CHAPTER VI

A MULTI CHANNEL BULK QUEUEING MODEL WITH VACATION.
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A MULTI CHANNEL BULK QUEUEING MODEL WITH VACATION.

This chapter is continuation of the previous chapter in which we did multi-server finite capacity queueing models with controllable rates. It consists of two sections. In section – I, we assume that there is bulk arrival and in the second section there is bulk service to the customer. Here we are taking two cases of bulk arrival i.e slower bulk arrival or bulk arrival with the faster rate. There are so many cases in which we can see this type of situation. i.e. after the completion of match there can be bulk arrival in faster rate from the stadium for taxi, buses, auto-rickshaw, while bulk arrival of customer in the stadium can be slow before the start of match. All assumptions are same as were in the previous chapter. i.e the arrival and service processes are correlated and follow a bivariate poisson process.

Section - I

Steady-State equations :- Let suppose that $P_n(0)\rightarrow$ represents the steady state probability that there are n customers in the system when system is in faster rate $P_n(1)\rightarrow$ deponents the steady state prob. That there are n customer in the system, when system is in the slower rate of arrivals.

Where it is assumed that $P_n(0)$ exists for $0 \leq n \leq h-1 \ P_n(1)\ r + 1 \leq n \leq M$

Then steady-state equations are given by

\[
(\sum_{c=1}^{n} a_c \lambda - e) P_0(0) = (\mu-e)P_1(0) \quad \ldots(6.1.1)
\]

\[
(\sum_{c=1}^{n} a_c \lambda+\eta\mu -(n+1)e) P_n(0) = (\sum_{c=1}^{n} a_c \lambda - e) P_{n-1}(0) + (n+1) (\mu-e)P_{n+1}(0)
\]

\[ 1 \leq n \leq c -1 \quad \ldots(6.1.2) \]

\[
(\sum_{c=1}^{n} a_c \lambda +c\mu +(n-c)\theta -(c+1)e) P_n(0) = [c(\mu-e) +(n+1-c)\theta] P_{n+1}(0) + (\sum_{c=1}^{n} a_c \lambda - e)
\]

\[ P_{n-1}(0)\quad c \leq n \leq r-1 \quad \ldots(6.1.3) \]

\[
(\sum_{c=1}^{n} a_c \lambda +c\mu +(r-c)\theta -(c+1)e] P_r(0) = (\sum_{c=1}^{n} a_c \lambda - e)P_{r-1}(0) +[c(\mu-e) + (r +1-c)\theta]
\]

\[ P_{r+1}(0) + [c(\mu-e) + (r +1-c)0] P_{r+1}(1) = 0 \quad \ldots(6.1.4) \]
\[
\sum_{c=1}^{n} a_c \lambda^{c\mu} + (n-c-\theta - (c+1)e) P_0(0) = [c(\mu-e) + (n+1-c)\theta] P_{n+1}(0) \quad r+1 \leq n \leq M-2
\] ....(6.1.5)

\[
\sum_{c=1}^{n} a_c \lambda_0 + c\mu + (M-1-c)\theta - (c+1)e = P_{R-1}(0) + \left( \sum_{c=1}^{n} a_c \lambda_0 - e \right) P_{M-2}(0)
\] ....(6.1.6)

\[
\sum_{c=1}^{n} a_c \lambda_{0}' + c \mu - (c+1)e + (r+1-c)\theta P_{r+1}(1) = [c(\mu-e) + (r+2-c)\theta] P_{r+2}(1)
\] ....(6.1.7)

\[
\sum_{c=1}^{n} a_c \lambda_{0}' + (\mu - (c+1)e) + (n-c)\theta P_n(1)
\]
\[
= [c(\mu-e) + (n+1-c)\theta] P_{n+1}(1) + \left( \sum_{c=1}^{n} a_c \lambda_{0}' - e \right) P_{n-1}(1)
\]
\[
r + 2 \leq n \leq M-1
\] ....(6.1.8)

\[
\sum_{c=1}^{n} a_c \lambda_{0}'' + c\mu + (c+1)e + (M-c)\theta \right) P_{R(1)}(1) = \left( \sum_{c=1}^{n} a_c \lambda_{0}' - e \right) P_{M-1}(1) + \left( \sum_{c=1}^{n} a_c \lambda_0 - e \right) P_{M-1}(0)
\]
\[+[c(\mu-e) + (M+1-c)\theta] P_{r+1}(1)
\] ....(6.1.9)

\[
\sum_{c=1}^{n} a_c \lambda_{0}' + c \mu - (c+1)e + (n-c)\theta P_n(1)
\]
\[
= \left( \sum_{c=1}^{n} a_c \lambda_{0}' - e \right) P_{n-1}(1) + [c(\mu-e) + (n+1-c)\theta] P_{n+1}(1)
\]
\[M + 1 \leq n \leq M - 1
\] ....(6.1.10)

\[
[c \mu - c e + (N-c)\theta] P_N(1) = \left( \sum_{c=1}^{n} a_c \lambda_{0}' - e \right) P_{N-1}(1)
\] ....(6.1.11)

From equation (6.1.1) and (6.1.2), expression \( P_n(0) \) recursively obtained as

\[
P_n(0) = \left( \frac{\sum_{c=1}^{n} a_c \lambda_{0} - e}{\mu - e} \right)^n \cdot \frac{P_0(0)}{n!}
\] ....(6.1.12)
Thus we can say (6.1.11) to (6.1.16) gives all steady state. Probabilities which are expressed in
terms of $P_0(0)$. The convergence of the sum of the prob. Implies that the steady state exists when
$F$ is less than 1.
**Characteristic of Model**

(i) The probabilities that the system is in state $i = 0, 1$

\[ P(0) = \sum_{n=0}^{N} P_n(0) \text{ faster rate of arrival} \]

\[ P(1) = \sum_{n=0}^{N} P_n(1) \text{ slower rate of arrival} \]

∴ The probability that the system is faster rate of arrival is

\[ P(0) = \sum_{n=0}^{c} P_n(0) + \sum_{n=c+1}^{r} P_n(0) + \sum_{n=r+1}^{M-1} P_n(0) \]

\[ = \left[ \sum_{n=0}^{c} \frac{c^n A^n}{n!} + \sum_{n=c+1}^{r} \frac{c^n A^n}{c!} \prod_{m=1}^{n-c} (1+mB) + c^r A^r \prod_{m=1}^{r-c} (1+mB) \left( 1 + \sum_{p=0}^{M-r-2} \frac{z(X)}{A} \right) \right] P_0(0) \]

where \( Z(X) = \prod_{m=0}^{X} \frac{(D+mB)}{A^X} \)

\[ \ldots \text{(6.1.17)} \]

(ii) The probability that system is empty can be calculated from normalizing condition

\[ P(0) + P(1) = 1 \]

Substituting the value $P(0)$ and $P(1)$ from equation (6.1.16) and (6.1.17)
\[ P_0(0) = \left[ \sum_{n=0}^{c} \frac{c^n A^n}{n!} + \frac{c^n}{c!} \sum_{n=c+1}^{r} \frac{A^n}{\prod_{m=1}^{n-c} (1 + mB)} + \frac{c^n S_r}{c!} \sum_{n=r+1}^{M-l} \frac{1}{z(n-r-1)} \left( \sum_{p=n-r+1}^{M-r-2} \frac{z(p)}{A} \right) \right] + \frac{1}{D} \left( \sum_{n=M+1}^{N} \frac{D}{H(n-r-1)} \sum_{p=0}^{M-r-2} \frac{H(p)}{F} \right)^{-1} + \frac{1}{D} \frac{c^n}{c!} \prod_{m=1}^{r-c} (1 + mB) \left( 1 + \sum_{p=0}^{M-r-2} \frac{z(p)}{A} \right) \]

(iii) The probability that number of unit in the system lies between \( r \) and \( M \) is obtained as

\[
p(r \leq n \leq M) = \sum_{n=r}^{M} P_n(0) + \sum_{n=r+1}^{M} P_n(1)
\]

\[
= c^r A^r P_0(0) + \left[ \sum_{n=r+1}^{M} \frac{D}{H(n-r-1)} \left( 1 + \sum_{p=0}^{M-r-2} \frac{H(p)}{F} \right) \right] + \frac{A}{D} \left( \sum_{p=0}^{M-r-2} \frac{z(p)}{A} \right) \]

(iv) The conditional probability that system is in faster rate of arrival when system size lies between \( r \) and \( M \) is
\[ P(0/r \leq n \leq M) = \frac{\sum_{n=0}^{M-1} P_n(0)}{p(r \leq n \leq M)}. \]

\[ \cdot \frac{c^c A^n P_0(0)}{c! \prod_{m=1}^{r-c} (1 + n m B)} + \frac{A^n P_0(0)}{c! \prod_{m=1}^{r-c} (1 + n m B)} \left( 1 + \sum_{p=0}^{M-n-2} \frac{Z(P)}{A} \right) \]

\[ = \frac{\sum_{n=r+1}^{M-1} \frac{1}{Z(n-r-1)} \sum_{p=0}^{M-r-2} Z(p)}{P(r \leq n \leq M)} \]

\[ \ldots(6.1.20) \]

where \( p(r \leq n \leq M) \) is given by (6.1.21) and \( P_0(0) \) is obtained from (6.1.20).

**Mean Queue Length** :- Let us – av. n. of customers in the system, then we have

\[ L_s = L_{s_0} + L_{s_1} \]

where

\[ L_{s_0} = \sum_{n=0}^{r} n P_n(0) + \sum_{n=r+1}^{M-1} n P_n(0) \]

and

\[ L_{s_1} = \sum_{n=r+1}^{M} n P_n(1) + \sum_{n=M+1}^{N} n P_n(1) \]

\[ = \left[ (r+1) + \sum_{n=r+2}^{M} \frac{nD}{H(n-r-1)} \left[ 1 + \sum_{p=0}^{n-r-2} \frac{H(P)}{F} \right] \right] \]

\[ + \sum_{n=M+1}^{N} \frac{nD}{H(n-r-1)} \left[ 1 + \sum_{p=0}^{M-r-2} \frac{D(p)}{F} \right] \]

\[ \times \frac{c^c A^{r+1}}{Dc! \prod_{M=1}^{r-c} (1 + n m B)} \frac{P_0(0)}{1 + \sum_{p=0}^{R-r-2} \frac{Z(p)}{A}} \]

\[ \ldots(6.1.21) \]

Substituting the values of \( L_{s_0} \) and \( L_{s_1} \) we obtained the value of \( L_s \) where \( P_0(0) \) is given by (6.1.21).

**Expected Waiting Time** :- Using Little’ formulae the expected waiting time of the customer in the system is
\[ W_s = \frac{L_s}{\lambda} \]

where \( \bar{\lambda} = \lambda_0 P(0) + \lambda_1 P(1) \)

where \( P(0) \) and \( P(1) \) are obtained from (6.1.19) and (6.1.20)

Section - II

In present section we are taking multi-server finite capacity queueing models with controllable rates, when there is bulk service providing to the customer.

Steady-State equations: Let suppose that \( P_n(0) \) represents the steady state probability that there are \( n \) customers in the system when system is in faster rate \( P_n(1) \) deponents the steady state prob. That there are \( n \) customer in the system, when system is in the slower rate of arrivals.

Where it is assumed that \( P_n(0) \) exists for \( 0 \leq n \leq h-1 \ P_n(1) \). \( r+1 \leq n \leq M \)

Then steady-state equations are given by

\[
(\lambda - e) P_0(0) = (\sum_{c=1}^{a_c} \mu - e) P_1(0) \quad \text{....(6.2.1)}
\]

\[
(\lambda + \eta e) P_n(0) = (\lambda + (n+1)e) P_{n+1}(0) + (n+1) (\sum_{c=1}^{a_c} \mu - e) P_{n+1}(0) \quad \text{1 \leq n \leq c-1} \quad \text{....(6.2.2)}
\]

\[
(\lambda + c \sum_{c=1}^{a_c} \mu + (n-c)\theta - (c+1)e) P_n(0) = [c(\sum_{c=1}^{a_c} \mu - e) + (n+1-c)\theta] P_{n-1}(0) + (\lambda - e) P_{n-1}(0) \quad \text{c \leq n \leq r-1} \quad \text{....(6.2.3)}
\]

\[
(\lambda + c \sum_{c=1}^{a_c} \mu + (r-c)\theta - (c+1)e) P_r(0) = (\lambda - e) P_{r-1}(0) + [c(\sum_{c=1}^{a_c} \mu - e) + (r + 1-c)\theta] P_{r+1}(0) + [c(\sum_{c=1}^{a_c} \mu - e) + (r + 1-c)\theta] P_{r+1}(1) = 0 \quad \text{....(6.2.4)}
\]

\[
[\lambda + c \sum_{c=1}^{a_c} \mu + (n-c)\theta - (c+1)e] P_0(0) = [c(\sum_{c=1}^{a_c} \mu - e) + (n+1-c)\theta] P_{n+1}(0) \quad \text{r+1 \leq n \leq M - 2} \quad \text{....(6.2.5)}
\]
\[ \lambda + c \sum_{e=1}^{n} a_e \mu + (M-1-c)\theta -(c+1)e = P_{R-1}(0) + (\lambda -e) P_{M-2}(0) \quad . \quad \ldots (6.2.6) \]

\[ \lambda'_{0} + c \sum_{e=1}^{n} a_e \mu -(c+1) e + (r+1-c)\theta \] \[ P_{r=1}(1) \]

\[ = [c(\sum_{e=1}^{n} a_e \mu -e) + (r+2-c)\theta] P_{r+2}(1) \quad \ldots (6.2.7) \]

\[ \lambda'_{0} + \sum_{e=1}^{n} a_e \mu -(c+1) e + (n-c)\theta \] \[ P_{n}(1) \]

\[ = [c(\sum_{e=1}^{n} a_e \mu -e) + (n+1-c)\theta] P_{n+1}(1) + (\lambda' -e) P_{n-1}(1) \]

\[ r + 2 \leq n \leq M-1 \quad \ldots (6.2.8) \]

\[ \lambda'_{0} + c \sum_{e=1}^{n} a_e \mu -(c+1) e + (M-c)\theta \] \[ P_{R(1)} \]

\[ = (\lambda' -e) P_{M-1}(1) + (\lambda -e) P_{M-1}(0) + [c(\sum_{e=1}^{n} a_e \mu -e)+(M+1-c)\theta] P_{r+1}(1) \]

\[ \ldots (6.2.9) \]

\[ \lambda'_{0} + \sum_{e=1}^{n} a_e \mu -(c+1) e + (n-c)\theta \] \[ P_{n}(1) \]

\[ = (\lambda' -e) P_{n-1}(1) + [c(\sum_{e=1}^{n} a_e \mu -e) + (n+1-c)\theta] P_{n+1}(1) \]

\[ M + 1 < n \leq M - 1 \quad . \quad \ldots (6.2.10) \]

\[ [c \sum_{e=1}^{n} a_e \mu -c e + (N-c)\theta] P_{N}(1) = (\lambda'_{0} -e) P_{N-1}(1) \quad . \quad \ldots (6.2.11) \]

From equation (6.2.1) and (6.2.2), expression \( P_n(0) \) recursively obtained as

\[ P_n(0) = \left( \frac{\lambda -e}{\sum_{e=1}^{n} a_e \mu -e} \right)^n \frac{P_0(0)}{n!} \quad \ldots (6.2.12) \]
From equation (6.2.4), we can write

\[
P_{r+1}(0) = \left( \frac{\lambda - e}{c\left(\sum_{e=1}^{n} a_e \mu - e\right)} \right)^n \frac{1}{\prod_{m=1}^{n-e} (1 + mB)} \cdot \frac{P_0(0)}{c! c^{n-e}} \quad \text{....(6.2.13)}
\]

\[
P_{r+1}(1) = \frac{\lambda - e}{c\left(\sum_{e=1}^{n} a_e \mu - e\right)} \frac{1}{\prod_{m=1}^{n-e} (1 + mB)} \cdot \frac{P_0(0)}{c! c^{n-e}}
\]

From equation (6.2.4), we can write

\[
P_{r+1}(0) = \left( \frac{\lambda - e}{c\left(\sum_{e=1}^{n} a_e \mu - e\right)} \right)^n \frac{1}{\prod_{m=1}^{n-e} (1 + mB)} \cdot \frac{P_0(0)}{c! c^{n-e}} \quad \text{....(6.2.14)}
\]

where

\[
D^D = \left( 1 + \frac{\lambda - e}{c\left(\sum_{e=1}^{n} a_e \mu - e\right)} \right) \frac{1}{\prod_{m=1}^{n-e} (1 + mB)} \cdot \frac{P_0(0)}{c! c^{n-e}}
\]

from equation (6.2.5), the expression \( r + 1 \leq n \leq M - 2 \) derived as

\[
P_n(0) = \frac{\lambda - e}{c\left(\sum_{e=1}^{n} a_e \mu - e\right)} \frac{1}{\prod_{m=1}^{n-e} (1 + mB)} \cdot \frac{P_0(0)}{c! c^{n-e}} \quad \text{....(6.2.15)}
\]

from equation (6.2.5) and (6.2.6), we will find the value \( P_{r+1}(1) \)
Therefore we can say (6.2.11) to (6.2.18) gives all steady state. Probabilities which are expressed in terms of $P_0(0)$. The convergence of the sum of the prob. Implies that the steady state exist when $F$ is less than 1.

**Characteristic of Model**

(i) The probabilities that the system is in state $i = 0, 1$
\[ P(0) = \sum_{n=0}^{N} P_n(0) \text{ faster rate of arrival} \]

\[ P(1) = \sum_{n=0}^{N} P_n(1) \text{ slower rate of arrival} \]

\[ \therefore \text{ The probability that the system is faster rate of arrival is} \]

\[ P(0) = \sum_{n=0}^{c} P_n(0) + \sum_{n=c+1}^{r} P_n(0) + \sum_{n=r+1}^{M-1} P_n(0) \]

\[ = \left[ \sum_{n=0}^{c} \frac{c^n}{n!} A^n + \frac{c^c}{c!} \sum_{n=c+1}^{r} \frac{A^n}{\prod_{m=1}^{n-c} (1+mB)} + \frac{c^c}{c!} \frac{A^r}{\prod_{m=1}^{r-c} (1+mB)} \right] P_0(0) \]

where \( Z(X) = \frac{\prod_{m=0}^{X} (D+mB)}{A^X} \)

(ii) The probability that system is empty can be calculated from normalizing condition

\[ P(0) + P(1) = 1 \]

Substituting the value \( P(0) \) and \( P(1) \) from equation (19) and (20)

\[ P_0(0) = \left[ \sum_{n=0}^{c} \frac{c^n}{n!} A^n + \frac{c^c}{c!} \sum_{n=c+1}^{r} \frac{A^n}{\prod_{m=1}^{n-c} (1+mB)} + \frac{c^c}{c!} \frac{A^r}{\prod_{m=1}^{r-c} (1+mB)} \right] + \]

\[ + \left[ 1 + \sum_{n=r+2}^{N} \frac{D}{H(n-r-1)} + \sum_{n=r+2}^{M} \frac{D}{H(n-r-1)} \sum_{p=0}^{n-r-2} \frac{H(p)}{F} \right. \]

\[ + \left. \sum_{n=M+1}^{N} \frac{D}{H(n-r-1)} \sum_{p=0}^{M-r-2} \frac{H(p)}{F} \right]^{-1} \]

\[ \times \left( \sum_{p=0}^{M-r-2} \frac{z(P)}{A} \right) \]

\[ \ldots (6.2.20) \]
(iii) The probability that number of unit in the system lies between \( r \) and \( M \) is obtained as

\[
p(r \leq n \leq M) = \sum_{n=r}^{M-1} P_n(0) + \sum_{n=r+1}^{M} P_n(1)
\]

\[
= c^r A^r P_0(0) \left[ \frac{1}{D} \sum_{n=r}^{M-1} \frac{1}{z(n-r-1)} \sum_{p=n-r}^{M-2} Z(p) + \frac{A}{D} \right] \left[ 1 + \sum_{n=r+2}^{M} \frac{D}{H(n-r-1)} \left( 1 + \sum_{p=0}^{n-r-2} \frac{H(p)}{F} \right) + \frac{A}{D} \right] + \sum_{p=0}^{R-r-2} \frac{Z(P)}{A}
\]

\[
= \frac{1}{1+\frac{A}{D}} + \frac{A}{D} \left[ 1 + \sum_{p=0}^{R-r-2} \frac{Z(P)}{A} \right]
\]

where \( P_0(0) \) is obtained from (6.2.21).

where \( p(r \leq n \leq M) \) is given by (6.2.20) and \( P_0(0) \) is obtained from (6.2.21).

**Mean Queue Length** :- Let us – av. n. of customers in the system, then we have

\[
L_s = L_{s_0} + L_{s_1}
\]

where

\[
L_{s_0} = \sum_{n=0}^{r} n P_n(0) + \sum_{n=r+1}^{M-1} n P_n(0)
\]

and \( L_{s_1} = \sum_{n=r+1}^{M} n P_n(1) + \sum_{n=M+1}^{N} n P_n(1) \)

Therefore,

\[
L_{s_0} = \sum_{n=0}^{c} \frac{n(cA)^n}{n!} P_0(0) + \frac{c^c}{c!} \sum_{n=c+1}^{r} \frac{n A^n}{\prod_{M=1}^{M-1}} + \frac{c^r A^r}{c!} \sum_{M=r+1}^{N-1} \frac{n}{z(n-r-1)} \sum_{p=n-r}^{M-2} Z(p) + \frac{P_0(0)}{} \]

\[
\sum_{p=0}^{r-c} \frac{Z(P)}{A} \left[ 1 + \sum_{p=0}^{M-r-2} \frac{Z(P)}{A} \right]
\]

and
\[ L_{S_0} = (r + 1)P_{r+1}(1) + \sum_{n=r+2}^{M} nP_n(1) + \sum_{n=M+1}^{N} nP_n(1) \]

\[ = \left[ (r + 1) + \sum_{n=r+2}^{M} \frac{nD}{H(n - r - 1)} \left( 1 + \sum_{p=0}^{n-r-2} \frac{H(p)}{F} \right) \right] + \sum_{n=M+1}^{N} \frac{nD}{H(n - r - 1)} \left( 1 + \sum_{p=0}^{M-r-2} \frac{D(p)}{F} \right) \times \frac{c^cA^{r+1}}{Dc! \prod_{M=1}^{r+c} \left( 1 + \sum_{p=0}^{R_r+c-2} \frac{Z(p)}{A} \right)} P_0(0) \]

\[ \ldots (6.2.22) \]

Substituting the values of \( L_{S_0} \) and \( L_{S_1} \) we obtained the value of \( L_s \) where \( P_0(0) \) is given by (6.2.22).

**Expected Waiting Time** :- Using Little’ formulae the expected waiting time of the customer in the system is

\[ W_s = \frac{L_s}{\lambda} \]

where \( \lambda = \lambda_0 P(0) + \lambda_1 P(1) \)

where \( P(0) \) and \( P(1) \) are obtained from (6.2.19) and (6.2.20)

**Numerical Analysis**

From table (6.1.1) it is observed that, the probability that the system is in state 0 is inversely proportional to the probability that the system is in state 1. The \( P_n(1) \) values is increasing when the parameter \( r \) is increasing for fixed values of the other parameters. \( P(0) \) is increasing function of \( M \) also. \( P_n(1) \) is decreasing function of \( \lambda' \) when other parameter remain fixed. from the table (6.2.1) we observe that the expected number in system is a decreasing function of mean dependence rate when other parameter remain fixed. The mean number of customers in system is an increasing function of ‘\( R \)’ for given values of the parameters.
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Different values are calculated by taking $\mu = 0.1$, $\lambda = 0.2$, $M = 0.4$

**Table 6.1.1**

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Different values are calculated by taking $\mu = 0.1$, $M = 0.4$

Table 6.2.1

This graph shows that queue length increases as arrival rate increases.

$\lambda = 0.5$  $\lambda = 0.4$  $\lambda = 0.2$
Graph 6.1.1

This graph shows that queue length decreases as service rate increases.

Graph 6.2.1
This graph shows that queue length increases as arrival rate increases.

Graph 6.2.2