Chapter 2

OVERVIEW OF FRACTURE MECHANICS OF CONCRETE

2.1 INTRODUCTION

Concrete structures contain cracks even under service loads. The presence of micro cracks and other flaws such as pores and voids act as potential sources for crack propagation leading to fracture under external loadings. Hence, the existence of cracks should be taken into account in predicting ultimate load capacity as well as behavior in service. In the limit state analysis, neither the tensile strength of concrete is taken into account nor the degradation in stiffness with crack growth and the stress concentration effects due to presence of cracks are accounted for. This assumption works well for concrete structures with low stresses. However, for many kinds of failure where the tensile capacity governs, a size effect has been observed which could not be explained either by the elastic or ultimate load theory. Such failure types are

- Structural failure of concrete beams.
- Shear and bond failure
- Cracking in heavy and mass structures.

To explain these failure types, it is necessary to take into account the complete load deformation curve in tension. The descending branch of the load deformation curve plays an important role in the application of fracture mechanics to concrete. Fracture mechanics can be used as a practical tool to get a better insight into the behavior of structural components. It can be used to consider the effect of size, bond between reinforcement and concrete, impact and impulsive loading, dynamic shear fracture, and predictions of deflections and ductility.

Both experimental and theoretical efforts are under way to understand the fracture behavior of concrete. This has also led to finding the important fracture parameters of concrete and also innovation of several theoretical concepts to model fracture in
concrete. The current chapter briefly discusses the history of development of fracture mechanics and highlights important works in fracture mechanics of concrete related to the present study.

2.2 HISTORICAL DEVELOPMENT OF FRACTURE MECHANICS

The first mathematical approach to fracture was attempted by Inglis (1913). The significance of intense and localized concentration of stress was first emphasized by him. He found that the stress near the tip of a crack or notch can be many times greater than the applied stress at distances far away. Inglis analysed the problem of thin glass plate having an elliptical hole in the middle in a different way. Theoretically, the plate was infinitely large and the hole very small in comparison. When the plate was pulled perpendicular to the elliptical hole (Fig. 2.1a) it was found that maximum stress \( \sigma_{y_{\text{max}}} \) occurred at the apex of the major axis (point A). Also, as the ratio of \( a/b \) gets bigger (the ellipse gets longer and thinner) the stress at point A becomes greater.

![Fig. 2.1 Plate with elliptical hole subjected to normal stress](image)

Inglis (1913) also observed that by pulling the plate in a direction parallel to the elliptical hole (Fig. 2.1b) does not produce higher stress at point A. This lead to the fact that a load perpendicular, not parallel, to the crack makes it grow. Further, he
gave the elastic solution for stress at the vertex of an elliptical hole in an infinite plate as

\[ \sigma_{y,\text{max}} = \sigma \left[ 1 + \frac{2a}{b} \right] \]  

(2.1)

where 'a' and 'b' are the major and minor semi-axis of the ellipse.

Expression (2.1) reveals that as the elliptical crack approaches a line crack (b=0) the stress at the vertex of the ellipse tends to infinity. This leads to a paradoxical conclusion that a cracked structure cannot sustain any loading. Hence to form a true notion of the strength of a body with a crack, it is insufficient to have a solution of one theory of elasticity or plasticity. Therefore fracture theories were proposed to take into account these limitations. Initially two important fracture theories namely the Griffith Fracture Theory (Energy Approach) and the Irwin’s Stress Intensity Approach (Stress field approach) were developed.

Subsequently many theories were developed to suit the fracture in different types of materials. A brief discussion of important theories is presented in the following discussion.

2.2.1 Griffith Fracture Theory (Energy Approach)

In 1920’s, Griffith (1921, 1924) concluded that in the presence of a crack the stress value cannot be used as a criterion of failure since the stress at the tip of a sharp crack in an elastic continuum is infinite no matter how small the applied load. He extended Inglis’ (1913) work by subjecting an infinite plate under tension where he stretched the elliptical hole into a crack. Experiments revealed that the real tensile strength of brittle materials is significantly lower than the theoretically predicted strength because they contained cracks. The cracks introduced high stress concentration near their tips in an elastic brittle material. Griffith considered transverse crack of length '2a' in an elastic infinite plate whose linear dimensions were much larger than its thickness (L >> t). A similar test on soft iron wire, about 2.5m long and 0.70mm in diameter revealed that scratches introduced on the
surface increased the stress in the wire by 3 or 4 times which made it yield earlier.
The test results indicated that materials with fractures, no matter how small the
fracture, act much differently than the same material without cracks. He said that for
a crack to grow, it was necessary that there should be enough potential energy in the
system to create new surface area of the crack.

Based on numerous experimental results, Griffith (1921) postulated the energy
approach. He considered an infinite plate of unit thickness which contained a crack
length $2a$ subjected to a uniform tensile stress $\sigma$ as shown in Fig. 2.2 a.

![Fig. 2.2 a) Cracked plate subjected to tension, b) Load-displacement curve](image)

The change in energy due to the introduction of a crack is

$$U - U_0 = -U_a + U_s \tag{2.2}$$

where $U$ and $U_0$ are the elastic energies of the cracked and uncracked plate
respectively; $U_a$ is the change in the strain energy due to introducing the crack in the
plate, which is equal to $(\pi \sigma^2 a^2)/E$; $U_s$ is the change in the elastic surface energy due
to formation of the crack surfaces, which is equal to $4ay_s$; $E$ is Young's modulus of
material; and $y_s$ is the elastic surface energy due to formation of a unit crack surface.
Hence, eq. (2.2) is rewritten as
The energy equilibrium condition for crack extension is obtained by minimizing the energy change with respect to change in the crack length $\delta a$ (Fig. 2.2b). This is done by setting the first derivative of eq. (2.3) with respect to crack length $a$ equal to zero. This results in the well known Griffith's relation relating the critical stress $\sigma_C$ and the critical half crack length $a_c$.

- For plane stress
  \[ \sigma_C = \sqrt{\frac{2E\gamma_s}{\pi a_c}} \]  \hspace{1cm} (2.4)

- For plane strain
  \[ \sigma_C = \sqrt{\frac{2E\gamma_s}{\pi a_c (1-\mu^2)}} \]  \hspace{1cm} (2.5)

where $\mu$ is Poisson's ratio.

The above results led Griffith to propose the energy criterion of failure, which serves as the basis of the classical linear elastic fracture mechanics (LEFM) theory. This theory is valid for perfectly brittle materials like glass.

### 2.2.2 Other Energy Theories

According to Griffith (1921), the crack will propagate if the energy available to extend the crack by a unit surface area equals the energy required to do so. He took this energy to be equal to $2\gamma_s$, where $\gamma_s$ is the specific surface energy of the elastic solid, representing the energy that must be supplied to break the bonds in the material microstructure and, thus, create a unit area of new surface. However, it was later realized that the energy actually required for unit crack propagation is much larger than this value, due to the fact that cracks in most materials are not smooth and straight but rough and tortuous, and are accompanied by microcracking, frictional slip, and plasticity in a sizable zone around the fracture tip. For this reason, the solid state specific fracture energy $2\gamma_s$ was replaced by a more general crack growth resistance curve, $R$, which, in the simplest approximation, is a constant. The determination of $R$ has been a basic problem in experimental fracture mechanics. The other essential problem of LEFM is the determination, for a given structure, of
the energy available to advance the crack by a unit area. This magnitude is called the energy release rate, and is usually called $G$.

The development of elastic fracture mechanics essentially occurred during 1940-1970, which was stimulated by some perplexing failures of U.S. naval ships during world-war II when they were docked in harbor. A Liberty tanker, split in two while moored in calm water (Fig. 2.3). Without warning and with a huge noise which was heard for at least a mile, the deck and sides of the vessel fractured. The ship was twenty-four hours old. During this period, advances in theoretical, numerical and experimental work were achieved in the field of LEFM. The theoretical work consisted of generalizing Griffith's ideas to any situation of geometry and loading, and to link the energy release rate $G$ to the elastic stress and strain fields. The essence of experimental work consisted in setting up test methods to measure the crack growth resistance $R$. In the energetic approach, the last theoretical step was the discovery of the $J$-integral by Rice (1968 a,b). This helped the circle relating the energy release rate to the stress and strain fields close to the crack tip for any elastic material, linear or not, and supplied a logical tool to analyse fracture for more general nonlinear behaviors. This concept provides the basis for the elasto-plastic fracture mechanics which deals with fracture of ductile materials.

2.2.3 Irwin's Stress Intensity Approach (Stress field approach)

Another major theoretical achievement in LEFM during this period was due to Irwin (1957), who introduced the concept of stress intensity factor $K$ as a parameter for the intensity of stresses close to the crack tip and related it to the energy release
rate. He reformulated LEFM problem in terms of the stress states in the material close to the crack tip and showed that his local approach was essentially equivalent to the Griffith's energetic approach. He stated that if the behavior of the material is isotropic and linear elastic except in a vanishingly small fracture process zone, the stress concentration has the same distribution close to the crack tip whatever the size, shape and specific boundary conditions of the body. Only the intensity of the stress concentration varies. For the same intensity, the stresses around and close to the crack tip are identical.

Irwin (1957) showed that for cracked structures of interest in engineering the stress field at the tip of the crack is singular which decreases in proportion to the inverse square root of the distance from the crack tip. He further concluded that all the possible modes of crack extension of a crack lying in a given plane could be described by the three basic modes of deformation which is discussed in section 1.4.2. Any general mode of cracking can be obtained by the superposition of three modes. The asymptotic expressions for the stress field around a crack tip in the three deformation modes are given by the following relations.

- Mode I \[ \sigma_{ij} = \frac{K_i}{\sqrt{2\pi r}} f_{ij}(r, \theta) \]  
- Mode II \[ \sigma_{ij} = \frac{K_{II}}{\sqrt{2\pi r}} f_{ij}(r, \theta) \]  
- Mode III \[ \sigma_{ij} = \frac{K_{III}}{\sqrt{2\pi r}} f_{ij}(r, \theta) \]

where \( \sigma_{ij} \) are the Cauchy stresses, \( K_i, K_{II} \) and \( K_{III} \) are the stress intensity factors corresponding to mode I, mode II and mode III respectively, \( r \) is the radial distance from the crack tip, \( \theta \) is the angle with respect to the plane of the crack, and \( f_{ij} \) are functions that are independent of the crack geometry and loading conditions. Further, for each mode the function \( f_{ij} \) is different which also varies with each type of stress. Since the quantity \( f_{ij} \) is dimensionless, the stress intensity factor is expressed in units of MPa \( \sqrt{m} \).
Irwin’s approach had the enormous advantage that the stress intensity factors are additive, while Griffith's energy release rates were not. However, his approach was limited to linear elasticity, while the concept of energy release rate was not.

**Relation between K and G**

Both the strain energy release rate $G$ and the stress intensity factor $K$ were the crack driving forces to open a crack, and hence a relationship between $G$ and $K$ was attempted. The relation for problems under mode I opening are given by

- For plane stress
  $$G_I = \frac{K_I^2}{E}$$  
  (2.9)

- For plane strain
  $$G_I = (1-\mu^2)\frac{K_I^2}{E}$$  
  (2.10)

Similarly, relations between $K$ and $G$ are also obtained for modes II and III.

**2.2.4 Application of LEFM to Cementitious Composites**

Most of the early research on fracture mechanics of concrete was based on the assumption that principles of LEFM were applicable. Several experiments were conducted to measure fracture parameters like critical stress intensity factor ($K_c$) or strain energy release rate ($G_c$). Both $K_c$ and $G_c$ were found to vary with notch length, crack length, size and shape of specimens, size and volume of coarse aggregates, methods of testing and mix variables. The inconsistencies observed with $K_c$ and $G_c$ were attributed to the following factors.

- Specimen dimensions.
- Notch sensitivity of concrete.
- Substantial slow crack growth prior to fracture.
- Formation of non-linear fracture process zone ahead of the crack tip.
- Strain softening response of concrete in tension.

It was also found that, LEFM principles were no longer applicable to concrete structures of medium and small size and hence it was not possible to characterize the process of fracture using a single parameter fracture criterion like $K_c$ and $G_c$. 

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2.2.5 Nonlinear Fracture Theories

LEFM was found to give good predictions when fracture was brittle. In reality, this was not the case for many practical situations. This led to the development of the various nonlinear theories. Apart from elasto-plastic fracture mechanics, two major fracture theories were developed, i.e., equivalent elastic crack models and cohesive crack models.

In equivalent crack models, the nonlinear zone is approximately simulated by stating that its effect is to decrease the stiffness of the body, which is approximately the same as increasing the crack length while keeping everything else elastic. This longer crack is called the effective or equivalent crack. Its treatment is similar to LEFM except that some rules have to be added to express how the equivalent crack extends under increased forces. For concrete, the fracture models proposed by Jenq and Shah (1985 a,b) and Bazant and co-workers are among the popular equivalent crack models which have led to the recommendations for fracture properties of concrete. The effect of size on fracture was also explored during 1980s. Bazant (1984a) developed approximate expression to relate the effect of structure size on the nominal strength of structures, and later extended not only for the prediction of failure of structures, but also as the basis of test recommendations for the determination of nonlinear fracture properties, including the fracture energy, the length of the fracture process zone, and the $R$-curve.

The cohesive crack models were developed to simulate the nonlinear material behavior near the crack tip. In these models, the crack is simulated to extend and to open while still transferring stress from one face to the other. The first cohesive model was proposed by Barenblatt (1959, 1962) with the aim to relate the macroscopic crack growth resistance to the atomic binding energy, while relieving the stress singularity. Barenblatt (1959, 1962) postulated that the cohesive forces were operative on only a small region near the crack tip, and assumed that the shape of the crack profile in this zone was independent of the body size and shape. Dugdale (1960) formulated a model of a line crack with a cohesive zone with constant cohesive stress (yield stress). Although formally close to Barenblatt’s, this
model was intended to represent a completely different physical situation: macroscopic plasticity rather than microscopic atomic interactions. In both models the stress singularity was removed. Dugdale's simple approach to plasticity gave good description of ductile fracture for materials having small size plastic zone. Later though a number of more elaborate cohesive crack models have been proposed with various names, all these models shared common features suggested by Barenblatt and Dugdale.

The fictitious crack model proposed by Hillerborg et al. (1976) needs a special mention. In general, all the foregoing fracture mechanics theories required a preexisting crack to analyze the failure of a structure or component, i.e., if there is no crack, neither LEFM nor EFM, equivalent crack models or classical cohesive crack models, could be applied. This is not so with Hillerborg's fictitious crack model. It is a cohesive crack in the classical sense described above, but it is more than that because it includes crack initiation rules for any situation (even if there is no pre-crack). This means it can be applied to initially uncracked concrete structures and describes all the fracture processes from no crack at all to complete structural breakage. It provided a continuous link between the classical strength-based analysis of structures and the energy-based classical fracture mechanics: cohesive cracks start to open as dictated by a strength criterion that naturally and smoothly evolves towards an energetic criterion for large cracks. With a view of providing a simple analytical solution, Ananthan (1989) developed the softening beam model based on fundamental equations of equilibrium for singly reinforced concrete beams. This model includes the softening response of concrete in tension.

During the above period, researchers also analyzed the crack propagation in concrete using numerical techniques like finite element method. The first attempt of using finite element to analyze crack propagation in concrete was by Rashid (1968) where the stiffness of the elements were reduced to zero when the stress reached the tensile strength of the material. Later, more sophisticated models were used with progressive failure of the elements by various researchers. However, even though some results were very encouraging, it later become apparent that numerical
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Analysis using these continuum models with softening yielded results strongly dependent on the size of the finite elements used in the mesh. To overcome this difficulty while keeping the continuum mechanics formulation, Bazant and Oh (1983) developed the crack band model in which the crack was simulated by a fracture band of a fixed thickness (a material property) and the strain was uniformly distributed across the band. Since the 1980s, a great effort, initiated by Bazant (1984b) with the imbricate continuum, was devoted to develop softening continuum models that can give a consistent general description of fracture processes without further particular hypotheses regarding when and how the fracture starts and develops. Inspite of many attempts by various researchers, there exist numerical difficulties associated with using generalized continuum models which prevent these models to be used for practical purposes. Moreover, sound theoretical analysis concerning convergence and uniqueness is still lacking, which keeps these models somewhat provisional. However, the generalizing power of these models is undeniable and they can provide a firm basis to extend some simpler and well accepted models.

2.3 CLASSIFICATION OF FRACTURE MECHANICS

The problems under fracture mechanics are classified into the following three groups.

- Linear – elastic fracture mechanics (LEFM)
- Elastic - plastic fracture mechanics (EPFM)
- Time - dependent fracture mechanics (TDFM)

When the stress-strain behavior and the load–displacement behavior are linear, LEFM can be used and relevant crack tip parameter is the stress intensity parameter \( K \). In this, the plastic zone is small in comparison to the crack size and other pertinent dimensions of the cracked body. When dominantly linear conditions can no longer be ensured due to large–scale plasticity, EPFM is used and relevant crack tip parameter is the \( J \)-integral. Finally, when the stress-strain behavior and the load–displacement behavior is time dependent due to either dynamic loading or due to
time dependent creep, the concepts of TDFM must be used. Both EPFM and TDFM theories are beyond the scope of the present study and hence are not discussed.

2.3.1 Linear Elastic Fracture Mechanics (LEFM)

The linear elastic fracture mechanics is the theory which explains the fracture mechanism (crack initiation and its propagation) of the elastic–brittle material which shows the linear behavior between the stress and strain within the elastic limit and fails suddenly at the elastic limit. In other words fracture mechanics in the absence of plastic effect is known as Linear Elastic Fracture Mechanics (LEFM). It considers infinitely small fracture zone in front of the crack. In LEFM the description of brittle fractures involves only one additional parameter (i.e., fracture toughness), besides the usual two elastic constants $E$ and $\mu$. Also in LEFM it is assumed that the stresses and strains in the vicinity of a sharp crack tip are very large and during the fracture process the entire body remains elastic and energy is only dissipated at a point (the crack tip).

2.3.2 Limitations of LEFM

Though LEFM predicts the fracture behavior of concrete it has its own limitations. They are as follows:

- Griffith (1921) fracture theory is only applicable to elastic homogeneous brittle materials, such as glass. Its later modification by Irwin (1957) is also applicable to elastic-plastic homogeneous materials, such as metals with limited ductility. The modified methods give a crude estimate of the length of fracture process zone in concrete.

- Stress and displacements predicted by LEFM are only valid, if the size of the plastic zone is small and within the singularity dominated zone. Proper results can be obtained when the plastic zone is small compared to the size of the structure. But in concrete there exists an extensive fracture process zone ahead of pre-existing notch/crack.
• If the plastic zone is relatively large, like ductile metals, then the singularity dominated zone is destroyed and the stresses and displacements predicted by LEFM have no meaning.

• For concrete, if only one parameter (fracture toughness) is considered, then the fracture toughness increases with increasing compressive strength or increasing strain rate. Concrete becomes more brittle when its compressive strength increases.

• In case of structures characterized by plane stress condition, the propagation of crack is very slow and unless it is determined the onset of instability cannot be characterized and hence LEFM becomes inapplicable for such conditions.

2.4 REVIEW OF LITERATURE

Encouraged by the success in the application of fracture mechanics to metals, many attempts were made to apply fracture mechanics to cementitious materials. A number of analytical, experimental and numerical studies are carried out by researchers to understand the fracture behavior of concrete. A brief review of important works related to the present study is discussed in the following section. The review has been divided into three parts, i.e., flexure, size effect and minimum reinforcement.

2.4.1 Flexure

Kaplan (1961) was the first to apply LEFM principles to concrete. Tests were conducted on concrete beams with crack simulating notches under three and four point loading arrangement on two different sizes of specimens. Two methods, i.e., analytical and experimental, were used to determine the critical strain energy release rate $G_C$ associated with the rapid extension of the crack. The $G_C$ values computed for beams with different notch depths subjected to different arrangement of load were found to be in good agreement. However, $G_C$ varied greatly with the beam size, i.e., smaller sized beams gave a higher value of $G_C$ when compared with larger beams. Slow crack growth in front of a crack tip prior to unstable crack propagation was pointed out to be the possible factor that influenced the results. It
was suggested that the critical strain energy release rate should be ascertained by suitable analytical and experimental procedures and using this value it may be possible to predict the fracture strength of concrete containing cracks.

Walsh (1972) studied the application of linear fracture mechanics to the fracture load of plain concrete specimens. The results of pre-cracked concrete specimens depended on the size of the specimen. Further, the results of twenty-four tests on cracked beams with depths ranging from 75 mm to 375 mm also showed similar trend. Further, to apply the LEFM principle, it was suggested that a minimum depth of 225 mm for beam specimens should be used for fracture testing of concrete.

Hillerborg et al. (1976) proposed a method of analysis of crack formation in concrete by means of fracture mechanics and finite elements. The method is popularly known as the fictitious crack model. In this model stresses were assumed to act across a crack as along as it is narrowly opened. The assumption was regarded as a way of expressing the energy absorption $G_C$ in the energy balance approach, but it is also in agreement with results of tension tests. As a demonstration the method was also applied to the bending of an unreinforced beam, which led to an explanation of the difference between bending strength and tensile strength, and of the variation in bending strength with beam depth. It was concluded that the approach seems to yield realistic results with regard to crack formation and propagation as well as failure even if a coarse element mesh was used.

Petersson (1981) conducted three point bending tests on notched beams to determine the fracture energy $G_F$. In addition two more different types of concrete specimens were also tested. Based on the test results it was concluded that $G_F$ was reasonably independent of specimen dimensions and could be considered as a useful material property for concrete.

Bazant and Oh (1983) proposed the crack band theory for concrete. The model is popularly known as blunt crack model. The model considers the material to exhibit a gradual strain-softening due to microcracking, and the aggregates were not
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necessarily small when compared with the structural dimensions. The Mode I fracture was modeled as a blunt smeared crack band, which was justified by the random nature of the microstructure. The material fracture parameters were characterized by only three parameters—fracture energy, uniaxial strength limit and width of the crack band, while the strain-softening modulus was a function of these parameters. A method to determine fracture energy was also given. The theory was verified by comparisons with numerous experimental data from the literature. The optimum value of crack band width was found to be about three times the aggregate size. The method of implementing the theory in a finite element code was also indicated. A simple formula was suggested to predict fracture energy from the tensile strength and aggregate size. The statistical analysis revealed that the theory was superior to LEFM as well as strength theory, and also that the principles of fracture mechanics can be applied to concrete.

Jenq and Shah (1984) showed the inapplicability of LEFM principles to concrete because of the nonlinear effects associated with crack propagation. It was concluded that the slow crack growth, closing pressure in the process zone and inelastic behavior of concrete need to be considered in order to apply LEFM to concrete. It was shown that $K_{IC}$ was a size independent fracture toughness parameter. It was reported that the crack surface formation mechanisms and energy absorbed by non-critical section were the factors causing the size effect on $G_F$.

Jenq and Shah (1985b) proposed the two parameter fracture model for concrete. The model considers the introduction of the nonlinear slow crack growth occurring in concrete beam prior to the peak load. The two parameters considered were the critical stress intensity factor $K_{ICS}$ calculated at the tip of the effective crack, and the elastic critical crack tip opening displacement $CTOD_C$. Based on the test results, the two parameters were found to be size dependent. The model was shown to calculate the maximum load (Mode I) of a structure of an arbitrary geometry. The model results were shown to compare well with experimental results under tension and flexure.
Nallathambi and Karihaloo (1986a) studied two existing models proposed by Hillerborg et al. (1976) and Bazant and Cedolin (1979) for predicting the load-deflection behavior of plain concrete from fracture energy measured in three-point bending. These models were shown to overestimate the peak load attainable by concrete beams. This major drawback was overcome by proposing a new model that accounts for the strain hardening in the material prior to the attainment of peak load. It was also argued that fracture energy in its present form was dependent on the specimen size and therefore it was not a reliable indicator of the fracture toughness of plain concrete.

Nallathambi and Karihaloo (1986b) proposed simple analytical expressions to determine the critical stress intensity factor $K_C$ and the critical energy release rate $G_C$ of plain concretes in three-point bending. These expressions incorporated the slow crack growth preceding fracture and the complex state of stress existing at a propagating crack front. The fracture toughness so determined was shown to be essentially independent of the test specimen dimensions and to depend only on the mix variables: It was also argued that unlike $K_C$ or $G_C$ the fracture energy $G_f$ of plain concrete was strongly dependent on the test specimen dimensions. It was suggested to express the fracture toughness of plain concretes through the specimen size independent parameters $K_C$ or $G_C$.

Malvar and Warren (1988) conducted tests on single edge notched beam under three-point bending to evaluate the fracture energy of concrete. The fracture energy was determined from the area under the complete load-deflection diagram. A nonlinear fictitious crack model was also implemented in a finite element analysis which showed good agreement with the experimental results. It was also shown that fracture energy changed with variation in notch depth and beam depth. This was attributed to the energy dissipation in the process zone which was not accounted in the analytical model.

Ananthan et al. (1989) studied the fracture behavior of notched and un-notched plain concrete beam under three-point bending using the softening beam model.
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The model was developed using the fundamental equations of equilibrium. The influence of structural size in altering the fracture mode from brittle fracture to plastic collapse was presented through the stress distribution across the uncracked ligament obtained by varying the strain softening modulus. It was shown that the length of the fracture process zone depends on the value of strain softening modulus. Several nonlinear fracture parameters, such as crack tip opening displacement, crack mouth opening displacement and fracture energy were computed for a wide variety of beam specimens. A simple procedure was also given to obtain quite accurately both the pre-peak and post-peak portions of load versus crack opening displacement. Also a procedure was also given to calculate the maximum peak load. The analytical results were reported to compare very well with experimental results.

Brincker and Dahl (1989) adopted the substructure method introduced by Petersson (1981) which was reformulated to analyse the three-point loading specimen in order to obtain complete load-displacement relations without significant truncation. To avoid the problem of instability caused by linearization of the softening in fracture zone, an alternate energy formulation was given so that it was possible to distinguish between stable and unstable situations. The method was used to give results using computer for multi-linear stress crack opening displacement. It was also concluded that a bi-linear stress-displacement approximation was sufficient when compared with other types.

Chaung and Mai (1989) investigated the flexural behavior of a beam to establish the correlation between the tensile and bending properties of strain-softening solids. It was found that knowing the complete uniaxial stress-strain relations, including the post-peak tension softening gives rise to enhanced bending strength in agreement with experimental data. It was also shown that conversely, given the bending responses together with the softening characteristics the complete tensile behavior can be determined. It was concluded that since bending experiments were easier to perform than uniaxial tensile tests, this well-defined correlation provides a feasible
means to obtain the entire tensile behavior of strain-softening solids such as concrete, rocks and ceramics.

Carpinteri (1990) applied a cohesive crack model to analyse the crack stability in elastic-softening materials. The shape of the load-displacement response was observed to change substantially when size-scale was varied though the geometrical shape of the structure was unchanged. The softening branch tends to become steeper with the increase in size scale. It was observed that a critical size scale does exist for which the softening slope was infinite. For this, the load carrying capacity also drastically decreased for relatively small displacement increments. For larger size-scales, the softening slope was observed to become positive and a cusp catastrophe appeared. The bifurcation point was determined using the simple LEFM condition $K_f = K_{IC}$.

Jenq and Shah (1991) examined different models for mode I crack propagation in concrete. It was proposed that notch sensitivity and size effect can be used as criteria to evaluate the validity of fracture models without difficulty. In addition, it was stated that a successful model should also satisfy a portability condition, namely the model should be applicable to structures of an arbitrary geometry. Discussion was also presented on the effects of different loading conditions.

Bazant (1992) arrived at the following conclusions after reviewing the various theoretical and experimental results on the size effect in reinforced concrete structures, discussing the size effect law and concept of brittleness number, and experimental results conducted on diagonal shear failure, punching shear failure, torsional failure and pullout failure. The experimental results were in excellent agreement with the theoretical size effect law. It was also pointed out that the test data on diagonal shear disagreed with the classical Weibull-type theory of size effect, thus strengthening the theoretical arguments against using this theory for the size effect in concrete structures whose maximum load was much larger than the cracking initiation load. Based on the results it was recommended that fracture mechanics size effect ought to be incorporated into the formula for the contribution
of concrete to the ultimate load capacity in brittle failures of concrete structures. It was also shown that such formulas can be based on brittleness number. For any given structure shape, it was possible to determine this number from size effect tests.

Gerstle et al. (1992a) studied the crack growth in flexural members using fracture mechanics approach. Several simplifying assumptions were introduced to the fictitious crack model which allows a reinforced concrete beam to be simulated in an approximate analytical approach. The elastic deformation of the beam was due to a moment $M$ over a band of length $2a$ where $a$ was the crack length. A method was developed to predict not only the crack widths and lengths, but also the strength and cracking stability of reinforced concrete beams. Design equations and charts were also presented. The formulation of an equation for size effect curve of an unreinforced, unnotched member was also discussed.

Hawkins (1992) discussed the role of fracture mechanics in reinforced concrete design. He stated that post-cracking behavior, which controls stiffness and durability at service loads especially for unidirectionally and lightly reinforced members, was closely tied to concrete properties (fracture softening) best established through displacement controlled fracture type tests. He opined that fracture mechanics concepts will not influence building code provisions for reinforced concrete until practitioners start using those concepts to improve their designs. To facilitate that action, efforts were needed to develop data that correlate the relative contributions of the fracture softening properties of different concretes to building code provisions that control designs in which only unidirectional reinforcement was desired. When that correlation has been established, and when appropriate fracture softening characteristics of different typical concretes have been determined through a comprehensive test program, then designers will need an ASTM Standard that they can use to check whether the fracture softening characteristics (of the actual concrete placed in their structure) used in their design were being achieved.
Ulfkjaer et al. (1994) extended the analytical model based on fictitious crack propagation developed for plain concrete beam to lightly reinforced concrete beam. Though the model was simple it was able to capture some distinctive effects, i.e., the change in behaviour from brittle to ductile when the reinforcement area increased, and from ductile to brittle when the size scale increased. Comparison with experimental results showed that the assumption of a linear softening relation was not adequate.

Ulfkjaer et al. (1995) proposed an analytical model for load-displacement curves of plain concrete beams. The fracture was modeled by a fictitious crack in an elastic layer around the midsection of a plain concrete beam under three-point loading. Outside the elastic layer the deformations were modeled by beam theory. A linear softening relation was used for the load-displacement relation. Results from the analytical model were compared with results of numerical analysis covering a wide range of beam sizes. It was concluded that the analytical models were in good agreement with the numerical results when the thickness of the elastic layer considered was half the beam depth. It was shown that the point on the load-displacement curve where the fictitious crack starts to develop and the point where the real crack starts to grow correspond to the same moment. Closed-form solutions for the maximum size of fracture zone and the minimum slope on the load-displacement curves were also presented.

Iyengar et al. (1997) proposed a method to determine the load-deflection curves of plain concrete notched beam in three-point bending using the softening beam model. The predicted load-deflection curves were compared with the ones from experiments and a satisfactory agreement was found. The method requires simple programming and was concluded to be cost effective.

Iyengar et al. (1998) developed an analytical model for fictitious crack propagation in concrete beams using a bilinear $\sigma$-$w$ relation. The model proposed was an extension of the earlier model developed by Ulfkjaer et al. (1995) which used a linear $\sigma$-$w$
relation. The results obtained from the bilinear $\sigma$-w relation agreed closely with the numerical results given by Brincker and Dahl (1989).

Christensen and Brincker (1999) investigated the failure behaviour of lightly reinforced concrete beams. A numerical model based on the fictitious crack approach according to Hillerborg et al. (1976) was established in order to estimate the load-deflection curve for lightly reinforced concrete beams. The debonding between concrete and reinforcement was taken into account by introducing a debonded zone with constant shear friction stress. Results were presented for material models representing normal strength concrete (two degrees of brittleness) and high strength concrete.

Elices et al. (2000) showcased the value of fracture mechanics tools in dealing with engineering fracture problems of concrete, either plain or reinforced in two parts. The first part covered the review of modeling concrete fracture in tension: the suitability of linear elastic fracture mechanics (LEFM) as an asymptotic approach was considered first, followed by an outline of classical fracture models based on stress-strain relations and their associated problems of non-objectivity. And, finally, cohesive process zone models based on stress-displacement relations were shown to be one of the simplest models capturing the essential features of fracture processes in concrete. The second part gave some practical examples of applications of fracture mechanics to concrete, mostly drawn from the authors' experience: the difference between strength and toughness in concrete was clearly shown in the example of piles. The size effect in flexural strength, unobtainable with classical strength theories, was accurately predicted with the cohesive process zone model. For plain concrete and large concrete structures such as dams, LEFM was proven suitable. Fracture in reinforced concrete was a more involved problem; nevertheless some promising results for lightly reinforced concrete beams were discussed. Also some comments on fracture of fiber reinforced concrete (FRC) and its application were presented.
Iyengar et al. (2001) compared the results obtained from the fictitious crack model and blunt crack model for a three-point loaded plain concrete beam. Different types of softening i.e., linear, bilinear and trilinear relations were used for concrete. The study showed that the results of the two models for plain concrete beams agreed well.

Iyengar and Raviraj (2001) studied the flexural behavior of plain concrete beam using the blunt crack model (BCM). A generalized power law (with an exponent $n$) was used for the post peak stress-strain relation. It was observed that for the particular case of $n=1.0$ (i.e., linear softening), the resulting equations were the same for both blunt crack model and fictitious crack model. Determination of moment curvature and load deflection curves were discussed. Calculations were given for the critical brittleness factor from the snapback considerations for the beam. A method was suggested for determining softening exponent $n$ and the elasticity co-efficient of the central layer $k$ from an experiment.

Iyengar et al. (2002) analysed the crack propagation in strain softening beams using the fictitious crack model. A beam under three-point bending was considered in the analysis. The stress-displacement relation assumed was a generlaised power law function. Relations for moment-curvature were given. The effect of softening exponent $n$ on the size effect and snap back behavior of beams were presented. Effect of length of central elastic layer $h$ on moment-rotation results was also given. A method to determine $n$ and $h$ from experiments was also suggested.

Raghuprasad et al. (2005) proposed a fracture mechanics model for the analysis of plain and reinforced high-performance concrete beams. This improved model was based on the fundamental equilibrium equation for the progressive failure of plain concrete beams. The concrete stress-strain relationship in tension was derived by calculating the peak tensile stress and softening modulus for different depths of beams on the basis of the fracture parameters obtained with the size effect law. The model used the peak tensile stress and the softening modulus, which varied with the size of the beam.
Vidyasagar and Raghuprasad (2008) presented a hybrid approach to obtain the fracture energy $G_F$ of concrete in a simpler way. In this approach one of the variables like the length of transition ligament was obtained experimentally using which $G_F$ could be obtained. The $G_F$ obtained from the proposed hybrid model was slightly larger than those mentioned in literature. However, the authors claim the values reported by the hybrid model could be considered more realistic.

Xu and Zhang (2008) used the new analytical model describing fracture behaviour on cracked concrete based on the conception of energy release rate. Fracture tests were conducted on three-point loading beams and wedge-splitting on compact tension specimens. Double-$G$ fracture parameters were determined from the experiments. Further the unstable fracture energy releases were calculated. A comparable result between the measured and calculated unstable fracture energy releases confirmed this assumption. In order to verify the feasibility of this new model, the effective double-$K$ fracture parameters converted by double-$G$ fracture parameters using $K = \sqrt{\frac{E}{G}}$ were compared with the double-$K$ fracture parameters calculated by double-$G$ fracture model. It was found that there was a good agreement. Further, the results of another two series of three-point bending beams carried out by Swartz and Refai (1989) were also collected to provide more experimental verification. It was concluded that the results obtained from the double-$G$ fracture model agreed well with those of double-$K$ fracture model.

### 2.4.2 Size Effect

Bazant (1984a) developed the famous size effect law (SEL) by studying the size effect in blunt fracture considering concrete, rock and metal. He explained that the fracture front in concrete, as well as rock, was blunted by a zone of microcracking, and in ductile metals by a zone of yielding. This blunting caused deviations from the structural size effect known from linear elastic fracture mechanics (LEFM). The size effect was studied first for concrete and rock structures, using dimensional analysis and illustrative examples. Fracture was considered to be caused by propagation of a crack band that had a fixed width at its front relative to the aggregate size. The
analysis was on the hypothesis that the energy release caused by fracture depended on both the length and the area of the crack band. The size effect was shown to consist of a smooth transition from the strength criterion for small sizes to LEFM for large sizes, and the nominal stress $\sigma_N$ at failure was found to decline with the structure size. A function was given which was verified by Walsh's (1972) test data. The results of reinforced concrete were also discussed. It was also noted that some known size effects which have been attributed to random strength variations within the structure should be explained by fracture mechanics, which gives a very different extrapolation to large structures.

Nallathambi et al. (1985) investigated various size effects in fracture of concrete by conducting a series of three-point bend tests on pre-cracked cement mortar and concrete beams. The tests were performed with a view to study the influence of pre-crack, aggregate and specimen sizes on the fracture of concrete. A simple formula based on the experimental data was proposed to account for all the three size dependent effects.

Bazant and Pfeiffer (1987) analysed a number of tests on the size effect due to blunt fracture. They proposed fracture energy as the specific energy required for crack growth in an infinitely large specimen. This definition eliminates the effects of specimen size, shape, and the type of loading on the fracture energy values. The problem was to identify correct size effect law to be used for extrapolation to infinite size. Bazant’s (1984a) simple size effect law was applicable for this purpose as an approximation. Indeed, very different type of specimens, including three point bend, edge-notched tension and eccentric compression specimens were found to yield approximately the same fracture energy values. The $G_F$ values found from the size effect approximately agreed with the values of fracture energy for the crack band model when the test results were fitted by finite elements.

Bosco et al. (1990a) investigated the fracture of reinforced concrete for scale effects and snap-back instability. Remarkable size-scale effects were theoretically predicted and experimentally confirmed in low reinforced high strength concrete beams. The
brittleness of the system was found to increase by increasing size-scale and/or
decreasing steel area. The tensile strength and toughness of concrete, usually
disregarded, were observed to be so high in some cases that the peak bending
moment overcame the bending moment of limit design (hyper strength). The drop in
the loading capacity could hide a virtual softening branch with positive slope (snack-
back), which was detected if the loading process was controlled through the crack
width.

Gerstle et al. (1992b) investigated the effect of reinforcement and initial crack length
on the fracture mechanics size effect in a lightly reinforced concrete specimen in
flexure using a simple analytical model. An unprenotched and/or reinforced beam
was found to exhibit a lower limit to the size effect; however, a prenotched
unreinforced beam demonstrated no lower limit in strength as the size increased. A
parametric study was carried to show the region which prominently exhibits size
effect.

Gerstle et al. (1992c) used the simple analytical model developed earlier by the
authors to study the size effect in reinforced flexural members. The model used the
fictitious crack concept to determine the crack growth in small beams and LEFM to
determine crack growth in large beams. The model captured size effect which
started with a high nominal strength for low values of normalised depth of beam
(small beam) and a low nominal strength for high values of normalised depth of
beam (large beam).

Planas and Elices (1992, 1993) showed that for large sizes, a notched specimen
behaves close to LEFM. They proved that the limiting size effect behavior was given
by an expression identical to that resulting from an equivalent elastic crack model. It
was also shown that for large enough sizes, the values of stresses and displacements
at points far from the cohesive zone were identical to those given by a suitable
chosen effective crack model.
Eo et al. (1994) observed experimentally that the flexural strength of unnotched plain concrete specimens initially decreased with an increase in specimen's depth, and subsequently reached a limiting value for very large specimens. They could not understand the factors dictating that limiting value. A FEM using a bilinear FCIM was made to investigate size effect predictions for normal strength concrete and high strength concrete three-point bending specimens with and without an initial crack. A simple size effect equation was proposed based on the numerical results and was compared with the prediction of equations proposed by others. It was concluded that fracture characteristics and notch characteristics need to be included in size effect relationships.

Guinea et al. (1994) computed the size effect on the maximum load for notched beams made of concrete characterized by cohesive cracking with bilinear softening. It was found that a limiting size $D_e$ can be defined for a sample so that when the size of the sample is less than $D_e$ the tail of the softening curve will be irrelevant for predicting the maximum load and, therefore, the concrete appears to be indistinguishable from a material with linear softening. Based on these results, a simplified procedure to compute the size effect curve was presented. The analysis was generalized to cover the usual specimen geometries.

Rokugo et al. (1995) investigated size effects and shape effects on the flexural strength of concrete through experiments and numerical analysis using fracture mechanics concepts. The flexural strengths of smaller specimens were found to be greater. Further, the flexural strength of specimens with circular cross section was found to be greater than with rectangular cross section which was attributed to the contribution of the tension softening properties of concrete. Using the numerical results, an equation was proposed to estimate the flexural strength of concrete beams. The estimated values of flexural strength were found to be in good agreement with the experimental results.
Morgan et al. (1997) showed that the size effect in flexure and shear strength for different plain and reinforced concrete beam sizes subjected to concentrated and uniformly distributed loads could be successfully predicted on the basis of nonlinear fracture mechanics. An advanced nonlinear analysis was carried out by computer simulation. Here, the fictitious crack model was adopted with two orthogonal rod elements which involved the nonlinear fracture mechanics through their constitutive model to simulate the discrete crack path and represent the localized crack zone. By combining the arc-length calculation technique with the fictitious crack model, it was shown that the post-peak behaviour could be predicted well even for snapback instability.

Planas et al. (1997) suggested a single function that fits the size effect curve for notched specimens of small and medium sizes. Further, for three point bending specimens with relatively deep notches \((a_0/D \geq 0.3)\), a single equation was suggested which worked well for small and medium sizes.

Load-deformation curves for reinforced concrete beams subjected to bending showed size effect due to tensile failure of the concrete at early stages in the failure process and due to compression failure of the concrete when the final failure took place. Brincker et al. (1999) modeled these effects using fracture mechanics concepts, and size effects of the models were studied and compared with experimental results.

Carpinteri et al. (2003) presented a dimensionless formulation of the bridged and the cohesive crack model for reproducing the constitutive flexural response of a reinforced concrete element with a nonlinear matrix. The nonlinearity of the matrix was modeled by considering a distribution of closing forces onto the crack faces which increased the fracture toughness of the cross-section with a shielding action. The peculiarity of the models consisted in the imposition of both the equilibrium and the compatibility conditions to the cracked element. The constitutive flexural response depended on three dimensionless parameters, which controlled the extension of the process zone and related to the reinforcement phases. Some
experimental and numerical examples were also presented and discussed in order to validate the consistence of the proposed approach.

Belgin and Şener (2008) investigated the size effect on failure of over reinforced concrete beams. The results of full-scale failure of singly reinforced four-point-bend beams of different sizes containing deformed longitudinal reinforcing bars were reported. The tests consisted of four groups with one-, two- and three-different size combinations. The specimens were made of concrete with a maximum aggregate size of 10 mm. The beams were geometrically similar in one-, two- and three-dimensions, and even the bar diameter and cover thicknesses were scaled in proportion. The reinforcement ratio was 3%. The results revealed the existence of a significant size effect, which could approximately be described by the size effect law previously proposed by Bazant (1984a). The size effect was found to be stronger in two-dimensional similarities than for one- and three-dimensional similarities.

Eswari and Sanakarasubramanian (2008) used a cohesive zone modeling to simulate analytically the size effect in plain concrete beams. The authors observed that the size effect in plain concrete beam was predominant and needs detailed study. The area under the load displacement curve for smaller specimen was found to be greater than larger size specimens indicating that large beams were more brittle than smaller beams.

Singh et al. (2008) studied size effect for the fracture energy of concrete structures using non-linear fracture mechanics. A new experimental programme for material characterization of softening behavior of concrete in compression and tension was described. The fracture energy evaluation on notched and unnotched, plain and reinforced, three-point loading beams were presented using conventional instrumentation, acoustic b-value analysis and high resolution image processing systems. A few case studies were also presented with numerical finite element cohesive crack and crack band models to illustrate the issues of mesh sensitivity.
2.4.3 Minimum Reinforcement

Bosco et al. (1990b) carried out a one-dimensional analysis to compute the minimum amount of reinforcement for high-strength concrete members in flexure. The amount of minimum reinforcement was assumed to be influenced by the conditions of simultaneous first cracking and steel yielding. The model utilizes a factor $N_p$, defined as brittleness number which is a measure of the ductility of the test. The brittleness number $N_p$ for a structure was found to increase by increasing the size and/or decreasing the steel content, which was similar to the actual trend. Experiments were also conducted to verify the influence of size and steel on brittleness of beam.

Carpinteri et al (1990) compared the experimental results and numerical simulations with theoretical results based on LEFM model. The theoretical model was found to provide a satisfactory estimate of the minimum percentage of reinforcement which depends on the scale and enables the element in flexure to prevent brittle failure. Though the minimum steel percentage provided by Eurocode 2 (1992) and ACI 318 (2005) were independent of beam depth, the relationship established by the brittleness number suggested for decreasing values with increasing depths.

Baluch et al. (1992) proposed a fracture mechanics based method to compute fracture moment for a reinforced concrete member subject to Mode I stress field in the presence of an edge crack. It was reported that the applicability of the method was not hindered by the condition of steel stress and the crack propagation at both phases of pre-yielding and post-yielding of steel could be predicted with reasonable accuracy. The proposed model also had the potential of serving as a crack-controlled design in which crack propagation was restricted to a permissible height. The method yielded an easy-to-use expression for the minimum reinforcement requirement in flexure ($\rho_{\text{min}}$) in terms of two variants, which showed dependency on concrete properties and beam geometry. This dependency of $\rho_{\text{min}}$ on the characteristic materials was significant in view of the increased use of high strength concrete.
Gerstle et al. (1992a) using the analytical model for crack growth in flexural members based on fictitious crack propagation developed charts for normalized applied moment $M$ versus normalized crack length $A$ for various values of reinforcement parameter $\alpha$ and material scale parameter for concrete $\beta$. They defined $A_{\text{stab}}$ as the normalized crack length beyond which cracking becomes unstable. Using the charts, the values of $A_{\text{stab}}$ for combination of $\alpha$ and $\beta$ values were chosen. It was concluded that for low values of $\beta$ and high values of $\alpha$, unstable cracking does not occur. Also from the charts, minimum $\alpha$'s (called as $\alpha_{\text{min}}$) for which the $M$- versus $-A$ curve increases monotonically were obtained, and was plotted against $\beta$. An equation was developed to fit the above curve which was proposed as a design aid to determine the minimum reinforcement ratios to prevent unstable cracking.

Hawkins and Hjorteset (1992) examined minimum reinforcement requirements for concrete flexural members using fictitious crack models, four different crack closure stress (CCS) distributions, and the results of physical test on slabs and beams of varying depth. It was concluded that a CSS straight-line model with an associated $G_F$ value based on measured material properties was appropriate provided account was taken of shrinkage effects and the way those effects influence cracking. It was stated that minimum reinforcement requirements should be based on gross section properties and the condition that the yield moment be greater than the cracking moment. It was shown that fracture mechanics principles demonstrate that minimum ratios depend on member depth, ratios of concrete to steel strength and concrete material properties.

Ruiz et al. (1996) demonstrated experimentally the influence of bond on peak load for flexural members considering a fixed steel ratio for two different types of reinforcement: ribbed bars with strong bond, and smooth bars with weak bond. They also verified the existence of double peak in load-displacement diagram obtained experimentally with their cohesive crack model. Based on the above results, a relation for minimum reinforcement in flexural members was suggested.
Ozbolt and Bruckner (1999) studied and discussed different aspects of the requirement for the minimum reinforcement ratio. The influence of the beam depth was investigated in more detail. Numerical analysis for reinforced concrete beams of different sizes was carried out using finite element method based on the nonlocal mixed constrained microplane model. Some test results were compared with the numerical results. It was concluded that the requirement on the minimum reinforcement depends on the beam size but also on the material properties as well as on the amount and type of the distributed reinforcement. To define the dependency between the minimum reinforcement and geometrical as well as material parameters in more detail, further theoretical and experimental studies were recommended.

Ferro et al. (2007) examined the problem of the assessment of minimum reinforcement in concrete members both theoretically and experimentally by the bridged crack model. The model used was demonstrated to be an efficient numerical tool for investigating the behavior of structural elements in bending, and allowed to show the minimum reinforcement percentage depended on the structural element size, and decreased with increasing beam depths. In the model, Linear Elastic Fracture Mechanics concepts were used to determine the equilibrium and the compatibility equations of a beam segment subjected to bending in presence of a mode I crack. Recently, the model was extended to include the presence of closing stresses as a function of the crack opening in addition to steel reinforcement closing traction. This allowed to characterize the mechanical behavior of fiber reinforced structural elements. A criterion to account for crushing in compression was also introduced, to bound from below (minimum reinforcement) and from above (maximum reinforcement) a region of stable and ductile mechanical behavior as a function of the mechanical properties as well as the size of the structural element. Some experimental results were also discussed.
2.5 SUMMARY

Literature review in this chapter is mainly focused on the previous works carried by various researchers related to fracture study on concrete beams. The basic aspects of study covered are flexure, size effect and minimum reinforcement of concrete beams. Considering the above review, the following aspects are noticed.

- The analytical models developed previously for concrete beams have not considered a more realistic stress variation for concrete in compression.
- The analytical models developed earlier are mostly based on fictitious crack propagation.
- Comparison of the analytical models for reinforced concrete beam considering the fictitious crack propagation and blunt crack propagation need to be made considering more aspects.
- The standard codes do not consider the size effect in concrete structures on nominal strength and ductility.
- Most of the codes have specification for minimum reinforcement of reinforced concrete beam as a function of grade of steel only and hence needs modification.
- The minimum reinforcements of other standard codes are compared with respect to minimum reinforcement obtained by previous researchers with their models based on fracture theories. However no comparison is made with reference to Indian code.

The present study finds importance based on the above factors. The following chapters presented in this work are focused towards achieving the above needs.