3.1 Introduction

A supply chain consists of parties involved, directly or indirectly, in fulfilling customer’s request. The supply chain includes not only the manufacturer and suppliers, but also transporters, warehouses, retailers and customers themselves. Within each organization, such as manufacturer, the supply chain includes all functions involved in receiving and satisfying customers demand. The customer is an integral part of the supply chain. In fact, the primary purpose of any supply chain is to satisfy customers demand and in the process generate profit for itself. The term supply chain conjures up images of product or supply moving from suppliers to manufacturers, manufactures to distributors, distributors to retailers, and retailers to customers in a chain. In reality, manufacturers receive material (raw material/ semi finished products) from several suppliers and after processing supplies to several distributors and retailers. Therefore supply chain is a real network.

A typical supply chain may involve a variety of stages. These include:

- Raw material suppliers
- Manufacturers
- Distributors
- Retailers
- Customers
Each stage in supply chain is connected through the flow of products information, and funds. These flows often occur in both directions and may be managed by one of the stages or an intermediary. The appropriate design of the supply chain depends on both the customers’ need and the roles played by stages involved.

In recent years, the supply chain network (SCN) problem has been gaining importance due to increasing competition in market. Firms are obliged to maintain high customer service levels while at the same time they are forced to reduce cost and maintain profit margins. Traditionally, planning, purchasing, manufacturing, marketing, and distribution organizations along the supply chain operate independently. These organizations have their own objectives and these are often conflicting. But, there is a need for a mechanism through which these different functions can be integrated together. Supply Chain Management is a strategy through which such integration can be achieved.

The network design problem is one of the most comprehensive strategic decision problem that need to be optimized for long term efficient operation of whole supply chain. It determines the number, location, capacity and type of plants, warehouses and retailers to be used. It also establishes distribution channels and the amount of materials and items to consume, produce and ship from suppliers to customers. SCN problem cover wide range of formulations ranged from simple single product type to complex multi-product one and from linear deterministic models to complex nonlinear ones.

A much more general form of the SCN model need to be considered if the entire supply chain network from the supplier to the customer is to be designed. We consider a supply chain in which suppliers send material to plants. Material is processed at plant and finished items send to warehouses
and in some cases directly to retailers. Warehouses send material to retailers as shown in fig.3.1.

![General Supply Chain Network](image)

In traditional supply chain management, the focus of the integration of SCN problem is usually on single objective such as minimum cost or maximum profit. For example, Jayaraman et al. [63], Jayaraman et al. [64], Yan et al. [119], Syam [107], Syarif et al. [108], Amiri [4], and Troung et al. [113] considered total cost of supply chain as an objective function in their studies. However there are no design tasks that are single objective problems. The design projects are usually involving trade-offs among different incompatible goals. Recently, Multi-objective optimization of SCN’s has been considered by different researchers in literature. Sabri et al.
[94] developed an integrated multi-objective supply chain model for strategic and operational supply chain planning under uncertainties of product, delivery and demand. Chen et al. [20] developed a multi-product, multi-stage and multi-period scheduling model for multi-stage SCN with uncertain demands and product prices. As objectives, fair profit distribution among all participants, safe inventory levels and maximum customer service levels, and robustness of decision to uncertain demands had been considered, and a two-phased fuzzy decision-making method was proposed to solve the problem.

In a real situation for a SCN problem, many input informations are not known precisely. At the time of making decisions, the value of many criteria and constraints are expressed in vague terms, such as very high in quality or low in price. Deterministic model cannot easily take this vagueness into account. In these cases the theory of fuzzy sets is one of the best tool for handling uncertainty. Fuzzy set theory is employed due to the presence of vagueness and imprecision of information in the SCN problem. A fuzzy multi-objective model has been developed for the supplier selection problem. This model provides a more realistic modeling structure by treating vagueness in the target values of the supply chain partner objectives. To reduce the computational burden, fuzzy modeling approach is used in this Chapter.

3.2 Mathematical Model

Supply Chain Network problem can be constructed in different forms. In this Chapter we construct a Multi-objective Supply Chain Network problem. The supply chain system in the problem consists of a manufacturer company
having multiple plants in different geographical regions, multiple
distribution centers (DC), retailers and customers/demand markets.

Below are the main characteristics and assumptions used in the problem
formulation:

- All demands of customers must be satisfied.
- Manufacturing plants are composed of multiple workstations.
- Locations of customers, material required to customers and
  plants are fixed and predefined.
- A pool of predetermined qualified suppliers is given.
- The production system at the manufacturer and each supplier is
  aggregated into a capacitated single stage system.
- The production and distribution systems considered are
  operationally connected and closely related with each other.
- Production capacities at the manufacturer and suppliers are
  estimated by considering a rough estimate of various
  contingencies.
- The customers do not wait to receive their orders in a future
  period. Therefore, inventory shortages are not allowed at each
  echelon.
- It is assumed that the different supply parties have been located
  in close proximity, so the procurement and distribution lead
  times are negligible.
Notations and Terminology

Following are the notations used in this Chapter.

- **Indices**
  - \( i \) - index of retailers \( i=1,2,\ldots,I \).
  - \( j \) - index of warehouses \( j=1,2,\ldots,J \).
  - \( k \) - index of plants \( k=1,2,\ldots,K \).
  - \( l \) - index of suppliers \( l=1,2,\ldots,L \).

- **Parameters**
  - \( D_i \) - Annual demand from \( i^{th} \) retailers.
  - \( A_k \) – Potential capacity of \( k^{th} \) plants.
  - \( B_l \) - Supply capacity of \( l^{th} \) suppliers.
  - \( E_j \) - Potential capacity of \( j^{th} \) warehouses.
  - \( G_{lk} \) - Cost of shipping one unit from supply source \( l \) to plant \( k \).
  - \( G_{kj} \) - Cost of producing and shipping one unit from plant \( k \) to warehouse \( j \).
  - \( G_{ki} \) - Cost of producing and shipping one unit from plant \( k \) to retailer \( i \).
  - \( G_{ji} \) - Cost of shipping one unit from warehouse \( j \) to retailer \( i \).
  - \( F_{kj} \) - Delivery time of shipping one unit from plant \( k \) to warehouse \( j \).
  - \( F_{ki} \) - Delivery time of shipping one unit from plant \( k \) to retailer \( i \).
  - \( F_{ji} \) - Delivery time of shipping one unit from warehouse \( j \) to retailer \( i \).
3.3 Multi-Objective Supply Chain Network Model

The objective function that minimizes variable costs of the supply chain network is given by

$$\min Z_1 = \sum_{l=1}^{L} \sum_{k=1}^{K} G_{lk} Y_{lk} + \sum_{k=1}^{K} \sum_{j=1}^{J} G_{kj} U_{kj} + \sum_{k=1}^{K} \sum_{i=1}^{I} G_{ki} V_{ki} + \sum_{j=1}^{J} \sum_{i=1}^{I} G_{ji} W_{ji}$$  \hspace{1cm} (3.1)

The objective function that minimizes delivery time of the supply chain network is given by

$$\min Z_2 = \sum_{k=1}^{K} \sum_{j=1}^{J} F_{kj} U_{kj} + \sum_{k=1}^{K} \sum_{i=1}^{I} F_{ki} V_{ki} + \sum_{j=1}^{J} \sum_{i=1}^{I} F_{ji} W_{ji}$$  \hspace{1cm} (3.2)

Subject to the constraints

$$\sum_{k=1}^{K} Y_{lk} \leq B_l, \quad \forall l.$$  \hspace{1cm} (3.3)

i.e. total quantity shipped from supplier to plant cannot exceed suppliers capacity.

$$\sum_{j=1}^{J} Y_{lk} - \sum_{j=1}^{J} U_{kj} - \sum_{i=1}^{I} V_{ki} \geq 0 \quad \forall k.$$  \hspace{1cm} (3.4)

i.e. the total quantity shipped from plant to warehouse and retailers cannot exceed quantity of raw material received.

$$\sum_{j=1}^{J} V_{ki} + \sum_{j=1}^{J} U_{kj} \leq A_k \quad \forall k.$$  \hspace{1cm} (3.5)
i.e. the quantity produced in factory cannot exceed its capacity.

\[ \sum_{k=1}^{K} U_{kj} - \sum_{i=1}^{J} W_{ji} \geq 0 \quad \forall j. \]  \hspace{1cm} (3.6)

i.e. the quantity shipped out of warehouse to retailers cannot exceed its capacity.

\[ \sum_{j=1}^{J} W_{ji} \leq E_j \quad \forall j. \]  \hspace{1cm} (3.7)

i.e. the quantity shipped through a warehouse cannot exceed its capacity.

\[ \sum_{j=1}^{J} W_{ji} + \sum_{k=1}^{K} V_{ki} \geq D_i \quad \forall i. \]  \hspace{1cm} (3.8)

i.e. the quantity shipped to retailers must cover the customer demand.

Nonnegative constraints are as follows:

\[
\begin{align*}
W_{ji} &\geq 0 \quad \forall i, j. \\
U_{kj} &\geq 0 \quad \forall k, j. \\
Y_{lk} &\geq 0 \quad \forall l, k. \\
V_{ki} &\geq 0 \quad \forall k, i.
\end{align*}
\]  \hspace{1cm} (3.9)

In classical LP, the violation of any constraints in model renders the solution infeasible, hence all constraints are considered to be of equal weights or importance. Before we develop a specific model of linear programming in a fuzzy environment. It should become clear that in contrast to classical Linear Programming, fuzzy linear programming is not uniquely defined type of model, many variations are possible, depending on the assumptions or features of the real situation to be modeled. Let us now turn to a first basic model for Fuzzy Linear programming (Supply Chain) problem. In this model, we shall assume
that the decision maker can establish an aspiration level \( \lambda \), for the value of the objective function. Decision Maker wants to achieve desired aspiration level.

### 3.4 Fuzzy Membership Function

The linear membership function for fuzzy objectives is given as

\[
\mu_{Z_m}(X) = \begin{cases} 
1 & \text{for } Z_m(X) \leq Z_{m}^{\text{PIS}} \\
\frac{Z_{m}^{\text{NS}} - Z_m(X)}{Z_{m}^{\text{NS}} - Z_{m}^{\text{PIS}}} & \text{for } Z_{m}^{\text{PIS}} \leq Z_m(X) \leq Z_{m}^{\text{NS}} \\
0 & \text{for } Z_m(X) \geq Z_{m}^{\text{NS}} 
\end{cases}
\]  

(3.10)

Here \( Z_{m}^{\text{NS}} \) is \( \max_m Z_m(x^*) \), \( Z_{m}^{\text{PIS}} \) is \( \min_m Z_m(x^*) \) and \( x^* \) is optimum solution.

A fuzzy constraint \( C \in X \) is a fuzzy subset of \( X \) characterized by its membership function \( \mu_{c_n}(x): x \rightarrow [0,1] \). The linear membership function for the fuzzy constraints is given by

\[
\mu_{c_n}(x) = \begin{cases} 
1 & \text{for } g_n(x) \leq b_n \\
\frac{(b_n + d_n - g_n(x))}{d_n} & \text{for } b_n \leq g_n(x) \leq b_n + d_n \\
0 & \text{for } b_n + d_n \leq g_n(x) 
\end{cases}
\]  

(3.11)

For all parameters \( n = 1, 2, \ldots, N \), where \( d_n \) is tolerance interval. These membership functions are illustrated in figure 3.2 and figure 3.3 respectively. \( Z_{m}^{\alpha} \) is the aspiration level that the decision maker wants to reach.
Fig. 3.2 Membership function of fuzzy objective functions

Fig. 3.3(a) Membership function of fuzzy constraints.

Fig. 3.3(b)
3.5 Fuzzy Multi-Objective Supply Chain Network Model

We formulate the fuzzy multi-objective supply chain network model:

Find X to satisfy

\[
\sum_{j=1}^{J} \sum_{k=1}^{K} G_{ik} Y_{lk} + \sum_{k=1}^{K} \sum_{j=1}^{J} G_{kj} U_{kj} + \sum_{k=1}^{K} \sum_{i=1}^{I} G_{ki} V_{ki} + \sum_{j=1}^{J} \sum_{i=1}^{I} G_{ji} W_{ji} < Z_1
\]

\[
\sum_{k=1}^{K} \sum_{j=1}^{J} F_{kj} U_{kj} + \sum_{j=1}^{J} \sum_{i=1}^{I} F_{ki} V_{ki} + \sum_{j=1}^{J} \sum_{i=1}^{I} F_{ji} W_{ji} < Z_2
\]

\[
\sum_{k=1}^{K} Y_{lk} < B_l, \quad \forall l.
\]

\[
\sum_{j=1}^{J} Y_{lk} - \sum_{j=1}^{J} U_{kj} - \sum_{i=1}^{I} V_{ki} > 0 \quad \forall k.
\]

\[
\sum_{j=1}^{J} V_{ki} + \sum_{j=1}^{J} U_{kj} < A_k \quad \forall k.
\]

\[
\sum_{k=1}^{K} U_{kj} - \sum_{i=1}^{I} W_{ji} > 0 \quad \forall j.
\]

\[
\sum_{j=1}^{J} W_{ji} < E_j \quad \forall j.
\]

\[
\sum_{j=1}^{J} W_{ji} + \sum_{k=1}^{K} V_{ki} > D_i \quad \forall i.
\]

\[
W_{ji} \geq 0 \quad \forall i,j.
\]

\[
U_{kj} \geq 0 \quad \forall k,j.
\]

\[
Y_{lk} \geq 0 \quad \forall l,k.
\]

\[
V_{ki} \geq 0 \quad \forall k,i.
\]

Crisp model for the above fuzzy model can be formulated as, find an equivalent crisp model by using a linear membership function for the initial
fuzzy model. We solve this problem by considering symmetric problem. i.e. there is no difference between the fuzzy goals and fuzzy constraints.

\[
\begin{align*}
\text{Maximize} & \quad \lambda \\
\text{Subject to the constraints} & \quad Z_m(X) + \lambda (Z_{\text{m}}^{\text{SS}} - Z_{\text{m}}^{\text{PS}}) \leq Z_{\text{m}}^{\text{SS}}, \quad m = 1, 2, \ldots, M \\
& \quad g_n(x) + \lambda d_n \leq (b_n + d_n), \quad n = 1, 2, \ldots, r. \\
& \quad g_n(x) - \lambda d_n \geq (b_n + d_n), \quad n = r+1, r+2, \ldots, N. \\
& \quad x \geq 0
\end{align*}
\]  

(3.12)

3.6 Model Algorithm

**Step 1:** Formulate the Multi-objective supply chain network problem.

**Step 2:** Consider the Multi-objective supply chain network problem as a single objective supply chain network problem. If our objective function is of minimization type we get the lower bound and solving this same objective function by considering maximization we get upper bound. If our objective function is of maximization type we get the upper bound and solving this same objective function by considering minimization we get lower bound.

**Step 3:** The values of the level of uncertainties for all the fuzzy parameters of constraints are taken as 10 percent of the deterministic model.

**Step 4:** Use these lower and upper bound values for crisp formulation and solve this problem to get fuzzy optimum solution.
3.7 Computational Experiment

As stated previously, we consider in our computational experiment a hypothetically constructed supply chain network consisting of manufacturing company having multiple plants in different geographical regions, multiple warehouses, retailers and customers. It is assumed that five raw material suppliers and four manufacturing plants produce single type of product. The distribution system contains six warehouses where products are temporarily stored and eight retailers from which the product are sold to several number of customers. It is also assumed that all of the manufacturing plants in the SC belong to same company, while the Distribution Centers and retailers are independent entities. We solve the following supply chain problem by using above mentioned algorithm.

Table 3.1: Transportation cost from supplier to plant in tonnage

<table>
<thead>
<tr>
<th>Suppliers</th>
<th>Plant</th>
<th>Suppliers</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>G1</td>
<td>G2</td>
</tr>
<tr>
<td>A</td>
<td>200</td>
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<td>B</td>
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<td>C</td>
<td>500</td>
<td>125</td>
</tr>
<tr>
<td>D</td>
<td>400</td>
<td>300</td>
</tr>
<tr>
<td>E</td>
<td>600</td>
<td>700</td>
</tr>
</tbody>
</table>
Table 3.2: Transportation and production cost of plant to retailers in tonnage.

<table>
<thead>
<tr>
<th>Plant</th>
<th>M1</th>
<th>M2</th>
<th>M3</th>
<th>M4</th>
<th>M5</th>
<th>M6</th>
<th>M7</th>
<th>M8</th>
<th>Capacity of Plant</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1</td>
<td>300</td>
<td>450</td>
<td>350</td>
<td>450</td>
<td>250</td>
<td>350</td>
<td>400</td>
<td>475</td>
<td>450</td>
</tr>
<tr>
<td>G2</td>
<td>350</td>
<td>500</td>
<td>300</td>
<td>375</td>
<td>275</td>
<td>375</td>
<td>475</td>
<td>450</td>
<td>300</td>
</tr>
<tr>
<td>G3</td>
<td>450</td>
<td>475</td>
<td>350</td>
<td>300</td>
<td>380</td>
<td>450</td>
<td>475</td>
<td>300</td>
<td>300</td>
</tr>
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<td>325</td>
<td>390</td>
<td>425</td>
<td>450</td>
<td>300</td>
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<td>80</td>
<td>60</td>
<td>50</td>
<td>95</td>
<td>97</td>
<td>78</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.3: Transportation and production cost of Plants to warehouses in tonnage.

<table>
<thead>
<tr>
<th>Plant</th>
<th>N1</th>
<th>N2</th>
<th>N3</th>
<th>N4</th>
<th>N5</th>
<th>N6</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1</td>
<td>300</td>
<td>150</td>
<td>200</td>
<td>200</td>
<td>125</td>
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<td>G2</td>
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</tr>
<tr>
<td>G3</td>
<td>550</td>
<td>150</td>
<td>200</td>
<td>300</td>
<td>250</td>
<td>300</td>
</tr>
<tr>
<td>G4</td>
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<td>350</td>
<td>300</td>
<td>175</td>
<td>300</td>
<td>305</td>
</tr>
</tbody>
</table>

Table 3.4: Transportation cost from warehouses to retailers in tonnage.

<table>
<thead>
<tr>
<th>Warehouses</th>
<th>M1</th>
<th>M2</th>
<th>M3</th>
<th>M4</th>
<th>M5</th>
<th>M6</th>
<th>M7</th>
<th>M8</th>
<th>Capacity of Warehouses</th>
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<td>175</td>
<td>170</td>
<td>198</td>
<td>183</td>
<td>170</td>
<td>150</td>
</tr>
<tr>
<td>N2</td>
<td>115</td>
<td>195</td>
<td>167</td>
<td>170</td>
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</tr>
<tr>
<td>N3</td>
<td>125</td>
<td>100</td>
<td>132</td>
<td>179</td>
<td>190</td>
<td>190</td>
<td>185</td>
<td>172</td>
<td>150</td>
</tr>
<tr>
<td>N4</td>
<td>130</td>
<td>165</td>
<td>137</td>
<td>180</td>
<td>195</td>
<td>175</td>
<td>190</td>
<td>173</td>
<td>200</td>
</tr>
<tr>
<td>N5</td>
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<td>147</td>
<td>190</td>
<td>196</td>
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<td>N6</td>
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<td>197</td>
<td>180</td>
<td>196</td>
<td>170</td>
<td>200</td>
</tr>
</tbody>
</table>
Table 3.5: Delivery time of item from plants to retailers.

<table>
<thead>
<tr>
<th>Plant</th>
<th>M1</th>
<th>M2</th>
<th>M3</th>
<th>M4</th>
<th>M5</th>
<th>M6</th>
<th>M7</th>
<th>M8</th>
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</thead>
<tbody>
<tr>
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<td>40</td>
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<td>75</td>
<td>80</td>
</tr>
<tr>
<td>G2</td>
<td>35</td>
<td>60</td>
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<td>40</td>
<td>25</td>
<td>48</td>
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<td>80</td>
</tr>
<tr>
<td>G3</td>
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<td>80</td>
<td>60</td>
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<td>85</td>
<td>60</td>
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<td>68</td>
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</tbody>
</table>

Table 3.6: Delivery time of item from plants to warehouses.

<table>
<thead>
<tr>
<th>Plants</th>
<th>N1</th>
<th>N2</th>
<th>N3</th>
<th>N4</th>
<th>N5</th>
<th>N6</th>
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<td>15</td>
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<td>28</td>
</tr>
<tr>
<td>G2</td>
<td>40</td>
<td>20</td>
<td>25</td>
<td>26</td>
<td>28</td>
<td>37</td>
</tr>
<tr>
<td>G3</td>
<td>55</td>
<td>60</td>
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<td>40</td>
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<tr>
<td>G4</td>
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<td>45</td>
<td>20</td>
<td>70</td>
<td>75</td>
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</table>

Table 3.7: Delivery time of item from warehouses to retailers.

<table>
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<tr>
<th>Warehouses</th>
<th>M1</th>
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<th>M4</th>
<th>M5</th>
<th>M6</th>
<th>M7</th>
<th>M8</th>
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</thead>
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</tr>
<tr>
<td>N2</td>
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<td>20</td>
<td>21</td>
<td>22</td>
<td>29</td>
<td>29</td>
<td>35</td>
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</tr>
<tr>
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<td>32</td>
<td>31</td>
<td>42</td>
<td>71</td>
</tr>
</tbody>
</table>
(1) LP formulation for obtaining Cost of Supply Chain Network problem using equation (3.1 to 3.8) and step 1.

Min $Z_1 = 200Y_{11} + 100Y_{12} + 150Y_{13} + 125Y_{14} + 300Y_{21} + 150Y_{22} + 200Y_{23} + 200Y_{24} + 500Y_{31} + 125Y_{32} + 225Y_{33} + 225Y_{34} + 400Y_{41} + 300Y_{42} + 250Y_{43} + 275Y_{44} + 600Y_{51} + 700Y_{52} + 300Y_{53} + 350Y_{54} + 300U_{11} + 150U_{12} + 200U_{13} + 200U_{14} + 125U_{15} + 300U_{16} + 400U_{21} + 125U_{22} + 225U_{23} + 250U_{24} + 275U_{25} + 320U_{26} + 550U_{31} + 150U_{32} + 200U_{33} + 300U_{34} + 250U_{35} + 300U_{36} + 650U_{41} + 350U_{42} + 300U_{43} + 175U_{44} + 300U_{45} + 305U_{46} + 300V_{11} + 450V_{12} + 350V_{13} + 450V_{14} + 250V_{15} + 350V_{16} + 400V_{17} + 475V_{18} + 350V_{21} + 500V_{22} + 300V_{23} + 375V_{24} + 275V_{25} + 375V_{26} + 475V_{27} + 450V_{28} + 450V_{31} + 475V_{32} + 350V_{33} + 350V_{34} + 300V_{35} + 380V_{36} + 450V_{37} + 475V_{38} + 500V_{41} + 450V_{42} + 325V_{43} + 400V_{44} + 425V_{45} + 390V_{46} + 425V_{47} + 450V_{48} + 150W_{11} + 190W_{12} + 162W_{13} + 175W_{14} + 170W_{15} + 198W_{16} + 183W_{17} + 170W_{18} + 115W_{21} + 195W_{22} + 167W_{23} + 170W_{24} + 180W_{25} + 180W_{26} + 184W_{27} + 171W_{28} + 125W_{31} + 100W_{32} + 132W_{33} + 179W_{34} + 190W_{35} + 190W_{36} + 185W_{37} + 172W_{38} + 130W_{41} + 165W_{42} + 137W_{43} + 180W_{44} + 195W_{45} + 175W_{46} + 190W_{47} + 173W_{48} + 140W_{51} + 170W_{52} + 147W_{53} + 190W_{54} + 196W_{55} + 165W_{56} + 195W_{57} + 176W_{58} + 175W_{61} + 155W_{62} + 150W_{63} + 100W_{64} + 197W_{65} + 180W_{66} + 196W_{67} + 170W_{68}

Subject to constraints
And all variables are nonnegative.
(2) LP formulation for obtaining Delivery time of Supply Chain Network problem using equation (3.1 to 3.8) and step 1 of model algorithm.

\[
\text{Min } Z_2 = 30U_{11} + 20U_{12} + 20U_{13} + 15U_{14} + 26U_{15} + 28U_{16} + 40U_{21} + 20U_{22} + 25U_{23} + 26U_{24} + 28U_{25} + 37U_{26} + 55U_{31} + 60U_{32} + 56U_{33} + 59U_{34} + 60U_{35} + 85U_{41} + 60U_{42} + 45U_{43} + 20U_{44} + 70U_{45} + 75U_{46} + 50V_{11} + 70V_{12} + 55V_{13} + 65V_{14} + 40V_{15} + 48V_{16} + 75V_{17} + 80V_{18} + 35V_{21} + 60V_{22} + 45V_{23} + 40V_{24} + 25V_{25} + 48V_{26} + 70V_{27} + 80V_{28} + 75V_{31} + 70V_{32} + 72V_{33} + 80V_{34} + 60V_{35} + 70V_{36} + 75V_{37} + 90V_{38} + 90V_{41} + 95V_{42} + 80V_{43} + 85V_{44} + 60V_{45} + 70V_{46} + 78V_{47} + 68V_{48} + 150W_{11} + 190W_{12} + 162W_{13} + 175W_{14} + 170W_{15} + 198W_{16} + 183W_{17} + 170W_{18} + 115W_{21} + 195W_{22} + 167W_{23} + 170W_{24} + 180W_{25} + 180W_{26} + 184W_{27} + 171W_{28} + 125W_{31} + 100W_{32} + 132W_{33} + 179W_{34} + 190W_{35} + 190W_{36} + 185W_{37} + 172W_{38} + 130W_{41} + 165W_{42} + 137W_{43} + 180W_{44} + 195W_{45} + 175W_{46} + 190W_{47} + 173W_{48} + 140W_{51} + 170W_{52} + 147W_{53} + 190W_{54} + 196W_{55} + 165W_{56} + 195W_{57} + 176W_{58} + 175W_{61} + 155W_{62} + 150W_{63} + 100W_{64} + 197W_{65} + 180W_{66} + 196W_{67} + 170W_{68}
\]

Subject to constraints (3.13)

LINDO optimization software is used for solving the Problem. Using the given parameter setting, we constitute the payoff table to the aim of defining the membership functions of the objectives. We take the minimum and maximum values of the efficient extreme solutions as the lower and upper bounds, respectively. The bounds are presented in Table 3.1.

Using the upper and lower bounds given in Table 3.8, the membership functions of the fuzzy objectives are defined. As illustrated in fig.3.2 and fig.3.3, trapezoidal type membership functions are used in this study.

Table 3.8: Lower and upper bounds on objectives are given below by using step 2 of model algorithm.

<table>
<thead>
<tr>
<th>Objectives</th>
<th>Lower bound</th>
<th>Upper bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>272574</td>
<td>1268380</td>
</tr>
<tr>
<td>Delivery time</td>
<td>23134</td>
<td>127640</td>
</tr>
</tbody>
</table>

74
(3) LP formulation for compromise solution is as follows by using step 3 and step 4 of model algorithm and equation (3.12)

Max $\lambda$

Subject to constraints

$$
200Y_{11}+100Y_{12}+150Y_{13}+125Y_{14}+300Y_{21}+150Y_{22}+200Y_{23}+200Y_{24}+500Y_{31}+125Y_{32} +225Y_{33}+225Y_{34}+400Y_{41}+300Y_{42}+250Y_{43}+275Y_{44}+600Y_{51}+700Y_{52}+300Y_{53} +350Y_{54}+300U_{11}+150U_{12}+200U_{13}+200U_{14}+125U_{15}+300U_{16}+400U_{21}+125U_{22}+225U_{23} +250U_{24}+275U_{25}+320U_{26}+550U_{31}+150U_{32}+200U_{33}+300U_{34}+250U_{35}+300U_{36} +650U_{41}+350U_{42}+300U_{43}+175U_{44}+300U_{45}+305U_{46}+300V_{11}+450V_{12}+350V_{13}+450V_{14} +250V_{15}+350V_{16}+400V_{17}+475V_{18}+350V_{21}+500V_{22}+300V_{23}+375V_{24}+275V_{25} +375V_{26}+475V_{27}+450V_{28}+450V_{31}+475V_{32}+350V_{33}+350V_{34}+300V_{35}+380V_{36}+450V_{37} +475V_{38}+500V_{41}+450V_{42}+325V_{43}+400V_{44}+325V_{45}+390V_{46}+425V_{47}+450V_{48} +150W_{11}+190W_{12}+162W_{13}+175W_{14}+170W_{15}+198W_{16}+183W_{17}+170W_{18}+115W_{21} +195W_{22}+167W_{23}+170W_{24}+180W_{25}+180W_{26}+184W_{27}+171W_{28}+125W_{31}+100W_{32} +132W_{33}+179W_{34}+190W_{35}+190W_{36}+185W_{37}+172W_{38}+130W_{41}+165W_{42}+137W_{43} +180W_{44}+195W_{45}+175W_{46}+190W_{47}+173W_{48}+140W_{51}+170W_{52}+147W_{53}+190W_{54} +196W_{55}+165W_{56}+195W_{57}+176W_{58}+175W_{61}+155W_{62}+150W_{63}+100W_{64}+197W_{65} +180W_{66}+196W_{67}+170W_{68}+995806\lambda \leq 1268380
$$

$$
30U_{11}+20U_{12}+20U_{13}+15U_{14}+26U_{15}+28U_{16}+40U_{21}+20U_{22}+25U_{23}+26U_{24}+28U_{25} +37U_{26}+55U_{31}+60U_{32}+56U_{33}+59U_{34}+60U_{35}+40U_{36}+85U_{41}+60U_{42}+45U_{43}+20U_{44}+70U_{45}+75U_{46}+50V_{11}+70V_{12}+55V_{13}+65V_{14}+40V_{15}+48V_{16}+75V_{17}+80V_{18}+35V_{21}+60V_{22} +45V_{23}+40V_{24}+25V_{25}+48V_{26}+70V_{27}+80V_{28}+75V_{31}+70V_{32}+72V_{33}+80V_{34}+60V_{35} +70V_{36}+75V_{37}+90V_{38}+90V_{41}+95V_{42}+80V_{43}+85V_{44}+60V_{45}+70V_{46}+78V_{47}+68V_{48} +150W_{11}+190W_{12}+162W_{13}+175W_{14}+170W_{15}+198W_{16}+183W_{17}+170W_{18}+115W_{21} +195W_{22}+167W_{23}+170W_{24}+180W_{25}+180W_{26}+184W_{27}+171W_{28}+125W_{31}+100W_{32} +132W_{33}+179W_{34}+190W_{35}+190W_{36}+185W_{37}+172W_{38}+130W_{41}+165W_{42}+137W_{43} +180W_{44}+195W_{45}+175W_{46}+190W_{47}+173W_{48}+140W_{51}+170W_{52}+147W_{53}+190W_{54} +196W_{55}+165W_{56}+195W_{57}+176W_{58}+175W_{61}+155W_{62}+150W_{63}+100W_{64}+197W_{65} +180W_{66}+196W_{67}+170W_{68}+995806\lambda \leq 1268380
$$

$$
Y_{11}+Y_{12}+Y_{13}+Y_{14}+100\lambda \leq 285
$$

$$
Y_{21}+Y_{22}+Y_{23}+Y_{24}+100\lambda \leq 575
$$

75
\[ \begin{align*}
Y_{31} + Y_{32} + Y_{33} + Y_{34} + 100\lambda & \leq 300 \\
Y_{41} + Y_{42} + Y_{43} + Y_{44} + 100\lambda & \leq 300 \\
Y_{51} + Y_{52} + Y_{53} + Y_{54} + 100\lambda & \leq 400 \\
Y_{11} + Y_{12} + Y_{13} + Y_{14} + U_{11} + U_{12} + U_{13} + U_{14} + U_{15} + U_{16} + V_{11} + V_{12} + V_{13} + V_{14} + V_{15} + V_{16} + V_{17} + V_{18} + 5\lambda & \geq 5 \\
Y_{12} + Y_{22} + Y_{23} + Y_{24} + Y_{25} + U_{22} + U_{23} + U_{24} + U_{25} + U_{26} + V_{21} + V_{22} + V_{23} + V_{24} + V_{25} + V_{26} + V_{27} + V_{28} + 100\lambda & \geq 100 \\
Y_{13} + Y_{23} + Y_{33} + Y_{34} + Y_{35} + U_{33} + U_{34} + U_{35} + U_{36} + V_{31} + V_{32} + V_{33} + V_{34} + V_{35} + V_{36} + V_{37} + V_{38} + 100\lambda & \geq 100 \\
Y_{14} + Y_{24} + Y_{34} + Y_{44} + Y_{45} + U_{44} + U_{45} + U_{46} + U_{47} + V_{41} + V_{42} + V_{43} + V_{44} + V_{45} + V_{46} + V_{47} + V_{48} + 5\lambda & \geq 5 \\
U_{11} + U_{12} + U_{13} + U_{14} + U_{15} + U_{16} + V_{11} + V_{12} + V_{13} + V_{14} + V_{15} + V_{16} + V_{17} + V_{18} + 100\lambda & \leq 550 \\
U_{21} + U_{22} + U_{23} + U_{24} + U_{25} + U_{26} + V_{21} + V_{22} + V_{23} + V_{24} + V_{25} + V_{26} + V_{27} + V_{28} + 10\lambda & \leq 310 \\
U_{31} + U_{32} + U_{33} + U_{34} + U_{35} + U_{36} + V_{31} + V_{32} + V_{33} + V_{34} + V_{35} + V_{36} + V_{37} + V_{38} + 100\lambda & \leq 400 \\
U_{41} + U_{42} + U_{43} + U_{44} + U_{45} + U_{46} + V_{41} + V_{42} + V_{43} + V_{44} + V_{45} + V_{46} + V_{47} + V_{48} + 100\lambda & \leq 400 \\
U_{11} + U_{12} + U_{13} + U_{14} + U_{15} + W_{11} + W_{12} + W_{13} + W_{14} + W_{15} + W_{16} + W_{17} + W_{18} + 5\lambda & \geq 5 \\
U_{12} + U_{22} + U_{23} + U_{24} + W_{21} + W_{22} + W_{23} + W_{24} + W_{25} + W_{26} + W_{27} + W_{28} + 5\lambda & \geq 5 \\
U_{13} + U_{23} + U_{33} + U_{34} + W_{31} + W_{32} + W_{33} + W_{34} + W_{35} + W_{36} + W_{37} + W_{38} + 10\lambda & \geq 10 \\
U_{14} + U_{24} + U_{34} + U_{44} + W_{41} + W_{42} + W_{43} + W_{44} + W_{45} + W_{46} + W_{47} + W_{48} + 5\lambda & \geq 5 \\
U_{15} + U_{25} + U_{35} + U_{45} + W_{51} + W_{52} + W_{53} + W_{54} + W_{55} + W_{56} + W_{57} + W_{58} + 5\lambda & \geq 5 \\
U_{16} + U_{26} + U_{36} + U_{46} + W_{61} + W_{62} + W_{63} + W_{64} + W_{65} + W_{66} + W_{67} + W_{68} + 10\lambda & \geq 10 \\
W_{11} + W_{12} + W_{13} + W_{14} + W_{15} + W_{16} + W_{17} + W_{18} + 100\lambda & \leq 250 \\
W_{21} + W_{22} + W_{23} + W_{24} + W_{25} + W_{26} + W_{27} + W_{28} + 5\lambda & \leq 180 \\
W_{31} + W_{32} + W_{33} + W_{34} + W_{35} + W_{36} + W_{37} + W_{38} + 100\lambda & \leq 250 \\
W_{41} + W_{42} + W_{43} + W_{44} + W_{45} + W_{46} + W_{47} + W_{48} + 10\lambda & \leq 210 \\
W_{51} + W_{52} + W_{53} + W_{54} + W_{55} + W_{56} + W_{57} + W_{58} + 100\lambda & \leq 275 \\
W_{61} + W_{62} + W_{63} + W_{64} + W_{65} + W_{66} + W_{67} + W_{68} + 100\lambda & \leq 300 \\
W_{11} + W_{21} + W_{31} + W_{41} + W_{51} + W_{61} + V_{11} + V_{21} + V_{31} + V_{41} + 5\lambda & \geq 95 \\
W_{12} + W_{22} + W_{32} + W_{42} + W_{52} + W_{62} + V_{12} + V_{22} + V_{32} + V_{42} + 10\lambda & \geq 60 \\
W_{13} + W_{23} + W_{33} + W_{43} + W_{53} + W_{63} + V_{13} + V_{23} + V_{33} + V_{43} + 5\lambda & \geq 85 \\
W_{14} + W_{24} + W_{34} + W_{44} + W_{54} + W_{64} + V_{14} + V_{24} + V_{34} + V_{44} + 10\lambda & \geq 70
\end{align*}\]
\[ W_{15}+W_{25}+W_{35}+W_{45}+W_{55}+W_{65}+V_{15}+V_{25}+V_{35}+V_{45}+5\lambda \geq 55 \]
\[ W_{16}+W_{26}+W_{36}+W_{46}+W_{56}+W_{66}+V_{16}+V_{26}+V_{36}+V_{46}+5\lambda \geq 100 \]
\[ W_{17}+W_{27}+W_{37}+W_{47}+W_{57}+W_{67}+V_{17}+V_{27}+V_{37}+V_{47}+5\lambda \geq 102 \]
\[ W_{18}+W_{28}+W_{38}+W_{48}+W_{58}+W_{68}+V_{18}+V_{28}+V_{38}+V_{48}+5\lambda \geq 83 \]

And all variables are nonnegative

Following is the solution results. The value of \( \lambda = 0.8378 \) that mean we reach the achievement level of 84%. Our supply chain decision be in the following table.

Table 3.9

<table>
<thead>
<tr>
<th>Objective Function</th>
<th>Optimum Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>434031</td>
</tr>
<tr>
<td>Delivery time</td>
<td>39064</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>0.8378</td>
</tr>
</tbody>
</table>

Table 3.10: Transported quantity from supplier to plant in tonnage.

<table>
<thead>
<tr>
<th>Supplier</th>
<th>Plants</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>G1</td>
<td>G2</td>
<td>G3</td>
</tr>
<tr>
<td>A</td>
<td>46.89</td>
<td>--</td>
<td>--</td>
<td>154.32</td>
</tr>
<tr>
<td>B</td>
<td>--</td>
<td>269.19</td>
<td>124.58</td>
<td>97.43</td>
</tr>
<tr>
<td>C</td>
<td>--</td>
<td>216.21</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>D</td>
<td>--</td>
<td>--</td>
<td>216.21</td>
<td>--</td>
</tr>
<tr>
<td>E</td>
<td>--</td>
<td>--</td>
<td>26.50</td>
<td>--</td>
</tr>
</tbody>
</table>
Table 3.11: Transported and produced quantity from plant to retailers in tonnage.

<table>
<thead>
<tr>
<th>Plant</th>
<th>Retailers</th>
<th>M1</th>
<th>M2</th>
<th>M3</th>
<th>M4</th>
<th>M5</th>
<th>M6</th>
<th>M7</th>
<th>M8</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1</td>
<td></td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>G2</td>
<td></td>
<td>--</td>
<td>--</td>
<td>57.43</td>
<td>--</td>
<td>59.189</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>G3</td>
<td></td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>78.37</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>G4</td>
<td></td>
<td>--</td>
<td>--</td>
<td>31.75</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
</tbody>
</table>

Table 3.12: Transported and produced quantity from plants to warehouses in tonnage.

<table>
<thead>
<tr>
<th>Plant</th>
<th>Warehouses</th>
<th>N1</th>
<th>N2</th>
<th>N3</th>
<th>N4</th>
<th>N5</th>
<th>N6</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1</td>
<td>N1</td>
<td>9.18</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>28.51</td>
<td>--</td>
</tr>
<tr>
<td>G2</td>
<td>N2</td>
<td>--</td>
<td>185</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>G3</td>
<td>N3</td>
<td>--</td>
<td>--</td>
<td>86.75</td>
<td>18.37</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>G4</td>
<td>N4</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>210.81</td>
<td>--</td>
<td>--</td>
</tr>
</tbody>
</table>

Table 3.13: Transported quantity from warehouses to retailers in tonnage.

<table>
<thead>
<tr>
<th>Warehouses</th>
<th>Retailers</th>
<th>M1</th>
<th>M2</th>
<th>M3</th>
<th>M4</th>
<th>M5</th>
<th>M6</th>
<th>M7</th>
<th>M8</th>
</tr>
</thead>
<tbody>
<tr>
<td>N1</td>
<td></td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>N2</td>
<td></td>
<td>99.18</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>76.62</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>N3</td>
<td></td>
<td>--</td>
<td>68.37</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>N4</td>
<td></td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>84.86</td>
<td>29.56</td>
<td>87.18</td>
<td>--</td>
</tr>
<tr>
<td>N5</td>
<td></td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>19.32</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>N6</td>
<td></td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
</tbody>
</table>
3.8 Conclusions

This chapter has proposed fuzzy linear programming model for supply chain network by integrating procurement, production and distribution planning activities into a multi-echelon supply chain network. The fuzzy model integrally handles all the epistemic uncertainty sources identified in supply chain network problem. This model has been tested by using numerical data. In this model, two objectives were considered: (1) Minimization of cost and (2) Minimization of total delivery time.

DM has demonstrated the effectiveness of fuzzy linear programming approach for SCN under uncertainty. After solving Multi-Objective Supply Chain Network problem we get a best compromise solution. The proposed fuzzy formulation is more effective than the deterministic methods for handling real situations where precise or certain information is not available for SCN. Additionally the fuzzy model behavior has been seen to be clearly superior than the deterministic model, and has obtained inferior total costs and delivery time.