CHAPTER V
MULTIPLE ATTRIBUTE DECISION MAKING
METHOD USED IN INVESTMENT COMPANY:
A FUZZY APPROACH

5.1. Introduction

In a MADM problem, the DM’s preference information is often used to rank alternatives. However, the DM’s judgment varies in form and depth. A DM may not indicate his/her preference at all or may represent his/her preference in the form of attributes or alternatives. Li [46] develops two approaches to solve fuzzy multi-attribute single-person decision making problems and fuzzy multi-attribute multi-person decision making problems respectively. In particular, triangular fuzzy numbers (fig.5.1) are used in their fuzzy linear programming model to assess alternatives with respect to qualitative attributes.

Grey theory is a new technique for performing prediction, relational analysis and decision-making in many areas. The application of grey relations methodology for defining the utility of an alternative is proposed as a method of multiple criteria COMplex PROportional ASsessment of alternatives with Grey relations (COPRAS-G). Chen and Hwang [15] proposed a new number ranking method, relative distance method is concept of Hamming distance that suppose a maximum fuzzy number and a minimum fuzzy number in term of obtaining their relative distance by calculating maximum and minimum fuzzy number from different alternative.

Finally, for checking the feasibility of proposed study we compared these two methods with a numerical example.
5.2. Model of problem

The aim of this study is to propose a technique for the choice and selection of investment product. The purpose is to be achieved by using various indicators of effectiveness, which have different dimensions, different significances as well as different directions of optimization.

The solving of each multiple attribute problem begins with constructing of decision making matrix.

\[
\begin{array}{c|ccc}
\text{Attributes} & C_1 & C_2 & \cdots & C_n \\
\hline
A_1 & x_{11} & x_{12} & \cdots & x_{1n} \\
A_2 & x_{21} & x_{22} & \cdots & x_{2n} \\
\cdots & \cdots & \cdots & \cdots & \cdots \\
A_m & x_{m1} & x_{m2} & \cdots & x_{mn} \\
\end{array}
\]

Where \( x_{ij} \) represents value of \( j^{th} \) attribute against \( i^{th} \) alternative.

5.3. Ranking of the alternatives applying COPRAS method

5.3.1. Determination of the attributes weights

In order to select the best alternative, it is necessary, to have formed the decision matrix, to perform the multiple attributes analysis. MCDM refers to making preference decisions on the alternatives in terms of multiple attributes. Typically, each alternative is evaluated on the established set/system of attributes.

To determine the weights of the attributes, the expert’s judgment method is applied Kendall [37] which has been successfully used in research by many authors. The importance of indicators was established according to the rating methods of these experts and also demonstrated the priorities of the user or owner. The significance of the attributes obtained by this method is presented in Table 5.1.
5.3.2. A method of multiple criteria complex proportional assessment with values determined in intervals – COPRAS-G

In many decisions the consequences of the alternative courses of action cannot be predicted with a certainty. The idea to COPRAS-G method comes from real conditions of decision-making and from applications of the Grey systems theory. This theory was originated by Deng [21] study of the relation degree among various attributes in an MCDM problem was studied. Deng [22] presented grey decision-making systems. It is useful mathematically when dealing with a system with limited information. Haq and Kannan [32] developed a hybrid normalized multi-criteria decision-making model for evaluating and selecting the vendor using Analytical Hierarchy Process and Fuzzy Analytical Hierarchy Process and an integrated approach of Grey Relational Analysis to a Supply Chain model. Lin et al. [47] presented an illustrative example of subcontractor selection by applying grey TOPSIS method.

The COPRAS method with attribute values is expressed in the form of interval. This method includes the following steps:

Step 1: Selection of the available set of the most important attributes, which describes alternatives;

Step 2: Preparing the decision-making matrix X:

\[
X = \begin{bmatrix}
[a_{11};b_{11}] & [a_{12};b_{12}] & \ldots & [a_{1n};b_{1n}] \\
[a_{21};b_{21}] & [a_{22};b_{22}] & \ldots & [a_{2n};b_{2n}] \\
\vdots & \vdots & \ddots & \vdots \\
[a_{m1};b_{m1}] & [a_{m2};b_{m2}] & \ldots & [a_{mn};b_{mn}]
\end{bmatrix} ; i = 1, m; j = 1, n, \tag{5.1}
\]

Where \( a_{ij} \) – lower limit, \( b_{ij} \) – upper limit.

Step 3: Determining weights of the attributes \( w_j \).
Step 4: Normalization of the decision-making matrix $\overline{X}$. The normalized values of this matrix ([54, 83]) are calculated as:

$$
\overline{a}_{ij} = \frac{a_{ij}}{\frac{1}{2} \left( \sum_{i=1}^{m} a_{ij} + \sum_{i=1}^{m} b_{ij} \right)} = \frac{2a_{ij}}{\sum_{i=1}^{m} a_{ij} + \sum_{i=1}^{m} b_{ij}} \quad i = 1, m; j = 1, n
$$

(5.2)

In formula (5.2) $a_{ij}$ is the lower values of the $j^{th}$ attribute in the $i^{th}$ alternative of a solution; $b_{ij}$ – the upper value of the $j^{th}$ attribute in the $i^{th}$ alternative of a solution; $m$ – the number of the alternatives compared with $n$ – the number of attributes. After this step we have normalized decision-making matrix:

$$
\overline{X} = \begin{bmatrix}
[a_{11}:b_{11}] & [a_{12}:b_{12}] & \cdots & [a_{1n}:b_{1n}] \\
[a_{21}:b_{21}] & [a_{22}:b_{22}] & \cdots & [a_{2n}:b_{2n}] \\
\vdots & \vdots & \ddots & \vdots \\
[a_{m1}:b_{n1}] & [a_{m2}:b_{m2}] & \cdots & [a_{mn}:b_{mn}]
\end{bmatrix}
$$

(5.3)

Step 5: Calculation of the weighted normalized decision matrix $\hat{X}$. The weighted normalized values $\hat{X}_{ij}$ are calculated as

$$
\hat{a}_{ij} = \overline{a}_{ij} \cdot w_j \\
\hat{b}_{ij} = \overline{b}_{ij} \cdot w_j
$$

(5.4)

In formula (5.4), $w_j$ is significance (weight) of the $j^{th}$ attribute. After this step we have weighted normalized decision-making matrix:
\[
\mathbf{X} = \begin{bmatrix}
[\hat{a}_{11};\hat{b}_{11}] & [\hat{a}_{12};\hat{b}_{12}] & \cdots & [\hat{a}_{1n};\hat{b}_{1n}] \\
[\hat{a}_{21};\hat{b}_{21}] & [\hat{a}_{22};\hat{b}_{22}] & \cdots & [\hat{a}_{2n};\hat{b}_{2n}] \\
\vdots & \vdots & \ddots & \vdots \\
[\hat{a}_{m1};\hat{b}_{m1}] & [\hat{a}_{m2};\hat{b}_{m2}] & \cdots & [\hat{a}_{mn};\hat{b}_{mn}] 
\end{bmatrix}. \quad (5.5)
\]

Step 6: Sum \( P_i \) of attribute values in which larger values are more preferable for each alternative:

\[
P_i = \frac{1}{2} \sum_{j=1}^{k} (\hat{a}_{ij} + \hat{b}_{ij}). \quad (5.6)
\]

In formula (5.6), \( k \) is number of attributes which must be maximized.

Step 7: Sum \( R_i \) of attribute values in which smaller values are more preferable for each alternative:

\[
R_i = \frac{1}{2} \sum_{j=k+1}^{n} (\hat{a}_{ij} + \hat{b}_{ij}); \quad j = k, n. \quad (5.7)
\]

In formula (5.7), \((k-n)\) is number of attributes which must be minimized.

Step 8: Determining the minimal value of \( R_i \):

\[
R_{\text{min}} = \min \{ R_i : i = i, m \}. \quad (5.8)
\]

Step 9: Calculation of the relative weight of each alternative \( Q_i \).

\[
Q_i = P_i + \frac{R_{\text{min}} \sum_{i=1}^{m} R_i}{R_i \sum_{i=1}^{m} \frac{R_{\text{min}}}{R_i}}, \quad (5.9)
\]
Formula (5.9) can be written as follows:

\[ Q_i = P_i + \frac{R_{\min} \sum_{i=1}^{m} R_i}{R_{\max} \sum_{i=1}^{m} \frac{1}{R_i}}, \quad (5.10) \]

Step 10: Determination of the optimality criterion \( K \):

\[ K = \max_i Q_i; i = 1, m. \quad (5.11) \]

Step 11: Determination of the project priority. The greater significance relative weight of alternative \( Q_i \), the higher is the priority (rank) of the project. The relative significance \( Q_i \) of project \( i \) indicates the satisfaction degree of the needs of the project participants. In case of \( Q_{\max} \), the satisfaction degree is the highest. The relative significance of other projects is less.

Step 12: Calculation of the utility degree of each alternative. The degree of project utility is determined by comparing the analyzed projects with the best one. The values of the utility degree are from 0 % to 100 % between the worst and the best alternatives. The utility degree \( N_i \) of each alternative \( i \) is calculated as

\[ N_i = \frac{Q_i}{Q_{\max}} \times 100\% \quad (5.12) \]

where \( Q_i \) and \( Q_{\max} \) are the significance of projects obtained from Eq.(5.10). The decision approach proposed in this section allows evaluating the direct and proportional dependence of the significance and
utility degree of alternatives in a system of attributes, weights and values of the attributes.

5.4. Fuzzy ranking using relative distance methodology

To obtain best alternative by using ranking fuzzy number we used the following steps:

Step 1: Define the evaluation matrix.

Step 2: To establish fuzzy evaluation matrix. For given a fuzzy value under the attribute of every alternative is converting into fuzzy vector. Then all vectors can be establishing fuzzy judgment matrix. It is described $\hat{A} = [\hat{a}_{ij}]$; $\hat{a} = (l, m, u)$ as fig. 5.1:

![Fig. 5.1 Triangular fuzzy number](image)

Step 3: To establish weight of every attribute by expert. According to environment factor, we give weight value of every attribute and represent fuzzy weight value by weight vector $\tilde{w}$.

Step 4: To develop weighted fuzzy matrix.

$$\tilde{R} = \hat{A} \otimes \tilde{w}^T = \begin{bmatrix} \hat{a}_{11} & \hat{a}_{12} & \cdots & \hat{a}_{1n} \\ \hat{a}_{21} & \hat{a}_{22} & \cdots & \hat{a}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{a}_{m1} & \hat{a}_{m2} & \cdots & \hat{a}_{mn} \end{bmatrix} \otimes \begin{bmatrix} \tilde{w}_1 \\ \tilde{w}_2 \\ \vdots \\ \tilde{w}_n \end{bmatrix}$$
\[
\begin{bmatrix}
\tilde{a}_{11} \otimes \tilde{w}_1 & \tilde{a}_{12} \otimes \tilde{w}_2 & \cdots & \tilde{a}_{1n} \otimes \tilde{w}_n \\
\tilde{a}_{21} \otimes \tilde{w}_1 & \tilde{a}_{22} \otimes \tilde{w}_2 & \cdots & \tilde{a}_{2n} \otimes \tilde{w}_n \\
\vdots & \vdots & \ddots & \vdots \\
\tilde{a}_{m1} \otimes \tilde{w}_1 & \tilde{a}_{m2} \otimes \tilde{w}_2 & \cdots & \tilde{a}_{mn} \otimes \tilde{w}_n
\end{bmatrix}
= 
\begin{bmatrix}
\tilde{r}_1 \\
\tilde{r}_2 \\
\vdots \\
\tilde{r}_n
\end{bmatrix}
(5.13)
\]

Step 5: Here we use relative distance ranking methodology by Hamming distance formula

\[
d(M,N) = \int_{-\infty}^{\infty} |\mu_M(x) - \mu_N(x)| dx \tag{5.14}
\]

Step 5.1.: Two fuzzy numbers as maximum and minimum fuzzy number \( \tilde{r}_{\text{max}} \) minimum fuzzy number \( \tilde{r}_{\text{min}} \), \([\beta_1, \beta_2]\) is an interval on the real number axis. When \( \alpha = 0 \), this interval can conclude every fuzzy number \( \tilde{A}_i \).

Maximum fuzzy number represented by triangular fuzzy number as: \( \tilde{r}_{\text{max}} \) and defined as:

\[
(\beta_1, \beta_1, \beta_2), G_{\mu_{\text{max}}}(x) = \frac{1}{\beta_2 - \beta_1} (1 - \beta_1) \tag{5.15}
\]

Minimum fuzzy number represented by triangular fuzzy number as: \( \tilde{r}_{\text{min}} \) and defined as:

\[
(\beta_1, \beta_1, \beta_2), G_{\mu_{\text{min}}}(x) = \frac{-1}{\beta_2 - \beta_1} (1 - \beta_2) \tag{5.16}
\]
Then for calculating fuzzy number $\tilde{A}_i$ from distance $d(\tilde{A}_i, \tilde{r}_{\text{max}})$ of maximum fuzzy number $\tilde{r}_{\text{max}}$ and $d(\tilde{A}_i, \tilde{r}_{\text{min}})$ of minimum fuzzy number $\tilde{r}_{\text{min}}$. Calculating relative distance as,

$$RD = \frac{d(\tilde{A}_i, \tilde{r}_{\text{min}})}{d(\tilde{A}_i, \tilde{r}_{\text{max}})}.$$  \hspace{1cm} (5.17)

There is no specific standard for defining any particular value between the intervals $[\beta_1, \beta_2]$; it will be confused. Here we used maximum and minimum fuzzy number as follows:

Fuzzy Max: 

$$\mu_{\text{max}}(x) = \begin{cases} x, & 0 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases} \quad (5.18)$$  

Fuzzy Min: 

$$\mu_{\text{min}}(x) = \begin{cases} 1 - x, & 0 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases} \quad (5.19)$$

This definition is used when information offered by fuzzy numbers. It is very easy to calculate and not confused with certain concrete method.

Step 6: Finally, by using ranking relative distance method selecting best alternative.
If $\bar{r}_i > \bar{r}_j$, , $i \neq j, i = 1, 2, ..., m; j = 1, 2, ..., n$ then $\bar{r}_i$ is the best alternative.

5.5. Numerical Example

Here we take an example of Investment Company who is planning to exploit a new type of product for better investment. There are five feasible alternatives $A_i (i = 1, 2, 3, 4, 5)$ and four attributes $C_j (j = 1, 2, 3, 4)$

- $C_1$: Investment amount ($10,000$)
- $C_2$: Expected net profit amount ($10,000$)
- $C_3$: Venture profit amount ($10,000$)
- $C_4$: Venture loss amount ($10,000$)

Case1: By COPRAS-G method

Among four attributes, $C_2$ and $C_3$ are of benefit type, $C_1$ and $C_4$ are of cost type. The fuzzy decision matrix

<table>
<thead>
<tr>
<th></th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>$C_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>[6; 6.5]</td>
<td>[6.1; 6.4]</td>
<td>[5.2; 5.8]</td>
<td>[0.5; 0.8]</td>
</tr>
<tr>
<td>$A_2$</td>
<td>[11.2; 11.7]</td>
<td>[7.4; 8.0]</td>
<td>[5.2; 5.8]</td>
<td>[1.7; 2.0]</td>
</tr>
<tr>
<td>$A_3$</td>
<td>[6.3; 7.0]</td>
<td>[5.2; 5.8]</td>
<td>[5.2; 5.8]</td>
<td>[1.4; 1.7]</td>
</tr>
<tr>
<td>$A_4$</td>
<td>[10.8; 11.5]</td>
<td>[6.3; 6.7]</td>
<td>[5.2; 5.8]</td>
<td>[1.5; 1.9]</td>
</tr>
<tr>
<td>$A_5$</td>
<td>[7.5; 7.8]</td>
<td>[4.8; 5.5]</td>
<td>[5.2; 5.8]</td>
<td>[0.9; 1.3]</td>
</tr>
</tbody>
</table>

$w = [\ (0.40) \ (0.30) \ (0.20) \ (0.10) \ ]$
Table 5.2: Weighted normalized decision making matrix according to a COPRAS-G method

$$
\begin{bmatrix}
C_1 & C_2 & C_3 & C_4 \\
A_1 & [0.056; 0.060] & [0.059; 0.062] & [0.041; 0.046] & [0.007; 0.012] \\
A_2 & [0.104; 0.108] & [0.071; 0.077] & [0.048; 0.051] & [0.025; 0.029] \\
X = A_3 & [0.058, 0.065] & [0.045; 0.049] & [0.035; 0.038] & [0.020; 0.025] \\
A_4 & [0.100; 0.106] & [0.061; 0.065] & [0.034; 0.038] & [0.022; 0.025] \\
A_5 & [0.069; 0.072] & [0.046; 0.053] & [0.032; 0.036] & [0.013; 0.019]
\end{bmatrix}
$$

Table 5.3: Decision results according to a COPRAS-G method

(\(R_i\) – Descending rank of alternatives. The smallest is the best)

<table>
<thead>
<tr>
<th>Alternative No.</th>
<th>Total sum maximizing normalizing indices (R_i)</th>
<th>Total sum minimizing normalizing indices (P_i)</th>
<th>Alternative’s significance</th>
<th>Alternative’s degree of efficiency (N_i)</th>
<th>Rank (R_i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.135</td>
<td>0.208</td>
<td>0.4834</td>
<td>100</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>0.266</td>
<td>0.247</td>
<td>0.3867</td>
<td>79.99</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>0.168</td>
<td>0.167</td>
<td>0.3883</td>
<td>80.33</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>0.255</td>
<td>0.198</td>
<td>0.3437</td>
<td>71.10</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>0.173</td>
<td>0.167</td>
<td>0.3823</td>
<td>79.08</td>
<td>4</td>
</tr>
</tbody>
</table>

According to the calculated results, alternative 1 is the best one (Table 5.3). The first alternative is also the best in terms of its utility degree that equals 100\%. The second alternative with utility degree 79.99\% has rank 3. The third alternative with utility degree 80.33\% has rank 2. The fourth alternative with the utility degree 71.10\% is the worst and has rank 5. The fifth alternative with utility degree 79.08 \% and has rank 4. Vector of optimality criterion values \(N_i\) is: \(N_i = [100; 79.99; 80.33; 71.10; 79.08]\)

According to the \(N_i\) alternatives rank as follows: 1 > 3 > 2 > 5 > 4.
Case 2: By Relative Distance method

The fuzzy evaluation matrix is

\[ \tilde{A} = \begin{bmatrix} 6 & \tilde{6} & 5 & \tilde{1} \\ 10 & \tilde{7} & 6 & \tilde{2} \\ 6 & \tilde{5} & 5 & \tilde{1} \\ 10 & \tilde{6} & 5 & \tilde{1} \\ \tilde{7} & \tilde{5} & 4 & \tilde{1} \end{bmatrix} \]

Weight vector is \( \tilde{w} = [\tilde{4} \ 3 \ 2 \ 1] \). Then the weighted fuzzy matrix by using formula (5.13) we get,

\[ \tilde{A} = \begin{bmatrix} 6 & \tilde{6} & 5 & \tilde{1} \\ 10 & \tilde{7} & 6 & \tilde{2} \\ 6 & \tilde{5} & 5 & \tilde{1} \\ 10 & \tilde{6} & 5 & \tilde{1} \\ \tilde{7} & \tilde{5} & 4 & \tilde{1} \end{bmatrix} \otimes \begin{bmatrix} 4 \\ 3 \\ 2 \\ 1 \end{bmatrix} \]

\( \tilde{r}_1 = (35, 47, 71), \tilde{r}_2 = (50, 75, 100), \tilde{r}_3 = (33, 50, 66), \tilde{r}_4 = (47, 69, 91), \tilde{r}_5 = (35, 52, 69) \)

For selecting best performance by relative distance method using (5.15)-(5.19), we maximize and minimize fuzzy values are given in table 5.4.

<table>
<thead>
<tr>
<th>Alternative</th>
<th>( \tilde{r}_{\text{max}} )</th>
<th>( \tilde{r}_{\text{min}} )</th>
<th>Relative distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.58</td>
<td>0.60</td>
<td>1.034</td>
</tr>
<tr>
<td>2</td>
<td>0.82</td>
<td>0.30</td>
<td>0.366</td>
</tr>
<tr>
<td>3</td>
<td>0.69</td>
<td>0.57</td>
<td>0.826</td>
</tr>
<tr>
<td>4</td>
<td>0.74</td>
<td>0.43</td>
<td>0.566</td>
</tr>
<tr>
<td>5</td>
<td>0.63</td>
<td>0.44</td>
<td>0.698</td>
</tr>
</tbody>
</table>

By applying relative distance method for ranking the alternatives preferences are: \( 1 > 3 > 5 > 4 > 2 \).
5.6. Conclusions

In multiple attribute modeling some attribute values, which deals with the future, must be expressed in the form of intervals. It can also be expressed in triangular form by relative distance method.

By comparing COPRAS-G method and relative distance method we get the best alternative i.e. 1. Hence the decision maker must choose the first alternative and his second priority will be 3\textsuperscript{rd} alternative. These methods can be applied to find the best optimal solution. It can be applied in many fields, such as economics, agriculture, geography, weather, earthquakes, science etc.

So COPRAS-G method is slightly easier to apply to find the solution among the selection of the alternatives to verify the feasibility and practicability of the approach. Also, by using ranking relative distance one can choose the best among the alternatives by presenting graphically.