CHAPTER-07

MODELLING AND PRICING OF RAINFALL DERIVATIVE

ABSTRACT:

Advent of weather derivatives is a recent development in agricultural risk management. One such product is index based or actual rainfall based insurance. The product is similar to an option, whereby a farmer/producer pays an upfront premium and is paid a predetermined amount linked to the amount of rainfall received and measured during a specified future period. This option is designed to protect the farmers’ income against scarce or excessive rainfall which is likely to have an adverse effect on the output and hence income.

This study proposes a model for calculating fair insurance premiums. The payoff is designed to compensate loss in real income rather than output. The model is applied to two agricultural commodities, wheat and rice. It is found that there is substantial loss in output in wheat production due to uncertain rainfall. In the case of rice, no such output loss occurs. A model he can be designed for compensation of loss of income due to production loss or for loss in real income. If applied for real income compensation, it can be designed for low premium and be affordable to small farmers too.
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7.1 CONVENTIONAL RAINFALL RISK MANAGEMENT

A large population is engaged in agriculture and related activities. In India, even in 2010, almost 55 per cent of employment is in the agriculture sector though it contributes only about 17 per cent to India’s GDP. Many of them are landless or have low land holding. Thus, a large population has income below national average.

Indian agriculture is rainfed and more than 63 per cent of crop is rain dependent. Rains are unpredictable and hence agricultural output varies from year to year. This has an adverse impact on farmers.

Suppose an economic agent holds a valuable asset whose value at a future time is uncertain and therefore he faces value risk. Farmers are unique producers since they face several risks over which they have no control. They take risks and invest in inputs at the beginning of the production cycle. Thereafter they are left to the uncertainties of the rainfall, climatic conditions, and pests. Most importantly, a farmer who grows specific crop on his land is exposed to the risk of losing a large portion of potential output due to uncertain rainfall. The risks they face are, both, loss of production and price risk. This impacts the income of the farmer. Since the proportion of labour force in agriculture is very large, there is further impact on demand for other goods and income and on total national income.

7.1.1 RAINFALL AND CROP OUTPUT

Annual output of different crops since 1951 is given in Annexure-A.4. Monthly rainfall since 1951 is given in Annexure-A.5.

Crops may be grown in either kharif or rabi season or both. Data on production separately for kharif and for rabi crops could not be obtained. Hence total annual output is indicated.
Output is not only a function of rainfall; it is also dependent on labour input, amount of fertilizers, seed quality, extent of mechanization, economies of scale, and amount of land irrigated. Aggregate data or disaggregated data are not reported for different products. Thus, for example, data on total agricultural labour may be available but they may not be available for rice, nor would data be available for different sizes of the farm. Since these data are important determinants of the output and since they vary over time, their contribution is determined by using a multiple regression with time as proxy for these inputs.

Water is an essential input for agricultural products. Requirement of water is different for different crops and to some extent for the soil type. Rains are the major supplier of water for agricultural and other uses. However, rainfall is a stochastic variable across years and also across days within a year. Unlike other inputs, input of water is not under the control of farmers. To reduce the variability of water supply and to ensure minimum supply, water is supplied through irrigation. Irrigation facility can be a substitute for rainfall. If rainfall is excessive, crop may get damaged with or without irrigation but if the rainfall is scarce, irrigation could substitute rainfall. Even when the rainfall is normal but not exactly as per requirement over time, irrigation helps. Hence amount of irrigated land is taken as a separate measure of input and its data is reported along with production data in Annexure A.3. However, even today in 2010, percentage of net sown area under cultivation is only about 40%. Erratic supply of water has an effect on output as the data and calculations prove.

The data show that there is a close relationship between difference in actual and expected output and difference in actual and expected rainfall. However, it is not just the total rainfall during a monsoon season which is important; even the distribution of the rainfall over the monsoon season is critical in determining total output. The monsoon season can be broadly divided into two phases: the sowing season (say first two months of a total of four month monsoon season); and the harvesting season (latter two months). During the first phase, rainfall should at least be a certain quantity and in the second phase, rainfall should be below a maximum
acceptable quantity. If the rainfall is below the threshold limit in the first phase, output will gradually be lower and if it is above a threshold level for the latter phase, again output will drop.

As observed, the monsoon plays a crucial role as major factor towards income generation, but it also brings in variability in income. Average annual rainfall in the monsoon season was 904.25 mm and standard deviation 94.52 mm in the sixty year period 1951-2010. The coefficient of variation of rains over 1951-2010 period was 10.45%. The reported data are for the entire country. However, the average and variance may be different in various states and regions as also for various crops. Effect on crops would be determined by regional variation. However, this study is based on macro data and is for designing a single product for a single crop for the entire country.

7.1.2 REMEDIAL RISK REDUCTION MEASURES

7.1.2.1 Conventional Measures

The government provides irrigation facility to reduce the risk of inadequate rainfall. These may not protect against heavy rains and floods. In any case, vast lands, even today, are un-irrigated. Water required for complete irrigation is simply too large.

At individual level, there is only limited remedy available to farmers against these risks. Farmers have used different strategies to reduce the risk, with limited success. Some of these strategies are conservation of water, use of seeds which do not require lot of water or are not affected severely due to certain amount of rain variation, use of different farming pattern and different crops, joint family system, storage of food-grains, off-farm income generation, and livestock for additional income. Another practice is to apply only a limited quantity of inputs and increase it as rainfall increases to optimal level. Most of these strategies work better for bigger farms and larger farmers.

However, farmers face significant problems in implementation of these strategies. One, a farmer may not have adequate funds, say, for storage or for buying inputs. In
the years when output is high, farmers may store the food grains or feed the same to the livestock. In the years of scarcity, the stored food can be used or livestock be sold. However, the stored food runs the risk of being destroyed and the livestock sold at lower prices during the crisis period as the supply may be higher than the demand.

7.1.2.2 Insurance

To mitigate or even eliminate the risk, one of the options is use of insurance. The rationale for the insurance is to protect the producer’s income or wealth.

An insurance product is offered to the beneficiary whose payoff is a function of certain defined event occurring in future. If the occurrence of the event can be predicted with certainty, there is no risk of holding the asset. However, if the event cannot be predicted with certainty, but its probability distribution is known, distribution of the stochastic payoff, and hence the expected value of the payoff can be determined. This understanding results in the offer of a product which, if purchased by the holder of the asset, helps in mitigating or eliminating the risk of holding the asset.

Insurance of assets – human as well as physical, in place or in the process of generation – is a standard method of protection against quantity risk. Life insurance, automobile insurance, factory insurance, insurance against potential damage to productive capacity are widely available products. The farmers also face the risk of output being less than expected and hence the income being less than expected. To guard against the risk of reduced production, crop insurance is available as an agricultural risk management tool the world over. Farmers sell their risk at a price, called insurance premium. But again, the risks faced by different farmers may not be correlated. The damage due to weather could be extensive or intensive.

7.1.2.2.1 Crop Insurance in India
The Government of India has offered Publicly Administered Insurance Scheme for farmers since 1972. The scheme was designed to protect the farmers against weather risk. The insurance offered was individual based.

**7.1.2.2.1.1 Comprehensive Crop Insurance scheme (CCIS)**

The Government, since 1985, started offering Comprehensive Insurance by way of Comprehensive Crop Insurance scheme (CCIS). CCIS was also individual based scheme. CCIS was offered from 1985 kharif Season to 1999 kharif season. The scheme was designed to protect the farmers and be financially viable. However, it never attained financial viability. During its regime, the ratio of claims to the premium paid was staggering 572 %, loss amounting to Rs. 18.4 billions. The reason is obvious: the premium charged was far too low compared to the claims made. (1-2% of the sum assured versus 9 % of sum assured). CCIS was compulsory for those farmers who had borrowed against crops in the states where the scheme was implemented (22 participating states). The threshold yield is the moving average yield of last 5 years. Those who do not borrow can also buy the insurance.

**7.1.2.2.1.2 National Agriculture Insurance scheme (NAIS)**

Since 1999-2000 rabi crop, CCIS has been replaced by The National Agriculture Insurance Scheme (NAIS). Under the scheme, the premium is different for different locations based on “Area Approach”, (and not individual based approach) and is on acturial basis. NAIS is compulsory in implementing states for the loanee (Punjab and Haryana do not participate as it is believed that the risks are low—probably a wrong assumption for all). Three different level of indemnity are available to account for diversity in climatic conditions. The insurance is subsidized (due to absence of financial viability).

Under NAIS, the farmers are not indemnified for the yield loss. Under the Area Approach used by NAIS, a set of plots in a given area is used for benchmarking the output in a given period. The farmers, who buy the insurance, are indemnified for the difference between the threshold yield and the yield on the selected area used
for benchmarking. This Area approach has several limitations: the farmer who actually suffers the loss may not be paid anything because the “benchmarked plots” did not suffer difference from the threshold, and vice versa. However the plus point of the scheme is the elimination of the moral hazard-a farmer will have no incentive to deliberately suffer a loss.

NAIS has not been a success story for several reasons. One reason is the problem of adverse selection. Two, those who have borrowed for production activity are forced to purchase insurance under NAIS. It is possible-as is now the case with futures markets- that the farmers/producers have better means for risk management. Diversification, use of futures markets, fragmenting land holding for different crops, increased off-farm employment, Savings are some of the other methods for risk management now available and used. For example, many of the farmers have their sons working in nearby cities, thus providing a welcome diversification.

Different farmers may have different utility functions (different risk appetite). Treating them with same yardstick is not an optimal solution. Again, only 16 states are participating in NAIS and some of the more prosperous states (though they may have farmers who need insurance) do not participate –adverse selection.

Indemnity under NAIS is not for loss in the yield per unit of land, as is already discussed above. Output from farms are compared with an “Area average”output. Indemnity is, thus, not individualized though the premium is. Farmers face uncertainty even after buying insurance, since there is uncertainty due to “Yield Difference”.

**7.1.2.2.1.3 Private Insurance Products**

Private crop insurance has limited existence the world over unless the private insurance is subsidized. In presence of subsidized government insurance, private insurance does not flourish. Another reason for low private participation is the control exercised by the government on many aspects of the agriculture (e.g., prices
of inputs (fertilizers) and outputs, inter-state and inter-nation trade, and so on). The control distorts the market and production.

The insurance companies face adverse selection problem-as with any insurance product. The farmers are more knowledgeable about the risks they face than either the government or the private players. The farmer who is likely to be affected most will take the maximum insurance whereas the farmer who is the least affected may not take any insurance. This will either make the product financially unviable or too costly to be welfare optimal.

However, in recent times many private insurance companies are offering insurance products against quantity risk due to weather.

### 7.1.2.2.1.4 Insurance in Other Countries

Different countries follow different crop insurance policies. Research (Turvey and Islam (1995)) has shown that the “Area approach” is neither efficient nor equitable. Most benefits accrue to those farmers whose production yield is similar to that of the “area Average”. Also, since the insurance is compulsory, reducing adverse selection but resulting in cross subsidization (better off farmers subsidizing worse off farmers).

### 7.1.2.3 Limitations of Conventional Measures

The conventional insurance products have, however, several limitations. With changing socio-economic dynamics, many conventional measures, like risk sharing within joint family, may not be available or have limited use. Insurance is costly, and there are associated problems of moral hazard and adverse selection.

Insurance is not necessarily the first best option. Better options could be one or combination of more than one alternative, subsidies for inputs, guaranteed minimum price, markets for trading risks, and so on. However, none of these could be for income protection, and if any, it is at a cost of private incentives.
7.2  **Weather Derivatives and Rainfall Insurance**

A new breed of insurance products, based on the principles of derivatives, have been developed and offered since 1997 for mitigating weather induced risk. In India such a product was pioneered by ICICI Lombard in 2003 and since then several institutions offer similar products. Though products based on the temperature as underlying weather variable dominate this market in USA, temperature based products are not currently available in India.

Weather derivatives are of recent origin, first offered in mid-1990s in USA. This is the time when energy and utility companies, faced with competition and varying revenues due to weather uncertainty wanted to hedge not only the price risk but also risk arising due to weather conditions. Weather (temperature) affected the demand for energy as also affected the price due to varying demand. The first deals were struck in 1996, the first major deal being one between Enron and Aquila Corporation in 1997 for temperature variation.

The underlying variable for the first offered products was temperature. Later products with underlying variable being rainfall, snowfall and humidity have also been offered. However, even today, weather derivatives are most prominent against temperature variation from the expected and its consequences. More than 90 % of weather insurance products are related to temperature though products against rainfall variation are steadily gaining ground.

Initially the market was Over the Counter market (OTC). Chicago Mercantile Exchange, USA was the first exchange to offer exchange traded weather derivative products in 1999. The market has grown rapidly since then. American Agrisurance Inc, NatSource Inc, World Wide Weather Insurance Inc., are some of the major companies which offer insurance products based on weather.
In India, the first weather derivative was offered by ICICI Lombard in 2003. Since then several financial institutions are offering these products. In India weather derivatives are OTC products.

**7.2.1 WEATHER DERIVATIVES**

Crop insurance is neither efficient nor equitable, and has problems of moral hazard and adverse selection.

In the last fifteen years there have been efforts, both by researchers and insurance companies to develop derivative based products which can provide insurance against adverse effects caused by uncertain weather. A weather derivative is a financial instrument that allows trading of risk arising from uncertainty of weather. They may be purchased and sold for speculative profits and/or to hedge the risk. Limitations of traditional insurance products are either absent or are significantly lower in the case of Weather derivatives.

Weather derivatives are based on a weather variable, such as temperature or rainfall, as the underlying. The underlying could also be snowstorm, etc. Its pricing is similar to the pricing of financial option. Pricing of weather derivative is not based on principle of arbitrage, however, since the underlying can not be traded. Added problem is the lack of price discovery since the market is illiquid and the product is OTC product. Hence, the weather derivatives are priced like insurance products, based on historical data and actuarially neutral pricing. The periodic movement (hourly/daily/monthly) of the weather variable over several time periods (years) is collected, and its distribution is determined. Alternately, the risk may be calculated for an index (say, monthly index of temperature). The effect of the variable or its relation to the agricultural output is determined the consequent loss of output is determined. Actuarially neutral premium is then determined. Variable is so chosen that its measurement is possible and verifiable. For example, rainfall and maximum temperature during the day.
Weather derivative products are options or swaps contracts. A buyer of the product pays a premium (similar to insurance premium / call or put option value) upfront. Contingent on occurrence of an event, pre-determined payoff is made by the seller to the buyer. An event is pre-defined. Event could be, for example, rainfall below a specified limit during sowing season. The limit value of the variable is set and is the threshold value, an exercise price in option language. If threshold /exercise value of rainfall is above the actual value of the rainfall, then the seller (that is the bank) will pay a pre-determined amount to buyer. The amount paid could be a function of value of the variable, say difference between exercise/threshold value and actual value. However, if the exercise value of rainfall is below the actual value, the seller is not required to pay anything to the buyer. Thus it is similar to a “Put Option”. Similarly, in the harvest period, if exercise value of rainfall is below the actual value of the rainfall then the seller will pay a pre-determined amount to buyer. However, if the exercise value of rainfall is above the actual value, the seller is not required to pay anything to the buyer. Thus it is similar to a “Call Option”. Similar products are designed for temperature as the underlying. Though this method is similar to “Area Approach”, its advantage lies in reduction in moral hazard, reduction in adverse selection and reduction in transaction cost.

### 7.2.1.1 Features of Weather Derivative

The weather derivative contracts are standardized contracts. They have the following features:

1. Basic option based pricing model for weather derivatives requires defining and evaluating values of the underlying variable, e.g., rainfall or temperature.
2. Underlying variable value and Period for which the underlying variable is considered. This could be a day, a month, a season (say, the entire rainy season), or a year.
3. The pre designated weather station where the variable data is collected for the specified period.
4. Exercise value of the weather variable
5. Cap and floor for the variable and/or cap on the amount of payment
6. Time to maturity
7. Riskless rate of interest
8. Volatility of the underlined weather variable
9. Tick size and amount of payment, in rupees, per unit of tick

Rainfall derivatives, in addition, are also characterized by the following:

- The rainfall derivatives are call options and/or put options.
- The payoffs may not depend on the amount of direct losses. The payoffs are related to average loss suffered on pre-designed production units within area. These may or may not equal the actual losses suffered by the farmer.
- Rainfall derivatives are instruments for hedging the quantity risk. Derivative products are generally instruments for hedging the price risk.

### 7.2.2 LIMITATIONS OF THE DERIVATIVE PRODUCT

Like any insurance product, this product also has its limitations.

The product is designed to compensate only for the loss in output due to inadequate or excessive rainfall. It is not for managing risks arising out of other sources of risk to the crop output, viz. pests, wrongful applications of the fertilizers, and poor quality of seeds. Farmers may not understand the limited scope of this product.

The product is not easy to understand for a layman. Uneducated farmer may have difficulty in appreciating the product and may not buy the product.

Farmer, who may purchase this product, may not understand how to calculate the rainfall and keep tab on it.

As for any other insurance product, private insurance players, at least in India, may not be trusted.
Selection of a weather station is not easy. A weather station should be representative receiver of the rainfall for the area covered. That may require large number of weather stations across the area. As of today such numbers of weather stations may not exist. The data at weather stations should be scientifically collected and collated. Trained manpower is required for the same. A large infrastructure and manpower is thus necessary for success.

The crop output is not perfectly correlated with the rainfall. Hence, the risk of loss of output may be replaced by the basis risk.

The payment is made on the basis of rainfall measured at a designated weather station. Rainfall on a particular farm could be different as it may not be located in close vicinity of the weather station. In such a case, actual loss suffered by a farmer could be different, substantially different, than that calculated. (For this reason, insurance is typically not sold to households more than 30 km from the rainfall station).

### 7.3 PURPOSE AND OBJECTIVES OF THE STUDY

Rainfall derivative insurance is being offered by more insurance companies across more regions and more products. However, the pricing of the products is not clear. This may partly be due to the fact that the products are proprietary and partly due to the fact that the products may not be optimally and rationally priced.

Three broad research questions are:

1. What is a suitable statistical model for forecasting rainfall?

2. What is a suitable structure for rainfall options contracts?

3. What is an appropriate pricing methodology for rainfall option contracts?

This study answers these questions for one product, wheat, by estimating the expected loss and designing an appropriate product.

Thus objectives of this study are several folds.
1. The study aims at quantifying the risk inherent in agricultural production due to uncertainty of rainfall. Quantification is in terms of quantity risk as well as income.
2. The study aims at modeling and designing a derivative product for insurance against rainfall uncertainty. This is done for one product-wheat, on all India basis.
3. The study aims at testing the designed model.

7.4 LITERATURE REVIEW—WEATHER RISK MANAGEMENT

Substantial research efforts have been directed at study of weather derivatives in the last fifteen years. Initially the research was mostly focused on the derivative insurance products for stochastic temperature which affected the demand for several products, e.g., energy products, and hence the revenue of companies supplying these products. Later research also included derivative insurance products to manage risk arising due to unpredictable rainfall. Hence, the research can broadly be divided in three groups:

A. Developments of Derivatives for Managing Risk
B. Research on Weather Derivatives
C. Research on Rainfall Insurance (specifically)

7.4.1. DEVELOPMENTS OF DERIVATIVES FOR MANAGING RISK

Derivatives are products whose price is a function of an underlying asset or variable. Taking simultaneous position in the underlying asset and the derivative product can help hedge the risk. Option, a derivative product, is concerned with the uncertainty of future outcomes resulting in risk of loss of the value of the underlying. Options, are priced so that expected profits are zero. Expected profits are function of the
stochastic payoff. Option value is dependent on possibility of favorable outcomes in the future which may result in positive payoff beyond the expected value. Bachelor (1900) is probably the first to analyze and determine the value of stock option. This was later extended by several economists. Sprenkle (1961, 1964) provides a formula for valuation of European call option considering log normal distribution of the price of the underlying stock, amount of risk aversion and the drift of the Brownian motion of the price. Samuelson and Merton (1969) use utility theory and developed a general equilibrium model for the option pricing. This was followed by the seminal work of Black and Scholes (1973). Original specification of the underlying stock price behavior was that the price distribution is log normal and that the price follows random walk and are weakly efficient. Rendleman et al (1979) and Cox et al (1979) use binomial distribution as an approximation of normal distribution, especially for discrete process.

Option valuation could also be based on actuarial methods. In actuarial method, the payoff at a future date is estimated as difference between expected value and actual value at a future date of the designated variable (say health) and its known effects on the value of the buyer, and risk averseness of the buyer.

7.4.2 WEATHER DERIVATIVES
Before weather derivatives were developed, several researchers had questioned the effectiveness of crop insurance. In extreme cases, especially, insurance products were not even available. Skees and Barnett (1999), in a conceptual paper discuss the challenges posed by catastrophic or systemic risk, with special emphasis on agricultural systems and rural population. Catastrophic risks-including droughts, floods, snow storms among others—are low frequency but result in heavy losses. The authors maintain that risk sharing markets for such risks are incomplete and/or inefficient. General strategies for managing risks, diversification, self-insurance, investment in loss mitigating capital, are generally not applicable to these types of risks. Hence, government intervention is required to create the markets or improve the efficiency. The government could directly provide the insurance or subsidize the private efforts. Capital markets could also create instruments, with or without government support, to share and mitigate or reduce these risks.

Derivative products have been developed for managing risks to value of assets arising from uncertain weather. When a product is designed to manage the risk from uncertain weather, there is no associated underlying variable price, and variables representing weather may have different distribution properties than those of, say, stocks. However, similarities are striking. As the asset price evolves in the future, based on its distribution, so does weather variable, e.g., rainfall. (Though, unlike asset price, weather variable is mean reverting.) Future values of weather variable can be estimated based on expectations, which in turn are driven by some distribution. Campbell et al (2003) assert that weather variables have large and unpredictable fluctuations resulting in adverse effects. Hence, if one wishes to mitigate the risk arising out of weather, expectations about its future values must be formed. Future values may be a function of several atmosphere related variables. But these are difficult to estimate for meaningful large horizon. Best prediction may only be for a few days whereas farmer may be exposed to risk of uncertain rainfall in, say, three months period. Hence, alternative approach of expectations based on past data may be used.
Striking feature of effects of weather is that the effect is a result of one time unexpected event (e.g. heavy rainfall in a day) and more often a result of accumulated value of the underlying weather variable over a period. Hence a weather derivative may be designed for a specific event or for an index, say of rainfall over a month. Campbell and Diebold (2003) have suggested that the adverse effect of weather is due to unexpected weather over several days or even a month rather than a single day. This necessitates that a weather derivative should be a function of value of weather index, though it can also be a function of specific event (very high temperature on a day resulting in high demand for energy).

Second feature of the weather derivative is that occurrence of a specific event does not directly affect the value of concerned individuals. The weather variable has direct effect on an intermediate variable that is the output of agriculture product or demand for energy product, etc., which in turn has effect on the welfare of the concerned individual. However, this is also indirect since weather does not act alone and acts in liaison with other variables, e.g., use of seeds and fertilizers. Hence an interesting issue is whether the derivative product should be designed for a value of a weather variable or for a value of production. In fact even a value of production may not be a relevant measure as there is an additional effect of price elasticity of demand. In fact, for this research I have designed a derivative product separately for the outcome of weather variable and for outcome of real income.

Market price of risk due to weather is difficult to assess because unlike stocks weather variable can not be marketed and hence the arbitrage model of pricing can not be used to price the option. Cao and Wei (2004) maintain that weather risk can not be hedged. Cao, Li and Wei (2004) simulate the option price and then add risk premium. Cao and Wei (2004) suggest that the risk can be assessed by linking the weather variable and economy and taking weather variable as a source of risk to the economy. Richards et al (2004) determine the correlation between temperature and aggregate output and uses Lucas general equilibrium model to determine market
price of weather (temperature) risk. A more simple approach is Burn analysis. Herein (German and Leonardi (2005)), option value is assessed as the average of the payoff on historical observations of weather variable.

Zeng, L. (2000) has discussed the fair value for an option based derivative. He has first discussed the type of contract that can be designed and offered and the payoff thereof. Fair value of the option contract depends on the demand and supply. However, true price for a weather derivative is difficult as there is no trading of the underlying asset and hence no arbitrage models, like Black Scholes do not really hold.

He has formulated basic option based pricing model for weather derivatives. He has defined $W$ to be the underling weather index or variable, $S$ for the strike price of a put or a call or a collar, $k$ as pre-negotiated tick value for a linear payout scheme, $P_0$ as the fixed payment, and $p$ as the premium for a call or a put.

Another method for pricing of the product is actuarial method. Actuarial method is based on the past data. This data is analyzed to identify distribution.

If the past data are sufficiently large, Monte Carlo simulation can also be run, using these data.

Expected cost of the contract to the offerer, $C$, equals,

$$C = E(x) + Oh,$$

where $x$=payout, $E(\cdot)$ is expectation operator and $Oh$ is the overhead expense.

Hence, for breakeven, total premium to be charged should equal $C$. $C$ of course would depend on $x$, which is turn would depend on probability distribution of the rainfall and the expected payoff.

Cumulative probability distribution function of the payout can be determined and plotted against expected payoff.
Actual price charged may be higher due to different risk tolerance of market players and need to pay for taking the risk to the risk averse seller. Since different sellers have different risk tolerance, in aggregate, average of the market is paid.

Brix et al (2002) discusses valuation of weather derivatives by statistical and acturial methods based on historical data has developed a general model of index weather derivative and assuming different distributions for weather variable, have derived the payoff function. Specifically, for temperature variable and for normal as well as negative binomial distribution, expected payoff is derived. The authors, after de-trending, have developed an autoregressive model for the variable itself, and the payoff function for such a model.

Jewson (2002) has developed models of pricing of weather derivatives as a combination of actuarial and arbitrage methods. He also calculates value at risk for a portfolio of weather derivatives.

Cao et al (2003) have discussed the weather derivative markets as it existed in 2003. It surveys the market, discusses the main products and their usages.

Jewson and Zervos (2003) have developed an analogus of Black-Scholes equations for weather derivatives. The authors have shown that the arbitrage free price of weather options is equal to the expected payoff under objective probabilities for a chosen stochastic process.
Cao and Wei (2004) have proposed a framework for valuation temperature derivatives. The model postulates daily temperature. They have proposed and implemented a valuation framework for temperature derivatives and studied the significance of the market price of weather risk. They have generalized the Lucas model of 1978 to include the weather as another fundamental source of uncertainty in the economy. Daily temperature is modeled by incorporating such key properties as seasonal cycles and uneven variations throughout the year. The temperature variable is related to the aggregate dividend or output through both contemporaneous and lagged correlations, as corroborated by the data. Numerical analysis shows that the market price of weather risk is significant for temperature derivatives.

Jewson (2004) has discussed qualitatively the pricing of weather derivatives. Richards et al (2004) presents a general method of pricing the weather derivatives, specifically for the temperature variation. The temperature is assumed to follow mean reverting Brownian motion with ARCH errors. The model is applied to crops in Fresno, California.

Jewson (2004) has discussed qualitatively the pricing of weather derivatives. Xu et al (2008) investigate the optimal design of weather bonds for the purpose of reinsurance of the weather derivatives. Johnson (2009) examines the value of the National Rural Employment Guarantee Act (NREGA) as a tool to mitigate or reduce the weather related risks. A parametric equation is tested for the significance of different variables on the income. Monthly data are collected from eight districts of Andhra Pradesh for the period 2007 (before NREGA) and 2008 (after NREGA implementation). Author concludes that NREGA does help reduce the risks arising from uncertain rainfall

### 7.4.3 Rainfall Insurance and Rainfall Derivatives
Crop insurance provides farmers insurance against possible loss of crops due to vagaries of monsoon. Insurance is generally offered by the governments and is subsidized, rather than actuarially priced. This is deemed necessary to protect the poor farmers.

Prior to the development of derivative products in late 1990s, insurance companies did offer insurance products for managing risk due to too little or too much rainfall. Such products are based on actuarial model. Determination of what length of past data is most appropriate for estimating future outcomes, what constitutes a threshold rainfall, what should be the centre whose rainfall reporting is to be considered for calculations, how to determine the loss function, are some of the issues that have been discussed in the literature. Karl (1988) and Changnon et al (1989) are illustrative of the complexity of designing appropriate insurance instruments against production and income loss in agricultural sector due to uncertainty of rains.

Changnon et al (1989) is one of the insurance pricing models for crops which accounts for stochastic rainfall. The study focuses on the climatic conditions, effects of the climatic conditions on crops and insurance needs of the farmers and develops a model based on the rainfall data from 1950 to 1984. The study finds that, on an average, the effects in the long run are within 5 percent of the average, however, short term effects are as high as 20 percent in several states of the USA. The study recommends that an insurance product for protection against uncertain rainfall should be designed using the most recent 25 year rainfall.

The research on rainfall derivatives has focused on three key areas:

1. Statistical or otherwise modeling of relevant weather variable, e.g., daily rainfall,
2. Quantification and determination of relationship between the rainfall and crop production, and
3. Development of theoretical pricing model which is consistent and empirically testing the same
Specifically for rainfall, the effect is due to different outcomes at different times in a single season. At the beginning of the season, shortfall in rainfall may have adverse effects whereas in the second half of the season it is the excess rainfall that has an adverse effect. However, the effect is combined on a single outcome, crop production in a given season. Hence the design of a derivative product has to have two parts or two different products.

Related to these issues is the determination of strike value. There is no consensus in the literature on the strike price for weather derivatives. A strike price is one beyond which the payment would be made to the holder of the option. There are several problems in this determination. First, the weather variable has only indirect effect on the value. Output from agricultural holding is not only a function of rainfall but also of several other inputs. Second, it is not clear as to what should be the compensation. Should it be for deviation of rainfall or deviation of output or deviation of income or deviation of real income? Thirdly, the rainfall does not follow normal or log normal distribution. The Black schools model is valid for log normal distribution of the underlying. Hence, several researchers (Alaton et al (2003), Cao, Li and Wei (2004), German and Leonardi (2005) are some examples) are silent on the strike price.

Similar is the problem of determining the tick size and tick value. Normally one unit of rainfall for a day or one unit of index rain for a period is chosen as tick size. But tick value could vary from region to region. Even for a region, tick value is different for a field near the measuring station and a field far away from measuring station.

Wilks and Wilby (1999) have reviewed the statistical weather models and their evolution over time. Turvey (1999) is probably the first published paper on pricing of rain insurance. He has developed a basic framework for pricing rainfall insurance. Specific event risk is defined as either shortage or excess of rainfall over a specified period. A farmer maximizes his profits form agricultural produce. The author defines a profit function
conditional on the event of rainfall. The profit is defined as:
\[ \pi(X/r) = P^*Y(X/r) - C(X/r) \] (7.1)

Where, X=Input set, \( P^* \)=Profit and r=rainfall

The farmer chooses the input set, X, such as to maximize the profit. I.e. set the marginal profit at zero. \( \delta\pi(X/r) = P^*\delta Y(X/r) - \delta C(X/r) \) (7.2).

Since this is a convex function and C(.) is a concave function, \( \frac{dp}{dr} > 0 \) for \( r<r^* \), \( \frac{dp}{dr} = 0 \) for \( r=r^* \) and \( \frac{dp}{dr} < 0 \) for \( r>r^* \).

Farmer needs a minimum profit \( \pi^* \). Then, the insurer can provide two options: a Call Option, a Put Option or a Collar (combination of both). The pricing of these options is:

\[ V_{\text{put}} = \int (\frac{dp}{dr}) (r^*-r) f(r) dr \quad \text{for } r<r^* \] (7.3)

\[ V_{\text{call}} = \int (\frac{dp}{dr}) (r^*-r) f(r) dr \quad \text{for } r>r^* \] (7.4)

Where \( f(r) \) is the probability distribution of rainfall.

The study tests this model for five different options on payoffs for rainfall in three regions of Ontario, Canada. The rainfall distribution is derived from rainfall data for the 1892-1996 period.

Author has calculated premium based on Burn rate Model, Normal Curve Model, and Maximum Payout model.

Jewson (2003) has developed a closed form expression for the pricing of the derivative product for rainfall insurance.

Cao, Li and Wei (2004) address the valuation aspect of weather option when the underlying variable is rainfall. They suggest that an approach focused on risk and return would be more meaningful. That is, the expected payout based on derived distribution of rainfall is, after adding for risk premium and profits, is the correct value of the price of an option.
Sinha et al (2005) have derived payoff based on the Jewson (2003) model and the model is tested for annual rainfall in Mexico City area. Data from 1952 to 2004 are used. The authors assume normal distribution of the rain and have calculated average and standard deviation from the past data. The following payoff is suggested:

For a long call, payoff is defined as:

\[
P(r) = \begin{cases} 
0 & \text{if } r \leq K \\
D(r-K) & \text{if } K \leq r \leq R \\
R & \text{if } r \geq R 
\end{cases}
\]

Where \( r \) = rain index, \( D \) is the tick \( R_1 \) and \( R_2 \) are the upper and lower limits of index, \( K \) is the strike price and \( R \) is the limit expressed in currency unit.

For a long put, payoff is defined as:

\[
P(r) = \begin{cases} 
0 & \text{if } r \geq K \\
D(r-K) & \text{if } K \geq r \geq R \\
R & \text{if } r \leq R 
\end{cases}
\]

Where \( r \) = rain index, \( D \) is the tick. \( R_1 \) and \( R_2 \) are the upper and lower limits of index, \( K \) is the strike price and \( R \) is the limit expressed in currency unit.

The three different strike prices of rainfall are suggested as:

\[
K_1 = \mu + 0.5 \sigma \\
K_2 = \mu + 3 \sigma \\
K_3 = \mu + 1.5 \sigma 
\]

The authors, assuming normal distribution, derive the expected payoff as:

\[
\mu_{X_{\alpha \lambda \gamma}} = D^* \sigma^* n_k + D^*(\mu-K)^*(1-N_k) \quad \text{--------- (7.5)}
\]
\[ \mu_{\text{Call}} = D^*\sigma^*n_k + D^*N^*(K-\mu) \quad (7.6) \]

\[ \mu_{\text{Swap (Collar)}} = D^*(\mu-K) \quad (7.7) \]

where, \( n_k \) = sample size upto \( k' \) and \( N_k \) = Cumulative distribution upto \( k' \)

The authors calculate the payoff using Burn analysis and Index Model.

Mubhoff et al (2006) have developed an option model with daily rainfall as the underlying weather variable and tested the same for application to crops in Brandenburg, Germany.

The precipitation amount is modeled as a sequence of continuous random variables with independent distributions. The authors estimate the parameters of the distribution from the data. The authors then use three different pricing methods: Historical Simulation (Burn Analysis), Index Value Simulation and Daily simulation to determine the risk exposure with and without rainfall insurance and compare with these three methods. Authors conclude that there is significant difference in valuation among the three methods.

A random variable \( X_t \), which is assumed to follow first order markov chain, is defined as:

\[ X_t = 0, \text{ if day } t \text{ is dry} \]
\[ =1, \text{ if day } t \text{ is wet} \]

Probability \( p_t \) that it will rain on day \( t \) is calculated as:
\[ p_t = p_{t-1}q^{t11} + (1-p_{t-1})q^{t01}, \ t=1, 2, ..., n \]
\( q^{t01} \) = probability of rain on day \( t \) given dryness on previous day \( t-1 \), and
\( q^{t11} \) = probability of rain on day \( t \) given wetness on previous day \( t-1 \).

Mixed exponential distribution is assumed for the daily rainfall.
Density function of \( y_t \), a random variable, denoting precipitation amount on day \( t \) is,
\[ f(y_t|x_t=1) = \left( \frac{\alpha_t}{\beta_t}\right) \exp\left(-\frac{y_t}{\beta_t}\right) + \left(1-\frac{\alpha_t}{\beta_t}\right) \exp\left(-\frac{y_t}{\beta_t}\right), \text{ with } 0<\alpha<1, 0<\beta<\gamma. \]

Where \( \alpha, \beta, \gamma \) are parameters of the mixed exponential distribution.

For purpose of estimation, this is simplified by using finite Fourier series.
Since the derivative is based on the recorded rainfall at a particular weather station, and since any agricultural field may be at a distance from such a station and where the rainfall may be different, the authors have used a spatial relationship of precipitation at distance $d$ from the weather station as,

$$\rho(d) = e_1 \exp(-e_2 d e_3)$$

Where $\rho(d)$ is the correlation coefficient between rainfall at distance $d$ from the weather station and rainfall at the weather station and $e_1, e_2, e_3$ are constant coefficients to be estimated.

Authors have developed a valuation model using option derivative. Rainfall data are collected for the period January 1, 1948 to August 31, 2004 at 23 evenly distributed weather stations. Correlation function, as discussed above, is determined. Three different option pricing methods are used: Burn Analysis, Index Value simulation and daily simulation.

Burn analysis is based on the historical data. For index value simulation, the authors use Excel-add-in Best-Fit to test for the most appropriate distribution. Using Chi Square test, Kolmogorov Smirnov test and the Anderson Darling test, they find the total rainfall during the first two months of monsoon follows Weibull Distribution and total rainfall in the subsequent period follows Erlang Distribution. Using this index, value of the index is randomly drawn for 50000 times. Discounted payout is determined for each value and the option price is determined as the average of these values.

Reliability of the three models is tested using standard errors of the estimate. A linear relationship is assumed between production of the crop and rainfall, discounting all other variables. Normalizing the production, hedge effectiveness is determined.

They conclude that there is significant difference in pricing of the product between different methods of pricing. Second, they conclude that risk reducing effect of the rainfall derivative is regionally stronger. That is, a single price can not apply for the entire region.
Muamba (2010) has examined the viability of rainfall insurance contracts for agricultural production in Northern Ghana.

Bandopadhyay et al (2010) have tested several rainfall insurance models for Burdwan District, West Bengal, India. Monthly rainfall data were collected for the period 1943-2002. The authors conclude that there is no trend movement. The monsoon period is divided in two periods: sowing season and harvesting season. This study assumes that the rainfall follows gamma distribution. The given data set is used to calculate the Maximum Likelihood Estimator (MLE) of the distribution. These are then used to determine the payoff distribution. Expected Payoff is determined as the expected value of the payoff distribution.

Prices for Call Option (for sowing season) and put option (for harvesting season) are determined based on the average calculated from the past data and setting upper limit and lower limit at (average+0.25 Standard Deviation) and (average-0.25 Standard Deviation). Premiums are also calculated using Simulation exercise and assuming gamma distribution for the rainfall. The authors do not consider the product market dynamics.

7.4.4 EXISTING RESEARCH GAPS AND RESEARCH AGENDA

The research till date is indicative of the need for further research. It is agreed that a derivative product for protection against uncertain rainfall could be useful and has potential for less developed countries. Different models have been developed and offered. However, at least on three counts further research is called for.

1. A derivative rainfall insurance product has to be designed separately for different regions. Though the pricing model may remain the same, rainfall characteristics could be different.
2. Different researchers have used different distributions for the rainfall. Normal distribution, exponential distribution, gamma distribution, weibull
distribution have been used. In different regions, rainfall pattern may be different and hence may follow different distribution. Choice of distribution determines the payoff. It is true that the expected payoff would depend on the distribution of rainfall in the future. However one may assume the past distribution to be replicated and which could be different for different regions though over a long period of time it could change. Hence it is necessary that the research be “localized”.

3. A pricing model for weather derivative is generally developed for a region. This approach is incorrect. Firstly, as is the case of most insurance products, a product has to be as simple and universal as possible. Even for insurance products, say for vehicles, accident characteristics are different in different regions based on the population density, behavioral norms, road conditions, etc. However, a single product is offered to all. Similarly, an attempt should be made to design a single product for a larger region. Localized product has great limitations. Suppose a single product is offered in one district. It is not necessary that different fields have the same rainfall and hence the same impact on crops as the representative field. The departure from the average could be large. Second, administration of different products for different regions may be administratively difficult and even the field staff may have difficulty in comprehension.

4. A model must offer different product for different crops. The requirements of rain are different for different crops. Also, the effects of abnormal rains could also be different.

5. The payoff has to be in terms of the loss function and not merely in terms of rainfall. It is important to determine the loss function which could be different for different crops.

6. One glaring omission in most research so far is that the product must be designed after accounting for effects of other variables. This is especially true for agriculture on two counts. One, as irrigation facility increases, the adverse effects of erratic rainfall decreases. The irrigation facility, in countries like India, is inadequate but is improving over the time.
7. For agricultural products the loss in crop output is not necessarily loss in the welfare of the farmer. It is possible that price of the product so changes, due to price elasticity of demand, that total revenue may not change to the extent that the output has changed. Hence, loss function needs to be carefully defined, which is not the case with research done so far.

This research specifically tries to fulfill the following needs:

1. This research tries to develop a one single universal derivative product, applicable for the entire country for two agricultural commodity.
2. The research does not consider one of the different distributions of rainfall used in the literature as given for this study. Actual past rainfall data are fitted to determine the most applicable distribution. The estimated distribution is then used for the purpose of deriving option value.
3. The effect of other variables on the output is taken into account by modifying the change in output before the effect of change in rainfall is estimated.
4. This research also determines the option value after considering price elasticity and accounting for its effects on the change in value and the income. Also considered is the true effect on the welfare by modifying the change in income by change in consumer price index. Thus, only real effect on the welfare is considered which the right approach is probably. This is necessary because the change in price of a product may not just be due to demand-supply mismatch but also due to general economic conditions.

7.5 MODELS AND METHODOLOGY

7.5.1 ACTUARIAL CROP INSURANCE MODEL

A farmer who holds the asset, land, has two alternatives: rent the land or till the land. If he tills the land, using his labour and other inputs, the output of his efforts are dependent on the amount of rainfall. An insurance product is offered to a farmer, at a price, whose payoff is a function of an event- specified rainfall-occurring. A
company which offers such a product prices the product such that its expected profits are zero or certain positive. The event, amount of rainfall, is stochastic; hence its probability distribution is required and is determined from the past data. It is assumed the rain, in future, will follow the same distribution with a known expected value.

The crop insurance is based on this model and is widely practiced. However there are several limitations of this product. One, the past distribution is not necessarily a true distribution for future rainfall. Second, the cost of such an insurance product is high and may not attract the farmer. The actuarial model is useful when the stochastic variable is truly unpredictable and random; Rainfall follows a pattern and has a tendency to move towards an average. Thus, if rainfall is average at time $t$, it may fairly be assumed that rainfall will be normal at time $t$. if so, this model fails to be efficient.

Even if the payment is made for extreme occurrence, this model may not be useful, since even extreme occurrence may be followed by normal occurrence with different and higher probability. Such a model is more useful for temperature based derivatives since temperatures are truly random and extreme temperature is not necessarily followed with predictable future values.

7.5.2 MODELS OF RAINFALL DERIVATIVES

The rainfall insurance model is based on the relationship between rainfall and crop output. Rainfall is an important input. Adequate rainfall is necessary for an optimal output given (optimal) other inputs. Deficit of rainfall at the beginning of the production cycle is observed to be harmful. Similarly, excessive rainfall in the second half of the production cycle is also harmful to the crop production. In the first phase of the production cycle, if the rainfall is not adequate, output may start dropping from the expected value and as shortfall of rain increases, shortfall in output could increase. (It is possible that decrease in output is increasing or decreasing function of the rainfall). Similar is the case in the second half of the
production cycle except that excessive rainfall is the cause of loss of output in this period.

The rainfall derivative product is designed to compensate for the loss in production due to deficient or excessive rainfall. The compensation may be limited and not unbounded. Thus insurance product is an option with a cap.

Since the two phases of production cycles have different requirements of rain, each phase should be separately considered and valued. A product, thus, could be separately offered for each phase or as a combined product for both phases: compensation for loss due to insufficient rains in first phase and compensation for the loss due to excessive rains in the second phase, with a cap on the total payment of the two phases together.

Both rainfall and output can be stochastic. A production function needs to be constructed, though it may at best be only a fair estimate since there could be variables which are difficult to measure (e.g., fertility of soil) or are altogether unknown and hence ignored. Also, the inputs may not be known for all “production units” (different production units, i.e., farmers, may use different quantity and non-optimal inputs), may be difficult to estimate/calculate (e.g., fertilizers of different grades may be used by different producers). Hence, the model may be formulated in parametric form to estimate effects of different inputs on the output. This is to be followed by determining the probability distribution of the rainfall, as that is the variable of interest. Such a distribution may be derived from the past data.

### 7.5.3 FEATURES OF THE RAINFALL DERIVATIVE

Any rainfall derivative model has to account for two stochastic models superimposed on each other:

- A model of rainfall
- A model of production – production function

A rainfall derivative is a product which has rainfall as the underlying variable. Hence, first, distribution of the rainfall and its expected value and standard
deviation need to be determined. Once distribution of rainfall is obtained, its relation to output of a specific product needs to be understood since compensation is for the loss of output and not for the loss of rainfall. Third, if the compensation is for the loss of income and not for the loss of output, relation between loss of output and price of the product has to be derived.

7.5.4 PRICING OF THE RAINFALL DERIVATIVE

The pricing of the derivative has four broad features.

1. Determination of the Distribution of the Rainfall

2. Determination of the linkage between rainfall and production

3. Determination of Loss Function

4. Product Design

7.5.4.1 Determination of the Distribution of the Rainfall

Since the rainfall is stochastic, distribution of the rainfall needs to be determined.

7.5.4.1.1 Determination from the past data

Determination of the specified value of the variable or a distribution from which the value will be selected.

(a) This could be average value from the past data, with attendant standard deviation, actuarial distribution of the past data). No specific distribution is established. Only two statistical parameters are determined and it is assumed that during the next period expected value of the rainfall will be the past average value and its standard deviation will be the standard deviation of the past.

Or
(b) A fitted distribution from the past data. The past data are collected and a distribution is fitted to these data. Such a distribution could be, say, normal distribution with its statistical parameters. It is assumed that the rainfall in the next period will also follow the same distribution with same statistical parameters.

Or

(c) Some researchers have used a pre specified widely used distribution rather than fitting a distribution to the past data. For example, researchers have used gamma or exponential distribution.

or

(d) The variable may auto-regress and autoregressive model may be used.

Or

(e) Value may also be based on parametric estimation. This could be an economic model.

### 7.5.4.1.2 Distribution from Simulation Model of the Past Data

The distribution derived from the sample may not give true population parameters. Hence, the raw data or the distribution as determined from the past data may be subjected to simulation. The averages of the simulated results are then taken as representing true population distribution and true population parameters.

### 7.5.4.2 Determination of the Linkage between Rainfall and Production

Unlike financial derivatives, where an unexpected event has direct impact on the underlying asset value, uncertain rainfall has only indirect value on the asset value. First, rainfall has an effect on the crop output. The change in crop output is then reflected in the change in value of the asset (land). Hence, before the effect on the output is determined, determination of the linkage between the weather variable
and output which generates revenue needs to be established. For example, demand for electricity/energy as a function of temperature: since in case of low/high temperature air-conditioning/heating may or may not be required). In the case of rainfall, relation between change in rainfall from the expected and change in output from the expected needs to be established.

7.5.4.3 Relationship between output and income
Ultimately the welfare is measured by change in income from the expected and hence this relationship is important in designing the product.

7.5.4.4 Actual pricing formula or a model.
The final step in the valuation of a derivative is actual pricing of the derivative.

This would include variable specification, model specification, payment terms, etc. In addition, especially for rainfall derivative, a two stage model is used and relationship between the two stages determined to arrive at the optimal product design.

7.5.5 Generic model of rainfall derivative
Rainfall derivative is generally designed as an option. A typical product may be combination of a call and put option, a collar.

Rainfall is the underlying variable. The place where the rainfall data would be collected and period over which it is to be considered are also specified.

The rainfall data is collected at a specified weather station at a location near the producing fields. The data may be compiled and classified as daily data, weakly data, monthly data or yearly data. A cumulative payout, for an assumed tick size and value, is calculated.

The expected cost to the seller equals present value of expected payout plus all incidental expenses. If time to maturity is T and riskless rate of interest is r, present value of incidental (includes administrative and selling expenses) expenses is A, then,
Total Cost to the seller = $C = e^{-rT} \cdot \text{Payout} + A = e^{-rT} \cdot Y + A$

Expected cost of the contract to the offerer, $C$, equals,

\[ E(C) = e^{-rT} \cdot E(Y) + E(A), \]

where \(Y=\) payout, \(E(.)\) is expectation operator and \(A\) is the incidental expense.

Hence, for breakeven, total premium to be charged should equal \(E(C)\). \(C\) of course would depend on \(Y\), which is turn would depend on probability distribution of the rainfall, and the payoff as function of rainfall, \(R\).

\[ E(C) = e^{-rT} \cdot E(f(R)) + A \]

Actual price charged may be higher due to different risk tolerance of market players. The seller needs to be compensated for taking the risk.

Suppose seller requires minimum amount of \(Q\) from the derivative business to satisfy the criteria of breakeven. Suppose there are \(Z\) acres of land under cultivation by the farmers who purchase the derivative product. Then the contact price per unit of land would be at least \(Q/Z\). This would be increased by amount of profit required, amount to be paid to induce risk averse seller to participate in the market and amount to take care of uncertainty that is possibly not captured.

The value of the derivative is also a function of the exercise/strike value. For different strike value, payoff would be different. Thus, both strike value and payoff per unit are to be jointly determined for optimality.

### 7.5.5.1 One Stage Rainfall Derivative Insurance Model

In a one stage generic model, a buyer of the option would be paid a specific predetermined amount per unit of rainfall (tick value) for every unit of rainfall short of the expected average value of the rainfall subject to a maximum payment, which may correspond to a limit on losses that may be compensated.

#### 7.5.5.1.1 The payment in case the option is exercised
Let \( W \) be the underlying weather index or variable, \( E \) be the strike price of a put or a call or a collar, \( k \) as tick value for a linear payout scheme, \( Y_0 \) be the fixed payment, and \( p \) be the premium for a call or a put.

The payout of an options contract by the seller to the buyer of the contract, \( P \), would then be,

\[
Y_{\text{swap}} = k(\text{W} - \text{E})
\]

\[
Y_{\text{call}} = k \cdot \max(\text{W} - \text{E}, 0) \quad \text{(payment if W exceeds E)}
\]

\[
Y_{\text{put}} = k \cdot \max(\text{E} - \text{W}, 0) \quad \text{(payment if E exceeds W)}.
\]

### 7.5.5.2 The Distribution of the Rainfall

Several models have been developed for pricing of rainfall derivative products. They can be classified as follows:

a. Actuarial model
b. Burn Analysis based on Historical Data and fitting a distribution
c. Assumptions of the distribution of the rainfall and use of the standard distributions-e.g., normal distribution or gamma distribution

#### 7.5.5.2.1 Actuarial method for Pricing of Weather Derivative

The easiest way is to assume that the rainfall will follow the past and the derivative product can be priced by using actuarial method. Actuarial method is based on statistical analysis of the historical data of the rainfall.

#### 7.5.5.2.2 Fitted Distribution

Instead of determining the statistical parameters from the past data and using those parameters for estimation, the data can be examined for the best fit distribution. The distribution population parameters can be estimated from the sample data. Future rainfall may then be assumed to follow the same distribution with same
parameters. Limitation of this method is that the future value of the rainfall may not follow the same distribution as the past.

7.5.5.2.3 Gamma Distribution/Exponential Distribution/Other Distributions

Some researchers have fitted rainfall data to a distribution and have found that the best fit is gamma distribution. Some have found that the best fit is exponential distribution. Others have assumed that either gamma or exponential distribution is a true distribution without testing for the best fit. Historical data is then fitted to the assumed distribution to find its parameters.

Berger and Thom (1949) were the first to propose that the climate variable (temperature, rainfall) follow gamma distribution. Wilks and Wilby (1999) have reviewed the developments in modeling of weather variables. They have suggested the usefulness of gamma and exponential distribution. Martin et al (2001) have modeled rainfall using Gamma distribution. Mubhoff et al (2006) have used exponential distribution. Bandopadhyay (2010) has used Gamma distribution for rainfall in Burdwan district, West Bengal, India.

Recent research has shown that gamma distribution is not necessarily the best fit distribution. Mubhoff (2006) has found Weibull Distribution to be best fit for rainfall in Germany.

7.5.5.2.4 Auto-Regressive Model

Rainfall may be modeled as regressing on itself. Auto regressive models assume that the rainfall in the next period would be a function of the past rainfall. Thus, for example, excessive rainfall is more likely to be followed by excessive rainfall. This may or may not be true. Hence, auto regressive models are not preferred.

7.5.5.2.5 Parametric Estimation

Rainfall may be estimated as a function of parameters such as temperature, with or without lagged effects. Ahijevich (2002) has concluded that the rainfall is a function
of longitude and time of the day. It is not certain that this can be generalized to all geographical regions. Research may indicate, in future, that some other parameters do influence the amount of rainfall but exclusivity of such parameters is doubtful.

7.5.5.2.6 Use of Monte Carlo Simulation for Determination of the Distribution

The distribution, as determined from the sample by any of the above methods, can be directly used for estimating true underlying population distribution and from that the expected rainfall.

Instead of directly using the estimated underlying distribution, one can simulate the past rainfall or simulate the estimated distribution. Major problem in using the data directly is that the sample may be small or the sample may not generate true population distribution. Hence, use of simulation could be helpful in generating true population parameters. Monte carlo simulation can generate a large number of distributions with different statistical parameters (say, 500 or 5000, etc., depending on the choice). Average of these values is taken as the true underlying distribution. The distribution is then used to determine expected payoff, for different strike value. If the past data are sufficiently large, Monte Carlo simulation can also be run, using these data. Simulation can be applied to daily data or an index.

7.5.5.3 Two Stage Model

Generally crop production can be divided into two stages which may be termed as sowing season and harvesting season. Rainfall requirement in these two stages is different. In the first stage, rainfall has to be certain minimum for average output to be achieved whereas in the second stage rainfall should not exceed a level, beyond which it is harmful. (The season may be divided in one more season: middle season or Maturing season). Hence one can not apply a single valuation model nor can one apply the same model for both stages. If there is an adverse impact in the first stage, payment would be made for rainfall below a threshold limit (a put option) whereas if the adverse impact is in the second stage, payment is made for rainfall above
another threshold (a call option). So a call option is offered, a put option is offered or a collar is offered. A farmer can choose either of them.

7.5.5.4 Relation between Stages in Two Stage Model

The production loss could be due to poor rainfall in 1st stage or excessive rainfall in second stage or both. However, payment is only one. Hence it is necessary to find correlation between the two stages. If they are significantly correlated, then the total payment has to discount the correlated damages.

7.5.5.5 Estimation of Production Function

There are two stochastic variables: One, rainfall and two, output per unit land. Production function or Output per unit land is a function of rainfall and other input variables, e.g., quality and quantity of seeds, capital used, quality of soil, fertilizers, pest control, etc. It is not correct to assume that average observed output in a set of selected fields is the expected output. The output must be adjusted for the other inputs as well as increase in productivity over time. (Productivity may just be a function of understanding of better application of the same inputs).

Output can be assumed to follow normal distribution with limit on the minimum. Then the output is a conditional outcome based on the amount of rainfall.

\[ Q_{it} = f\left(F_{it}, L_{it}, K_{it}, P_{it}, r_t, R_t, S_{it}\right) \] \hspace{1cm} (7.8)

To simplify, assume that the output follows a linear production function with a minima and maxima. Then, conditional expected output can be determined, with its statistical parameters. The model is written as:

\[ Q_{it} = a + b*F_{it} + c*L_{it} + d*K_{it} + e*P_{it} + f*(E(r_t) - r_t) + g*(E(R_t) - R_t) + h*S_{it} + \tilde{\theta}_{it} \] \hspace{1cm} (7.9)

Where,

\[ Q_{it} = \text{Production of } i\text{th product at time } t \]

\[ F_{it} = \text{Fertilizer input at time } t \text{ for } i\text{th product} \]
L_{it} = labour input at time t for i\textsuperscript{th} product

K_{it} = Capital, like tractors, used at time t for i\textsuperscript{th} product

P_{it} = Pesticides used at time t for i\textsuperscript{th} product

r_t = Cumulative rainfall at time t for sowing season

E (r_t) = Expected value of cumulative rainfall r_t , in first stage of the rain season, sowing season

R_t = Cumulative rainfall at time t for harvesting season, rainfall in the second stage of the rain season

E (R_t) = expected value of R_t

S_{it} = Quality of seeds at time t for harvesting season for i\textsuperscript{th} product

T = time

a,b,c,d,e,f,g,h = Constants (real numbers)

Total output is a function of land area used for the crop. Land area could be different from year to year. Hence a better measure is output per acre, productivity. That is the measure used in this study.

Irrigation facility, if available, adds to the availability of water, to the certainty of water availability, and makes land more productive. Hence one more variable, percentage of land under irrigation, is added.

The amount of other inputs, e.g., fertilizers, is not identical across all the farmers. Not only the quantity per acre is different, but also the quality. Data for each farmer is neither available, nor desirable for simplicity of the model. These data are assumed to be captured by a time variable T. T, thus, incorporates all inputs other
than rainfall and irrigation, and also accounts for improvement in technology and labour productivity.

The model is rewritten as:

\[ Q_{it} = f(T_{it}, I_{it}, r_t, R_t, S_{it}) \]

\[ Q_{it} = a + b^*T + c^*I_{it} + d^*(E(r_t) - r_t) + e^*(E(R_t) - R_t) + e_i \quad (7.10) \]

where,

\( T \) = time variable, with 0 value for the first period.

\( I_{it} \) = Area under irrigation at time \( t \) for \( i \)th product

This analysis of estimating a production function has to be done separately for each crop.

The analysis is useful as follows:

- Using the production function, expected output can be determined for given rainfall. This would generate a distribution for the past data which then can be used to derive an output distribution.
- The production function informs whether the chosen independent variables are statistically significant.
- From the estimated production function, one can derive the effect of irrigation, technology and rainfall on the output. These data can then be used to predict future expected output.

**7.5.5.5 Determining Expected Output for Given Rainfall**

The expected output can be derived from the past data. Average output for average rainfall can be determined and different output at different level of rainfall can also be determined from these data.

**7.5.5.7 Estimating Loss of Income and Payoff**
The function of payoff can be protection against income loss, or protection against yield, or lump-sum protection against risk. (In similarity, life protection, in case of an event, could be protection against loss of future potential income due to loss of income generating capacity due to loss of life, or, protection against temporary loss of output due to illness, or, some lump-sum payment based on actuarially neutral insurance.)

Based on the rainfall distribution and associated output, and price of the product, loss of income at different rainfall levels can be calculated and payoff determined for each level of rainfall.

7.5.5.8 Specifying Model Parameters

The model parameters are set so that the expected net payoff is zero. This requires specifying tick size and tick value, threshold rainfall below (above) which payment would be made, and the maximum payment that would be made.

The model may be designed for different assumptions on rainfall distribution, production function, and income loss. Each of these models would require pricing for each stage separately or combined pricing.

7.5.5.9 Levels of Insurance

Three levels of insurance can be offered:

1. A low price insurance (under assumption that farmers with small holdings will take this insurance
2. A medium price insurance
3. A high price insurance

The low price insurance can compensate for the loss of real income. The medium price insurance may compensate for production loss below 50% of the expected output. The high price insurance may compensate for loss below 75% of production and/or income.
7.6 DESIGNED MODEL OF RAINFALL INSURANCE

The model for this research is designed as follows.

7.6.1 BASICS OF MODEL

The model uses an option based derivative as an insurance product offered to a farmer.

- A farmer buys insurance at a price, a premium at the beginning of the production cycle.
- The production cycle is divided in two parts: Sowing season, and Harvesting Season.
- Payment for a given period is based on the pre-decided level of cumulated rainfall during the season.
- For the sowing period,
  - At the expiry of the production cycle, the farmer will be paid as per pre-decided formula or none.
- For the harvesting Period,
  - At the expiry of the production cycle, the farmer will be paid as per pre-decided formula or none.
- The payoff is event specific—in this case the event is total cumulated rainfall during a pre-specified period. The function of payoff is modeled as protection against income loss, and protection against yield.
- Herein, the assumption is that the insurance is designed:
  1. Only for loss limited to a specified period
  2. For loss against expected real income or production loss against expected output (thus dynamics of the product markets are not ignored – price is assumed to be WPI index for the specific product and CPI index for farm labour for required expenditure (normalized).
  3. One level of insurance is offered.
7.6.2 Payoff in the First Phase

Payoff, \( P(r) \), for the first period, is designed as follows.

Let the output follow a distribution as follows:

Payoff \( P(r) \), for the first period, is designed as follows.

**First Phase Output as a Function of Rainfall**

**Figure-7.1**
Is assumed that the market is competitive and every farmer in the market faces the same rainfall distribution. (This assumption is to simplify the model. However, in reality, when there are several agro-climatic zones, which all grow the same product, this assumption may not be valid.)

Let \( r_t \) be the cumulative rainfall during the period and the production \( Q_t \).

Model-1: Normalized Price

Price is normalized to 1; hence, income and output are the same. If \( I_t \) denotes the income in period \( t \), \( I_t = Q_t \).

Assume that the output declines gradually to some level \( Q_0 \), for cumulative rainfall below a level, say, \( r_1 \), and \( (Q_t(r_1) - Q_t(r_0)) = \frac{1}{2} \) is likely to be a small number. That is,

\[
\frac{1}{2} = f(F_t, L_t, K_t, P_t, r_1, S_t) - f(F_t, L_t, K_t, P_t, r_0, S_t) \approx 0 \quad \text{(7.11)}
\]

(Note: Of course, if rainfall is low, amount of other inputs used will also be low. However, that is ignored since the overall effect on the difference in output is likely to be small at that low level of rainfall.)

Then,

Expected Income = \( E(Q_t * P_t) = E(Q_t) \) (if \( P_t = 1 \), normalized)

Expected Real Income =

\[
E(Q_t) = Q_0 * p(r < r_{1,1}) + \frac{1}{2}(Q_1 - Q_0) * p(r_{1,1} \leq r < r_{1,2}) + \frac{1}{2}(Q_2 - Q_1) * p(r_{1,2} \leq r < r_{1,3}) + (Q_2) * p(r > r_{1,3}) \quad \text{(7.12)}
\]

\( r_{1,1} \) = threshold low rainfall below which the output is at least \( Q_0 \).

\( r_{1,2} \) = threshold low rainfall below which the output is at least \( Q_0 / r_{1,1} \).

\( r_{1,3} \) = threshold low rainfall below which the output is at least \( Q_0 \).

Let the payoff from the insurance = \( f \) (rainfall)
Payoff is not designed to compensate for the difference between ex-ante expected output and actual total output. Because, if so, producers will be required to pay to the insurance company for the output in excess of total output. Output is event specific. But output is also stochastic given an event. Thus, for the same rainfall actual output may be different in different years due to other inputs and uncertainty of production function.

Hence, the design of the insurance product follows options approach: If the buyer wants to exercise the option, he may do so but he is not forced. Thus, if the payoff is the difference between expected output and actual output, and if the output exceeds the expected output, buyer (farmer) may not exercise the option at all. Alternately, the product is so designed that payoff is a certain number(s), but is non-negative. But if the purpose of the insurance is to compensate for the loss in output due to rainfall, then total compensation will raise an issue of moral hazard- a producer has no incentive to invest in other inputs and work hard.

Hence the product is designed for only partial compensation. Compensation should be is a function of several parameters:

1. What is the minimum required standard of living?
2. What is the size of a typical family and its age distribution?
3. Is there additional need for savings-say for unexpected medical expenses?
4. What is the probability that the rainfall will be lower than expected the next year?
5. Is there a correlation between the effects of the 1st phase and effects of the 2nd phase of the rainfall?

Actual design, thus, could be a complex exercise, difficult to operationalize.

Let the minimum guaranteed (gross) real income be associated with output $Q@$. Then the compensation = $Q@ - Q_a$ for $Q_a < Q@$

$$= 0 \quad \text{for } Q_a \geq Q@$$
Q@ could be (for the production function as shown in figure 1 above) equal to Q1, above Q1 or below Q1. Q1 is a “stabilized output”, or one may say (though mathematically incorrect unless that level of rainfall is the mode), most likely output. For the purpose of this study, it is assumed that r1,2 is the mode and the producers design their inputs for this level of rainfall (and NOT expected rainfall). A producer will apply other inputs with an assumption that the rainfall will be the mode of the distribution.

If the actual output is above the output associated with mode, Q1, then there is no need for payoff.

7.6.2.1 Income Compensation Model with Normalized Price

Suppose the output is below Q1. It is not necessary that the farmer is in distress simply because the output is less than the “mode” output. Distress may not even arise if the price elasticity of demand is -1. This needs to be established before any insurance product is designed. Since, the markets are all integrated, and not correlated; it is assumed that all the producers around the world do not face simultaneous risk of loss. Hence, price is normalized 1 and consequently the distress if there is loss of production.

How much loss can be accepted before compensation, as discussed earlier, required very detailed analysis. Such an analysis, if undertaken, is dynamic in nature as real personal incomes grow with time. Also, compensation and subsidies in other areas of expenditure and income also need to be examined. Alternately, some arbitrary cutoff may be made.

In this study, maximum amount of expected output, below which payoff is made is set at about 66.67% (2/3rd) of “mode” output, i.e., Q1. (This is arbitrarily done).

The output, below which payment is fixed at a minimum level, is set at Q0. Q0 is derived from the past data as that level of output below which the slope of incremental output/incremental rain is less than 0.1. (10°radian).
The payoff, in between these two extremes, would be proportional to the loss in production, which is assumed to be linear with the rainfall.

### 7.6.2.2 The Payoff

The payoff is, thus, designed, albeit arbitrarily, as follows:

1. \[ Y(r) = 0.25*Q_1 \] for \( r = r_{1,1} + (r_{1,1}-r_{1,2})*0.75 \] \[ -----(7.13) \]
2. \[ Y(r) = +0.75 Q_1-Q_0+ (r_{2-r})/(r_{2-r_1})*(Q_0-0.50*Q_1) \] \[ -----(7.14) \]
3. \[ Y(r) = 0.75Q_1-Q_0 \] for \( r < r_{1,1} \) \[ -------------------(7.15) \]
4. \[ Y(r) = 0 \] for \( r > r_{1,2} \) \[ -------------------(7.16) \]

OR, in terms of standard deviation,

1. \[ Y(r) = 0.25*Q_1 \] for \( r = (r_{av}-2*(\text{Std. Deviation})+0.75*(r_{av}-r_{1,2}) \) \[ -------(7.17) \]
2. \[ Y(r) = +0.75Q_1-Q_0+ (r_{2-r})/(r_{2-r_1})*(Q_0-0.50*Q_1) \] \[ -------(7.18) \]
3. \[ Y(r) = 0.75Q_1-Q_0 \] for \( r < r_{av}-2*(\text{Std. Deviation}) \) \[ -------------------(7.19) \]
4. \[ Y(r) = 0 \] for \( r > r_{av}-1*(\text{Std. Deviation}) \) \[ -------------------(7.20) \]

### Premium:

The payoff is only to those who buy the policy. In fact a case can be made that the rainfall insurance is compulsory for all, in view of adverse selection issues and consequent increase in the premium. Also, larger the base more is the true actuarial value of loss due to inadequate rain.

Suppose one ignores this issue. Then, for a typical farmer (producer), premium should be actuarially neutral. I.e. \( \text{Premium} = \text{Expected payoff} = \)

\[
Y = +Pr(r=r_{1,1}+(r_{1,1}-r_{1,2})*0.75)*0.30*Q_1+Pr(r_{1,2}<r<r_{1,1})*(0.75Q_1-Q_0+ (r_{1,2}-r)/(r_{2-r_1})*(Q_0-0.50*Q_1)+Pr(r<r_{1,1})*(0.75*Q_1-Q_0)+Pr(r>r_{1,2})*0 \] \[ -----(7.21) \]
Now, this is actuarially neutral. However, to this one must add administrative expenses. Hence, revised premium = \( P = 1.25 \times S \) \( \ldots \ldots \) (7.22)

### 7.6.3 Model Illustration

The moot question is why should a farmer buy such an insurance product? The only reason is that it will smoothen his income flow. However, if the cost of the insurance is very high compared to his income, he will not buy the insurance.

Suppose his expected annual income in the next eight years (assumed to be one cycle of rainfall) is:

100, 90, 50, 45, 110, 85, 90, 93. Suppose past data shows that the most likely occurrence is 90. Also assume that output equals rainfall.

Then, as per this model,

\[ Q_2 = 100 \]
\[ Q_1 = 90 \]
\[ Q_0 = 50 \]

Let probability (over a long period) be 0.15 (for \( r < r_{1,1} \)), 0.20 (for \( r > r_{1,2} \)), 0.20 (for \( r_{1,2} \)), and 0.45 (for \( r_{1,1} < r < r_{1,2} \)).

\[ r_{1,1} = 50 \text{ and } r_{1,2} = 90 \]

Let cutoff be 75% and 25% for payout.

Then, Payoff would be as follows:

\[ r = r_{1,1} + (r_{1,1} - r_{1,2}) \times 0.75 = 50 + 40 \times 0.75 = 80 \] \( \ldots \ldots \) (7.23)

The payoff is, thus, designed as follows:

\[ Y(r) = 0.25 \times Q_1 = 0.25 \times 90 = 22.5 \text{ for } r = 80 \] \( \ldots \ldots \) (7.24)
6. \( Y(r) = +0.75Q_1 - Q_0 + \frac{(r_2 - r_2 r_1)}{(r_2 - r_1)}(Q_0 - 0.50*Q_1) = 0.75*90 - 50 + \frac{(90 - r)}{(90 - 50)} \)

\[ = 17.5 + \frac{(90 - r)}{40} \quad (7.25) \]

7. \( Y(r) = 0.75Q_1 - Q_0 = 17.5 \) for \( r < 50 \)  \( \quad \text{--------------------------(7.26)} \)

8. \( Y(r) = 0 \) for \( r > 90 \)  \( \quad \text{--------------------------(7.27)} \)

Expected Payoff = \( \sum (\text{Probability}_k \times \text{Payoff}_k) \)

\[
= 0.15 \times 17.5 + 0.01 \times \left[ 0.45 \times \frac{(90 - r)}{40} \right] \left[ 90 \right]^{50} + 0.20 \times 22.5 + 0.20 \times 0 \\
= 2.625 + 0.45 \times 8 + 4.5 + 0 = 7.125 + 3.6 = 10.725 \quad (7.28) \\
\]

Hence, a farmer who buys inputs and produces agricultural output, with a most likely production of 90, will pay 10.725 as premium towards rainfall insurance.

(Actuarially fair vale less excluding administrative expenses)

Farmer’s income would be as follows:

<table>
<thead>
<tr>
<th>TABLE-7.1</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ILLUSTRATIVE CALCULATIONS-RAINFALL DERIVATIVE INSURANCE</strong></td>
</tr>
<tr>
<td>Rainfall</td>
</tr>
<tr>
<td>-------------------</td>
</tr>
<tr>
<td>Below 50</td>
</tr>
</tbody>
</table>
### Table

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Between 50 and 90</td>
<td>0.45</td>
<td>50 to 90 (Linear)</td>
<td>10.725</td>
<td>39.275 to 79.275</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>90</td>
<td>0.20</td>
<td>90</td>
<td>10.725</td>
<td>79.275</td>
<td>4.5</td>
</tr>
<tr>
<td>4</td>
<td>Above 90</td>
<td>0.20</td>
<td>Av 100 (assumed)</td>
<td>10.725</td>
<td>89.275</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>Expected/Av</td>
<td>7.5+31.5+18+20 =77</td>
<td></td>
<td>10.725</td>
<td>66.275</td>
<td>73.4</td>
</tr>
<tr>
<td>6</td>
<td>Range</td>
<td>50 to 100</td>
<td></td>
<td></td>
<td></td>
<td>47.275 to 89.275</td>
</tr>
</tbody>
</table>

The effect of insurance purchase is as follows:

- The range of income is reduced by about 20%. That is, the variability of income is significantly reduced. Though it is largely at the upper end, it is also significant at the lower end.
- Though average income is reduced, the reduction is only about 5%.

### 7.6.4 Payoff in the Second Phase

Let the output follow a distribution as follows for the second phase:
Second Phase:

Let the output follow a distribution as follows for the second phase:

\[
\begin{align*}
\text{Output} & \quad Q_0 \quad Q_1 \quad Q_2 \\
0 & \quad \text{Rav} \quad R_1 \quad R_2 \quad R_3 \quad (=2R) \\
\end{align*}
\]

Rainfall mm (Cumulative for phase-II)

SECOND PHASE OUTPUT AS A FUNCTION OF RAINFALL

FIGURE-7.2

R = Cumulative rainfall for 2nd stage

The payoff in the second phase is designed as follows:

The payoff is, thus, designed, albeit arbitrarily, as follows:

9. \[ Y(R) = 0.25*Q_1 \quad \text{for} \quad r = R_1+(R_2-R_1)*0.75 \quad \text{------}(7.29) \]
10. \[ Y(R) = +0.75Q_1-Q_0+ \frac{(R_2-\text{Rav})}{(R_2-R_1)}(Q_0-0.50*Q_1) \quad \text{-------}(7.30) \]
11. \[ Y(R) = 0.75Q_1-Q_0 \quad \text{for} \quad R > R_1 \quad \text{---------------------------}(7.31) \]
12. \[ Y(R) = 0 \quad \text{for} \quad R > R_2 \quad \text{---------------------------}(7.32) \]

OR, in terms of standard deviation,

1. \[ Y(R) = 0.25*Q_1 \quad \text{for} \quad r = (\text{R}_{av}*\text{(Std. Deviation)})-0.75*(R_2-\text{Rav}) \quad \text{------}(7.33) \]
2. \[ Y(R) = +0.75Q_1-Q_0+ \frac{(R_2-\text{Rav})}{(R_2-R_1)}(Q_0-0.50*Q_1) \quad \text{-------}(7.34) \]
3. \[ Y(R) = 0.75Q_1-Q_0 \quad \text{for} \quad r < \text{R}_{av}+2*(\text{Std. Deviation}) \quad \text{---------}(7.35) \]
4. \[ Y(R) = 0 \quad \text{for} \quad r < \text{R}_{av}+1*(\text{Std. Deviation}) \quad \text{---------}(7.36) \]
The payoff is designed based on the assumption that the output will at least be certain minimum, no matter how much is the rainfall. This amount is $Q_0$. Compensation is bounded above by the output at rainfall of expected rainfall plus 2 standard deviation of the rainfall.

No payment is made if the rainfall is more than the expected but less than (expected rainfall plus 1 standard deviation).

7.7 RESULTS AND ANALYSIS

7.7.1 DATA COLLECTION AND COMPILATION

The data was primarily collected from the website of Ministry of Agriculture, the Government of India, Meteorological Department, and Central Statistical Organization.

Data for the rainfall (monthly data) and annual output of crops for the entire country are collected. Ideally, statewise data (or even crop area wise) and daily rainfall data should be analyzed and output for kharif and rabi crop should be analyzed separately. But these detailed data were not available and hence annual country-wise data are analyzed. Such analysis has its own merits. Instead of offering different products for different crops and different regions, one single product is offered, as is done for all insurance products. Variations can be introduced later.

Data are compiled for the period 1951 to 2009 (59 years).

- Rainfall

Monthly rainfall data are compiled. The rainfall in June and July are added and the total is termed as “rainfall in the 1st phase”. Similarly, rainfall in August and September are added and the sum is termed as “rainfall in the 2nd phase”. These are the primary data points for the study.
Crop output

Annual crop output for fifteen major crops, including bajra, jowar, rice and wheat, for the years 1951 to 2009 are collected and compiled. Similarly, data on area under cultivation, area under irrigation facility and per hectare yield data are also collected and compiled.

Price Indices

Since the study focuses on income compensation model, two price series are required. One, wholesale price index (WPI), for the crop under study is collected. These are annual price indices, published by the government. The reported price series have different base period. Hence they are normalized to a single base period. Also, data of annual consumer price index (CPI) for agricultural worker are collected and compiled with a uniform base period. (Same base as for WPI).

7.7.2 DISTRIBUTION OF RAINFALL

The distribution of annual rainfall for the period 1951 to 2009 (59 years) has following statistical parameters. These are calculated for Phase 1 and Phase 2 separately. Phase 1 is June 1st to July 31st. Phase 2 is August 31st to September 30th.

| TABLE-7.2 | 
| --- | --- | 
| **PARAMETERS OF ANNUAL RAINFALL, 1951-2009** | 
| Period | Mean, Cm | Standard Deviation, Cm |
| Phase I (June-July) | 454.2 | 53.7 |
| Phase II (Aug-Sept) | 431.7 | 63.4 |
The actual sample rainfall data over 59 years are used to determine best-fit probability distribution. The rainfall in the first phase follows Log-logistics 3P distribution, with \( \alpha=7.8144 \) and \( \beta=2.7573 \) and \( \gamma=-2.7573 \).

The rainfall in the second phase follows Generalized Extreme value distribution with parameters \( k=0.19036, \mu=406.3 \) and \( \sigma=61.121 \).

These distributions are then are simulated. 5000 simulations of the first phase result in a best fit distribution of Burr 4P distribution with parameters \( \alpha=2.9312, \beta=1.0239, \gamma=0.00313 \) and \( k=1.0919 \).

Similarly, 5000 simulations of the second result in the best fit distribution of Freschet 3P distribution with parameters \( \alpha=2.0609, \beta=2.0579, \gamma=2.0472 \).

Thus now there are three sets of summary:

(i) The summarized statistical parameters for actual data and

(ii) Summarized statistical parameters for fitted distribution for each phase

(iii) Statistical parameters of the distribution derived from 5000 simulation for each phase.

### 7.7.3 Analysis of Derivative for Wheat Production

#### 7.7.3.1 Model-1: Payoff based on Rainfall Distributions

#### 7.7.3.1.1 Design and payoff for Compensation against Rainfall Deficit in 1st Phase

Change in output (equivalent to yield in this study) per change in unit rainfall is determined from the past data. Average rainfall and average output are calculated.
Assuming linear relationship, change in output from the average is regressed against Change in rainfall from the average. This is then used to calculate expected payoff for different distributions as per the following methodology.

**7.7.3.1.2 Product pricing for different Rainfall distributions**

Payoff for rainfall deficit in the first period is calculated as:

\[ Y = (\text{change in productivity per unit of rainfall deficit}) \times (-\text{actual rain} + \text{average rain} - 1 \times \text{standard deviation of rain}) \times (\text{actual rainfall} < \text{average rain} - 1 \times \text{standard deviation}) \]

\[ Y = 0 \text{ for (actual rainfall} > \text{average rain} - 1 \times \text{standard deviation}) \]

\[ Y = (\text{change in production for unit of rainfall deficit}) \times (2 \times \text{standard deviation of rain} - 1 \times \text{standard deviation of rain}) \times (\text{actual rainfall} < \text{average rainfall} - 2 \times \text{standard deviation}) \]

Different distributions result in different expected payoff.

Thus

\[ Y = 6.95 \text{ for actual data} \]

\[ Y = 7.61 \text{ for fitted distribution} \]

\[ Y = 6.48 \text{ for distribution derived from simulation} \]

The product is priced based on the expected payoff. For the purpose of pricing, a factor of 1.25 is applied to account for administrative costs and for profits. (On ad hoc basis)

\[ P = \text{expected payoff} + \text{Administrative cost} + \text{Profit} \]

On ad hoc basis \( P_1 = 8.69 \text{ for actual data} \)
= 9.51 for fitted distribution

= 8.10 for distribution derived from simulation

The pricing so determined is for the loss of production and not loss of income. To determine the loss of income, the price as determined above should be multiplied by the expected average wholesale price.

**7.7.3.1.3 Payoff for Excess Rainfall in 2\textsuperscript{nd} stage**

Payoff for the 2\textsuperscript{nd} stage is determined in similar manner as the payoff for the 1\textsuperscript{st} stage except that for the second stage payoff is for excessive rains. Since in this phase payment is made for excessive rain, formula for payoff is different from that of the 1\textsuperscript{st} phase.

Payoff for rainfall deficit in the 2\textsuperscript{nd} phase is designed as:

\[ Y = (\text{Expected Production for average rainfall}) \times (\text{change in production for unit of rainfall deficit}) \times (\text{actual rainfall - average rainfall - 1*standard deviation of rain}) \text{ if } (\text{average rain + 2*standard deviation} > \text{actual rainfall} > \text{average rain + 1*standard deviation}) \]

\[ Y = 0 \text{ if } (\text{actual rainfall} < \text{average rain + 1*standard deviation}), \]

\[ Y = (\text{Expected Production for average rainfall}) \times (\text{change in production for unit of rainfall deficit}) \times (2*\text{standard deviation of rain} - 1*\text{standard deviation of rain}) \text{ if } (\text{actual rainfall} > \text{average rain + 2*standard deviation}) \]

Different distributions result in different expected payoff.

Thus \( Y = 9.67 \) for actual data

\( Y = 10.31 \) for fitted distribution
\[ Y = 8.93 \text{ for distribution derived from simulation} \]

The product is priced based on the expected payoff. For the purpose of pricing, a factor of 1.25 is applied to account for administrative costs and for profits. (On adhoc basis)

Price in 2\textsuperscript{nd} Stage = \( P_2 = \text{expected payoff} + \text{Administrative cost} + \text{Profit} \)

On ad hoc basis \( P_2 = 12.08 \) for actual data

\[ \begin{align*}
&= 12.89 \text{ for fitted distribution} \\
&= 11.16 \text{ for distribution derived from simulation}
\end{align*} \]

The pricing so determined is for the loss of production and not loss of income. To determine the loss of income, the price as determined above should be multiplied by the expected average wholesale price.

\textbf{7.7.3.2 Model-2: Product Pricing Using Black-Scholes Model}

\textbf{7.7.3.2.1 Pricing of the product-1\textsuperscript{st} Phase-Using Black-Scholes Option Model}

In the first phase, a farmer is offered a put option with exercise value being average rainfall-1 standard deviation of rainfall. If rainfall is below this exercise value, farmer would exercise the option and seller would pay the difference between the (expected value-1 standard deviation) and actual value. Since payment can not be boundless, the option is bounded at average rainfall-2 standard deviation.

Hence the product is Sale of One put option with exercise value at 25 \% shortfall (assumed at 1 standard deviation) + Purchase of one put option with exercise value at 50 \% shortfall (assumed at 2 standard deviation)

\textbf{7.7.3.2.2 Pricing of the product-2\textsuperscript{nd} Phase-Using Black-Scholes Option Model}
In the second phase, a farmer is offered a call option with exercise value being average rainfall+1 standard deviation of rainfall. If rainfall is above this exercise value, farmer would exercise the option and seller would pay the difference between the (expected value+1 standard deviation) and actual value. Since payment can not be boundless, the option is bounded at average rainfall+2 standard deviation.

Hence the product is Sale of One call option with exercise value at 25 % excess (assumed at 1 standard deviation) +Purchase of call option with exercise value at 50 % shortfall (assumed at 2 standard deviation)

### 7.7.3.2.3 Results

The Black-Scholes parameters are, for aggregate rains of June and July:

- Riskless rate of interest = Current (December, 2010) treasury bill rate for 180 days
- Time to maturity: 180 days/6 months (the product will be offered in April for payment on October 1st)
- Underlying value: Expected rainfall
- 1st phase Exercise value: A put option with Exercise value = \( \mu(rainfall) - 1* \sigma(rainfall) \) or A put option with Exercise value = \( \mu(rainfall) - 2* \sigma(rainfall) \)
- 2nd phase Exercise value: A call option with Exercise value = \( \mu(rainfall) + 1* \sigma(rainfall) \) or A call option with Exercise value = \( \mu(rainfall) + 2* \sigma(rainfall) \)
- Volatility: Volatility of the rainfall

Product price is set at 1.25*(option value) (arbitrarily, to account for other expenses and profits).
### TABLE -7.3

**OPTION PRICING OF RAINFALL DERIVATIVE FOR WHEAT**

<table>
<thead>
<tr>
<th>Particulars</th>
<th>Phase I</th>
<th>Phase II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Underlying value</td>
<td>454.19</td>
<td>431.73</td>
</tr>
<tr>
<td>Exercise value-Put/Call</td>
<td>400.45</td>
<td>495.17</td>
</tr>
<tr>
<td>Exercise Value-Call/Put</td>
<td>346.72</td>
<td>558.60</td>
</tr>
<tr>
<td>Volatility</td>
<td>53.73306%</td>
<td>63.43906%</td>
</tr>
<tr>
<td>Time to maturity</td>
<td>6 months</td>
<td>6 months</td>
</tr>
<tr>
<td>Riskless rate of interest</td>
<td>7.95%</td>
<td>7.95%</td>
</tr>
<tr>
<td>Value of call</td>
<td>5.37</td>
<td>6.52</td>
</tr>
<tr>
<td>Product price (125 % of option value)</td>
<td>6.71</td>
<td>8.15</td>
</tr>
</tbody>
</table>

---

### 7.7.3.3 Model-3: Income Loss Distribution Model

The production function is specified as:

\[ Q_{it}=f \left( F_{it}, L_{it}, K_{it}, P_{it}, r_t, R_t, S_{it} \right) \quad \text{------(7.37)} \]

The model is written as:

\[ Q_{it}=a+b*F_{it}+c*L_{it}+d*K_{it}+e*P_{it}+f*(E(r_t)-r_t)+g*(E(R_t)-R_t)+h*S_{it}+\bar{\theta}_{it} \quad \text{------(7.38)} \]

Where,

\[ Q_{it} = \text{Production of } i^{th} \text{ product at time } t \]
\( F_{i,t} = \) Fertilizer input at time \( t \) for \( i^{th} \) product

\( L_{i,t} = \) labor input at time \( t \) for \( i^{th} \) product

\( K_{i,t} = \) Capital, like tractors, used at time \( t \) for \( i^{th} \) product

\( P_{i,t} = \) Pesticides used at time \( t \) for \( i^{th} \) product

\( r_t = \) Cumulative rainfall at time \( t \) for sowing season

\( E(r_t) = \) Expected value of \( r_t \), rainfall in first half of the rain season

\( R_t = \) Cumulative rainfall at time \( t \) for harvesting season, rainfall in the second half

\( E(R_t) = \) expected value of \( R_t \)

\( S_{i,t} = \) Quality of seeds at time \( t \) for harvesting season for \( i^{th} \) product

\( T = \) time

\( a,b,c,d,e,f,g,h = \) Constants (real numbers)

Total output is a function of land area used for the crop. Land area could be different from year to year. Hence a better measure is output per acre, productivity. That is the measure used in this research.

Irrigation facility, if available, adds to the availability of water, to the certainty of water availability, and makes land more productive. Hence one more variable, percentage of land under irrigation is added.

The amount of other inputs, e.g., fertilizers, is not identical across all the farmers. Not only the quantity per acre is different, but also the quality. Data for each farmer is neither available, nor desirable for simplicity of the model. These data are assumed to be captured by a time variable \( T \). \( T \), thus, incorporates all inputs other than rainfall and irrigation, and also accounts for improvement in technology and labor productivity.
Since focus is on the deviation from the average, the model is rephrased in difference terms.

Dependent variable = Change in productivity

Independent variable = Change in time (which is constant = 1)

Independent variable = Change in irrigated land

Independent variable = Change in deviation of rainfall from mean

The multiple regression is run with sample size of 58 (1952 to 2009). The values from the regression in the first phase and second phase are tabulated below:

<table>
<thead>
<tr>
<th>Sr No.</th>
<th>Independent Variable</th>
<th>Effect on Output per Unit Change</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1&lt;sup&gt;st&lt;/sup&gt; Phase</td>
</tr>
<tr>
<td>1</td>
<td>Change in Productivity</td>
<td>36.12</td>
</tr>
<tr>
<td>2</td>
<td>Change in % of Irrigated Land</td>
<td>1.43</td>
</tr>
<tr>
<td>3</td>
<td>Deviation of Rain from expected</td>
<td>-0.5167</td>
</tr>
</tbody>
</table>

7.7.3.3.1 Estimating Loss of Income

Two output series are now available. One, the actual output series, and second, the expected output series, for each crop.

Wholesale price index (WPI) for the entire time period under study is compiled for each i<sup>th</sup> product (crop). From the data on output and WPI index, income per acre is determined. Consumer price index (CPI) is also obtained from the published data.
Output multiplied by the ratio of WPI to CPI gives the real income from the crop produced.

\[
\text{Loss/gain in Expected income} = \text{difference between expected income and actual income} \\
= (Q_{i,t} - E(Q_{i,t})) \times (WPI_{i,t}/CPI_{t})
\]

Underlying assumption in this approach is that the basic consideration in providing a cheap, affordable product is to compensate for the loss in real income due to unpredictable rainfall, and not loss of production. Hence I believe that this approach would yield a better and more acceptable product.

The income loss per unit of change in rainfall from the expected is determined from the change in income divided by the change in rainfall from the expected for actual data. From the data income loss per unit of rainfall is derived as:

Income loss in Phase 1 per unit of rainfall shortfall: Rs. 0.3677

Income loss in Phase 2 per unit of excess rainfall = Rs. 0.2153

These losses, as discussed earlier, denote real welfare loss (in money terms) due to uncertain rainfall.

**7.7.3.3.2 Payoff for rainfall deficit in the first phase**

Payoff for rainfall deficit during the first phase, as income compensation, is structured as:

\[
\text{Payoff} = Y = (\text{change in income for unit of rainfall deficit}) \times (\text{actual rain - average rain - 1}\sigma\text{standard deviation of rain}) \text{ for } (\text{actual rainfall < average rain - 1}\sigma\text{standard deviation}),
\]
Payoff = \( Y = 0 \) for (actual rainfall > average rain - 1 \( \sigma \) * standard deviation)

Maximum Payoff = \( Y = (\text{change in real income for unit of rainfall deficit}) \times (2 \sigma \text{standard deviation of rain} - 1 \sigma \text{standard deviation of rain}) \)

Average Payoff is calculated at 1.5 standard deviation. This equals = \( 53.73 \times 1.5 \times 0.37 \)
\( = 29.82 \text{ Rs} \)

With 25 \% mark up, actual price at which the insurance product should be offered = Rs. 37.26 per unit of land for wheat.

**7.7.3.3 Payoff for rainfall deficit in the second phase**

Payoff for rainfall excess during the second phase, as income compensation, is structured as:

Payoff = \( Y = (\text{change in income for unit of excess rainfall}) \times (-\text{actual rain} + \text{average rain} - 1 \sigma \text{standard deviation of rain}) \) for (actual rainfall > average rain + 1 \( \sigma \) * standard deviation).

Payoff = \( Y = 0 \) for (actual rainfall < average rain + 1 \( \sigma \) * standard deviation)

Maximum Payoff = \( Y = (\text{change in real income for unit of rainfall deficit}) \times (2 \sigma \text{standard deviation of rain} - 1 \sigma \text{standard deviation of rain}) \)

Average Payoff is calculated at 1.5 standard deviation. This equals = \( 63.43 \times 1.5 \times 0.22 \)
\( = 16.93 \text{ Rs} \)

With 25 \% mark up, actual price at which the insurance product should be offered = Rs. 21.15 per unit of land for wheat.

**7.7.3.4 Summary of Results for Different Models**

Results from different models are compiled in table T-7.5 below.
### Table 7.5

**Summary of Pricing of Rainfall Derivative Insurance Product: Wheat**

<table>
<thead>
<tr>
<th>No.</th>
<th>Model</th>
<th>Price -1st Phase</th>
<th>Price-2nd Phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Actual data as Distribution</td>
<td>8.69</td>
<td>12.08</td>
</tr>
<tr>
<td>2</td>
<td>Fitted Distribution</td>
<td>9.51</td>
<td>12.89</td>
</tr>
<tr>
<td>3</td>
<td>Distribution derived from 5000 Simulation</td>
<td>8.10</td>
<td>11.16</td>
</tr>
<tr>
<td>4</td>
<td>Black schools Model*</td>
<td>6.71</td>
<td>8.15</td>
</tr>
<tr>
<td>5</td>
<td>Income Compensation Model</td>
<td>Rs. 37.26</td>
<td>Rs.21.15</td>
</tr>
</tbody>
</table>

Notes: * = The results are numbers which need to be multiplied by current price for conversion to value in rupees.

It is observed that results are model dependent. This is natural since each model has different objective and/or different assumptions.

### 7.7.4 Analysis for Rice

Similar exercise is carried out for rice. Annual production data for rice are compiled. The effect of the unexpected rainfall is assessed on the rice production by running a multiple regression on the change in productivity from the expected as a function of change in technology (time as a proxy), change in irrigation facility and departure of rain from the expected value. The regression results show that the effect of the unexpected rainfall on the output was not significant. Further analysis is not warranted.
The reasons for such a result could be many. Firstly, it is possible that the departure of rainfall from the expected really does not affect the output. This may be due to the fact that rice is grown only when sufficient rain is received. Secondly, there could be a data problem. Thirdly, it is possible that rice is grown only when the rain uncertainty is low. Fourthly rice may be grown only where irrigation is assured. In that case uncertain rainfall may not have significant adverse effects.

7.8 CONCLUSIONS

The study is probably the first of its kind which looks at compensation for loss in real income rather than only at loss in income and/or production. However, the study is limited in several ways.

The study clearly demonstrates that an affordable derivative product for managing risks arising from uncertain rainfall can be designed and offered.

Such a product will cost less than Rs. 10 in terms of production loss per unit of land. If the product is designed using option pricing model, the price could be even less.

Interestingly, a product can be directly designed for real income loss to a farmer. This approach would thus be a welfare optimal and could even be cheaper.

It is not necessary to believe that the production loss is equivalent to an income loss, especially for agricultural products. A more thorough analysis of real income effects could reduce the cost of the product without adverse effects on welfare.

A similar analysis for rice proves that the rainfall derivative product has to be crop specific. Rainfall has different effects on different crops. Also a derivative product will have less usefulness when the land is well irrigated.

Without loss of generality it can be said that a uniform product can be designed for all geographical regions.

7.9 LIMITATIONS OF THE STUDY
1. The study is done on national level data. It does not look at data disaggregated at regional level. The regional variation in rainfall, other inputs, irrigation facility and differential costs of inputs are not considered.

2. The study considers annual production rather than kharif production and rabi production separately.

3. Only two agricultural products are studied though there are several others, some of them widely consumed.

4. The study considers consumer price index as proxy for expenditure. This could be true only for landless labourers (for whom the index may be constructed). The owner farmer may not buy many of the products of the index—e.g., milk products, food grains, etc. Hence the index may overstate the expenditure. Consumer price is higher than the wholesale price at which the farmer notionally sells these products, which he actually consumes.

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