CHAPTER 7
7.1 Nomenclature

$C_p$ : Specific heat at constant pressure

$erfc$ : Complementary error function

$g$ : Acceleration due to gravity

$G_r$ : Thermal Grashof number

$k$ : Dimensionless porosity factor

$k_1$ : Thermal conductivity of the fluid

$k'$ : Porosity factor

$k^*$ : Mean absorption coefficient

$M$ : Dimensionless magnetic intensity

$N$ : Radiation parameter

$p$ : Pressure

$p_r$ : Prandtl number

$q_r$ : Radiative heat flux in the y-direction

$t$ : Time

$T$ : Temperature of the fluid near the plate

$T_w$ : Temperature of the plate

$T_\infty$ : Temperature of the fluid far away from the plate

$u$ : Velocity of the fluid in the x-direction

$u_0$ : Velocity of the plate

$U$ : Dimensionless velocity

$y$ : Coordinate axis normal to the plate

$y'$ : Dimensionless coordinate axis normal to the plate

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Greek symbols

\( \alpha \) : Thermal diffusivity
\( \beta \) : Volumetric coefficient of thermal expansion
\( \beta_0 \) : Magnetic induction
\( \mu \) : Coefficient of viscosity
\( \nu \) : Kinematic viscosity
\( \theta \) : Dimensionless temperature
\( \rho \) : Density
\( \sigma \) : Stefan-Boltzmann constant
\( \tau \) : Dimensionless skin-friction

7.2 Introduction:

Mixed convection in porous medium has important applications mainly in soil physics, geothermal energy extraction, chemical engineering and in several biological systems. Radiation in heat transfer accounts in high temperature applications in plasma physics, liquid metal floors, magneto hydrodynamic accelerators and in power generation systems. The problem assumes greater significance in the heat and mass transfer and plays an important role in the design of nuclear and chemical reaction chambers, gas turbines, design of turbine machinery and turbine blades. Also the problem is of prime importance in several industrial and environmental situations viz: heating and cooling chambers, chemical reactors, fossil fuel combustion and energy processes, evaporation of water from large open reservoirs, astrophysical flows, solar power energy and space vehicle re-entry, radiative heat transfer plays a significant role.

In all above applications, understanding the nature of flow entities and convective heat transfer characteristics are of primary requirements to examine the problem in detail. Further, the problem of effect of porosity and time on moving infinite vertical plate with uniform heat flux is applicable in situations demanding the efficient design of propulsion devices for aircraft and naval engineering.

The thermal radiation effects of a optically thin gray gas bounded by a stationary vertical plate was studied by England and Emery [81] while, Cheng and Minkowyz [88] examined free convective heat transfer characteristics for vertical
plate in porous medium and obtained analytical expressions for the boundary layer thickness, local and overall heat flux. Subsequently, Cheng [90] had investigated the mixed convection on inclined surface using boundary layer approximation theory. The effects of transverse magnetic field on the heat transfer characteristics in porous medium was analysed by Ramana Rao, et al. [89]. They brought out the effects of porous parameter on temperature and Nusslet number. Later, Soundalgkar and Takhar [82] examined the radiative free convective flow of an optically thin grey gas past a semi-infinite vertical plate. Thereafter, the radiation effect on mixed convection along an isothermal vertical plate was studied by Hossain and Takhar [83].

In all above situations, a stationary vertical plate was considered. The effect of thermal radiations and free convection past a moving infinite vertical plate was examined by Raptis and Perdikis [84]. Later, Chamkha [95] studied the effects of thermal radiation and magnetic field on natural convection heat transfer from an inclined plate embedded in variable porosity medium, while Merkin and Pop [96] obtained the similarity solutions for mixed convective conditions considering wall temperature variations. Thereafter, Aly et al. [92] had numerically investigated the existence and uniqueness of a vertically flowing fluid past a vertical surface in porous medium by considering variable wall temperature under mixed convective conditions. Subsequently, Muthucumaraswamy and Ganesan [85] presented the case of radiation effects on moving infinite vertical plate with variable temperature. The governing equations of motion were solved by the Laplace Transform method. Sobha and Ramakrishna [91] presented the effects of magnetic field on heat transfer in porous medium under natural convection.

The effect of magnetic field on the mixed convective flow over a heated vertical plate in porous medium for buoyancy aiding and opposing flow conditions was studied by Ramakrishna and Sobha [93]. The effects of MHD mixed convection heat transfer from a vertical surface with radiation conduction interaction for both transverse and induced magnetic field under isothermal surface conditions was presented by Rabhi A. Damesh [94]. Inclusion of radiation component in the conservation energy equation makes it complicated due to non-linear partial differential equations. Recently Chandrakala and Antony Raj [86] studied the effects of thermal radiations on the flow past a semi-infinite vertical isothermal plate with uniform heat flux in the presence of transversely magnetic field. The problem of
unsteady natural convection flow past an impulsively started infinite vertical plate with uniform heat flux in the presence of radiation has not received much of attention from contemporary researchers. Therefore, an attempt has been made by Chandrakala [87].

Flow in a porous medium is an ordered flow in a disordered geometry. The transport process of fluid through a porous medium involves two substances, the fluid and the porous matrix. Therefore, it will be characterized by specific properties of these two substances. A porous medium usually consists of a large number of interconnected pores each of which is saturated with the fluid. The exact form of the structure, however, is highly complicated and differs from medium to medium. A porous medium may be an either an aggregate of a large number of particles such as sand or gravel or solid containing many capillaries such as porous rock. When the fluid percolates through a porous material, because of the complexity of microscopic flow in the force, the actual path of an individual fluid particle cannot be followed analytically. In all such situations one has to consider the gross effect of the phenomenon represented by a macroscopic view applied to the masses of fluid, large compared to the dimensions of the pore structure of the medium. The process can be described in terms of equilibrium of forces. The driving force necessary to move a specific volume of fluid at a certain speed to a porous medium is in equilibrium with the resistive force generated by internal friction between the fluid and the pore structure. The simplest model for flow through porous medium is the one dimensional model derived by Darcy.

7.3 Mathematical Formulation

In the present paper, the flow of an incompressible viscous radiating fluid past an impulsively started infinite vertical porous plate with uniform heat flux under transversely applied magnetic field is examined. The flow is considered to be laminar and Ohmic effects arising out of the situation are assumed to be very small and are negligible. In the cartesian frame of reference the \( x' \)-axis is taken along the plate in the vertical direction and the \( y' \)-axis is taken normal to the plate. Initially, the plate and fluid are assumed to be at the same temperature in a stationary condition. At time \( t > 0 \), the plate is given an impulsive motion in the vertical direction against the gravitational field with constant velocity \( u_0 \). At the same time, the heat is supplied
from the plate to the fluid at uniform rate. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. Then by usual Boussinesq’s approximation, the unsteady flow is governed by the following equations.

\[ \frac{\partial u}{\partial t'} = g \beta (T - T_\infty) + \nu \frac{\partial^2 u}{\partial y'^2} - \frac{\nu}{k} u + \frac{\sigma \beta_0^2}{\rho} u \]  

(7.1)

\[ \rho C_p \frac{\partial T}{\partial t'} = k_i \frac{\partial^2 T}{\partial y'^2} - \frac{\partial q}{\partial y'} \]  

(7.2)

where the Rosseland approximation is used, which leads to

\[ q_r = -\frac{4 \sigma}{3 k_r} \frac{\partial T^4}{\partial y'} \]  

(7.3)

The initial and boundary conditions are as follows

\[ t' \leq 0, \quad u = 0, \quad T = T_\infty \quad \text{for all} \quad y' \]  

\[ t' > 0, \quad u = u_0, \quad \frac{\partial T}{\partial y'} = -\frac{q}{k_i} \quad \text{at} \quad y' = 0 \]  

(7.4)

\[ u = 0, \quad T \rightarrow T_\infty \quad \text{as} \quad y' \rightarrow \infty \]

We assume that, the temperature differences within the flow are sufficiently small such that \( T^4 \) may be expressed as a linear function of the temperature. This is accomplished by expanding \( T^4 \) in a Taylor series about \( T_\infty \) and neglecting higher order terms, thus:

\[ T^4 \approx 4 T_\infty^3 T - 3 T_\infty^4 \]  

(7.5)

By using equations (7.4) and (7.5), equation (7.2) reduces to

\[ \rho C_p \frac{\partial T}{\partial t'} = k_i \frac{\partial^2 T}{\partial y'^2} + \frac{16 \sigma T_\infty^3}{3 k_r} \frac{\partial^2 T}{\partial y'^2} \]  

(7.6)

On introducing the following non-dimensional quantities
Equations (7.1) to (7.6), leads to:

\[
\frac{\partial U}{\partial t} = G_r \theta + \frac{\partial^2 U}{\partial y^2} - \frac{U}{k} + MU \quad (7.8)
\]

\[
3NP_r \frac{\partial \theta}{\partial t} = (3N + P_r) \frac{\partial^2 \theta}{\partial y^2} \quad (7.9)
\]

While the initial and boundary conditions in non-dimensionless form are

\[
t \leq 0: \quad U = 0, \quad \theta = 0, \quad \text{for all}\ y
\]
\[
t > 0: \quad \frac{\partial \theta}{\partial y} = -1, \quad \text{at}\ y = 0
\]
\[
U = 0, \quad \theta \to 0 \quad \text{as}\ y \to \infty \quad (7.10)
\]

Equations (7.8) and (7.9), subject to the boundary conditions (7.10), are solved by the usual Laplace Transform technique and the solutions are derived as follows.

\[
\theta = \sqrt{\frac{4t}{a\pi}} \exp\left(-\frac{ay^2}{4t}\right) - \text{erfc}\left(\sqrt{\frac{4y^2}{4t}}\right) \quad (7.11)
\]

\[
U = \frac{y}{2\sqrt{\pi}} \int_{t}^{w} \left[ p^\frac{3}{2} \exp\left(-\frac{y^2}{4p} - \frac{p}{w}\right) + \frac{CG_r (t - p)^{\frac{3}{2}}}{2} \sqrt{w(a-1)} \exp\left(\frac{p}{w(a-1)}\right) \text{erf}\left(\frac{p}{\sqrt{w(a-1)}} - 2\sqrt{\frac{p}{\pi}}\right) \right] \text{d}p \quad (7.12)
\]

**Skin-friction:** The dimensionless shearing stress on the surface of a body, due to the fluid motion, is known as skin-friction and is defined by the Newton's law of viscosity.
The skin-friction is \( \tau = \left( \frac{\partial u}{\partial y} \right)_{y=0} \)

\[
= \frac{1}{2\sqrt{\pi}} \int_{0}^{\frac{1}{w}} \left[ p^{\frac{1}{2}} \exp\left(-\frac{p}{w}\right) + \frac{wG_{r}(t-p)^{\frac{1}{2}}}{\sqrt{a}} \left\{ \sqrt{w(a-1)} \exp\left(\frac{p}{w(a-1)}\right) e^{r} \sqrt{\frac{p}{w-1}} \right. \right] dp \\
\left\{ \exp\left(\frac{t-p}{w}\right) - \sqrt{a} \right\} \right] dp
\]

(7.13)

where \( a = \frac{3NP_{r}}{3N+P_{r}} \) and \( w = \frac{k}{1-Mk} \)

7.4 Results and conclusions:

7.4.1 Figure 7.1, figure 7.2, figure 7.3 and figure 7.4 illustrates the effect of the magnetic field on the velocity. In each of these situations it is observed that as increase in the magnetic field contributes to the increase in the fluid velocity. From figure 7.1 and figure 7.2 it is concluded that, partially such an increase can also be attributed to increase in the value of \('a'\). From figure 7.1 and figure 7.3 it is noticed that, while all other parameters remaining constant, the increase in velocity can also be attributed to the time parameter \('t'\). It holds good for figure 7.2 and figure 7.4 also.
Figure 7.1: Effect of magnetic field on velocity

Figure 7.2: Effect of magnetic field on velocity
Figure 7.3: Effect of magnetic field on velocity

Figure 7.4: Effect of magnetic field on velocity
7.4.2 The effect of magnetic intensity near the boundary layer region has been illustrated in figure 7.5 and figure 7.6. In each of these illustrations, it is observed that, as the magnetic field intensity increases, a back flow is noticed. Such a back flow can be attributed to the fact that the driving force required for the fluid to drain along the plate is not that sufficient and as a result of which the fluid is trapped in the pores of the fluid bed and also at times the seepage occurs.

Figure 7.5: Effect of magnetic field on velocity
7.4.3 The combined effect of magnetic intensity and time are examined in illustrations figure 7.7 and figure 7.8. In each of these figures it is noticed that for the constant magnetic intensity, as 't' increases the fluid velocity decreases. Further, it is also noticed that the decrease in fluid velocity can also be attributed to the Prandtal number (Pr) and Nusselet number (N) which inturn are related to 'a'. It can be concluded that, increase in 'a' also contributes to the decrease in the fluid velocity.
Figure 7.7: Effect of time on velocity

Figure 7.8: Effect of time on velocity
7.4.4 The effect of time on temperature field is illustrated in figure 7.9, figure 7.10 and figure 7.11. In all these illustrations, it is noticed as we move away from the bounding surface the temperature in the fluid medium decreases. Further, it is noticed that as time parameter 't' increases, the temperature also increases. Also, it is observed that the temperature of the fluid medium is dependent on 'a'. And it is noticed that as 'a' increases, the temperature decreases.

Figure 7.9: Effect of 'a' on temperature profiles
Figure 7.10: Effect of ’a‘ on temperature profiles

Figure 7.11: Effect of ’a‘ on temperature profiles
7.4.5 The effect of magnetic field on is shown in figure 7.12 and figure 7.13. As the magnetic field intensity increases, the skin-friction increases. In each of these situations it is seen that not much of variation is seen till 50% of the time. However, the effect is more predominant at higher values of time.

Figure 7.12: Effect of magnetic field on skin-friction
Figure 7.13: Effect of magnetic field on skin-friction

7.4.6 The consolidated effect of time and magnetic intensity is shown in Figure 7.14 and Figure 7.15. From the illustrations it is noticed that as 't' increases the skin-friction also increases. Further, as the applied magnetic intensity is increased the dispersion in the profiles is not that significant as compared for the smaller values of magnetic intensity.
Figure 7.14: Effect of time on skin-friction

Figure 7.15: Effect of time on skin-friction