Chapter III

Development of Training Strategy and Tools
CHAPTER – III

DEVELOPMENT OF TRAINING STRATEGY AND TOOLS

3.1 RATIONALE BEHIND DEVELOPMENT TRAINING STRATEGY

Mathematics education is a quest unending and exciting, fascinating and thrilling, that promises growth and development in a systematic, harmonious and disciplined manner. Mathematics education of the highest quality requires mathematics teachers of the highest quality. Prior knowledge is essential for subsequent learning. It has been widely accepted that subject matter expertise is critical for effective teaching, especially in secondary and post secondary education.

- Expert teaching is a complex phenomenon that is comprised of expertise in multiple domains, including knowledge of the content area, pedagogy and students (Shulman, 1986a) as well as a honed perceptions and a rich skill set. Yet expert teaching also appears to substantiate many of the general claims made by experts in general (e.g. Berliner, 1986; Borko & Livingstone, 1989; Chi, Feltovich & Glaser, 1981; Ericsson & Lehmann, 1996; Leinhardt & Greeno, 1986). Expert teachers differ from novices in different dimensions. They notice different things about the classroom environment, they do more planning and plan things differently than novices, and they more deeply organize their knowledge of content, student and pedagogy in ways that readily facilitate lesson planning and teaching (Borko & Livingstone, 1989).

While the importance of content knowledge for teaching has long been acknowledged, only in the past few years has the educational community became concerned with specific knowledge effective teachers possess for ‘how to teach content to novices’; the research work to teach content/pedagogy to novice mathematics teachers is very scanty.

- At the beginning of his teacher education, a young student/prospective teacher of mathematics faces problems which does not remind him at all on what he has worked on when he was at school; of course he forgets those things quickly and completely, but, if then, after he has passed his exams, the
individual starts working as a teacher, he is expected to teach exactly the elementary mathematics in a school adequate manner. As he i.e. the pupil teacher is nearly unable to connect the elementary mathematics to his university mathematics, in most cases, he will soon come back to the old-fashioned teaching traditions and his university education remains more or less a romantic memory which has no impact on his teaching. This is referred to as “Double Discontinuity” on the part of a novice mathematics teacher. Shulman (1987a) introduced the term “Pedagogical Content Knowledge” (PCK) to describe the “blending of content and pedagogy into an understanding of how particular topics, problems or issues are organized, represented and adapted to the diverse interests and abilities of learner for instruction”. The investigator could not locate any study related to content and pedagogy of teaching in the field of mathematics.

The study of mathematics equips students with knowledge, skills, and habits of mind that are essential for successful and rewarding participation in an information- and technology-based society. Such a society requires individuals, who are able to think critically about complex issues, analyse and adapt to new situations, solve problems of various kinds, and communicate their thinking effectively. *Competence in mathematics* means the ability to understand, judge, do, and use mathematics in a variety of intra- and extra-mathematical contexts and situations in which mathematics plays or could play a role. The investigator could not locate any study conducted on mathematical competencies for prospective mathematics teachers in Indian situations.

Literature on competency-based education and tools and techniques employed in pre-service and in-service teacher training reveals that very little work has been done in developing mathematical competencies among prospective teachers. Although in other countries various efforts have been made in this area, in Indian conditions the work is too scanty. Also, the investigator could not locate any scale for competency assessment of prospective mathematics teachers in Indian situations.
Review of literature reveals that, in the present context, every field is emphasizing on implementation of competency-based programs. Various fields like medical, industry, distance learning professionals, etc are utilizing this approach in assessing the performance of their employees. The investigator felt the need to develop such training strategy specifically for exit competencies requisite for prospective mathematics teachers and assess their performance.

Hence a need was felt by investigator to develop a competency – based strategy for training of prospective mathematics teachers, suitable for Indian conditions. The central idea underneath the Mathematics Competency Based Training Strategy (MCBTS) is that prospective mathematics teachers should be able to demonstrate expertise in requisite Mathematical Exit Competencies for mathematics teaching up to the set criterion level. The most fundamental idea of strategy is that prospective mathematics teachers should focus their attention upon desired performance / behavioural indicators in teaching mathematics. Therefore, at one time, a prospective mathematics teacher is concerned with one Mathematical Exit Sub-Competency which may be defined as a set of components and each component is defined as a set of desired performance/behavioural indicators. The operational elements of Mathematics Competency Based Training Strategy (MCBTS) are:-

- Identification of Mathematical Exit Competencies.
- Carefully identified/recognized, selected and verified Mathematical Exit Sub-Competencies for prospective mathematics teachers.
- Assessment criterion for assessing Mathematical Exit Competencies and Exit Sub-Competencies of prospective mathematics teachers.
- Instructional strategy for training of prospective mathematics teachers.
- Evaluation of each of the Mathematical Exit Competencies and Exit Sub-Competencies specified.
- Participants progress through the instructional program at their own rate by demonstrating the attainment of the specified Exit Sub-Competencies.
- Development of Mathematics Teaching Competency Assessment Scale (MTCAS).
3.2 DEVELOPMENT OF MATHEMATICS COMPETENCY BASED TRAINING STRATEGY (MCBTS)

A competency-based training strategy for prospective mathematics teachers was developed in a systematic manner. The Mathematics Competency Based Training Strategy (MCBTS) was based on set of finally selected Mathematical Exit Competencies and Mathematical Exit Sub-Competencies. The prospective mathematics teachers were expected to acquire the desired performance level up to the set criterion level in each of the Mathematical Exit Sub-Competencies. Following steps were followed in order to develop the Mathematics Competency Based Training Strategy (MCBTS):

- Identification of Mathematical Exit Competencies.
- Identification of Mathematical Exit Sub-Competencies.
- Specification of key factors/components associated with each Mathematical Exit Sub-Competency.
- Specification of desired performance/behavioural indicators against components of each Mathematical Exit Sub-Competency.
- Description of selected Mathematical Exit Competencies and Mathematical Exit Sub-Competencies.
- Establishing reliability and validity of observation schedules and rating scales.
- Setting criterion level.
- Development of Mathematics Teaching Competency Assessment Scale (MTCAS).

3.2.1 IDENTIFICATION OF MATHEMATICAL EXIT COMPETENCIES

The investigator reviewed available mathematical literature; held discussions with mathematics teachers, teacher educators and students; referred literature available at various websites of internet (www.altavista.com, www.cec.sped.org/index.html, www.augie.edu/dept/educ/andrews/teachereducation_handbook.doc); and from investigators own personal mathematics teaching experience; the investigator identified three Mathematical Exit Competencies.
The three Mathematical Exit Competencies identified and taken in the strategy were: Mathematics Content Competencies, Mathematics Process Competencies and Mathematical Pedagogical Competencies.

3.2.2 IDENTIFICATION OF MATHEMATICAL EXIT SUB-COMPETENCIES

Corresponding to each Mathematical Exit Competency, Mathematical Exit Sub-Competencies were identified, listed and selected in the strategy. The investigator made an extensive review of literature available; consulted other mathematics teachers, mathematics teacher educators and mathematics students; literature available at internet; considered Indian situations; and investigators own mathematical background and teaching experience; and found certain Mathematical Exit Sub-Competencies respective to each Mathematical Exit Competency which were considered to be significant for mathematics teachers. The investigator enlisted total fifteen Mathematical Exit Sub-Competencies for prospective mathematics teachers. The enlisted Mathematical Exit Sub-Competencies respective to three Mathematical Exit Competencies were as under:-

MATHEMATICS CONTENT COMPETENCIES

- Writing Instructional Objectives
- Mathematics Content Knowledge
- Illustrating with Examples
- Selection and Organization of Mathematics Content
- Mathematical Connections

MATHEMATICS PROCESS COMPETENCIES

- Mathematical Communication
- Questioning and Response Management
- Mathematical Problem Solving
- Evaluation
- Mathematical Representations
- Reasoning and Proving
- Critical Thinking

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MATHEMATICAL PEDAGOGICAL COMPETENCIES

- Black Board Writing
- Multiple Instructional Strategies
- Pedagogical Knowledge competency

Out of these fifteen Mathematical Exit Sub-Competencies, ten Mathematical Exit Sub-Competencies were shortlisted to be included in the Mathematics Competency Based Training Strategy (MCBTS). The justification behind selection of these Mathematical Exit Sub-Competencies was that these were found to be more relevant to teaching of mathematics and supportive in effective teaching learning process in mathematics. Also, the short duration of B.Ed course forced the investigator to restrict its study to ten Mathematical Exit Sub-Competencies. The ten Mathematical Exit Sub-Competencies included in Mathematics Competency Based Training Strategy (MCBTS) were as under:

MATHEMATICS CONTENT COMPETENCIES

- Mathematics Content Knowledge
- Illustrating with Examples
- Selection and Organization of Mathematics Content
- Mathematical Connections

MATHEMATICS PROCESS COMPETENCIES

- Mathematical Communication
- Questioning and Response Management
- Mathematical Problem Solving
- Evaluation

MATHEMATICAL PEDAGOGICAL COMPETENCIES

- Black Board Writing
- Multiple Instructional Strategies
The Mathematical Exit Competencies and Mathematical Exit Sub-Competencies, along with symbols are shown in table below.

Table 3.1
Showing Mathematical Exit Competencies and Mathematical Exit Sub-Competencies, along with symbols

<table>
<thead>
<tr>
<th>S.NO.</th>
<th>Mathematical Exit Competencies</th>
<th>Mathematical Exit Sub-Competencies</th>
<th>Symbols</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Mathematics Content Competencies</td>
<td>• Mathematics Content Knowledge</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Illustrating with Examples</td>
<td>A2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Selection and Organization of Mathematics Content</td>
<td>A3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Mathematical Connections</td>
<td>A4</td>
</tr>
<tr>
<td>2</td>
<td>Mathematics Process Competencies</td>
<td>• Mathematical communication</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Questioning and Response</td>
<td>B2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Management</td>
<td>B3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Mathematical Problem Solving</td>
<td>B4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Evaluation</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Mathematical Pedagogical</td>
<td>• Black Board Writing</td>
<td>C1</td>
</tr>
<tr>
<td></td>
<td>Competencies</td>
<td>• Multiple Instructional Strategies</td>
<td>C2</td>
</tr>
</tbody>
</table>

3.2.3 SPECIFICATION OF KEY FACTORS/COMPONENTS ASSOCIATED WITH EACH MATHEMATICAL EXIT SUB-COMPETENCY

The associated components/key factors against each Mathematical Exit Sub-Competency were specified, in terms of analysis of each Mathematical Exit Sub-Competency and assessable factors related to each Mathematical Exit Sub-Competency. Also, the related literature was consulted for listing the components/key factors associated to each Mathematical Exit Sub-Competency. The listed components/ key factors were then rated in terms of their significance in a classroom teaching learning process.
3.24 SPECIFICATION OF DESIRED PERFORMANCE / BEHAVIOURAL INDICATORS AGAINST COMPONENTS OF EACH MATHEMATICAL EXIT SUB-COMPETENCY

Competencies, in order to be measurable, are described in terms of 'behavioural indicators'. Competencies are measured from observable behaviour. 'Behavioural indicators' are the behaviours exhibited by the prospective teachers. 'Behavioural indicators' describe directly observable behaviour or specific evidence of an individual’s competency. Since it indicates the behaviour, it includes a verb, includes contextual information to make meaning of the action and describe only a limited behaviour or evidence. Also required is to look at the motive/intent that causes the behaviour. Behaviour without intent is not a competency.

Each ‘behavioural indicator’ should:

❖ Describe directly observable behaviour or other specific evidence of the individual’s competency.
❖ Describe just one piece of behaviour or evidence.
❖ Not to be duplicated under two or more competencies.
❖ Include a verb phrase.
❖ Include enough contextual information to make the action meaningful.

On the basis of desired performances or behavioural indicators on the part of a mathematics teacher in a classroom situation; according to mathematical literature available; and above discussed facts, the desired performances/behavioural indicators against components of each Mathematical Exit Sub-Competency were specified. For this, desired performances/behavioural indicators were listed and rated in terms of their frequency of occurrence and their significance in a mathematics classroom teaching learning process. These listed factors were then stated in terms of desired performances/behavioural indicators against components of each Mathematical Exit Sub-Competency.
3.2.5 DESCRIPTION OF SELECTED MATHEMATICAL EXIT
COMPETENCIES AND MATHEMATICAL EXIT SUB-
COMPETENCIES

A MATHEMATICS CONTENT COMPETENCIES

In the Mathematical Exit Competency, ‘Mathematics Content Competencies’, four Mathematical Exit Sub-Competencies were identified: - Mathematics Content Knowledge, Illustrating with Examples, Selection and Organization of Mathematics Content and Mathematical Connections.

❖ Mathematics Content Knowledge (A1)

To teach all students according to today’s standards, teachers need to understand subject matter deeply so that they can help students create useful cognitive maps, relate one idea to another, and address misconceptions. Teachers need to see how ideas connect across fields and to everyday life. This kind of understanding provides a foundation for pedagogical content knowledge that enables teachers to make ideas accessible to others (Shulman, 1987). In Shulman’s theoretical framework, teachers need to master two types of knowledge: (a) content, also known as "deep" knowledge of the subject itself, and (b) knowledge of the curricular development. Content knowledge encompasses what Bruner (as cited in Shulman, 1992) called the "structure of knowledge"—the theories, principles, and concepts of a particular discipline. Especially important is content knowledge that deals with the teaching process, including the most useful forms of representing and communicating content and how students best learn the specific concepts and topics of a subject. "If beginning teachers are to be successful, they must wrestle simultaneously with issues of pedagogical content (or knowledge) as well as general pedagogy (or generic teaching principles)” (Grossman, as cited in Ornstein, Thomas, & Lasley, 2000).

Knowledge may be considered in terms of content (factual where, who, what, and when kind of knowledge) and context (factual information embedded in a connected space which provides the knowledge with meaning and provides the knower with a logical inferential basis for making wise judgment before acting or accurate predictions based on factual information). The novice teacher can readily pick up content knowledge, which is the material of text books, but in the absence of
context and experience the novice teacher has difficulty in making judgments in the application of the content knowledge. The student finds a context for content knowledge by developing researching skills in which to embed the content knowledge and through experience becomes an expert in making judgments.

The components of Mathematical Exit Sub-Competency ‘Mathematics Content Knowledge’ along with desired performance / behavioural indicators from prospective mathematics teachers are given below in table 3.2.

Table 3.2
Shows Components and Desired Performance Associated With Mathematical Exit Sub-Competency ‘Mathematics Content Knowledge’

<table>
<thead>
<tr>
<th>Component</th>
<th>Desired Performance/Behavioural Indicators</th>
</tr>
</thead>
<tbody>
<tr>
<td>While performing the exit sub-competency ‘Mathematics content knowledge’ student teacher:-</td>
<td></td>
</tr>
<tr>
<td>Making subject content meaningful</td>
<td>Defines, explains, examplifies and relates concepts correctly; states axioms and formulae correctly; demonstrates efficiency in drawing and computation.</td>
</tr>
<tr>
<td>Constructing Mathematics content knowledge</td>
<td>Remembers factual information; demonstrates the ways in which new knowledge is created; leads students to construct new knowledge; enables students to solve mathematics problems; give meaningful learning experiences to students; uses logic, reasoning in developing mathematics knowledge.</td>
</tr>
</tbody>
</table>

❖ **Illustrating with Examples (A2)**

The act of illustrating refers to the act of making clear and distinct; also, that which illustrates using a comparison or example intended to make clear or to remove obscurity. In a subject like mathematics, concrete examples will really help students to understand the concepts to be taught in class. A novice mathematics teacher needs
to be competent in use of these examples in class so that they are maximally effective. He may confront questions like:

Should I go over the general concept first, and then show examples to illustrate what I just talked about?

Or

Would it be more effective to go over the examples first, and then guide students toward the general concept or principle?

Do I make sure they understand the components before moving on to the big picture?

Or

Do I start with the big picture and then go over the specific components?

It’s important to think about HOW and WHEN a teacher is going to present examples. In mathematics one of the significant approaches is Inductive vs. Deductive Approaches.

With an inductive approach, a teacher shows students a series of examples and non-examples, then guide them toward noticing a pattern and coming up with the generalization or concept rule. In use of deductive approach, a teacher tells students the rule, then gives them examples and asks them when it applies and when it doesn’t. Using examples like these, challenges students to think about how new principles relate to what they already know, and helps them apply it to new situations. Inductive reasoning, in particular, requires students to mentally search through their prior knowledge, and it often leads to the ability to apply the concept or strategy to a wider variety of contexts.

The components of Mathematical Exit Sub-Competency ‘Illustrating with Examples’ along with desired performance / behavioural indicators from prospective mathematics teachers are given below in table 3.3.
### Table 3.3

Shows Components and Desired Performance Associated With Mathematical Exit Sub-Competency ‘Illustrating with Examples’.

<table>
<thead>
<tr>
<th>Component</th>
<th>Desired Performance/Behavioural Indicators</th>
</tr>
</thead>
<tbody>
<tr>
<td>Appropriate examples</td>
<td>Gives simple examples; examples in accordance with apperceptive mass of students, age, mental level of students; arousing curiosity and maintaining interest in class; give concrete examples from the experience of pupils.</td>
</tr>
<tr>
<td>Modes of illustrating- visual, auditory and tactual</td>
<td>Gives visual examples using objects, charts, models, pictures, graphs, etc.; oral examples (auditory); and, tactual examples like use of laboratory apparatus, equipments; based on learning by doing; relevant to age, grade, mental level of students and topic taught.</td>
</tr>
<tr>
<td>Inducto- Deducto approach</td>
<td>Uses inductive approach in beginning, i.e., gives examples to arrive at generalizations/concept; state the generalizations; ask students to give more examples.</td>
</tr>
</tbody>
</table>

VARIOUS MATHEMATICS CONTENT

Selection and Organization of Mathematics Content (A3)

Mathematics content builds coherently from grade-to-grade; students have the opportunity to build upon their learning year-to-year; each year, the content increases in complexity and is appropriately challenging. The units or modules includes lessons, activities, assignments, and projects that are multi-days; emphasize the connections between mathematical concepts, and promote the attainment of several, rather than just one, instructional objectives. After ensuring that content is useful, well-written, and in a format that is suitable for the learners, it is important to ensure that the content is clearly organized. In some cases, the content under a particular topic can be organized in multiple ways to accommodate multiple learners. A novice mathematics
teacher should be competent enough to select and organize content in order to maximize learning in class.

The components of Mathematical Exit Sub-Competency ‘Selection and Organization of Mathematics Content’ along with desired performance / behavioural indicators from prospective mathematics teachers are given below in table 3.4.

<table>
<thead>
<tr>
<th>Component</th>
<th>Desired Performance/Behavioural Indicators</th>
</tr>
</thead>
<tbody>
<tr>
<td>Selecting worthwhile content</td>
<td>Select content relevant to topic, predefined objectives; adequate to achieve objectives; accurate content.</td>
</tr>
<tr>
<td>Selecting appropriate content</td>
<td>Select content relevant to age, grade, mental level of students and in accordance with time available.</td>
</tr>
<tr>
<td>Organizing content appropriately</td>
<td>Organize content logically – simple to complex, concrete to abstract, known to unknown, empirical to rational.</td>
</tr>
</tbody>
</table>

❖ Mathematical Connections (A4)

An emphasis on Mathematical Connections helps students recognize how ideas in different areas are related. Students should come both to expect and to exploit connections, using insights gained in one context to verify conjectures in another. For example, elementary school students link their knowledge of the subtraction of whole numbers to the subtraction of decimals or fractions. Middle school students might collect and graph data for the circumference (C) and diameter (d) of various circles. They could extend their previous knowledge in algebra and data analysis to recognize that the values nearly form a straight line, so C/d is between 3.1 and 3.2 (a rough estimation of π).
Of all of the reform recommendations being made by the National Council of Teachers of Mathematics, making Mathematical Connections is among the more difficult to achieve, yet is so helpful in motivating students in the early grades. Mathematical Connections can relate mathematical topics to students' daily lives and to other mathematical topics but are probably most important in relating mathematics to other curriculum areas. These connections help students understand mathematics better and see it as a useful and interesting subject to study.

The opportunity to experience mathematics in context is important. Students should connect mathematical concepts to their daily lives, as well as to situations from science, the social sciences, medicine, and commerce. For example, high school students worked with a drug store chain to determine where it should locate a new pharmacy in their neighborhood on the basis of analyses of demographic and economic data. Students should recognize the value of mathematics in examining personal and societal issues. Mathematical Connections provides a forum for exploring the interplay between mathematics and other disciplines. This includes, but is not limited to, the relationship between mathematics and humanities, art, literature, religion, and physical sciences. Mathematics has a tradition of presenting results in a logical and deductive style which shows that how every fact is derived from the previous ones. For too long, professional development of secondary school teachers has meant either taking yet another university mathematics course (with no connection to what we teach) or taking a workshop about mathematics (with no mathematics in it at all). But the foundations of high school mathematics go deep enough to involve sophisticated thinking, hard problems, and subtle connections, while staying connected to the kind of mathematics we talk about with our student's every day.

The components of Mathematical Exit Sub-Competency ‘Mathematical Connections’ along with desired performance / behavioural indicators from prospective mathematics teachers are given below in table 3.5.
Table 3.5

Shows components and desired performance associated with Mathematical Exit Sub-Competency ‘Mathematical Connections’

<table>
<thead>
<tr>
<th>Component</th>
<th>Desired Performance/Behavioural Indicators</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relating mathematics knowledge within the subject</td>
<td>Apply the knowledge of arithmetic, algebra, geometry and trigonometry in solving problems from either of the branches.</td>
</tr>
<tr>
<td>Relating mathematics knowledge with real life</td>
<td>Give examples from real life according to age, grade, mental level of students; demonstrate the use of mathematics in solving real and daily life problems.</td>
</tr>
</tbody>
</table>

B  MATHEMATICS PROCESS COMPETENCIES:

In the Mathematical Exit Competency, namely, ‘Mathematical Process Competencies’; four Mathematical Exit Sub-Competencies were identified: - Mathematical Communication, Questioning and Responses Management, Mathematical Problem Solving and Evaluation.

❖  Mathematical Communication (B1)

Effective communication is now seen as a skill that middle school student should demonstrate in all subject areas, not just language, arts and social science courses (Kist, 2003). Indeed, mathematics is increasingly seen as a field in which effective communication is essential as both a learning process and an outcome. Principles and Standards for School Mathematics (PSSM), a guide published by the National Council Of Teachers Of Mathematics outlining essential components for improving the quality of school mathematics programs, lists communication as one of the five process standards that students will need to function effectively in the twenty first century. The PSSM document elaborates that communication is an essential part of mathematics and mathematics education because it is a “way of sharing ideas and
clarifying understanding. Through communication, ideas become objects of reflection, refinement, discussion and amendment. The communication process helps build meaning and permanence for ideas and makes them public” (NCTM, 2000).

In 1989, the National Council of Teachers of Mathematics (NCTM) published the Curriculum and Evaluation Standards for School Mathematics. This document began a widespread shift in the way many educators think, write, and talk about mathematics education in the United States. NCTM suggested that educators should approach the teaching of mathematics in new ways and proposed changes in both content (e.g., number sense and numeration, geometry, and probability) and processes (e.g., problem solving, communication, reasoning, and connections). Suggesting a broader definition and use of communication in the mathematics classroom, NCTM (1989) called for an increase in students' reading, writing, discussing, representing, and modeling mathematics, because, "as students communicate their ideas, they learn to clarify, refine, and consolidate their thinking." Teachers who embraced the standards sought ways to shift the emphasis in their classrooms from talking and writing as answer-giving to talking and writing as sense-making.

In April 2000, NCTM released its revised "standards document," Principles and Standards for School Mathematics (PSSM). In PSSM, communication as a standard maintained its prominent status. Now, however, the forms emphasized in the communication-process standard are verbal and written, while the symbolic, graphic, pictorial, and gestural forms are emphasized separately in a representation-process standard (NCTM 2000). The separation is not a true bifurcation; instead, it underscores the important role that representation plays in the communication of and about mathematics. NCTM (2000) explicitly acknowledged the link between the two: "Representations should be treated as essential elements ... in communicating mathematical approaches, arguments, and understandings to one's self and to others." Representation can, but does not have to, have a communicative purpose.

The components of Mathematical Exit Sub-Competency ‘Mathematical Communication’ along with desired performance / behavioural indicators from prospective mathematics teachers are given below in table 3.6.
Table 3.6
Shows Components and Desired Performance Associated With Mathematical Exit Sub-Competency ‘Mathematical Communication’

<table>
<thead>
<tr>
<th>Component</th>
<th>Desired Performance/Behavioural Indicators</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demonstrating familiarity with mathematical language</td>
<td>While performing the exit sub-competency ‘Mathematical Communication’ student teacher: Decodes and interprets symbolic, pictorial, diagrammatic, graphic and formal mathematical language.</td>
</tr>
<tr>
<td>Using relevant mathematical language</td>
<td>Uses mathematical symbols, pictures, diagrams, graphs according to age, grade and mental level of students.</td>
</tr>
<tr>
<td>Making effective use of mathematical language</td>
<td>Translates from natural language to formal/symbolic mathematical language (symbols, pictures, diagrams, graphs and formulae) and vice versa. Handles and manipulates appropriately statements and expressions containing symbols, pictures, diagrams, graphs and formulae; selects, applies and translates among mathematical representations to solve problems.</td>
</tr>
<tr>
<td>Using listening skills</td>
<td>Listen to students using appropriate body language like eye contact, nodding, facial expressions and physical gestures. Listen patiently even if the student is wrong; clarify their doubts empathetically; listen to alternative solutions given by students and accept the right ones.</td>
</tr>
<tr>
<td>Interacting with students</td>
<td>Deliberately changes the interaction style from one pattern to another: - teacher-pupil interaction and pupil-pupil interaction.</td>
</tr>
</tbody>
</table>

**Questioning and Response Management (B2)**

Asking questions is a fundamental part of finding information and for subtle (and otherwise) persuasion. A question is any sentence which has an interrogative form or function. In classroom settings, questions put by the teacher are defined as instructional cues or stimuli that convey to students the content elements to be learned and directions for what they are to do and how they are to do it.
A variety of purposes of teachers' classroom questions emerge from analysis of the literature, including:

- To develop interest and motivate students to become actively involved in lessons
- To evaluate students' preparation and check on homework or seatwork completion
- To develop critical thinking skills and inquiring attitudes
- To review and summarize previous lessons
- To nurture insights by exposing new relationships
- To assess achievement of instructional goals and objectives
- To stimulate students to pursue knowledge on their own

These purposes are generally pursued in the context of classroom recitation, defined as a series of teacher questions, each eliciting a student response and sometimes a teacher reaction to that response. Within these recitations, students follow a series of steps (consciously or unconsciously) in order to produce responses to the questions posed. These steps include:

- Attending to the question
- Deciphering the meaning of the question
- Generating a covert response (i.e., formulating a response in one's mind)
- Generating an overt response; and often
- Revising the response (based on teacher probing or other feedback)

Questioning can......

- arouse curiosity
- stimulate interest in the topic
- clarify concepts
- emphasize key points
- enhance problem-solving ability
- encourage students to think at higher cognitive levels
- motivate student to search for new information
- ascertain students’ knowledge level to aid in modifying instruction

The components of Mathematical Exit Sub-Competency ‘Questioning and Response Management’ along with desired performance / behavioural indicators from prospective mathematics teachers are given below in table 3.7
Table 3.7
Shows Components and Desired Performance Associated With Mathematical Exit Sub-Competency ‘Questioning and Response Management’

<table>
<thead>
<tr>
<th>Component</th>
<th>Desired Performance/Behavioural Indicators</th>
</tr>
</thead>
<tbody>
<tr>
<td>Structuring of question</td>
<td>While performing the exit sub-competency ‘Questioning and Response Management’ student teacher:— Ask lesson developing questions which are: relevant to content and mental level of students; grammatically correct; clear, specific and unambiguous; brief; purposeful; in proper sequence and simple language/terminology; and with only one correct response.</td>
</tr>
<tr>
<td>Questioning technique</td>
<td>Ask thought provoking questions with: appropriate speed; difficulty level of questions according to cognitive level of students; pause after every question; posed with audible and clear voice; sufficient time to think about and respond; no unnecessary repetition of questions and answers; no elliptical questions; no rhetorical questions.</td>
</tr>
<tr>
<td>Types of questions</td>
<td>Ask lower order questions like recognition and recall type questions; middle order questions like comparing, explaining the relationships and application of knowledge already gained; higher order questions involving analysis, synthesis, inductive and deductive reasoning.</td>
</tr>
<tr>
<td>Prompting</td>
<td>Modify/simplify the questions by breaking them into smaller questions; give clues/hints to lead the student to correct response.</td>
</tr>
<tr>
<td>Seeking further information</td>
<td>Seek for more information after incomplete or partially correct answer; ask questions like “state in your own words”, “state in other words”, “anything else”, “explain in detail”, etc.</td>
</tr>
<tr>
<td>Refocusing</td>
<td>Ask students to relate the knowledge gained to new situations; to compare; to establish relationships; state and restate problem.</td>
</tr>
<tr>
<td>Redirection</td>
<td>Ask same question to a number of students; maximize pupil participation by organizing discussion, brain storming, etc.</td>
</tr>
<tr>
<td>Increasing critical awareness</td>
<td>Elicit a rationale for the pupils’ answer by using words like ‘how’ and ‘why’.</td>
</tr>
</tbody>
</table>
Problem solving forms part of thinking. Considered the most complex of all intellectual functions, problem solving has been defined as higher-order cognitive process that requires the modulation and control of more routine or fundamental skills (Goldstein & Levin, 1987). It occurs if an organism or an artificial intelligence system does not know how to proceed from a given state to a desired goal state. It is part of the larger problem process that includes problem finding and problem shaping. Problem solving is an integral part of all mathematics learning. In everyday life and in the workplace, being able to solve problems can lead to great advantages. However, solving problems is not only a goal of learning mathematics but also a major means of doing so. Problem solving should not be an isolated part of the curriculum but should involve all Content Standards. Problem solving means engaging in a task for which the solution is not known in advance. Good problem solvers have a "mathematical disposition"—they analyze situations carefully in mathematical terms and naturally come to pose problems based on situations they see. For example, a young child might wonder, how long would it take to count to a million?

Good problems give students the chance to solidify and extend their knowledge and to stimulate new learning. Most mathematical concepts can be introduced through problems based on familiar experiences coming from students' lives or from mathematical contexts. For example, middle-grades students might investigate which of several recipes for punch giving various amounts of water and juice is "fruitier." As students try different ideas, the teacher can help them to converge on using proportions, thus providing a meaningful introduction to a difficult concept.

Students need to develop a range of strategies for solving problems, such as using diagrams, looking for patterns, or trying special values or cases. These strategies need instructional attention if students are to learn them. However, exposure to problem-solving strategies should be embedded across the curriculum. Students also need to learn to monitor and adjust the strategies they are using as they solve a problem.

Teachers play an important role in developing students' problem-solving dispositions. They must choose problems that engage students. They need to create an
environment that encourages students to explore, take risks, share failures and successes, and question one another. In such supportive environments, students develop the confidence they need to explore problems and the ability to make adjustments in their problem-solving strategies.

The components of Mathematical Exit Sub-Competency ‘Mathematical Problem Solving’ along with desired performance / behavioural indicators from prospective mathematics teachers are given below in table 3.8.

Table 3.8
Shows components and desired performance associated with Mathematical Exit Sub-Competency ‘Mathematical Problem Solving’.

<table>
<thead>
<tr>
<th>Component</th>
<th>Desired Performance/Behaviour Indicators</th>
</tr>
</thead>
<tbody>
<tr>
<td>While performing the exit sub-competency ‘Mathematical Problem Solving’ student teacher :-</td>
<td></td>
</tr>
<tr>
<td>Posing a problem</td>
<td>Pose various kinds of problems: instructional, routine and non routine problems.</td>
</tr>
<tr>
<td>Helping students to identify the information given and information that needs to be determined</td>
<td>Asks students to reread and restate the problems ; uses questioning/prompting to help them to describe their observations and state what is given and what goal is to be achieved.</td>
</tr>
<tr>
<td>Helping students to develop repertoire of strategies for problem solving</td>
<td>Ask the students: to prepare a model, picture or diagram; to guess and check which information, formula or relationship is required to solve the problem; enable them to synthesize the facts; present problem in simpler terms; working backward and using logical reasoning.</td>
</tr>
<tr>
<td>Helping students to check suitability of strategy</td>
<td>Uses correcting, suggesting, providing analogies, critical awareness, seeking further information and prompting to check the suitability of strategy.</td>
</tr>
<tr>
<td>Helping students to execute the chosen strategy</td>
<td>Ask students to do necessary calculations, draw pictures, use words and symbols to represent the steps in carrying out the plan, look back at the solution to check the reasonableness of the answer.</td>
</tr>
<tr>
<td>Helping students to analyze the applied strategy</td>
<td>Verify the result obtained.</td>
</tr>
</tbody>
</table>
Evaluation (B4)

Education and evaluation are inter-related processes. Evaluation is as old as the process of education itself. Education in its wider sense implies not only acquisition of knowledge, but also development of abilities, skills, personality qualities which are important in individual’s personal and social life. The function of evaluation in education is to provide a systematic assessment of the development of these qualities as an outcome of educational endeavour. Without the benefit of evaluation, one is unable to determine how well the programme has achieved its goal. Evaluation also serves as the basis for the improvement of the way activities are carried out. Evaluation is also an effort at discovering whether certain activities have led to desired effects or outcomes.

Evaluation in education means describing something, in terms of selected attributes and judging the degree of acceptability or suitability of that which has been described. The something which is described can be any aspect of the educational scene. In broadest sense, the term evaluation is defined as a systematic assessment of the value or worth of ‘something’. This ‘something’ could be a programme or a segment of a programme, a technique or strategy used, educational materials or a situation prevalent in a particular community. It is also defined as the collection and use of information to make decisions about an educational programme. An evaluation can be an important tool in improving the quality of a program if it is integrated into the fabric of an educational program rather than added on after the fact.

Evaluation/assessment of student’s work is critically important part of every subject. In the past, evaluation tasks were designed to measure and demonstrate how much student has learnt of what the faculty member decreed they should know in a unit. Evaluation is also concerned with ranking and comparing students according to their achievements in qualitative terms. The judgment of evaluators was assumed to be definitive statements of a student’s ability. The current focus of evaluation is to link evaluation tasks with anticipated learning outcomes and criteria of competence. According to Boud (in Armstrong and Conrad, 1995), good assessment:-
- Is designed to assess a broader range of student’s abilities, e.g. problem solving, critical thinking, effective communication, group coordination.

- Along with feedback, shapes learning in positive and negative ways, e.g. promotes rote learning or learning in depth.

- Involves student’s in the assessment process and develops student’s capacity to monitor their own performance, e.g. by understanding the criteria used in performance.

- Pay more attention to self assessment as a result of concern for reflective and self-sustained learning.

- Assess not only what student does not know but also what they can do.

- Should reflect desired learning outcomes and have a beneficial effect on the learning process.

- Should promote search for meaning and understanding, self-directed and independent learning.

- Should provide contextualized, complex challenges; not fragmented, static bits or tasks.

- Should expand learning opportunities to include active collaboration with others leading to assessment of projects produced by group of students (Mentkowski et al, 1991).

The components of Mathematical Exit Sub-Competency ‘Evaluation’ along with desired performance / behavioural indicators from prospective mathematics teachers are given below in table 3.9.
Table 3.9
Shows Components and Desired Performance Associated With Mathematical Exit Sub-Competency ‘Evaluation’.

<table>
<thead>
<tr>
<th>Component</th>
<th>Desirable performance/Behavioural indicators</th>
</tr>
</thead>
<tbody>
<tr>
<td>Apperceptive mass judgment</td>
<td>While performing the exit sub-competency ‘Evaluation’ student teacher:-</td>
</tr>
<tr>
<td>Observing general weaknesses</td>
<td>Diagnose pre-requisite entry behavior of students and their general weaknesses and misconceptions in mathematics by asking questions, making objective observations, analyzing and oral and written responses.</td>
</tr>
<tr>
<td>Identify causes of weaknesses</td>
<td>Enlist general weaknesses in mathematics by questioning and continuous observations.</td>
</tr>
<tr>
<td>Prepare remedial strategy</td>
<td>Do questioning and analyze the oral/written responses; maximize pupil participation in classroom teaching learning process.</td>
</tr>
<tr>
<td>Techniques of evaluation</td>
<td>Plan suggestive measures for improving upon weaknesses in mathematics like review questions, practice questions, drill work, etc.</td>
</tr>
<tr>
<td>Test construction</td>
<td>Ask various types of questions like short answer, long answer, open ended, completion type, selection type, etc. in written/oral form at regular intervals.</td>
</tr>
<tr>
<td>Test administration</td>
<td>Frame questions according to objectives, difficulty level of the content, mental level of students, content taught in class and proper distribution of marks.</td>
</tr>
<tr>
<td>Scoring the test</td>
<td>Give proper instruction and sets time.</td>
</tr>
<tr>
<td>Analyzing the scores</td>
<td>Prepare scoring key and give marks for each step</td>
</tr>
<tr>
<td>Reporting the results</td>
<td>Assess the performance of every student according to objectives and enlist weaknesses.</td>
</tr>
<tr>
<td></td>
<td>Informing the students about their achievement, provide reinforcement and provides remedies for removing weaknesses.</td>
</tr>
</tbody>
</table>
C MATHEMATICAL PEDAGOGICAL COMPETENCIES:

In the Mathematical Exit Competency, namely, ‘Mathematical Pedagogical Competencies’; two Mathematical Exit Sub-Competencies were identified: - Black Board Writing and Multiple Instructional Strategies.

❖ Black Board Writing (C1)

Black board is said to be the right hand of a teacher. It is the simplest, suitable, convenient, and widely used teaching aid employed in teaching and learning of mathematics. The Blackboard is a teacher’s best visual aid when question arises that involves the whole class. A mathematics teacher can organize discussions on it, emphasize major points, makes note of page numbers for assignments, and work with examples by writing problems. The skill of using the Blackboard refers to the technique that enables a teacher to use Blackboard in teaching learning process as effectively as possible for proper realization of stipulated teaching learning objectives.

Teachers use Blackboard as a platform for sharing notes, uploading class materials and taking tests for a while now, but there is still a lot to learn. Elementary and high school students are getting in on the action and developers continue to create new tools and accessories that make blackboard better than ever. A mathematics teacher is not just conveying information, he or she teaches to think mathematically, and teaches by example, in real time. It is crucially important to be in full control of timing and tempo of narrative.

The components of competency ‘Black Board Writing’ along with desired performance/behavioural indicators from prospective mathematics teachers are given below in table 3.10.
Table 3.10
Shows Components and Desired Performance Associated With Mathematical Exit Sub-Competency ‘Black Board Writing’.

<table>
<thead>
<tr>
<th>Component</th>
<th>Desired Performance/Behavioural indicators</th>
</tr>
</thead>
<tbody>
<tr>
<td>Check physical aspects of blackboard</td>
<td>Checks physical conditions like smoothness, light availability in room, availability of chalk and duster, ventilation in room, glare on the blackboard.</td>
</tr>
<tr>
<td>Neat presentation on blackboard</td>
<td>Write in straight parallel lines, no overwriting and rubbing with hands.</td>
</tr>
<tr>
<td>Write in proper sequence</td>
<td>Write points in proper sequence; simple and concise language; highlights main important points; Adequate spacing between two words; Adequate spacing between two lines; and retaining only relevant matter on the blackboard.</td>
</tr>
<tr>
<td>Legibility of Handwriting</td>
<td>Checks that: Every letter is distinct; Slant of each letter (nearly vertical); Size of capital letters; Size of letters; Thickness of lines.</td>
</tr>
<tr>
<td>Appropriateness of the Written Work</td>
<td>Content should be in simple words; underline to draw attention and focus on a line; use coloured chalks; simple and clear illustrations and diagrams.</td>
</tr>
<tr>
<td>Use Black board effectively</td>
<td>Check before the use whether the blackboard is clean; Check after use whether the blackboard is clean; Avoided squeaking noise of the chalk while writing; Applied adequate pressure while writing on the blackboard; no mistakes either in spelling or illustration/diagrams; stand at an angle of 45° with the blackboard and face towards students; speaks content while writing on blackboard.</td>
</tr>
</tbody>
</table>
Multiple Instructional Strategies (C2)

Instructional strategies are the methods that are used in the lesson to ensure that the sequence or deliveries of instruction help students learn. The term “effective instructional strategies” refers to the fact that the student’s performance improves when the instructional strategies are used. This allows teacher to be confident about how to apply the strategies in their classrooms. The teacher understands and uses a variety of instructional strategies to encourage student’s development of critical thinking, problem solving and performance skills. Teacher deals daily with many complexities, including differences among their students in terms of abilities, attitudes and learning preferences. For these widely varying needs of the students, there are multiple goals and objectives to be met, including those dealing with content, basic skills, problem solving, attitudes, dispositions and critical thinking skills. It is clear that no routine or “pet” teaching approach can effectively meet all of these needs. Effective teachers draw from a wide repertoire of instructional strategies and models, adjusting their choices to meet their intended objectives and the needs of particular students.

There are many instances in which the most efficient and effective way to teach certain kinds of knowledge is through expository teaching or teacher-directed step by step learning. In such cases direct instruction, presentations, and skills practice are appropriate. However, there are many other times when methods that appear time consuming yield the greatest results in the long run. When students are given the time and material to be active investigators, they are able to construct a basic framework of knowledge within which to expand their understanding.

Learning for understanding often requires experimentation, problem solving, collaboration, and manipulation of physical objects. Therefore, teachers need models of teaching that include inquiry learning, cooperative learning, concept attainment and class discussions. These models have as a goal the formation of cognitive structures including concepts, generalizations, dispositions, and understanding rather than simple attainment of specific facts or mastery of discrete skills. As teachers will be
able to understand the wide variety of instructional strategies available, they will better able to choose and combine them to integrate affective and cognitive development and to educate for understanding, both of content and of self.

The major dimensions of Multiple Instructional Strategies are as follows:-

- **Belonging:** By using multiple instructional strategies, all children feel they are part of the classroom learning environment. The pre-service teacher’s selection of instructional strategies (class instructions, peer mediated instructions, etc) impacts students sense of belonging.

- **Mastery:** The pre-service teacher’s use of multiple instructional strategies will develop critical thinking, problem solving and performance skills in the prospective teachers. Teachers must be aware that there are multiple goals that will be addressed among diverse learners.

- **Independence:** The use of multiple instructional strategies by pre-service teachers offers students choices in how and what to learn and encourages students to be active, to investigate and construct framework of knowledge.

- **Generosity:** A spirit of generosity is evident through the pre-service teacher’s selection of specific strategies that will promote student’s giving, helpfulness, and caring that encourages giving knowledge to others.

The components of Mathematical Exit Sub-Competency ‘Multiple Instructional Strategies’ along with desired performance / behavioural indicators from prospective mathematics teachers are given below in table 3.11
<table>
<thead>
<tr>
<th>Component</th>
<th>Desired Performance / Behavioural Indicators</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uses appropriate instructional strategy</td>
<td>While performing the exit sub-competency ‘Multiple instructional strategies’ student teacher: - Use instructional strategy relevant to topic, objectives, age, grade, pre-conceptions and apperceptive mass of students.</td>
</tr>
<tr>
<td>Employs instructional strategies suitable to arouse and maintain interest of students in mathematics</td>
<td>Use instructional strategies based on learning by doing and individual differences; make use of concrete material and relevant examples/puzzles/games.</td>
</tr>
<tr>
<td>Use instructional strategies to help students think mathematically</td>
<td>Use questioning for developing critical thinking, problem solving and analytic thinking; organizes open discussions and use refocusing and redirection for promoting divergent thinking; uses cooperative learning to develop discussing abilities.</td>
</tr>
</tbody>
</table>
| Uses inductive-deductive approach             |  - Use inductive reasoning by: giving relevant examples; providing opportunity for logical analysis of problem; enlist sufficient examples to observe and analyze; giving reasons; identifying logic beneath problem; and making generalizations.  
  - Use deductive reasoning by providing opportunity to: identify common facts; to differentiate; to conclude; to justify; to test and verify the solutions/conclusions by applying to similar related problems; to use sense-experiences to accept judgments.  
  - Make inductive deductions through critical thinking, creative investigation and logical thinking by organizing discussion, demonstrating/experimentation and observing analogies. |
| Uses analytic-synthetic approach               |  - Make analysis of problem by breaking them into smaller parts; observing the parts; beginning with what is to be proved; organizing and reorganizing the problem; asking higher order questions like ‘how’, ‘why’, ‘what do you observe’, etc.  
  - Synthesize data to move from known to unknown by questioning, interpreting abstract concepts, consolidating information and recall the previously known concepts.                                                                                      |
| Uses heuristic approach                       | Employ inquiry training by promoting critical investigation through questioning, testing possible solutions; provide opportunities to give suggestions, to reason out, exchange ideas with peers, discover, conclude and generalize.                                                                                                                                    |
3.2.6 CRITERION LEVEL

80/80 criterion level was set for the learner to exhibit desired performance.

3.2.7 OBSERVATION SCHEDULES AND RATING SCALES

Observation schedules were developed by investigator for ascertaining the acquisition of ten Mathematical Exit Sub-Competencies, identified and selected by investigator; according to the set 80/80 criterion level. Rating Scales were also developed by investigator for ascertaining the acquisition of three Mathematical Exit Competencies, identified and selected by investigator; according to the set 80/80 criterion level. The statements used in the observation schedules and rating scales were in tune with the key factors associated with the desired performance. These are seven point rating scales assessing the performance of mathematics teachers; ranging from “0” for “extremely poor” to “6” for “excellent”.

Ten observation schedules were developed to ascertain the acquisition of each Mathematical Exit Sub-Competency up to the set criterion level of 80/80.

Three rating scales were developed for assessment of prospective mathematics teacher’s performance on three Mathematical Exit Competencies, identified and selected by investigator.

Inter-observer reliability was calculated. The inter-observer reliability coefficients ranged from 0.85 to 0.91. The correlations were found to be significant at 0.01 level of significance. Hence the results may be considered as reliable.

The observation schedules and rating scales were prepared in one to one correspondence with the components associated with the Mathematical Exit Sub-Competencies. At every stage of its development; various mathematical exit competencies and the corresponding key factors/desired performances were discussed with mathematics teachers and mathematics teacher educators. Hence the results may be considered as valid.

3.2.8 DEVELOPMENT OF MATHEMATICS TEACHING COMPETENCY ASSESSMENT SCALE (MTCAS)

The investigator could not locate any tool that could assess Mathematics Teaching Competency among mathematics teachers/prospective mathematics teachers, in Indian conditions. The investigator could locate general teaching competency scales available in Indian conditions, namely, General Teaching
Competency Scale (GTCS) by Passi and Lalitha, 1994 and Baroda General Teaching Competence Scale. Literature available was also referred by the investigator.

A seven point scale was developed by the investigator to fulfill the purpose of assessment of Mathematics Teaching Competency among mathematics teachers/prospective mathematics teachers. The following are the details of the scale:-

The scale was developed by the investigator after methodical review of available mathematical literature, research studies and views of experts in the field and her personal experience. The scale was used to gather data related to the Mathematics Teaching Competency of prospective mathematics teachers at the pre-test and post-test levels.

The scale has been prepared keeping in view the all-inclusive dimensions of desired behaviours of a mathematics teacher. It is very easy to administer.

3.2.8.1 Construction of the scale

In all, there are sixty nine items respective to seven dimensions which encompass the mathematical pedagogy and learning in a mathematics classroom. There are five items in dimension I, fifteen items in dimension II, fourteen items in dimension III, seven items in dimension IV, nine items in dimension V, eight items in dimension VI and eleven items in dimension VII; thereby making sixty nine items in the scale. The items are framed such that the focal point is the desired performance of a mathematics teacher. It is a seven point rating scale assessing the performance of mathematics teachers; ranging from “0” for “extremely poor” to “6” for “excellent”.

The scale has following seven dimensions:-

I Planning
   i. Objectives
   ii. Content

II Knowledge of Mathematics and Mathematical Process

III Problem Solving Ability

IV Use Multiple Instructional Strategies

V Evaluation

VI Management of Supportive Learning Environment and Time
VII Attitude towards Teaching and Mathematics

The description of these dimensions is as under:-

I. Planning

The dimension refers to the planning desired on the part of a mathematics teacher before entering into the classroom. The dimension includes:-

(i) State the learning objectives for each topic
   a. Operationally defined objectives
   b. Attainable objectives
   c. Worthwhile objectives

(ii) Select and organize content effectively
   a. Selection
   b. Logical continuity and psychological organization

II. Knowledge of Mathematics and Mathematical Process

The dimension deals with teacher’s knowledge of mathematics and mathematical processes. It includes:-

- To define, explain, exemplify and relate concepts, principles and relationships;
- Adopting inductive deductive approach;
- Remember factual information and to demonstrate the ways in which new knowledge is created, to help students construct knowledge by engaging them in meaningful learning experiences;
- Appreciate and demonstrate how mathematics knowledge is created, organized and interconnected to different branches of mathematics, to daily life, recreation and other disciplines;
- Demonstrate pedagogical knowledge by employing various methodologies/techniques of teaching mathematics;
- Decoding and interpreting symbolic, pictorial, diagrammatic, graphic and formal mathematical language and understanding its relationship to natural language;
Translate from natural language to formal symbolic mathematics language;
Handling and manipulating statements and symbolic expressions; select, apply and translate among mathematical representations to solve problems;
Recognize reasoning and proof as fundamental aspects of mathematics; investigate and evaluate mathematical arguments and proofs;
Select and use various types of reasoning and methods of proof;
Employ verbal, non-verbal, media and technological communication techniques to foster active inquiry and supportive interaction in the classroom;
Employs patient and consistent listening skills;
Model mathematical reasoning by thinking aloud and encourage students to think aloud;
Provide feedback to students on their use of terminology;
Use the mathematical language to express ideas precisely and coherently.

III. Problem Solving Ability

The dimension discusses problem solving ability of a mathematics teacher. It has been prepared keeping in view the aspects of problem solving involved in mathematics and its processes. The dimension includes:-

- Apply the process of mathematical problem solving i.e. apply and adopt a variety of appropriate strategies to solve problems.
- Develop the ability to lead classes in mathematical problem solving & in developing in depth conceptual understanding and to help students develop and test/evaluate mathematical generalizations and relationships.
- To give opportunities to students to connect mathematical ideas & develop problem solving ability.
- Pose various kinds of problems: instructional problems, routine problems and non routine problems.
- To help students to develop and extend a repertoire of strategies and methods they can apply when solving various kinds of problems.
Ask students to re-read and re-state the problem, uses questioning/prompting to help them to describe their observation, identify the information given and information that needs to be determined, talk about the problem to understand it better.

Ask students to relate the problem to similar problems solved in the past, consider possible strategies, and select a strategy or combination of strategies.

Ask the students to: make model, picture, diagram looking for a pattern; guessing and checking which information, formula, relationship is required to solve the problem and synthesizing the facts; making an organized list, making a table or chart; working backward, using logical reasoning; to develop repertoire of strategies for problem solving.

Uses correcting, suggesting, providing analogies, critical awareness, seeking further information and prompting to enable the students to check the suitability of a strategy.

Execute the chosen strategy, do necessary calculations, draw pictures, use words and symbols to represent the steps in carrying out the plan.

Look back at the solution to check the reasonableness of the answer.

Use knowledge and skills to make connections within & between various contexts and synthesize the facts.

Monitor and reflect on the process of mathematical problem solving i.e. to enable the student to recognize when a particular technique is not fruitful, switch to a different strategy rethink the problem and search for related content knowledge to develop a spirit of enquiry.

Provide opportunities for use of critical thinking skills (estimating, evaluating, classifying, assuming, recognizing relationships, giving opinions with reasons, making generalizations).

IV. Use Multiple Instructional Strategies

The dimension assesses the various instructional strategies used by a mathematics teacher in a classroom. Multiple instructional strategies used in a
mathematics classroom makes classroom environment conducive for teaching learning process. The dimension ‘Multiple instructional strategies’ includes:-

- Relevant to topic, objectives, age, grade and mental level of students.
- Demonstrate understanding and employ a variety of evidence-based instructional strategies to encourage students development of critical thinking problem solving, performance skill; students sense of belongingness.
- Employs instructional strategies according to pre conceptions and background knowledge of students.
- Employs instructional strategies based on learning by doing (use of laboratory method), individual differences and select appropriate concrete material to arouse and maintain interest of students in mathematics.
- Use instructional strategies to develop critical awareness, reasoning and thinking mathematically: open discussion promoting divergent thinking; refocusing / redirecting to encourage convergent thinking; probing questioning to develop analytical and critical thinking; co-operative learning to discuss abilities.
- Maintains logical order and use appropriate explaining links while giving instructions.
- Use related content/ material to encourage students to do positive thinking.

V. Evaluation

The dimension refers to the evaluation of mathematics students and self assessment /reflection of teachers. It includes:-

- Ask questions to assess students learning.
- Reflect upon students responses/suggestions during every class and enlist positive/ negative points for further improvements.
- Accept student’s comments and suggestions, demand feedback from students to reflect upon his teaching.
- Seeks feedback from supervisors and accept their suggestions.
Diagnoses pre-requisite’ entry behaviour of students and their general weaknesses and misconceptions in mathematics by asking questions, objective observation, analyzing oral and written responses and gives a series of specially constructed problems/ exercises/ test to measure sub skills involved in a process.

Arrange varied types of assignments/homework according to content and students learning to facilitate retention, develop thinking and problem solving ability.

Give a quiz/test to check learners understanding during teaching proceedings.

Give a complete summary/revision at the end of a lesson.

Prepare remedial programs/ strategies according to nature of weaknesses, clarify the concepts with appropriate examples and by comparing (refocusing) mathematical concepts/ ideas using inductive-deductive approach, appraises the students and parents about their achievement.

VI. Management Of Supportive Learning Environment And Time

The dimension refers to the management of learning environment and time in the classroom teaching learning course of action. It assesses:-

Illustrates concepts and principles with the help of simple relevant examples through appropriate media (verbal and non verbal) to arouse and maintain interest of students.

Uses questioning, intentional silence/ pausing, varying stimuli like movements, gestures, changing speech patterns, changing interaction styles, oral visual switching and focusing attention of students to secure and maintain their attention and participation.

Makes use of verbal and nonverbal reinforcers on their (students) use of appropriate mathematical language and problem solving strategy to secure pupil participation, interest and motivation.

Use probing questions (prompting, seeking further information, refocusing, redirection and increasing critical awareness to develop reasoning, critical thinking, problem solving and performance skills).
- Does appropriate division of time for previous knowledge testing, announcement of topic, introduction of topic, presenting new content, posing problems, evaluating student’s achievement/ progress and assignment/ home work.
- Writes neatly, legibly and accurately the content in natural as well as mathematical language i.e. appropriate use of diagrams, symbols, pictures, mathematical equipment, etc. to secure attention of students.
- Teach lesson with appropriate speed and voice.
- Encourage students to express their ideas through asking questions, let students participate in teaching activities, let students work in pairs and cultivate their discussing abilities, give students clear response from their reflections or feedback to build two-way communication between teachers and students and ensure a supportive learning environment.

VII. Attitude Towards Teaching And Mathematics

The dimension deals with attitude of mathematics teachers towards teaching and their attitude towards mathematics. It includes:-

- Teach with confidence, patience, interest, resourcefulness and innovation.
- Develop heuristic attitude among students through pupil activity.
- Uses inductive approach in the beginning; followed by deductive approach.
- Does skilful questioning in order to arouse and maintain student’s interest in class.
- Listens to responses given by students empathetically and deals responses effectively.
- Elaborates every step towards solution of mathematical problem.
- Exhibits willingness to accept alternate strategies used by students to the solutions of mathematics problems.
- Use related materials /facts/ examples to enable students to appreciate role of mathematics in life and other disciplines as well.
- Promote pupils interest/ attitude in mathematics by developing willingness to persist, to investigate, to reason and explore alternative solutions.
➢ Use positive /negative re-inforcers at right time to give feedback to students and to motivate them.

➢ Provides new information/knowledge using various strategies and accuracy.

3.2.8.2 Reliability of the Scale

As this is an observation tool, more appropriate type of reliability is the inter-observer reliability. For this reason, inter-observer reliability was calculated. A total of ten prospective mathematics teachers were observed by three teacher educators (raters). The correlation between the ratings of raters ranged from 0.767 to 0.792. All these correlations were found to be significant at 0.01 level of significance.

3.2.8.3 Validity of the Scale

The scale has content validity since at every stage of its development; various Mathematical Exit Sub-Competencies and the corresponding key factors / components and desired performances / behavioural indicators were discussed with experts, mathematics teachers and mathematics teacher educators. The assessment scale has maintained a traditional notion of content validity by focusing on a task and rubric which are representative of day to day tasks of a novice mathematics teacher.