PREFACE
In mathematics, a manifold is a topological space that near each point resembles Euclidean space. More precisely, each point of an $n$-dimensional manifold has a neighbourhood that is homeomorphic to the Euclidean space of dimension $n$. Lines and circles, but not figure eights, are one-dimensional manifolds. Two-dimensional manifolds are called as surfaces, include plane, sphere, and torus, which can all be realized in three dimensions, but also the Klein bottle and real projective plane which cannot. The concept of a manifold is central to many parts of geometry and modern mathematical physics because it allows more complicated structures to be described and understood in terms of the relatively well-understood properties of Euclidean space. Manifolds naturally arise as solution sets of systems of equations and as graphs of functions and have additional features. One important class of manifolds is the class of differentiable manifolds. This differentiable structure allows calculus to be done on manifolds. A Riemannian metric on a manifold allows distances and angles to be measured. Symplectic manifolds serve as the phase spaces in the Hamiltonian formalism of classical mechanics, while four-dimensional Lorentzian manifolds model spacetime in general relativity.

In the study of Riemannian geometry, constant sectional curvature of manifolds and Einstein manifolds play prominent roles; It is known that, in general, the coordinate space
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for a dynamical system is a Riemannian manifold. In order to build a theory of dynamical systems, we need the appropriate tools. Thus, we use a purely geometrical treatment for problems in physics or mechanics. The Riemannian manifolds are smooth manifolds equipped with Riemannian metrics (smoothly varying choices of inner products on tangent spaces). This is the branch of modern differential geometry in which "geometric" ideas, in the familiar sense of the word, come to the fore. It is the direct descendant of Euclid’s plane and solid geometry, by way of Gauss’s theory of curved surfaces in space, and it is a dynamic subject of contemporary research.

In differential geometry, our spaces are equipped with an additional structure, a (Riemannian) metric, and some important concepts we encounter are distance, geodesics, the Levi-Civita connection, and curvature. In differential topology, important concepts are the degree of a map, intersection theory, differential forms, and deRham cohomology. In differential geometry, vector is a linear operator whose inputs are functions. At each point, the output of the operator is the directional derivative of the function in the direction.

Curvature and all related questions which surround curvature, constitute the central object of study in differential geometry. A small circle has large curvature and a large circle has small curvature. As the radius of the circle approaches infinity, the circle locally looks more and more like a straight line, and the curvature approaches to 0. If one were walking along a great circle on a very large sphere (like the earth) one would be perceive the space to be locally flat.

We use the terms, Riemann curvature tensor or RiemannChristoffel tensor to express curvature of Riemannian manifolds named after Bernhard Riemann and Elwin Bruno Christoffel. It associates a tensor to each point of a Riemannian manifold (i.e., it is a tensor field), that measures the extent to which the metric tensor is not locally isometric to
a Euclidean space. The curvature tensor can also be defined for any pseudo-Riemannian manifold, or indeed any manifold equipped with an affine connection. It is a central mathematical tool in the theory of general relativity, the modern theory of gravity, and the curvature of spacetime is in principle observable via the geodesic deviation equation. The curvature tensor represents the tidal force experienced by a rigid body moving along a geodesic in a sense made precise by the Jacobi equation. The curvature tensor is given in terms of the Levi-Civita connection by the following formula:

\[ R(X, Y)Z = \nabla_X \nabla_Y Z - \nabla_Y \nabla_X Z - \nabla_{[X,Y]} Z \]  

(0.0.1)

where \([X, Y]\) is the Lie bracket of vector fields. For each pair of tangent vectors \(X, Y\), \(R(X, Y)\) is a linear transformation of the tangent space of the manifold. It is linear in \(X\) and \(Y\), and so defines a tensor.

The Ricci curvature tensor, named after Gregorio Ricci-Curbastro, represents the amount by which the volume of a geodesic ball in a curved Riemannian manifold deviates from that of the standard ball in Euclidean space. As such, it provides one way of measuring the degree to which the geometry determined by a given Riemannian metric might differ from that of ordinary Euclidean n-space. The Ricci tensor is defined on any pseudo-Riemannian manifold, as a trace of the Riemann curvature tensor. Like the metric itself, the Ricci tensor is a symmetric bilinear form on the tangent space of the manifold.

The Ricci curvature is determined by the sectional curvatures of a Riemannian manifold, but generally contains less information. Indeed, if \(\xi\) is a vector of unit length on a Riemannian \(n\)-manifold, then \(Ric(\xi, \xi)\) is precisely \((n - 1)\) times the average value of the sectional curvature, taken over all the 2-planes containing \(\xi\). There is an \((n - 2)\)-dimensional family of such 2-planes, and so only in dimensions 2 and 3 does the Ricci tensor determine the full curvature tensor. A notable exception is when the manifold is
given a priori as a hypersurface of Euclidean space. The second fundamental form, which determines the full curvature via the Gauss-Codazzi equation, is itself determined by the Ricci tensor and the principal directions of the hypersurface are also the eigendirections of the Ricci tensor. Thus tensor was introduced by Ricci for this reason. If the Ricci curvature function $Ric(\xi, \xi)$ is constant on the set of unit tangent vectors $\xi$, the Riemannian manifold is said to have constant Ricci curvature, or to be an Einstein manifold. This happens if and only if the Ricci tensor $Ric$ is a constant multiple of the metric tensor $g$.

It is known that, an Einstein manifold is a Riemannian or pseudo-Riemannian manifold whose Ricci tensor is proportional to the metric. These manifolds are named after Albert Einstein because this condition is equivalent to saying that the metric is a solution of the vacuum Einstein field equations (with cosmological constant), although the dimension, as well as the signature, of the metric can be arbitrary, unlike the four-dimensional Lorentzian manifolds usually studied in general relativity. If $M$ is the underlying $n$-dimensional manifold and $g$ is its metric tensor the Einstein condition means that

$$Ric = kg$$

(0.0.2)

for some constant $k$, where Ric denotes the Ricci tensor of $g$. Einstein manifolds with $k = 0$ are called Ricci-flat manifolds. Simple examples of Einstein manifolds include, manifold with constant sectional curvature, Euclidean space, which is flat, is a simple example of Ricci-flat, hence Einstein metric. The $n$-sphere, $S^n$, with the round metric is Einstein with $k = n - 1$. Hyperbolic space with the canonical metric is Einstein with negative $k$.

Semi-symmetric metric connection plays important role in the study of Riemannian manifolds. There are various physical problems involving the semi-symmetric metric connection. For example, if a man is moving on the surface of the earth always facing one
definite point, say Jerusalem or Mekka or the North pole, then this displacement is semi-symmetric and metric[72]. Again during the mathematical congress in Moscow in 1934 one evening, mathematicians invented the "Moscow displacement". The streets of Moscow are approximately straight lines through the Kremlin and concentric circles around it. If a person walks in the street always facing the Kremlin, then this displacement is semi-symmetric and metric.

Apart from the conformal curvature tensor, the conharmonic curvature tensor K is another important tensor from the differential geometric point of view. K is invariant under the action of the conformal transformations of Riemannian manifold M which preserve, in a certain sense, real harmonic functions on M, and which therefore are called transformations. These transformations form a subgroup of the conformal transformation group. A Riemannian manifold M is related to a Euclidean space of the same dimension by a conharmonic transformation, and then is said to be conharmonically Euclidean, if and only if K vanishes identically.

The entire work represented in the thesis has been partitioned into 5 chapters.

Chapter-1

The first chapter is introductory and includes the definitions and preliminaries of contact manifolds, Sasakian manifolds, Kenmotsu manifolds, α-Sasakian manifolds, β-Kenmotsu manifolds, trans-Sasakian manifolds, ε-almost contact manifolds, (ε, δ)-trans-Sasakian manifolds and almost C(λ) manifolds. and also consist of definitions of weakly symmetric Riemannian manifolds, weakly concircular symmetric Riemannian manifolds, weakly pseudo-projective symmetric Riemannian manifolds, and Semi-symmetric metric connection on Kenmotsu manifolds and related formulas.
Chapter-2

In the first four sections of this chapter we studied the generalized $\phi$-recurrent and generalized concircular, conformal, projective, pseudo-Projective $\phi$-recurrent Sasakian manifolds. Next section consist of 3-dimensional locally generalized $\phi$-recurrent Sasakian manifolds. In the next section we studied generalized $\phi$-recurrent and generalized concircular, conformal $\phi$-recurrent $\alpha$-Sasakian manifolds. In the last two sections we have studied the generalized $\phi$-recurrent and generalized quasi-conformal $\phi$-recurrent Para-Sasakian manifolds.

Chapter-3

In the first two sections we studied the generalized $\phi$-recurrent and generalized concircular $\phi$-recurrent $\beta$ Kenmotsu manifolds. In the next section we discussed projectively semi-symmetric Kenmotsu Manifolds with Respect to Semi-symmetric Metric Connection connection. In the next section we studied $\phi$-projectively flat Kenmotsu manifolds with respect to semi-symmetric metric connection. In the next two sections we extended the same discussion for the pseudo-projective curvature tensor. In the next section we have shown that A conharmonically flat Kenmotsu manifolds admits semi-symmetric metric connection is an $\eta$-Einstein manifold with respect to semi-symmetric metric connection. The result is illustrated with an example. In the last section we discussed $\phi$-conharmonically flat Kenmotsu manifolds with respect to semi-symmetric metric connection.

Chapter-4

In the first two sections we have proved that A conformal flat 3-dimensional trans-Sasakian manifold is an $\eta$-Einstein manifold and 3-dimensional trans-Sasakian manifold is conformally flat if and only if it is conformally semi-symmetric. In the next section we have shown that three dimensional trans-Sasakian manifold satisfying $\mathcal{R}(X, Y) \cdot \overline{C} = 0$ is an
Einstein manifold of scalar curvature $6(\alpha^2 - \beta^2)$. In the next section we discussed weakly concircular symmetric three-dimensional trans-Sasakian manifolds. In the next section we discussed weakly concircular Ricci symmetric three-dimensional trans-Sasakian Manifolds. In the next section we extended the discussion for weakly pseudo projective symmetric trans-Sasakian manifolds. In the last two sections we studied the generalized $\phi$-Recurrent and generalized concircular $(\epsilon, \delta)$- trans-Sasakian Manifolds.

Chapter-5

This chapter is devoted to the study on almost $C(\lambda)$manifolds. In the first section we studied the generalized $\phi$-recurrent almost $C(\lambda)$manifolds. In the next two sections we studied generalized concircular and conformal $\phi$-recurrent almost $C(\lambda)$manifolds. In the next section we have discussed Conservative concircular almost $C(\lambda)$ manifolds. In the last section we have discussed the irrotational concircular almost $C(\lambda)$ manifolds.

Finally, the thesis ends with a short list of bibliography.