CHAPTER 2

Publications based on this Chapter;


• *Flow over a stretching sheet of a dusty fluid with radiation effect*, COMMUNICATED.
Chapter 2

Boundary layer flow and heat transfer of a dusty fluid over a stretching sheet

2.1 Introduction

The boundary layer flow and heat transfer over a stretching sheet have been studied many researchers in the recent past because in the fluid dynamics over a stretching sheet is an important in many practical applications such as extrusion of plastic sheet, glass blowing, drawing plastic film, paper production, metal spinning and cooling of the metallic plate in a bath. Many authors investigated some mathematical results in the heat transfer problem. Sakiadis [1] has initiated the study of boundary layer problem generated by a continuous solid surface moving with a constant velocity. Crane [2] initiated the analytical study of boundary layer flow due to a stretching sheet. Grubka and Bobba [3] carried out heat transfer studies by considering the power-law variation of surface temperature. Vajravelu and Roper [4] gave a solution for flow and heat transfer in a second grade fluid over a stretching sheet. Tsai et.al [5] extended the work of [4] and studied the unsteady stretching surface with non-uniform heat source. Abel et.al [6, 7] have studied
the boundary layer flow and heat transfer in a viscoelastic fluid over a stretching sheet with prescribed surface temperature (PST case) and prescribed heat flux (PHF case). Further they studied radiation effect in a heat transfer analysis over a stretching sheet. Ishak et al [8] have obtained the solution to unsteady laminar boundary layer over a continuously stretching permeable surface. Abdul Aziz [9] obtained the numerical solution for laminar thermal boundary over a flat plate with a convective surface boundary condition using the symbolic algebra software Maple.


To study the two-phase flows, in which solid spherical particles are distributed in a fluid are of interest in a wide range of technical problems, such as flow through packed beds, sedimentation, environmental pollution, centrifugal separation of particles, and blood rheology etc., The study of the boundary layer flow of fluid-particle suspension flow is
important in determining the particle accumulation and impingement of the particle on the surface. In view of these applications, Chakrabarti [16] analyzed the boundary layer flow of a dusty gas. Datta and Mishra [17] have investigated boundary layer flow of a dusty fluid over a semi-infinite flat plate. Further, many mathematicians in these fields have been studied such as Evgeny and Sergei [18], XIE Ming-liang et.al [19], Palani et.al [20], Agranat [21] and Vajravelu et.al [22].

Motivated by the above studies, this chapter deals with the study of boundary layer flow and heat transfer of a dusty fluid over a stretching sheet with the effects of non-uniform heat source/sink and radiation. Heat transfer analysis are examined for two types of boundary conditions, namely (i) wall is maintained surface temperature and (ii) wall is maintained heat flux. Here, the governing partial differential equations are transformed into ordinary differential equations using similarity transformation and are solved numerically by Runge Kutta Fehlberg fourth-fifth order method with the help of Maple. In the present chapter we analyze the effect of physical parameters like fluid particle interaction parameter, non-uniform heat source/sink parameter, radiation parameter, number density, Prandtl number and Eckert number are shown graphically.

2.2 Formulation and Solution of the Problem

A steady two dimensional laminar flow of an incompressible viscous dusty fluid due to stretching sheet is considered. The sheet is coinciding with the plane \( y = 0 \), with the flow being confined to \( y > 0 \). Two equal and opposite forces are applied along the \( x \)-axis, so that the sheet is stretched, keeping the origin fixed as shown in figure 2.1. The dust
particles are assumed to be spherical in shape and uniform in size.

![Schematic diagram of the flow.](image)

Figure-2.1: Schematic diagram of the flow.

The boundary conditions for the flow problem are given by

\[
\begin{align*}
    u &= U_w(x), \quad v = 0 \quad \text{at} \quad y = 0, \\
    u &\to 0, u_p \to 0, v_p \to v, \rho_p \to \omega \rho \quad \text{as} \quad y \to \infty,
\end{align*}
\]

where \( U_w(x) = cx \) is the stretching sheet velocity, \( c > 0 \) is the stretching rate, \( \omega \) is the density ratio. To convert the governing equations into a set of similarity equations, now we introduce the following transformation as,

\[
\begin{align*}
    u &= cx f'(\eta), \quad v = -\sqrt{\nu c} f(\eta), \quad \eta = \sqrt{\frac{c}{\nu}} y, \\
    u_p &= cx F(\eta), \quad v_p = \sqrt{\nu c} G(\eta), \quad \rho_r = H(\eta),
\end{align*}
\]

which are identically satisfies (1.2.2). Substituting (2.2.2) into (1.2.3) to (1.2.6) in the absence of magnetic field, gravitational force and free stream region, one can obtain the
following non-linear ordinary differential equations,

\[ f''''(\eta) + f(\eta)f'''(\eta) - [f'(\eta)]^2 + \gamma \beta H(\eta)[F(\eta) - f'(\eta)] = 0, \]  
(2.2.3)

\[ G(\eta)F'(\eta) + [F(\eta)]^2 + \beta[F(\eta) - f'(\eta)] = 0, \]  
(2.2.4)

\[ G(\eta)G'(\eta) + \beta[f(\eta) + G(\eta)] = 0, \]  
(2.2.5)

\[ H(\eta)F'(\eta) + H(\eta)G'(\eta) + G(\eta)H'(\eta) = 0. \]  
(2.2.6)

where a prime denotes differentiation with respect to \( \eta \) and \( \gamma = \frac{mN}{\rho} \), \( \tau = \frac{m}{K} \) is the relaxation time of the particle phase, \( \beta = \frac{1}{c} \) is the fluid particle interaction parameter and \( \rho_r = \frac{\rho}{\rho} \) is the relative density.

The boundary conditions defined as in (2.2.1) will becomes,

\[ f(\eta) = 0, \quad f'(\eta) = 0 \text{ at } \eta = 0, \]

\[ f'(\eta) = 0, \quad F(\eta) = 0, \quad G(\eta) = -f(\eta), \quad H(\eta) = \omega \text{ as } \eta \to \infty. \]  
(2.2.7)

If \( \beta = 0 \), the analytical solution of (2.2.3) was given by Cracn [2] as

\[ f(\eta) = 1 - e^{-\eta}, \]  
(2.2.8)

obviously.

### 2.3 Heat Transfer Analysis

The space and temperature dependent internal heat generation/absorption (non-uniform heat source/sink) can be expressed as

\[ q'' = \left( \frac{kU_w(x)}{\chi v} \right) [A^*(T_w - T_{\infty})f'(\eta) + B^*(T - T_{\infty})], \]  
(2.3.1)
where $A^*$ and $B^*$ are the parameters of the space and temperature dependent internal heat generation/absorption. It is to be noted that $A^*$ and $B^*$ are positive to internal heat source and negative to internal heat sink, $\nu$ is the kinematic viscosity.

Using the Rosseland approximation for radiation [14], radiation heat flux is simplified as

$$q_r = -\frac{4\sigma^* T^3}{3k^*} \frac{\partial T}{\partial y}, \quad (2.3.2)$$

where $\sigma^*$ and $k^*$ are the Stefan-Boltzman constant and the mean absorption coefficient respectively. Assuming that the temperature differences within the flow such that the term $T^4$ may be expressed as a linear function of the temperature, we expand $T^4$ in a Taylor series about $T_\infty$ and neglecting the higher order terms beyond the first degree in $(T - T_\infty)$ we get

$$T^4 \approx 4T_\infty^3 T - 3T_\infty^4. \quad (2.3.3)$$

Substituting equations (2.3.1), (2.3.2) and (2.3.3) in (1.6.1), then it reduces to

$$\rho c_p \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = \left( k + \frac{16\sigma^* T_\infty^3}{3k^*} \right) \frac{\partial^2 T}{\partial y^2} + q'' + \frac{Nc_p}{\tau_T} (T_p - T) + \frac{N}{\tau_v} (u_p - u)^2. \quad (2.3.4)$$

The solutions of the equations (2.3.4) and (1.6.2) depends on the nature of the prescribed boundary conditions. We employ two types of heating process as follows:

1. PST (Prescribed Power law Surface Temperature),
2. PHF (Prescribed Power law Heat Flux).
CASE-1: Prescribed Surface Temperature (PST-Case)

For this heating process, the boundary conditions in case of prescribed power law surface temperature are of the form

\[ T = T_w = T_\infty + A \left( \frac{x}{l} \right)^2 \text{ at } y = 0, \]

\[ T \rightarrow T_\infty, \ T_p \rightarrow T_\infty \text{ as } y \rightarrow \infty, \]  \hspace{1cm} (2.3.5)

where \( T_w \) and \( T_\infty \) denote the temperature at the wall and at large distance from the wall respectively, \( A \) is a positive constant, \( l = \sqrt{\frac{\xi}{\zeta}} \) is a characteristic length.

Defining the non-dimensional fluid phase temperature \( \theta(\eta) \) and dust phase temperature \( \theta_p(\eta) \) as

\[ \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \ \ \ \theta_p(\eta) = \frac{T_p - T_\infty}{T_w - T_\infty}, \]  \hspace{1cm} (2.3.6)

where \( T - T_\infty = A \left( \frac{x}{l} \right)^2 \theta(\eta) \).

Using (2.3.5) and (2.3.6) into (1.6.2) and (2.3.4), we get

\[ (1 + Nr) \theta'(\eta) + Pr[f(\eta)\theta'(\eta) - 2f'(\eta)\theta(\eta)] + \frac{NPr}{\rho c_{T}}[\theta_p(\eta) - \theta(\eta)] \]

\[ + \frac{Ec}{\rho v}[f'(\eta) - f(\eta)]^2 + A f'(\eta) + B^* \theta(\eta) = 0, \]  \hspace{1cm} (2.3.7)

\[ 2F(\eta)\theta_p(\eta) + G(\eta)\theta_p'(\eta) + \frac{c_p}{c_{cm} r_T} [\theta_p(\eta) - \theta(\eta)] = 0, \]  \hspace{1cm} (2.3.8)

where \( Pr = \frac{\mu_s}{k} \) is the Prandtl number, \( Ec = \frac{\sigma^2}{\lambda c_p} \) is the Eckert number, \( Nr = \frac{16s^* T_r^3}{3ak^*} \) is the radiation parameter.

The boundary conditions (2.3.5) will take the form

\[ \theta(\eta) = 1 \text{ at } \eta = 0, \]

\[ \theta(\eta) \rightarrow 0, \ \ \theta_p(\eta) \rightarrow 0 \text{ as } \eta \rightarrow \infty. \]  \hspace{1cm} (2.3.9)
CASE-2: Prescribed Heat Flux (PHF-Case)

The power law heat flux on the wall surface is considered to be a quadratic power of $x$ in the form

$$-k \frac{\partial T}{\partial y} = q_w = D \left( \frac{x}{l} \right)^2 \text{ at } y = 0,$$

$$T \to T_\infty, \quad T_p \to T_\infty \text{ as } y \to \infty, \quad (2.3.10)$$

where $D$ is the constant. On the other hand define a non-dimensional fluid phase temperature $g(\eta)$ and dust phase temperature $g_p(\eta)$ as,

$$g(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad g_p(\eta) = \frac{T_p - T_\infty}{T_w - T_\infty}, \quad (2.3.11)$$

where $T_w - T_\infty = \frac{D}{k} \left( \frac{x}{l} \right)^2 \sqrt{\frac{\nu}{\varepsilon}}$.

Equations (1.6.2) and (2.3.4) on using (2.3.11) can be transformed in terms of $g(\eta)$ and $g_p(\eta)$ as

$$N Pr (1 + Nr)g''(\eta) + Pr[f(\eta)g'(\eta) - 2f'(\eta)g(\eta)] + \frac{N Pr}{\rho c T T} [g_p(\eta) - g(\eta)]$$

$$+ \frac{N Pr Ec}{\rho v_T} [F(\eta) - f'(\eta)]^2 + A^* f'(\eta) + B^* g(\eta) = 0, \quad (2.3.12)$$

$$2F(\eta)g_p(\eta) + G(\eta)g_p'(\eta) + \frac{c_p}{\rho c m T_T} [g_p(\eta) - g(\eta)] = 0, \quad (2.3.13)$$

where $Ec = \frac{kP_{e^{\nu/2}}}{Dc_{p\nu^2}}$ is the Eckert number, and the boundary conditions (2.3.10) will becomes

$$g'(\eta) = -1 \text{ at } \eta = 0,$$

$$g(\eta) \to 0, \quad g_p(\eta) \to 0 \text{ as } \eta \to \infty. \quad (2.3.14)$$
2.4 Physical Quantities

Our interest lies in investigation of the flow behavior and heat transfer characteristics by analyzing the non-dimensional local shear stress ($\tau_w$) and Nusselt number ($Nu$). These non-dimensional parameters are defined as

$$\tau_w = \frac{\tau^*}{\mu bx \sqrt{b/\nu}} = f''(0)$$

where $\tau^* = -\mu \left( \frac{\partial u}{\partial y} \right)_{y=0}$

$$Nu = \frac{-h}{T_w - T_\infty} \frac{T_y}{T_y} = \begin{cases} \theta'(0), & \text{PST Case,} \\ 1/g(0), & \text{PHF Case.} \end{cases}$$

2.5 Numerical Solution

The exact solution do not seem feasible for a complete set of equations (2.2.3) to (2.2.6), (2.3.7), (2.3.8), (2.3.12), (2.3.13) with appropriate boundary conditions given in (2.2.7), (2.3.9) and (2.3.14) because of the non-linear form of the momentum and thermal boundary layer equations. This fact forces one to obtain the solution of the problem numerically. Appropriate similarity transformation is adopted to transform the governing partial differential equations of flow and heat transfer into a system of non-linear ordinary differential equations. The resultant boundary value problem is solved numerically using Runge-Kutta-Fehlberg-fourth-fifth order method using symbolic algebra software Maple (see Aziz [9]).

In order to check the accuracy of our present method, a comparison of wall temperature gradient $\theta'(0)$ with those reported by Grubka and Bobba [3], Abel and Mahesha [7] and Chen [23] for various values of Prandtl number is given in Table 2.1. A comparison is also made with dimensionless temperature gradient $-\theta'(0)$ for various values of Prandtl number and $B^*$ those reported Vajravelu and Roper [22] and Tsai et al. [5] are presented
in Table 2.2, this give confidence that the numerical results obtained are accurate, the present results agreed very well with those of previous investigations. The results of thermal characteristics at the wall values of the temperature gradient function $\theta'(0)$ in PST case and temperature function $g(0)$ in PHF case are documented in Table 2.3. It reveals that the effects of increasing the values of $\beta$ and $Pr$ is to decreases the wall temperature gradient function $\theta'(0)$ and temperature function $g(0)$, and the effect of increasing the values of $Ec$, $A^*$, $B^*$, $Nr$, $N$ is to increases the temperature gradient function $\theta'(0)$ and temperature function $g(0)$ for both PST and PHF cases.

### 2.6 Results and discussion

In the present chapter, we studied the boundary layer flow of a dusty fluid over a stretching sheet in the presence of non-uniform heat source/sink and radiation. The boundary layer equations for momentum and heat transfer are solved numerically. The temperature profile $\theta(\eta)$ and $\theta_p(\eta)$ in the PST Case and $g(\eta)$ and $g_p(\eta)$ in the PHF Case are depicted graphically. Here six parameters that arises in the study are fluid particle interaction parameter $\beta$, Prandtl number $Pr$, Eckert number $Ec$, space dependent heat source/sink $A^*$, temperature dependent heat source/sink $B^*$, Number density $N$ and radiation parameter $Nr$.

It is evident from the figure 2.2 that the flow is parabolic in nature and we can see that the flow of fluid particles is parallel to that of dust. The velocity of both fluid and dust particles, which are nearer to the axis of flow, move with the greater velocity. Further observation shows the effect of fluid particle interaction parameter $\beta$ on velocity...
components of the fluid velocity \( f'(\eta) \) and particle velocity \( F(\eta) \) i.e., if \( \beta \) increases we can find the decrease in the fluid phase velocity and increase in the dust phase velocity. Also it reveals that for the large values of \( \beta \) i.e the relaxation time of the dust particle decreases then the velocities of both fluid and dust particles will be the same.

Figures 2.3(a) and 2.3(b), are the graphical representation of the temperature for PST and PHF cases, for different values of \( \beta \) versus \( \eta \). We infer from these figures that temperature of the fluid and dust particle decreases with increase in \( \beta \) respectively. We have used throughout our thermal analysis the values of \( \tau_v = \tau_T = 0.5, c_p = c_m = 0.2, \rho = 0.5 \) and \( c = 1 \).

Figures 2.4(a) and 2.4(b) represent the temperature profiles versus \( \eta \) for different values of \( Pr \). By analyzing the graphs it reveals that the effect of increasing the \( Pr \) is to decrease the temperature distribution in the flow region in both PST and PHF cases, which implies that momentum boundary layer is thicker than the thermal boundary layer. In PST case both the fluid and dust phase temperatures asymptotically approaches to zero in free stream region. This is similar in PHF case also.

Figures 2.5(a) and 2.5(b) are plotted for the temperature distribution for PST and PHF case respectively, for different values of \( Ec \). We observe that the effect of increasing values of Eckert number is to increase the wall temperature in PST case and PHF case of both the fluid and dust phase temperatures. This is due to fact that heat energy is stored in the fluid due to the frictional heating.

Temperature profiles of the fluid and dust particle across the thermal boundary layer in the PST and PHF case are shown in figures 2.6(a) and 2.6(b) for several values of \( A^* \), it can be seen that the thermal boundary layer generates the energy, and this causes the
temperature profiles increase with increases of $A^*(>0)$ and decreases with increase of $A^*(<0)$.

The effect of temperature dependent heat source/sink parameter $B^*$ on heat transfer of the fluid and dust particle is demonstrated in figures 2.7(a) and 2.7(b) for the cases PST and PHF respectively. These graphs illustrates that energy is released for increasing values of $B^*(>0)$ which causes the temperature to increase both in PST and PHF, where as energy is absorbed for decreasing values of $B^*(<0)$ resulting the temperature to drop significantly near the boundary layer.

Figures 2.8(a) and 2.8(b) depict the temperature profiles $\theta(\eta), \theta_p(\eta)$ for the PST case and $g(\eta), g_p(\eta)$ for the PHF cases respectively. These figures shows the thermal radiation on temperature distributions in both cases, it observed that the increase in the thermal radiation parameter $N\tau$ produces a significant increases in the thickness of the thermal boundary layer fluid so the temperature distribution increases with increasing the value of $N\tau$.

Figures 2.9(a) and 2.9(b) are plotted for the temperature profiles for different values of Number density of the dust particle $N$. It can be seen that the temperature profiles of fluid and dust particle decreases with increase of $N$. In all the figures it is shown that the fluid phase temperature is higher than that of dust phase and it indicates that the fluid particle temperature is parallel to that of dust phase for both PST and PHF cases.
2.7 Conclusions

The governing equations for a steady, boundary layer flow of an incompressible dusty fluid over a stretching sheet were formulated. The governing partial differential equations is converted into set of ordinary differential equations using similarity transformations. The influence of the parameters $Pr$, $Ec$, $A^*$, $B^*$, $Nr$ and $N$ on dimensionless temperature profiles were examined. The following results are listed below.

- The effect of space and temperature dependent heat source/sink parameters is to generate temperature for increasing positive values and absorb temperature for decreasing negative values. Hence space and temperature dependent heat sinks are better suited for cooling purposes.

- Effect of thermal radiation parameter is increase with increases temperature profile of both fluid and dust phases.

- The rate of heat transfer $\theta'(0)$ and $g(0)$ decreases with increasing the Prandtl number. While it increases with increasing the radiation parameter and Eckert number.

- The effect of Prandtl number is to decreases the thermal boundary layer thickness.

- Fluid particle interaction parameter increases both the fluid and dust phase temperature decreases.

- Fluid phase temperature is higher than the dust phase temperature.

- The PHF boundary condition is better suited for effective cooling of the stretching sheet.
### Table-2.1: Comparison results for the wall temperature gradient $-\theta'(0)$ in the case of $\beta = 0, Ec = 0, Nr = 0, A^* = 0, B^* = 0$ and $N = 0$.

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### Table-2.2: Comparison results for the dimensional temperature gradient $\theta'(0)$ in the case of $\beta = 0, Nr = 0, N = 0$ and $A^* = 0$.

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### Table-2.3: Wall temperature gradient $\theta'(0)$ and temperature function $g(0)$ for different values of the parameters $\beta$, $Ec$, $A^*$, $Pr$, $B^*$, $Nr$ and $N$.

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Chapter-2: Boundary layer flow and heat transfer of a dusty fluid over a stretching sheet

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Figure-2.2: Variation of fluid velocity ($f'$) and particle velocity ($F$) components for several values of $\beta$. 
Chapter-2: Boundary layer flow and heat transfer of a dusty fluid over a stretching sheet

Figure-2.3(a): Effect of fluid-particle interaction parameter $\beta$ on temperature distribution (PST).

Figure-2.3(b): Effect of fluid-particle interaction parameter $\beta$ on temperature distribution (PHF).
Figure-2.4(a): Effect of Prandtl number (Pr) on temperature distribution (PST).

Figure-2.4(b): Effect of Prandtl number (Pr) on temperature distribution (PHF).
Chapter 2: Boundary layer flow and heat transfer of a dusty fluid over a stretching sheet

Figure 2.5(a): Effect of Eckert number \((Ec)\) on temperature distribution (PST).

Figure 2.5(b): Effect of Eckert number \((Ec)\) on temperature distribution (PHF).
Figure-2.6(a): Effect of non-uniform heat source/sink ($A^*$) on temperature distribution (PST).

Figure-2.6(b): Effect of non-uniform heat source/sink ($A^*$) on temperature distribution (PHF).
Figure-2.7(a): Effect of non-uniform heat source/sink ($B^*$) on temperature distribution (PST).

Figure-2.7(b): Effect of non-uniform heat source/sink ($B^*$) on temperature distribution (PHF).
Chapter 2: Boundary layer flow and heat transfer of a dusty fluid over a stretching sheet

Figure 2.8(a): Effect of radiation parameter ($Nr$) on temperature distribution (PST).

Figure 2.8(b): Effect of radiation parameter ($Nr$) on temperature distribution (PHF).
Chapter-2: Boundary layer flow and heat transfer of a dusty fluid over a stretching sheet

Figure-2.9(a): Effect of \( N \) on temperature distribution (PST).

Figure-2.9(b): Effect of \( N \) on temperature distribution (PHF).