are subjected to pulsating excitations they may fail well before the static bifurcation load leading to dynamic instability, hence a number of studies have focused on this aspect. The dynamic stability of simply supported cylinders under periodic axial and pressure loadings has been treated by Bolotin [14], Yao [87], and Wood and Koval [88], while that of a vertical cylinder with one end clamped and the other end free subjected to sinusoidal base motion, was studied by Vijayaraghavan and Evan-Iwanowski [89]. Nagai and Yamaki [90, 91] studied parametric oscillations in cylindrical shells using Donnell’s shallow shell equations. While the reference cited above was completely based on analytical approaches, Basar [40] and Eller [92] employed finite element method for the stability analysis of shell structures under parametric excitations.

In the present chapter the dynamic instability of thin shells under uniform periodic compressive force is investigated. Shell model is assumed to be linearly elastic, isotropic and homogenous. Finite element method is employed for dynamic stability analysis. Degenerated curved shell elements [97] are used for the formulation of finite element matrices. The next section explains in brief the formulation of degenerated shell elements.

5.2 Formulation of degenerated shell elements

In this section, the underlying basic ideas in the formulation of degenerated curved shell element are described in brief. Figure 5.1 shows a degenerated four-node shell element. The main assumptions made in the formulation of curved shell element degenerated from the 3D solid are,

1. Normals to the middle surface before deformation remain straight even after deformation. Same assumption is valid for thick shells as well.
2. Stress component normal to the shell mid-surface is constrained to be zero and eliminated from the constitutive equations.

Typically a nodal point of a degenerated shell element have three displacements \( u, v, w \) in the global directions \( x, y, z \) and two normal rotations \( \alpha_i \) and \( \beta_i \).

5.2.1 Element geometry

Let \( \xi, \eta \) be the two curvilinear coordinates in the middle plane of the shell and \( \zeta \) a
linear coordinate in the thickness direction. Further $\xi, \eta, \zeta$ vary between -1 and +1 on the respective faces of the element. Global coordinates of an arbitrary point in the element can be written in terms of curvilinear coordinates in the form

$$
\begin{bmatrix}
x \\
y \\
z \\
\end{bmatrix} = \sum N_i(\xi, \eta, \zeta) \begin{bmatrix} x_i \\
y_i \\
z_i \\
\end{bmatrix} + \sum N_i(\xi, \eta) \frac{f_i}{2} V_{3i}
$$

(5.1)

where $V_{3i}$ is a vector along the thickness direction and is given by

$$
V_{3i} = t_i \begin{bmatrix} l_{3i} \\
m_{3i} \\
n_{3i} \\
\end{bmatrix} \quad \text{where} \quad \begin{bmatrix} l_{3i} \\
m_{3i} \\
n_{3i} \\
\end{bmatrix} = \frac{1}{t_i} \begin{bmatrix} x_j - x_k \\
y_j - y_k \\
z_j - z_k \\
\end{bmatrix}
$$

(5.2)

in which $l_{3i}$, $m_{3i}$ and $n_{3i}$ are the direction cosines of the midsurface normal and $t_i$ is the shell thickness at the node $i$.

Vectors $V_{1i}$ and $V_{2i}$ shown in Figure 5.1(b) are perpendicular to each other and to $V_{3i}$. These vectors are used to define directions of nodal rotation degree of freedom $\alpha_i$ and $\beta_i$. Directions of $\alpha_i$ and $\beta_i$ may differ from node to node in a single element and may differ between elements at a node the elements share with each
other. Before elements are assembled, each element matrices must be transformed to suit a global set of degree of freedom at structure nodes.

A vector $V_{1i}$ can be obtained by describing it normal to both $V_{3i}$ and the global $y$ direction by writing the cross product $V_{1i} = j \times V_{3i}$, where $j$ is a unit vector in the $y$ direction. The last vector would be then $V_{2i} = V_{3i} \times V_{1i}$. If $j$ and $V_{3i}$ are parallel to each other, the above calculation fails, in that case use, $V_{2i} = V_{3i} \times i$ and $V_{1i} = V_{2i} \times V_{3i}$. We can define the matrix of direction cosine of vectors $V_{2i}$ and $V_{3i}$ as

\[
\begin{bmatrix}
\mu_i = \begin{bmatrix}
-\frac{V_{2i}}{V_{2i}} & \frac{V_{3i}}{V_{3i}}
\end{bmatrix} = \begin{bmatrix}
-l_{2i} & l_{1i} & 0 \\
-m_{2i} & m_{1i} & 0 \\
n_{2i} & n_{1i} & 0
\end{bmatrix}
\end{bmatrix}
\]

(5.3)

5.2.2 Displacement field

The displacement of an arbitrary point on the midsurface of the shell element can be written as

\[
\begin{bmatrix}
u \\
v \\
v
\end{bmatrix} = \sum N_i \begin{bmatrix}
u_i \\
v_i \\
w_i
\end{bmatrix} + \sum N_i \zeta \frac{\epsilon}{2} \begin{bmatrix}
u_i \\
v_i \\
w_i
\end{bmatrix}
\]

(5.4)

5.2.3 Strain-displacement relation

In order to make the shell assumption of zero normal stress on the surface $\zeta = \text{constant}$ to obey, the strain components should be defined in terms of local system of axes. Thus, at any point on this surface a normal $\zeta'$ is erected and two other orthogonal axes $x'$ and $y'$ tangent to it are considered, the significant strain components of interest are given by

\[
\epsilon' = \begin{bmatrix}
\epsilon_{x'} \\
\epsilon_{y'} \\
\gamma_{x'y'} \\
\gamma_{x'\zeta'} \\
\gamma_{y'\zeta'}
\end{bmatrix} = \begin{bmatrix}
\frac{\partial u'}{\partial x'} \\
\frac{\partial v'}{\partial y'} \\
\frac{\partial u'}{\partial y'} + \frac{\partial v'}{\partial x'} \\
\frac{\partial w'}{\partial x'} + \frac{\partial u'}{\partial \zeta'} + \frac{\partial v'}{\partial \zeta'}
\end{bmatrix}
\]

(5.5)
where \( u'\), \( v'\) and \( w'\) are the displacement components in the local direction axes \( x'\), \( y'\) and \( z'\) respectively. These local derivatives are obtained from the global derivatives of the displacements \( u\), \( v\) and \( w\). The stress components corresponding to these strain components are defined as follows

\[
\sigma' = \begin{bmatrix} \sigma_{x'} & \sigma_{y'} & \tau_{x'y'} & \tau_{x'z'} & \tau_{y'z'} \end{bmatrix}^T = D'\varepsilon' = D'[B]\{d\}
\]

where \( D'\) is the constitute matrix of size 5×5, given by

\[
D' = \frac{E}{1-\nu^2} \begin{bmatrix}
1 & \nu & 0 & 0 & 0 \\
\nu & 1 & 0 & 0 & 0 \\
0 & 0 & \frac{1-\nu}{2} & 0 & 0 \\
0 & 0 & 0 & \frac{(1-\nu)\alpha}{2} & 0 \\
0 & 0 & 0 & 0 & \frac{(1-\nu)\alpha}{2}
\end{bmatrix}
\]

in which \( E \) and \( \nu \) are Young’s modulus and Poisson’s ratio respectively. The factor \( \alpha \) included in the last two shear terms is taken as 5/6 and its purpose is to improve the shear displacement approximation.

And

\[
[B] = \begin{bmatrix} B_1 & B_2 & B_3 & B_4 \end{bmatrix}
\]

\[
\{d\} = \{d_1\ d_2\ d_3\ d_4\}^T
\]

Where the vector \( \{d_i\} \) of degrees of freedom is defined as

\[
\{d_i\} = \{u_i\ v_i\ w_i\ \alpha_i\ \beta_i\}
\]

And the strain displacement matrix \( B \) is given by
5.2.4 Jacobian matrix

The \([B]\) matrix is defined in terms of the displacement derivatives with respect to local Cartesian coordinates \((x', y', z')\). It is required to follow two sets of transformations before the element matrices are assembled with respect to the curvilinear coordinates \((\xi, \eta, \zeta)\).

First, the derivatives with respect to the global \((x, y, z)\) directions are obtained by using the matrix relation

\[
\frac{\partial N_i}{\partial x'} \frac{\partial N_i}{\partial y'} = \begin{bmatrix}
-0.5\zeta t_i \frac{\partial N_j}{\partial x'} l_{2i} & 0.5\zeta t_i \frac{\partial N_j}{\partial x'} n_{2i} \\
0 & -0.5\zeta t_i \frac{\partial N_j}{\partial y'} m_{2i} & 0.5\zeta t_i \frac{\partial N_j}{\partial y'} n_{2i} \\
0 & 0 & -0.5\zeta t_i \frac{\partial N_j}{\partial z'} n_{2i} \end{bmatrix}
\]

\[(5.11)\]

where \([J]\) is the Jacobian matrix and given by,

\[
\begin{bmatrix}
\frac{\partial N_i}{\partial x} & \frac{\partial N_i}{\partial y} & \frac{\partial N_i}{\partial z} \\
\frac{\partial N_i}{\partial x'} & \frac{\partial N_i}{\partial y'} & \frac{\partial N_i}{\partial z'} \\
\frac{\partial N_i}{\partial x} & \frac{\partial N_i}{\partial y} & \frac{\partial N_i}{\partial z}
\end{bmatrix} = [J]^{-1}
\]

\[(5.12)\]
The derivatives \( \frac{\partial u}{\partial \xi}, \frac{\partial v}{\partial \eta}, \ldots \) etc. are obtained using Eq. (5.4)

Second, after establishing the direction cosines \([T_{dc}]\) of local axes, the global derivatives of displacement \(u, v,\) and \(w\) are transformed to the local derivatives of the local orthogonal displacements by the transformation

\[
\begin{bmatrix}
\frac{\partial u'}{\partial \xi'} & \frac{\partial v'}{\partial \xi'} & \frac{\partial w'}{\partial \xi'} \\
\frac{\partial u'}{\partial \eta'} & \frac{\partial v'}{\partial \eta'} & \frac{\partial w'}{\partial \eta'} \\
\frac{\partial u'}{\partial \zeta'} & \frac{\partial v'}{\partial \zeta'} & \frac{\partial w'}{\partial \zeta'}
\end{bmatrix}
= \begin{bmatrix}
\frac{\partial u}{\partial x} & \frac{\partial v}{\partial x} & \frac{\partial w}{\partial x} \\
\frac{\partial u}{\partial y} & \frac{\partial v}{\partial y} & \frac{\partial w}{\partial y} \\
\frac{\partial u}{\partial z} & \frac{\partial v}{\partial z} & \frac{\partial w}{\partial z}
\end{bmatrix} \begin{bmatrix}
\frac{\partial u}{\partial \xi} & \frac{\partial v}{\partial \xi} & \frac{\partial w}{\partial \xi} \\
\frac{\partial u}{\partial \eta} & \frac{\partial v}{\partial \eta} & \frac{\partial w}{\partial \eta} \\
\frac{\partial u}{\partial \zeta} & \frac{\partial v}{\partial \zeta} & \frac{\partial w}{\partial \zeta}
\end{bmatrix}
\]  

(5.14)

The direction cosines needed in \([T_{dc}]\) are the direction cosines of the vectors \(V_1, V_2\) and \(V_3\).

5.2.5 Element stiffness matrix

The element stiffness matrix for a four noded isoparametric element is given by

\[
\left[ K^e \right]_{20 \times 20} = \int_{-1}^{1} \int_{-1}^{1} \left[ B^T \right]_{20 \times 6} \left[ D \right]_{6 \times 6} \left[ B \right]_{6 \times 20} \left| J \right| d\xi d\eta d\zeta
\]  

(5.15)

5.2.6 Element mass matrix

The element mass matrix for the four noded isoparametric element is given by
where $\rho$ is the density of the material.

5.2.7 Element geometric stiffness matrix

The geometric stiffness matrix is defined based on the constant terms of Cauchy stresses as,

\[
\begin{bmatrix}
K^e_i \\
K^e_{i+1} \\
K^e_{i+2}
\end{bmatrix} = \int_{-1}^{1} \int_{-1}^{1} \begin{bmatrix}
B^e_{NL} \\
B^e_{NL} \\
B^e_{NL}
\end{bmatrix} \begin{bmatrix}
\hat{\sigma}_0 \\\n\hat{\sigma}_0 \\
\hat{\sigma}_0
\end{bmatrix}_{9,9} \begin{bmatrix}
B^e_{NL} \\
B^e_{NL} \\
B^e_{NL}
\end{bmatrix} \frac{J}{d\xi d\eta d\zeta} (5.17)
\]

where

\[
\begin{bmatrix}
B^e_{NL} = \begin{bmatrix}
B^e_{NL} & B^e_{NL} & B^e_{NL}
\end{bmatrix}
\end{bmatrix}
\]

and matrix $[B^e_{NL}]$ is formed as shown below

\[
\begin{bmatrix}
\frac{\partial N_i}{\partial x'} & 0 & 0 & -0.5t_i l_{i2} \left( \zeta \frac{\partial N_i}{\partial x'} + \frac{\partial N_i}{\partial x} \right) & -0.5t_i l_{i1} \left( \zeta \frac{\partial N_i}{\partial x'} + \frac{\partial N_i}{\partial x} \right) \\
\frac{\partial N_i}{\partial y'} & 0 & 0 & -0.5t_i l_{i2} \left( \zeta \frac{\partial N_i}{\partial y'} + \frac{\partial N_i}{\partial y} \right) & -0.5t_i l_{i1} \left( \zeta \frac{\partial N_i}{\partial y'} + \frac{\partial N_i}{\partial y} \right) \\
\frac{\partial N_i}{\partial z'} & 0 & 0 & -0.5t_i l_{i2} \left( \zeta \frac{\partial N_i}{\partial z'} + \frac{\partial N_i}{\partial z} \right) & -0.5t_i l_{i1} \left( \zeta \frac{\partial N_i}{\partial z'} + \frac{\partial N_i}{\partial z} \right) \\
0 & \frac{\partial N_i}{\partial x'} & 0 & -0.5t_i m_{i2} \left( \zeta \frac{\partial N_i}{\partial x'} + \frac{\partial N_i}{\partial x} \right) & -0.5t_i m_{i1} \left( \zeta \frac{\partial N_i}{\partial x'} + \frac{\partial N_i}{\partial x} \right) \\
0 & \frac{\partial N_i}{\partial y'} & 0 & -0.5t_i m_{i2} \left( \zeta \frac{\partial N_i}{\partial y'} + \frac{\partial N_i}{\partial y} \right) & -0.5t_i m_{i1} \left( \zeta \frac{\partial N_i}{\partial y'} + \frac{\partial N_i}{\partial y} \right) \\
0 & \frac{\partial N_i}{\partial z'} & 0 & -0.5t_i m_{i2} \left( \zeta \frac{\partial N_i}{\partial z'} + \frac{\partial N_i}{\partial z} \right) & -0.5t_i m_{i1} \left( \zeta \frac{\partial N_i}{\partial z'} + \frac{\partial N_i}{\partial z} \right) \\
0 & 0 & \frac{\partial N_i}{\partial x'} & -0.5t_i n_{i2} \left( \zeta \frac{\partial N_i}{\partial x'} + \frac{\partial N_i}{\partial x} \right) & -0.5t_i n_{i1} \left( \zeta \frac{\partial N_i}{\partial x'} + \frac{\partial N_i}{\partial x} \right) \\
0 & 0 & \frac{\partial N_i}{\partial y'} & -0.5t_i n_{i2} \left( \zeta \frac{\partial N_i}{\partial y'} + \frac{\partial N_i}{\partial y} \right) & -0.5t_i n_{i1} \left( \zeta \frac{\partial N_i}{\partial y'} + \frac{\partial N_i}{\partial y} \right) \\
0 & 0 & \frac{\partial N_i}{\partial z'} & -0.5t_i n_{i2} \left( \zeta \frac{\partial N_i}{\partial z'} + \frac{\partial N_i}{\partial z} \right) & -0.5t_i n_{i1} \left( \zeta \frac{\partial N_i}{\partial z'} + \frac{\partial N_i}{\partial z} \right)
\end{bmatrix} \]

The Cauchy stress tensor $[\hat{\sigma}_0]$ is defined as follows
The stiffness, mass and geometric stiffness matrices given by Eq. (5.15), Eq. (5.16) and Eq. (5.17) respectively are computed by numerical integration employing Gaussian integration. Like plate element, thin shell also has shear locking problem, to reduce this problem reduced integration technique is carried out.

5.3 Governing equation for stability

Application of finite elements to a thin shell under parametric loading yields the following equilibrium equation

\[ M \ddot{q} + C \dot{q} + (K + P(t)K_G)q = 0 \]  

(5.22)

where \( M, C, K \) and \( K_G \) are the global assembled mass, damping, elastic stiffness and geometric stiffness matrices respectively. All these matrices are square symmetric matrices of order \( N \), the number of degrees of freedom of the system and \( q \) is the nodal displacement column vector of order \( N \). For the dynamic stability analysis a thin cylindrical shell under axial periodic compressive force \( p(t) \) is considered. A perfect thin-walled circular cylindrical shell of radius \( R \), length \( L \) and thickness \( t \) made of an elastic, homogenous and isotropic material with Young’s modulus \( E \), Poisson ratio \( \nu \) and mass per unit area \( \rho \) is considered for stability analysis. The cylindrical shell is clamped at the bottom and at the top surface a
uniform periodic compressive force is applied as shown in Figure 5.2. The matrices $M$, $K$ are obtained by finite element formulation as derived in the previous section and $C$ is obtained from the mass matrix by multiplying the mass matrix with proportionality constant. To calculate $K_G$ initially, plane stress analysis under the uniform compressive edge loading is carried out to obtain initial stress distribution and from these initial stress distribution $K_G$ matrix is evaluated.

![Figure 5.2: Cylindrical shell under uniform periodic compressive force](image)

**5.4 Free vibration of cylindrical shell**

The natural frequencies and free vibrational modes of a circular cylindrical shell can be obtained by solving the eigenvalue problem

$$ \left[ K - \omega^2 M \right] \phi = 0 \quad (5.23) $$

where $\omega$ is the natural frequency of vibration and $\phi$ gives the normal modes of vibration of the shell. The vibrational modes of a circular cylindrical shell can be classified as the cos $\theta$-type modes for which there is a single cosine wave of deflection in the circumferential direction, and as the cos $n\theta$-type modes for which the deflection of the shell involves a number of circumferential waves higher than 1. These circumferential cos $n\theta$-type modes can be further denoted as beam-type
modes because the shell behaves like a vertical cantilever beam across the length. Figure 5.3 shows the vertical nodal patterns and circumferential modes for a circular cylindrical shell.

Figure 5.3: Circular cylindrical shell vibrational modes (a) Vertical nodal patterns (b) Circumferential nodal patterns

5.5 Dynamic stability of cylindrical shell

Using the modal transformation Eq. (5.22) can be transformed into the following form [31]

\[
\ddot{\xi}_{in} + 2\eta_{in}\omega_{in}\dot{\xi}_{in} + \omega_{in}^2\xi_{in} + \varepsilon\cos\Omega t \sum_{n=1}^{N} d_{ijmn}\xi_{in} = 0 \quad (5.24)
\]

where \(\varepsilon\) is a small parameter, \(i, j\) corresponds to frequency number of beam type and \(m, n\) corresponds to circumferential mode number. The stability of Eq. (5.24) can be carried out using Hsu’s stability criteria as given in section 4.5 of chapter 4. Two different cylindrical tanks i.e. a tall tank and a broad tank are taken for dynamic
stability studies. The geometrical data for the tall and broad cylindrical shells used for the analysis are given in Table 3.1.

<table>
<thead>
<tr>
<th>Shell data</th>
<th>Tall shell</th>
<th>Broad shell</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$ (Radius)</td>
<td>7.32 m</td>
<td>18.13 m</td>
</tr>
<tr>
<td>$L$ (Length)</td>
<td>21.96 m</td>
<td>12.20 m</td>
</tr>
<tr>
<td>$t$ (Thickness)</td>
<td>0.0254 m</td>
<td>0.0254 m</td>
</tr>
<tr>
<td>$E$ (Young’s modulus)</td>
<td>206.7 GPa</td>
<td>206.7 GPa</td>
</tr>
<tr>
<td>$\nu$ (Poisson ratio)</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>$\rho$ (Shell mass density)</td>
<td>$7.84 \times 10^3$ Kg/m$^3$</td>
<td>$7.84 \times 10^3$ Kg/m$^3$</td>
</tr>
</tbody>
</table>

A program is written to obtain the required matrices of shells. Shell is discretized with isoparametric four noded elements. Elements elastic stiffness, geometric stiffness and mass matrices are obtained using Gaussian reduced integration and these elements are assembled into global matrices. After obtaining the global matrices, the appropriate boundary conditions are applied. A free vibration problem is solved and the global matrices are converted into the form given in Eq. (5.24) using the modal transformations and then stability chart is plotted using Hsu’s criteria given in Eq. (4.56 – 4.58).

5.5.1 Dynamic stability of tall and broad cylindrical shell

Table 5.2 shows the first ten cos $n\theta$-type modes of first and second beam mode frequencies obtained for tall and broad shell. Figure 5.4 shows the mode shapes for the frequencies given in Table 5.2 for a tall tank. Figure 5.5 and Figure 5.6 shows the stability chart of tall and broad circular cylindrical shells obtained using Hsu’s conditions respectively. The stability chart is limited to 30 Hz along the frequency axis. The stability charts have regions of simple resonance and combination resonance of sum type. A damping parameter of 0.01 is considered for all the modes for both the shells. To plot the stability charts first ten cos $n\theta$-type
Figure 5.4: Mode shapes of tall tank
modes of first and second beam mode are taken. Three types of resonance regions are observed. Simple resonance regions for each first two axial modes of \( \cos n\theta \)-type modes and combination resonance of sum type between the first two axial modes of respective \( \cos n\theta \)-type modes. It can be observed from the stability charts that tall shell is more prone to dynamic buckling under compressive force because there are many instable regions compared to broad shell and the instability regions are wider compared to instability regions of broad shell. Combination resonance regions of difference types are not observed in this case.

### Table 5.2: Natural frequencies of tall and broad shell

<table>
<thead>
<tr>
<th></th>
<th>Tall shell Frequency (Hz) axial mode</th>
<th>broad shell Frequency (Hz) axial mode</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( i = 1 )</td>
<td>( i = 2 )</td>
</tr>
<tr>
<td>( n )</td>
<td>( \omega_{1n} )</td>
<td>( \omega_{2n} )</td>
</tr>
<tr>
<td>0</td>
<td>57.73</td>
<td>109.09</td>
</tr>
<tr>
<td>1</td>
<td>19.17</td>
<td>56.13</td>
</tr>
<tr>
<td>2</td>
<td>8.37</td>
<td>33.39</td>
</tr>
<tr>
<td>3</td>
<td>4.44</td>
<td>20.78</td>
</tr>
<tr>
<td>4</td>
<td>3.19</td>
<td>14.00</td>
</tr>
<tr>
<td>5</td>
<td>3.48</td>
<td>10.36</td>
</tr>
<tr>
<td>6</td>
<td>4.73</td>
<td>8.80</td>
</tr>
<tr>
<td>7</td>
<td>6.61</td>
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<tr>
<td>8</td>
<td>9.08</td>
<td>10.41</td>
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<td>9</td>
<td>12.22</td>
<td>13.06</td>
</tr>
<tr>
<td>10</td>
<td>12.35</td>
<td>21.29</td>
</tr>
</tbody>
</table>

### 5.6 Summary

The dynamic stability of bottom clamped cylindrical shells under uniform periodic compressive force is obtained using Hsu’s stability criteria. The governing Mathieu-Hill equation is obtained by employing finite element formulation. 3D degenerated four noded shell elements are used for the formulation of global system matrices. For the considered tall tank and broad tank, it was found that tall shell is more prone to dynamic instability compared to broad shell.
Figure 5.5: Stability chart of a tall shell under axial periodic compressive force

Figure 5.6: Stability chart of a broad shell under periodic axial compressive force