CHAPTER IV
A COMPARATIVE APPROACH FOR FMADM METHOD
USING GREY RELATED ANALYSIS AND
FUZZY IMPROVED AHP

4.1. Introduction

In the two decades, there has been a great deal concerning with
decision theory and multiple attribute decision making. Presently decision
making is conducted in a highly dynamic environment, involving
complex tradeoffs and high levels of uncertainty. Practical decision
problems involve uncertainty with respect to all elements of the basic
decision making model [59]. The uncertainty and fuzziness inherent in
decision making makes the use of precise numbers problematic in
multiple attribute models. We proposed two approaches: first proposed
approach is grey relational theory and second is improved Analytical
Hierarchy Process (AHP). Methods applying intervals have included, use
of interval numbers as the basis for ranking alternatives [24, 66], error
analysis with interval numbers [84], use of linear programming and
object programming with feasible regions bounded by interval numbers
[6, 33], use of interval number ideal alternatives to rank alternatives by
their nearness to the ideal.

The method of grey related analysis as a means to reflect
uncertainty in multiple attribute models through interval numbers is
presented. Grey system theory was developed by Deng [17], based upon
the concept that information is sometimes incomplete or unknown. The
intent is the same as with factor analysis, cluster analysis, and
discriminant analysis, except that those methods often don't work well when sample size is small and sample distribution is unknown [72]. With grey related analysis, interval numbers are standardized through norms, which allow transformation of index values through product operations. The method is simple, practical, and demands less precise information than other methods. Grey related analysis and TOPSIS [48, 83, 85] both use the idea of minimizing a distance function. However, grey related analysis reflects a form of fuzzification of inputs, and uses different calculations, to include different calculation of norms. Feng and Wang [28], applied grey relation analysis to select representative criteria among a large set of available choices, and then used TOPSIS for outranking.

The method of grey related analysis is used to solve the problem by interval fuzzy numbers. This method standardizes inputs through norms of interval number vectors. Interval valued indices are used to apply multiplicative operations over interval numbers.

AHP was presented [63] as a way to take subjective human inputs in a hierarchy and convert these to a value function. This method has proven extremely popular. Saaty [63] used the eigenvector approach to reconcile inconsistent subjective inputs. Lootsma proposed a different scaling method in his REMBRANDT system [49]. Salo and Hamalainen [65] published their interval method using linear programming over the constrained space of weights and values as a means to incorporate uncertainty in decision maker inputs to AHP hierarchies. The problem of synthesizing ratio judgments in groups was considered very early in AHP [1], and the geometric mean was found to provide satisfactory properties. Fuzzy AHP was proposed as another way to reflect uncertainty in subjective inputs to AHP in the same group context [7-9]. Simulation has been presented as a way to rank order alternatives in the context of AHP values and weights [45]. Other multiple criteria methods besides AHP
have considered fuzzy input parameters. A multiple attribute method involving fuzzy assessment for selection has been given in the airline safety domain [2] and for multiple criteria selection of employees [27]. Sensitivity in multiple attribute models with fuzzy inputs [91].

Under many conditions, crisp data are inadequate to model real-life situations, since human judgments and preferences, are often vague and cannot be estimated with exact numerical value. The decisions mostly depend on expert knowledge, experience and favoritism [71]. To avoid subjective interference, difference deriving weight principle is provided here. The basic principle is that the weight co-efficient is the measure of attribute differentiation between the target and other attribute. The weight of the original information should come directly from an objective environment. The relative attribute weight can be determined by the relative attribute’s information.

When experts are not able to give exact numerical values to express their opinions, a more realistic alternative option is to use the trapezoidal fuzzy number. It can be used to convert an exact variant into a fuzzy variant.

The experts’ opinions are expressed in trapezoidal fuzzy numbers. To obtain the consensus of the experts, we adopt fuzzy attribute evaluation space to adjust the fuzzy rating of every expert. A fuzzy symmetrical matrix is constructed by referring to covariance definition of random variables and the aggregate fuzzy numbers can be obtained by the corresponding approximate fuzzy eigenvector. The final results become ranking fuzzy number problems. Finally, these two methods were compared by taking a numerical example.
4.2. Proposed Methodologies

4.2.1. The Grey Related Analysis methodology

4.2.1.1. Basic terminology: Grey related analysis has been used in a number of applications. Here, use the concept of the norm of an interval number column vector, the distance between intervals, product operations, and number-product operations of interval numbers.

Let $a=[a^-,a^+]=\{x|a^- \leq x \leq a^+, a^- \leq a^+, a^-, a^+ \in R\}$. We call $a=[a^-,a^+]$ an interval number. If $0 \leq x \leq a^+$, we call interval number $a=[a^-,a^+]$ a positive interval number. Let $X=([a_1^-,a_1^+], [a_2^-,a_2^+], \ldots, [a_n^-,a_n^+])^T$ be an n-dimension interval number column vector.

**Definition 4.1:** If $X=([a_1^-,a_1^+], [a_2^-,a_2^+], \ldots, [a_n^-,a_n^+])^T$ is an arbitrary interval number column vector, the norm of $X$ is defined here as,

$$\|X\| = \max \left( \max\left( |a_1^-|, |a_1^+| \right), \max\left( |a_2^-|, |a_2^+| \right), \ldots, \max\left( |a_n^-|, |a_n^+| \right) \right)^T$$

**Definition 4.2:** If $a=[a^-,a^+]$ and $b=[b^-,b^+]$ are two arbitrary interval numbers, the distance from $a=[a^-,a^+]$ to $b=[b^-,b^+]$.

$$|a-b| = \max\left( |a^- - b^-|, |a^+ - b^+| \right).$$

**Definition 4.3:** If $k$ is an arbitrary positive red number, and $a=[a^-,a^+]$ is an arbitrary interval number, then $k \cdot [a^-,a^+] = [ka^-,ka^+]$ will be called the number product between $k$ and $a=[a^-,a^+]$.

**Definition 4.4:** If $a=[a^-,a^+]$ is an arbitrary interval number, and $b=[b^-,b^+]$ are arbitrary interval numbers; we shall define the interval number product $[a^-,a^+], [b^-,b^+]$ as follows:

1. When $b^+ > 0$ $[a^-,a^+],[b^-,b^+] = [a^- b^-, a^+ b^+]$,
(2) When $b^+ < 0$ \[[a^-,a^+]\cdot[b^-,b^+]=\left[a^b-,a^b+b^\right].$ 

If $b^+ = 0$, the interval reverts to a point, and thus we would return to the basic crisp model.

**4.2.1.2. Algorithm for Grey Related Analysis**

Suppose that multiple attribute decision making problem with interval numbers has $m$ feasible alternatives $X_1 + X_2 + \ldots + X_m$, $n$ indices, the weight value $w_j$ of index $G_j$ is uncertain, but we know that $w_j \in [c_j,d_j]$. Here, $0 \leq c_j \leq d_j \leq 1$, $j = 1, 2, \ldots, n$, $w_1 + w_2 + \ldots + w_n = 1$, the index value of $j^{th}$ index $G_j$ of feasible alternative $X_i$ is an interval number $[a^-,a^+]$, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$. When $c_j = d_j$, $j = 1, 2, \ldots, n$, the multiple attribute decision making problem with interval numbers is an interval valued multiple attribute decision making problem with crisp weights. When $a_j^-, a_j^+$, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$, the alternative scores over criteria are crisp. The principle and steps of this method are given below.

**Step 1:** Construct decision matrix $A$ with index number of interval numbers. If the index value of $j^{th}$ index $G_j$ of feasible alternative $X_i$ is an interval number $[a^-,a^+]$, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$, decision matrix $A$ with index number of interval numbers is defined as follows,

$$A = \begin{bmatrix}
[a_{11}^-, a_{11}^+] & [a_{12}^-, a_{12}^+] & \ldots & [a_{1n}^-, a_{1n}^+] \\
[a_{21}^-, a_{21}^+] & [a_{22}^-, a_{22}^+] & \ldots & [a_{2n}^-, a_{2n}^+] \\
\vdots & \vdots & \ddots & \vdots \\
[a_{m1}^-, a_{m1}^+] & [a_{m2}^-, a_{m2}^+] & \ldots & [a_{mn}^-, a_{mn}^+]
\end{bmatrix}$$  

**Step 2:** Transform "contrary index" into positive index.

The index is called a positive index if a greater index value is better. The index is called a contrary index if a smaller index value is better. We may
transform contrary index into positive index if $j^{th}$ index $G_j$ is contrary index

$$[b_j^-, b_j^+] = [-a_j^+, -a_j^-] \quad i=1,2,\ldots,m$$  \hspace{1cm} (4.2)

Without loss of generality, in the following, we supposed that all the indexes are "positive indexes".

Step 3: Standardize decision matrix $A$ with index number of interval numbers to gain standardizing decision matrix $R=[r_j^-, r_j^+]$.

If mark the column vectors of decision matrix $A$ with interval-valued indexes with $A_1, A_2, \ldots, A_n$, the element of standardizing decision matrix $R=[r_j^-, r_j^+]$ is defined as the following:

$$[r_j^-, r_j^+] = \left[ \begin{array}{c} a_j^- \\ A_j^- \\ a_j^+ \\ A_j^+ \end{array} \right], \quad i=1,2,\ldots,m, \quad j=1,2,\ldots,n.$$  \hspace{1cm} (4.3)

TOPSIS uses the root mean square to evaluate distance. Grey related analysis uses a different norm, based on minimization of maximum distance.

Step 4: Calculate interval number weighted matrix $C=(c_j^- \times c_j^+)$.

The formula for calculation of the interval number weighted matrix $C=(c_j^- \times c_j^+)$ is,

$$C = \left[ c_j^-, c_j^+ \right]_{\text{max}} = [c_j^-] \cdot [r_j^-, r_j^+] \cdot i=1,2,\ldots,m, \quad j=1,2,\ldots,n.$$  \hspace{1cm} (4.4)

Step 5: Determine reference number sequence.

The vector for the reference number sequence is determined as the set of optimal weighted interval values associated with each of the $n$ attributes. $U_0 = ([u_0^-(1), u_0^+(1)], [u_0^-(2), u_0^+(2)], \ldots, [u_0^-(n), u_0^+(n)])$ is called a reference number sequence if $u_0^-(j) = \max_{1 \leq i \leq m} c_i^-$, $u_0^+(j) = \max_{1 \leq i \leq m} c_i^+$, $j=1,2,\ldots,n$
Step 6: Calculate the connection between the sequence composed of weight interval number standardizing index value of every alternative and reference sequence.

The connection coefficient $\xi(i, k)$, between the sequence composed of weight interval number standardizing index value of every alternative $U = ([c_{1i}, c_{1i}^+], [c_{2i}, c_{2i}^+], ..., [c_{ni}, c_{ni}^+])$ and reference number sequence $U_0 = ([u_1^0(1), u_2^0(1)], [u_2^0(2), u_2^0(2)], ..., [u_n^0(n), u_n^0(n)])$ is calculated.

The formula of $\xi(i, k)$ is,

$$\xi(i, k) = \frac{\min_j \min_k \left[ u_0(k), u_0^+(k) \right] - \left[ c_{ik}, c_{ik}^+ \right] + \rho \max_j \max_k \left[ u_0(k), u_0^+(k) \right] - \left[ c_{ik}, c_{ik}^+ \right]}{\left[ u_0(k), u_0^+(k) \right] - \left[ c_{ik}, c_{ik}^+ \right] + \rho \max_j \max_k \left[ u_0(k), u_0^+(k) \right] - \left[ c_{ik}, c_{ik}^+ \right]}$$

(4.5)

The resolving coefficient $\rho \in (0, +\infty)$ is used. The smaller $\rho$, the greater its resolving power usually; $\rho \in [0, 1]$. The value of $\rho$ reflects the degree to which the minimum scores are emphasized relative to the maximum scores. A value of 1.0 would give equal weighting. After calculating $\xi(i, k)$, the connection between $i^{th}$ alternative and reference number sequence will be calculated according to the following formula

$$r_i = \frac{1}{n} \sum_{k=1}^{n} \xi(i, k), \quad i = 1, 2, ..., m.$$  

(4.6)

Step 7: Determine optimal alternative

The feasible alternative $X_t$ is optimal by grey related analysis if $r_i = \max_{1 \leq i \leq m} r_i$. 

86
4.2.2. Improved AHP Methodology

4.2.2.1. Ranking fuzzy numbers procedure

Each method appears to have some advantages as well as disadvantage [44]. We emphasize a fuzzy number ranking procedure by a simple method. This procedure can be introduced as follows:

- **Intuition ranking method.** From membership function curves of fuzzy numbers, many fuzzy numbers can easily be ranked by intuition ranking method. Lee and Li [44] pointed out that human intuition would favour a fuzzy number with the characteristics of higher mean value and at the same time, lower spread.

- **If it’s ordering cannot be ranked by the intuition ranking method.** Rank fuzzy numbers represents by $\alpha$-cut method [48], fuzzy mean and spread [44], or other methods. Here, we use the defuzzification value of the trapezoidal fuzzy number to do the necessary rank orderings.

4.2.2.2. The defuzzification value of the trapezoidal fuzzy number

**Definition 4.5.** For a trapezoidal fuzzy number $\Lambda=(a_1,a_2,a_3,a_4)$, its defuzzification value [13] is defined to be:

$$C = \frac{(a_1 + a_2 + a_3 + a_4)}{4} \quad (4.7)$$

![Fig. 4.1. Trapezoidal fuzzy number](image)
4.2.2.3. The basic concepts of fuzzy numbers and relative operations

Special cases of fuzzy numbers include crisp real numbers and intervals of real numbers, the defuzzification value of the trapezoidal fuzzy number. Although there are many shapes of fuzzy numbers, the trapezoidal shapes are used most often for representing fuzzy numbers. The following definition describes the membership function of trapezoidal fuzzy numbers and the operations on them.

**Definition 4.6.** For a trapezoidal fuzzy number \( \tilde{A}=(a_1,a_2,a_3,a_4) \),

\( a_1 < a_2 < a_3 < a_4 \) (if \( a_2 = a_3 \), \( \tilde{A} \) is a triangular fuzzy number), its membership function is

\[
\lambda(\lambda) = \frac{(x-a_1)}{(a_2-a_1)} \quad x < a_1
\]

\[
\frac{(x-a_1)}{(a_2-a_1)} \quad a_1 \leq x \leq a_2
\]

\[
1 \quad a_2 \leq x \leq a_3
\]

\[
\frac{(x-a_4)}{(a_3-a_4)} \quad a_3 \leq x \leq a_4
\]

\[
0 \quad x > a_4
\]

(4.8)

Let \( \tilde{A}=(a_1,a_2,a_3,a_4) \) and \( \tilde{B}=(b_1,b_2,b_3,b_4) \) be any two positive trapezoidal fuzzy numbers. Then the operations \( \{+,-,\cdot,|\} \) are defined by [11]:

\( \tilde{A} (+) \tilde{B} = (a_1,a_2,a_3,a_4)(+) (b_1,b_2,b_3,b_4) \)

\( = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4) \)

\( \tilde{A} (-) \tilde{B} = (a_1,a_2,a_3,a_4)(-) (b_1,b_2,b_3,b_4) \)

\( = (a_1 - b_1, a_2 - b_2, a_3 - b_3, a_4 - b_4) \)

\( \tilde{A} (\cdot) \tilde{B} = (a_1,a_2,a_3,a_4) (\cdot) (b_1,b_2,b_3,b_4) \)

\( = (a_1 \cdot b_1, a_2 \cdot b_2, a_3 \cdot b_3, a_4 \cdot b_4) \)

\( \tilde{A} (|) \tilde{B} = (a_1,a_2,a_3,a_4) (|) (b_1,b_2,b_3,b_4) \)

\( = (a_1 | b_1, a_2 | b_2, a_3 | b_3, a_4 | b_4) \)

(4.9)
4.2.2.4. Algorithm for Improved AHP methodology

Analysis of the hierarchy process breaks down complicated decision-making problems into several hierarchies through merging quality analysis and quantity analysis. Quantification of subjective judgment is provided for every hierarchy and analysis. The comprehensive decision weights for each alternative are calculated by weight sum. For the classic multiple attribute decision making, eigenvectors can be used to determine the weights of attributes [63]. This method can be used for fuzzy multiple attribute decision making. The problem is to construct a fuzzy attribute evaluation matrix and calculate the eigenvector. The proposed fuzzy multiple attribute decision-making method will focus on to develop decision matrix information and define weights of all attributes based on regular AHP and the multi-attribute eigenvector decision-making method. n-dimensional fuzzy evaluation matrix of attribute $C_j = (j = 1, 2, ..., n)$ will be constructed by referring to covariance definition of random variables. The eigenvector of fuzzy attribute evaluation matrix are not calculated directly from a fuzzy eigenvector equations but are approximated. These eigenvectors could be adopted as optimize one-dimensional space to determine the weight of a multiattribute. It is obvious that actual difference could be calculated by eigenvector of fuzzy attribute evaluation space. $m$ points (or vector) of $\mathbb{R}^m$ (decision matrix) will be projected into one-dimensional space via eigenvector, so that all of these points will be very well dispersed. Finally, the decision-making becomes the fuzzy number ranking. The detail steps for improved fuzzy AHP method are described as follows:

Step 1: Build up hierarchy structure model. Set $\hat{A} = \{A_1, A_2, ..., A_m\}$ as alternatives from which decision makers have to choose. This set consists of alternatives hierarchy. Set $\hat{C}_j (j = 1, 2, ..., n)$ as attributes from which the
performance of alternatives is measured. This set consists of attribute hierarchy.

Step 2: Construct decision matrix. For the alternative \( A_i = (i = 1, 2, \ldots, m) \),
\[
\tilde{h}_{ij} \forall i, j = 1, 2, \ldots, m, j = 1, 2, \ldots, n, \text{ is rating of alternative } A_i \text{ with respect to } j^{th} \]
\((j = 1, 2, \ldots, n) \text{ attribute } C_j \). So the decision matrix can be expressed as can
be expressed by trapezoidal fuzzy numbers as \( \tilde{H} = (\tilde{h}_{ij})_{m \times n} \) where \( \tilde{h}_{ij} \) can
be expressed by trapezoidal fuzzy numbers as \( \tilde{h}_{ij} = (a_{ij}, b_{ij}, c_{ij}, d_{ij}) \)
\((a_{ij} < b_{ij} < c_{ij} < d_{ij}, i = 1, 2, \ldots, m, j = 1, 2, \ldots, n) \).

**Definition 4.7.** Represent \( \tilde{h}_{ij} \)'s the fuzzy decision matrix by trapezoidal
fuzzy numbers, we obtain
\[
\begin{bmatrix}
    C_1 & C_2 & \ldots & C_n \\
    A_1 \begin{bmatrix}
    \tilde{h}_{11} & \tilde{h}_{12} & \ldots & \tilde{h}_{1n} \\
    \vdots & \vdots & \ddots & \vdots \\
    \tilde{h}_{n1} & \tilde{h}_{n2} & \ldots & \tilde{h}_{nn}
    \end{bmatrix}
    & (a_{11}, b_{11}, c_{11}, d_{11}) & (a_{12}, b_{12}, c_{12}, d_{12}) & \ldots & (a_{1n}, b_{1n}, c_{1n}, d_{1n}) \\
    A_2 \begin{bmatrix}
    \tilde{h}_{21} & \tilde{h}_{22} & \ldots & \tilde{h}_{2n} \\
    \vdots & \vdots & \ddots & \vdots \\
    \tilde{h}_{n1} & \tilde{h}_{n2} & \ldots & \tilde{h}_{nn}
    \end{bmatrix}
    & (a_{21}, b_{21}, c_{21}, d_{21}) & (a_{22}, b_{22}, c_{22}, d_{22}) & \ldots & (a_{2n}, b_{2n}, c_{2n}, d_{2n}) \\
    \vdots & \vdots & \ddots & \vdots & \vdots \\
    A_m \begin{bmatrix}
    \tilde{h}_{m1} & \tilde{h}_{m2} & \ldots & \tilde{h}_{mn} \\
    \vdots & \vdots & \ddots & \vdots \\
    \tilde{h}_{n1} & \tilde{h}_{n2} & \ldots & \tilde{h}_{nn}
    \end{bmatrix}
    & (a_{m1}, b_{m1}, c_{m1}, d_{m1}) & (a_{m2}, b_{m2}, c_{m2}, d_{m2}) & \ldots & (a_{mn}, b_{mn}, c_{mn}, d_{mn})
\end{bmatrix}
\]

Step 3: Construct n-dimensional fuzzy attribute evaluation space.

n-dimensional fuzzy attribute \( \{C_j, (j = 1, 2, \ldots, n)\} \) evaluation matrix will be
constructed by referring to covariance definition of random variables.

(1) Construct centrally normalized matrix \( \tilde{\Phi} \). Fuzzy variant standardizing
includes type consistency and normalization. Type consistency is
preferred to normalization although both are necessary. The new centrally
normalized algorithm suitable for fuzzy sample set is given as follows.

The linear scale transformation is used to transform the various attributes’
scale into a comparable scale. Therefore, we can obtain the centrally
normalized fuzzy decision matrix denoted by \( \tilde{H} \).
For formula (4.10), fuzzy average $\tilde{E}_j$ can be expressed by trapezoidal fuzzy numbers, i.e., $\tilde{E}_j = [a'_j, b'_j, c'_j, d'_j]$. Fuzzy variance $S_j$ can be expressed by trapezoidal fuzzy numbers, i.e., $S_j = [a''_j, b''_j, c''_j, d''_j]$ where:

\[
\begin{align*}
    a_j &= \frac{1}{m} \sum_{i=1}^{m} a_{ij} \quad j = 1, 2, \ldots, n \\
    b_j &= \frac{1}{m} \sum_{i=1}^{m} b_{ij} \quad j = 1, 2, \ldots, n \\
    c_j &= \frac{1}{m} \sum_{i=1}^{m} c_{ij} \quad j = 1, 2, \ldots, n \\
    d_j &= \frac{1}{m} \sum_{i=1}^{m} d_{ij} \quad j = 1, 2, \ldots, n \\
    a_j &= \left[ \frac{1}{m} \sum_{i=1}^{m} (a_{ij} - a'_j)^2 \right]^{\frac{1}{2}} \quad j = 1, 2, \ldots, n \\
    b_j &= \left[ \frac{1}{m} \sum_{i=1}^{m} (b_{ij} - b'_j)^2 \right]^{\frac{1}{2}} \quad j = 1, 2, \ldots, n \\
    c_j &= \left[ \frac{1}{m} \sum_{i=1}^{m} (c_{ij} - c'_j)^2 \right]^{\frac{1}{2}} \quad j = 1, 2, \ldots, n \\
    d_j &= \left[ \frac{1}{m} \sum_{i=1}^{m} (d_{ij} - d'_j)^2 \right]^{\frac{1}{2}} \quad j = 1, 2, \ldots, n
\end{align*}
\]

From formula (4.11), the centrally normalized algorithm for the trapezoidal fuzzy number as:

\[
\tilde{\phi}_{ij} = \left[ \begin{array}{cccc}
    a_{ij} - a'_{ij} & b_{ij} - b'_{ij} & c_{ij} - c'_{ij} & d_{ij} - d'_{ij} \\
    a''_{ij} & b''_{ij} & c''_{ij} & d''_{ij} \\
\end{array} \right] \\
= \left[ \begin{array}{cccc}
    \alpha_{ij} & \beta_{ij} & \gamma_{ij} & \delta_{ij} \\
\end{array} \right] 
\]

where $\alpha_{ij} = \frac{a_{ij} - a'_j}{d'_j}$, $\beta_{ij} = \frac{b_{ij} - b'_j}{c'_j}$, $\gamma_{ij} = \frac{c_{ij} - c'_j}{b'_j}$, $\delta_{ij} = \frac{d_{ij} - d'_j}{a''_j}$
(2) Positive migration for centrally normalized matrix. For applications in actual decision-making, the weight should, in general, be positive so that the decision could be easily understood. Based on Frobenius theory in linear algebra, we know that the eigenvector related to the maximum eigen value of matrix $H$ should be positive when matrix $H$ is positive. By this we mean that all of the elements of $H$ are greater than 0.

If the n-dimensional fuzzy attribute evaluation space is established directly from $\tilde{\phi}_{ij}$ by referring to covariance definition, there should exist both zero and negative fuzzy numbers in such a space. Therefore, it is necessary to migrate the co-ordinate origin for each attribute so that all of elements in n-dimensional evaluation space are positive. Positive migration of the co-ordinate origin for each attribute is calculated and placed to eliminate the negative fuzzy numbers and small positive number $\epsilon$ is used to eliminate zero for the fuzzy centrally normalized matrix.

Set $\tau_j = \min_{i=1,2,\ldots,m} \alpha_{ij}, j = 1,2,\ldots,n$, then we can obtain $\tilde{r}_{ij}$ as:

$$
\begin{bmatrix}
\tilde{r}_{ij} = \tilde{\phi}_{ij} - \alpha_{ij} - \beta_{ij} - \gamma_{ij} - \delta_{ij} \\
\tilde{r}_{ij} = [\alpha_{ij} - \tau_i + \epsilon, \beta_{ij} - \tau_j + \epsilon, \gamma_{ij} - \tau_i + \epsilon, \delta_{ij} - \tau_j + \epsilon]
\end{bmatrix}
$$

If $-\tau_j \leq 0 i=1,2,\ldots,m, j=1,2,\ldots,n$

The positive fuzzy centralization decision matrix $\tilde{R}_{mn}$ can be expressed as:

$$
\tilde{R}_{mn} =
\begin{bmatrix}
\tilde{r}_{11} & \tilde{r}_{12} & \ldots & \tilde{r}_{1n} \\
\tilde{r}_{21} & \tilde{r}_{22} & \ldots & \tilde{r}_{2n} \\
\ldots & \ldots & \ldots & \ldots \\
\tilde{r}_{m1} & \tilde{r}_{m2} & \ldots & \tilde{r}_{mn}
\end{bmatrix}
$$

(3) Construct n-dimensional fuzzy attribute evaluation matrix. Referring to covariance matrix definition, the n-dimensional fuzzy attribute evaluation space can be established based on formula (4.12).
For positive fuzzy centralization decision matrix $\tilde{R}_{mn}$, let $\tilde{C}{\mathbf{\tilde{O}}R} = \tilde{R}^T \tilde{R}$, then $\tilde{C}{\mathbf{\tilde{O}}R}$ is fuzzy symmetry matrix. From formula (4.12), $\tilde{C}{\mathbf{\tilde{O}}R}$ can be expressed as:

$$\tilde{C}{\mathbf{\tilde{O}}R} = \tilde{R}^T \tilde{R} = \tilde{R}_{mn} = \begin{bmatrix}
\tilde{r}_{11} & \tilde{r}_{12} & \cdots & \tilde{r}_{1n} \\
\tilde{r}_{21} & \tilde{r}_{22} & \cdots & \tilde{r}_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
\tilde{r}_{m1} & \tilde{r}_{m2} & \cdots & \tilde{r}_{mn}
\end{bmatrix}^T \begin{bmatrix}
\tilde{r}_{11} & \tilde{r}_{12} & \cdots & \tilde{r}_{1n} \\
\tilde{r}_{21} & \tilde{r}_{22} & \cdots & \tilde{r}_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
\tilde{r}_{m1} & \tilde{r}_{m2} & \cdots & \tilde{r}_{mn}
\end{bmatrix}$$

$$= \begin{bmatrix}
\sum_{i=1}^{m} \tilde{r}_{kl} \tilde{r}_{k1} & \sum_{i=1}^{m} \tilde{r}_{kl} \tilde{r}_{k2} & \cdots & \sum_{i=1}^{m} \tilde{r}_{kl} \tilde{r}_{kn} \\
\sum_{i=1}^{m} \tilde{r}_{k2} \tilde{r}_{k1} & \sum_{i=1}^{m} \tilde{r}_{k2} \tilde{r}_{k2} & \cdots & \sum_{i=1}^{m} \tilde{r}_{k2} \tilde{r}_{kn} \\
\vdots & \vdots & \ddots & \vdots \\
\sum_{i=1}^{m} \tilde{r}_{kn} \tilde{r}_{k1} & \sum_{i=1}^{m} \tilde{r}_{kn} \tilde{r}_{k2} & \cdots & \sum_{i=1}^{m} \tilde{r}_{kn} \tilde{r}_{kn}
\end{bmatrix}$$

(4.13)

For the positive fuzzy numbers $\tilde{A}$ and $\tilde{B}$, from formula (4.9), following formula (4.14) can be obtained:

$$\tilde{A} + \tilde{B} = \tilde{B} + \tilde{A}$$

$$\tilde{A} \tilde{B} = \tilde{B} \tilde{A}$$

(4.14)

$\therefore \tilde{r}_j$ is positive fuzzy number, $\sum_{i=1}^{n} \tilde{r}_{kl} \tilde{r}_{k1}$ is positive fuzzy number, where $i=1,2,\ldots,m, j=1,2,\ldots,n$.

Based on formula (4.14), $\sum_{i=1}^{n} \tilde{r}_{kl} \tilde{r}_{kj} = \sum_{i=1}^{n} \tilde{r}_{kj} \tilde{r}_{ki}$ where $i=1,2,\ldots,m, j=1,2,\ldots,n$.

so $\tilde{C}{\mathbf{\tilde{O}}R}$ is positive fuzzy symmetry matrix. n-dimensional fuzzy attribute $\bigl[ C_j, (j=1,2,\ldots,n) \bigr]$ evaluation matrix would be constructed by the above $\tilde{C}{\mathbf{\tilde{O}}R}$. We can ensure that all of the weights of the eigenvector related to the maximum eigen value of n-dimensional positive fuzzy symmetry matrix could be positive by referring to Frobenius theory.
Step 4: Eigenvector of m-dimensional fuzzy attribute evaluation space. The eigenvector of $\text{COR}$ is not calculated directly from a fuzzy eigenvector equation but is approximated. Approximated eigenvector of m-dimensional fuzzy evaluation space $\text{COR}$ can be calculated by product and root method or geometry average method. This eigenvector could be adopted as a disperse projection factor of such n-dimensional fuzzy attribute evaluation matrix in the proposed method. Therefore, importance of attribute-related alternatives could be ordered objectively based on this eigenvector.

**Definition 4.8.** Fuzzy eigenvector of fuzzy symmetry matrix with n rank $\text{COR}$ can be obtained by product and root method (geometry average method) as

$$
\tilde{W}_i = \left( \frac{\alpha_i}{\delta}, \frac{\beta_i}{\gamma}, \frac{\gamma_i}{\beta}, \frac{\delta_i}{\alpha} \right) \text{ for } i = 1, 2, ..., m
$$

(4.14)

Where

$$
\alpha_i = \left( \frac{\prod_{j=1}^{n} \alpha_{ij}}{\prod_{k=1}^{m} \alpha_{ik}} \right)^{\frac{1}{n}} \text{ for } i = 1, 2, ..., m \quad \alpha = \sum_{i=1}^{m} \alpha_k
$$

$$
\beta_i = \left( \frac{\prod_{j=1}^{n} \beta_{ij}}{\prod_{k=1}^{m} \beta_{ik}} \right)^{\frac{1}{n}} \text{ for } i = 1, 2, ..., m \quad \beta = \sum_{i=1}^{m} \beta_k
$$

$$
\gamma_i = \left( \frac{\prod_{j=1}^{n} \gamma_{ij}}{\prod_{k=1}^{m} \gamma_{ik}} \right)^{\frac{1}{n}} \text{ for } i = 1, 2, ..., m \quad \gamma = \sum_{i=1}^{m} \gamma_k
$$

$$
\delta_i = \left( \frac{\prod_{j=1}^{n} \delta_{ij}}{\prod_{k=1}^{m} \delta_{ik}} \right)^{\frac{1}{n}} \text{ for } i = 1, 2, ..., m \quad \delta = \sum_{i=1}^{m} \delta_k
$$

Eigenvector $\tilde{W}_i$ of m-dimensional fuzzy attribute evaluation space can be calculated by Definition 4.8. Such a $\tilde{W}_i$ has not embodied the relative importance of attribute $C_i$ but it does represent the maximum disperse
degree of project factor for \( \{ \tilde{h}_{ij} \} \) overall. So this \( \tilde{W}_i \) can be selected as the attribute’s weight for improved fuzzy AHP algorithm.

Step 5: Comprehensive weights of alternatives for all attributes. Comprehensive weights of alternatives for all attributes can be derived by projection n vectors to such one-dimensional space. The decision judgment value \( \tilde{W}(A) \) can be calculated by using \( \tilde{W}_i \) and formula (4.10):

\[
\tilde{W}(A) = \tilde{H}(\cdot)\tilde{W}^T = \begin{bmatrix}
\tilde{h}_{11} & \tilde{h}_{12} & \ldots & \tilde{h}_{1n} \\
\tilde{h}_{21} & \tilde{h}_{22} & \ldots & \tilde{h}_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
\tilde{h}_{m1} & \tilde{h}_{m2} & \ldots & \tilde{h}_{mn}
\end{bmatrix}
\begin{bmatrix}
\tilde{W}_1 \\
\tilde{W}_2 \\
\vdots \\
\tilde{W}_n
\end{bmatrix}
\]

\[
= \begin{bmatrix}
(a_{11}b_{11}c_{11},d_{11}) & (a_{12}b_{12}c_{12},d_{12}) & \ldots & (a_{1n}b_{1n}c_{1n},d_{1n}) \\
(a_{21}b_{21}c_{21},d_{21}) & (a_{22}b_{22}c_{22},d_{22}) & \ldots & (a_{2n}b_{2n}c_{2n},d_{2n}) \\
\vdots & \vdots & \ddots & \vdots \\
(a_{m1}b_{m1}c_{m1},d_{m1}) & (a_{m2}b_{m2}c_{m2},d_{m2}) & \ldots & (a_{mn}b_{mn}c_{mn},d_{mn})
\end{bmatrix}
\begin{bmatrix}
\tilde{W}_1 \\
\tilde{W}_2 \\
\vdots \\
\tilde{W}_n
\end{bmatrix}
\]

(4.15)

Step 6: Comprehensive ranking. Definition 4.5 can be used for comprehensive weights of alternatives for all attribute ranking. On the other hand, fuzzy variant ranking methods provided in literature [11, 38, 48] can also be adopted for such comprehensive ranking.

4.3. Numerical Example

The theoretical approach outlined in Section 4.2 has been applied to the material selection for Wind turbine blade. The various alternatives and attributes related to material of wind turbine blade are tabulated below:
Table 4.1

<table>
<thead>
<tr>
<th>Alternative</th>
<th>Stiffness (GPA)</th>
<th>tensile strength (g/cm²)</th>
<th>density (g/cm³)</th>
<th>elongation at break (%)</th>
<th>Max temp (Mpa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td>30</td>
<td>190</td>
<td>7.5</td>
<td>15</td>
<td>550</td>
</tr>
<tr>
<td>Aluminium</td>
<td>10</td>
<td>90</td>
<td>2.7</td>
<td>12</td>
<td>400</td>
</tr>
<tr>
<td>Glass-E</td>
<td>73</td>
<td>3500</td>
<td>2.54</td>
<td>3</td>
<td>350</td>
</tr>
<tr>
<td>Carbon</td>
<td>350</td>
<td>4000</td>
<td>1.75</td>
<td>1.8</td>
<td>500</td>
</tr>
<tr>
<td>Aramid</td>
<td>120</td>
<td>3600</td>
<td>1.45</td>
<td>11</td>
<td>250</td>
</tr>
</tbody>
</table>

Case-I: The grey related analysis

We shall analyze this example with the method of grey related analysis to multiple attribute decision making problems with interval numbers. Assume a multiple attribute decision making problem for selection of materials related to the wind turbine blades and are tabulated as the interval number decision matrix A contains decision maker estimates of alternative performances on different scales as follows:

Table 4.2. Transform "contrary index" into positive index

<table>
<thead>
<tr>
<th>Properties</th>
<th>Stiffness (GPA)</th>
<th>Tensile strength (Mpa)</th>
<th>Density (g/cm³)</th>
<th>Elongation at break (%)</th>
<th>Max temp</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td>[25,35]</td>
<td>[180,200]</td>
<td>[6, 9]</td>
<td>[10, 20]</td>
<td>[530, 580]</td>
</tr>
<tr>
<td>Aluminium</td>
<td>[5,15]</td>
<td>[80,100]</td>
<td>[1.2,4.2]</td>
<td>[10, 15]</td>
<td>[380, 420]</td>
</tr>
<tr>
<td>Glass-E</td>
<td>[67,78]</td>
<td>[3400,3600]</td>
<td>[1, 4]</td>
<td>[1, 5]</td>
<td>[330, 380]</td>
</tr>
<tr>
<td>Carbon</td>
<td>[345,355]</td>
<td>[3900,4100]</td>
<td>[0.5, 3]</td>
<td>[1, 2]</td>
<td>[470, 530]</td>
</tr>
<tr>
<td>Aramid</td>
<td>[115,125]</td>
<td>[3500,3700]</td>
<td>[0.5,2]</td>
<td>[9, 12]</td>
<td>[220, 280]</td>
</tr>
</tbody>
</table>
The weights $w_1, w_2, w_3, w_4, w_5$ of attributes $G_1, G_2, G_3, G_4, G_5$ are uncertain, but the experts can specify the following weight ranges:

$w_1 \in [0.10, 0.10], w_2 \in [0.20, 0.20], w_3 \in [0.20, 0.20], w_4 \in [0.30, 0.30], w_5 \in [0.40, 0.40].$

Without loss of generality, we suppose that all the index values are positive.

(1) Standardize the interval number decision matrix $A$. Let $A_1, A_2, A_3, A_4, A_5$ denote the close interval column vector of index interval number decision matrix $A$, respectively, then

$$||A_1|| = 355, ||A_2|| = 4100, ||A_3|| = 9, ||A_4|| = 20, ||A_5|| = 580.$$

Standardizing the interval number decision matrix converts the initial divergent measures to a common 0-1 scale. Here, we obtain matrix $R$ as follows:

$$R =\begin{bmatrix}
[0.0704, 0.0986] & [0.0439, 0.0487] & [0.6666, 1.0000] & [0.5000, 1.0000] & [0.9138, 1.0000] \\
[0.0141, 0.0423] & [0.0195, 0.0244] & [0.1333, 0.4666] & [0.5000, 0.7500] & [0.6552, 0.7241] \\
[0.1887, 0.2197] & [0.8293, 0.8781] & [0.1111, 0.4444] & [0.0500, 0.2500] & [0.5689, 0.6552] \\
[0.9718, 1.0000] & [0.9512, 1.0000] & [0.0555, 0.3333] & [0.0500, 0.1000] & [0.8103, 0.9138] \\
[0.3239, 0.3521] & [0.8536, 0.9024] & [0.0555, 0.2222] & [0.4500, 0.6000] & [0.3793, 0.4827]
\end{bmatrix}$$

(2) Calculate the interval number weighted decision matrix $C$ by multiplying the weight intervals by matrix $R$.

$$C = \begin{bmatrix}
[0.0070, 0.0098] & [0.0088, 0.0097] & [0.1333, 0.2000] & [0.1500, 0.3000] & [0.3650, 0.4000] \\
[0.0014, 0.0042] & [0.0039, 0.0048] & [0.0266, 0.0933] & [0.1500, 0.2250] & [0.2621, 0.2896] \\
[0.0188, 0.0219] & [0.1658, 0.1756] & [0.0222, 0.0888] & [0.0150, 0.0750] & [0.2276, 0.2621] \\
[0.0972, 0.1000] & [0.1902, 0.2000] & [0.0111, 0.0666] & [0.0150, 0.0300] & [0.3241, 0.3655] \\
[0.0324, 0.0352] & [0.1707, 0.1805] & [0.0111, 0.0444] & [0.1350, 0.1800] & [0.1517, 0.1931]
\end{bmatrix}$$
(3) Determine the reference number sequence $U_0$.

$U_0 = (\{0.972,0.1000\}, \{0.1902,0.2000\}, \{0.1333,0.2000\}, \{0.1500,0.2250\}, \{0.3655,0.40\})$

(4) Calculate the connection between the sequence composed of weighted interval number standardizing index value of every alternative and reference number sequence.

<table>
<thead>
<tr>
<th></th>
<th>$G_1$</th>
<th>$G_2$</th>
<th>$G_3$</th>
<th>$G_4$</th>
<th>$G_5$</th>
<th>$\min \Delta_i(k)$</th>
<th>$\max \Delta_i(k)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta_1(k)$</td>
<td>0.1804</td>
<td>0.3717</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.3717</td>
</tr>
<tr>
<td>$\Delta_2(k)$</td>
<td>0.1926</td>
<td>0.3815</td>
<td>0.2174</td>
<td>0.075</td>
<td>0.2138</td>
<td>0.075</td>
<td>0.3815</td>
</tr>
<tr>
<td>$\Delta_3(k)$</td>
<td>0.1565</td>
<td>0.0488</td>
<td>0.2231</td>
<td>0.36</td>
<td>0.2758</td>
<td>0</td>
<td>0.36</td>
</tr>
<tr>
<td>$\Delta_4(k)$</td>
<td>0</td>
<td>0</td>
<td>0.2556</td>
<td>0.405</td>
<td>0.0759</td>
<td>0</td>
<td>0.405</td>
</tr>
<tr>
<td>$\Delta_5(k)$</td>
<td>0.1296</td>
<td>0.039</td>
<td>0.2778</td>
<td>0.135</td>
<td>0.4207</td>
<td>0</td>
<td>0.4207</td>
</tr>
</tbody>
</table>

Let $\Delta_i(k) = \left[u_{ik}^0, u_{k}^0 \right] - \left[c_{ik}, c_{ik}^+ \right]$. The connection coefficient $\xi_i(k)$ (a distance function) is then calculated by the formula as follows,

$$\xi_i(k) = \frac{\min \min_{i,k} \Delta_i(k) + \rho \max \max_{i,k} \Delta_i(k)}{\Delta_i(k) + \rho \max \max_{i,k} \Delta_i(k)}$$

In the example, $\rho = 0.5$. When $\xi_i(k)$ is determined, $\min \min_{i,k} \Delta_i(k)$ and $\max \max_{i,k} \Delta_i(k)$ will be calculated as follows.

From the above table 4.3, we know that $\min \min_{i,k} \Delta_i(k) = 0$, $\max \max_{i,k} \Delta_i(k) = 0.4207$. This is used in the connection coefficient formula to identify distances (larger values mean greater distance).
\(\xi_1 = (0.5383, 0.3614, 1.0000, 1.0000, 1.0000)\)
\(\xi_2 = (0.5233, 0.3554, 0.4918, 0.7372, 0.4959)\)
\(\xi_3 = (0.5734, 0.8117, 0.4853, 0.3688, 0.4327)\)
\(\xi_4 = (1.0000, 1.0000, 0.4514, 0.3418, 0.7348)\)
\(\xi_5 = (0.6187, 0.8436, 0.4309, 0.6091, 0.3333)\)

By these results, we know that the connection between every alternative and reference number sequence is, respectively
\(r_1 = 0.7799, r_2 = 0.5187, r_3 = 0.5344, r_4 = 0.7056, r_5 = 0.5671.\)

Ranking feasible alternatives from largest to smallest \(r_i\) the rank order of feasible alternatives is \(X_i, X_4, X_5, X_3, X_2.\)

**Case-II: The Improved AHP Methodology**

Step 1: Build up hierarchy structure. \(A_1, A_2, A_3, A_4, A_5\) are five selectable material alternatives; they form the alternative hierarchy. Stiffness (GPA) \((C_1)\), tensile strength (Mpa) \((C_2)\), density ((g/cm²)) \((C_3)\), elongation at break (%) \((C_4)\), maximum temperature \((C_5)\) are five attributes; they form the attribute hierarchy.

Step 2: Construct decision matrix. Normalized decision matrix \(\tilde{H}\) is:

\[
\begin{pmatrix}
(0.08, 0.08, 0.08, 0.08) & (0.05, 0.05, 0.05, 0.05) & (1, 1, 1, 1) & (1, 1, 1, 1) & (1, 1, 1, 1) \\
(0.03, 0.03, 0.03, 0.03) & (0.02, 0.02, 0.02, 0.02) & (0.36, 0.36, 0.36, 0.36) & (0.08, 0.08, 0.08) & (0.73, 0.73, 0.73, 0.73) \\
(0.21, 0.21, 0.21, 0.21) & (0.87, 0.87, 0.87, 0.87) & (0.34, 0.34, 0.34, 0.34) & (0.02, 0.02, 0.02, 0.02) & (0.64, 0.64, 0.64, 0.64) \\
(1, 1, 1, 1) & (1, 1, 1, 1) & (0.23, 0.23, 0.23, 0.23) & (0.12, 0.12, 0.12, 0.12) & (0.09, 0.09, 0.09, 0.09) \\
(0.34, 0.34, 0.34, 0.34) & (0.09, 0.09, 0.09, 0.09) & (0.19, 0.19, 0.19, 0.19) & (0.73, 0.73, 0.73, 0.73) & (0.45, 0.45, 0.45, 0.45)
\end{pmatrix}
\]
Step 3: Construct 5-dimensional fuzzy attribute evaluation space.

Table 4.5. Construct centrally normalized matrix $\tilde{\phi}_{ij}$

$$\tilde{\phi}_{ij} = \begin{bmatrix}
(-0.7182, -0.7182, -0.7182, -0.7182) & (-1.1843, -1.1843, -1.1843, -1.1843) & (1.9518, 1.9518, 1.9518, 1.9518) \\
(-0.8606, -0.8606, -0.8606, -0.8606) & (-1.2528, -1.2528, -1.2528, -1.2528) & (-0.2168, -0.2168, -0.2168, -0.2168) \\
(-0.3476, -0.3476, -0.3476, -0.3476) & (0.6904, 0.6904, 0.6904, 0.6904) & (-0.2846, -0.2846, -0.2846, -0.2846) \\
(1.9036, 1.9036, 1.9036, 1.9036) & (0.9876, 0.9876, 0.9876, 0.9876) & (-0.6235, -0.6235, -0.6235, -0.6235) \\
(0.0227, 0.0227, 0.0227, 0.0227) & (0.7590, 0.7590, 0.7590, 0.7590) & (-0.7929, -0.7929, -0.7929, -0.7929) \\
(1.2385, 1.2385, 1.2385, 1.2385) & (1.3012, 1.3012, 1.3012, 1.3012) & \\
(0.6624, 0.6624, 0.6624, 0.6624) & (-0.0819, -0.0819, -0.0819, -0.0819) & \\
(-1.0656, -1.0656, -1.0656, -1.0656) & (-0.5430, -0.5430, -0.5430, -0.5430) & \\
(-1.2961, -1.2961, -1.2961, -1.2961) & (0.8402, 0.8402, 0.8402, 0.8402) & \\
(0.4608, 0.4608, 0.4608, 0.4608) & (-1.1564, -1.1564, -1.1564, -1.1564) &
\end{bmatrix}$$

Table 4.6. Positive migration for centrally normalized matrix ($\varepsilon = 0.01$)

$$\tilde{R} = \begin{bmatrix}
(0.1524, 0.1524, 0.1524, 0.1524) & (0.0785, 0.0785, 0.0785, 0.0785) & (2.7547, 2.7547, 2.7547, 2.7547) \\
(0.01, 0.01, 0.01, 0.01) & (0.01, 0.01, 0.01, 0.01) & (0.5861, 0.5861, 0.5861, 0.5861) \\
(0.523, 0.523, 0.523, 0.523) & (0.8256, 0.8256, 0.8256, 0.8256) & (0.5183, 0.5183, 0.5183, 0.5183) \\
(1.053, 1.053, 1.053, 1.053) & (1.1228, 1.1228, 1.1228, 1.1228) & (0.1794, 0.1794, 0.1794, 0.1794) \\
(0.8933, 0.8933, 0.8933, 0.8933) & (0.8943, 0.8943, 0.8943, 0.8943) & (0.01, 0.01, 0.01, 0.01) \\
(2.5446, 2.5446, 2.5446, 2.5446) & (1.3012, 1.3012, 1.3012, 1.3012) & \\
(1.9685, 1.9685, 1.9685, 1.9685) & (1.4445, 1.4445, 1.4445, 1.4445) & \\
(0.2405, 0.2405, 0.2405, 0.2405) & (0.9834, 0.9834, 0.9834, 0.9834) & \\
(0.01, 0.01, 0.01, 0.01) & (2.3666, 2.3666, 2.3666, 2.3666) & \\
(1.7669, 1.7669, 1.7669, 1.7669) & (0.01, 0.01, 0.01, 0.01) &
\end{bmatrix}$$

Table 4.7. Construct five-dimensional fuzzy evaluation matrix

$$\text{COR} = \begin{bmatrix}
(2.2636, 2.2636, 2.2636, 2.2636) & (2.4250, 2.4250, 2.4250, 2.4250) & (0.8946, 0.8946, 0.8946, 0.8946) \\
(2.4250, 2.4250, 2.4250, 2.4250) & (2.7489, 2.7489, 2.7489, 2.7489) & (0.8604, 0.8604, 0.8604, 0.8604) \\
(0.8946, 0.8946, 0.8946, 0.8946) & (0.8604, 0.8604, 0.8604, 0.8604) & (8.2328, 8.2328, 8.2328, 8.2328) \\
(2.1222, 2.1222, 2.1222, 2.1222) & (2.0093, 2.0093, 2.0093, 2.0093) & (8.3075, 8.3075, 8.3075, 8.3075) \\
(3.4606, 3.4606, 3.4606, 3.4606) & (3.7132, 3.7132, 3.7132, 3.7132) & (9.5701, 0.01, 0.01, 0.01) \\
(2.1222, 2.1222, 2.1222, 2.1222) & (3.4606, 3.4606, 3.4606, 3.4606) & \\
(2.0093, 2.0093, 2.0093) & (3.7132, 3.7132, 3.7132, 3.7132) & \\
(8.3075, 8.3075, 8.3075, 8.3075) & (9.5701, 0.9834, 0.9834, 0.9834) & \\
(10.3165, 10.3165, 10.3165, 10.3165) & (16.6498, 16.6498, 16.6498, 16.6498) &
\end{bmatrix}$$
Step 4: Eigenvector of five-dimensional fuzzy attribute evaluation space. Approximated eigenvector $\tilde{W}_i$ of five-dimensional fuzzy evaluation space can be calculated based on product and root method.

$$\tilde{W}_i = \begin{bmatrix} (0.0998, 0.0998, 0.0998, 0.0998) \\ (0.1037, 0.1037, 0.1037, 0.1037) \\ (0.1698, 0.1698, 0.1698, 0.1698) \\ (0.2682, 0.2682, 0.2682, 0.2682) \\ (0.3585, 0.3585, 0.3585, 0.3585) \end{bmatrix}$$

Step 5: Comprehensive weights $\tilde{W}_i$ of alternative for all attributes.

$$\tilde{W}(A) = \begin{bmatrix} (0.8096, 0.8096, 0.8096, 0.8096) \\ (0.5425, 0.5425, 0.5425, 0.5425) \\ (0.4519, 0.4519, 0.4519, 0.4519) \\ (0.6009, 0.6009, 0.6009, 0.6009) \\ (0.5166, 0.5166, 0.5166, 0.5166) \end{bmatrix}, \quad W(A) = \begin{bmatrix} 0.8096 \\ 0.5425 \\ 0.4519 \\ 0.6009 \\ 0.5166 \end{bmatrix}$$

Step 6: It is obvious that the ranking result is: $A_1 > A_4 > A_2 > A_5 > A_3$.

4.4. Conclusions

In this Chapter, the method of grey related analysis for multiple attribute decision making problem with interval number is given with fuzzy input parameters. It applies the traditional method of grey related analysis. It is apparent that the proposed improved fuzzy AHP algorithm based on fuzzy eigenvector of fuzzy attribute evaluation space is more efficient than others. It has good objectivity and resolution.

The method reflects decision maker or group uncertainty concerning multiple criteria decision input parameters. The method presented here is simple, practical, and requires less rigid input from decision makers. Weight inputs are entered as fuzzy interval numbers. Alternative performance scores also can be entered as general interval.
fuzzy numbers. Both weight and performance scores are standardized, and composite utility value ranges obtained. When the above two methods are compared with each other, grey related analysis is more simple and less tedious and solved in less time duration.

In the final selection of material for the wind turbine blade by these two methods in the above order of the ideal solutions, best alternative found is Steel. But Alloy Steel has lower compressive strength, poor machineability, poor environmental stability and poor temperature strength. Therefore, second best alternative i.e. Carbon fiber material $A_4$ is selected as the connection co-efficient indicate that the Carbon is better than the other four alternatives.