Chapter 7

Conclusion

The main aim of this thesis is to study fuzzy automata under max-min, max-product and max-average compositions and to extend Myhill-Nerode theorem of finite automata to these compositions.

Chapter 1 gives preliminaries about fuzzy logic and fuzzy automata. Several results concerning max-min, max-product and max-average compositions have been proved in this chapter. When a result is not true for max-product and for max-average composition, same has been illustrated by example.

In chapter 2, we have introduced the concept of fuzzy regular languages and have shown that if L is a fuzzy regular language (under max-min composition), then every $\alpha - \text{cut}_\alpha (\alpha \in [0,1])$ is a regular language. This fact has enabled us to give a characterization of fuzzy regular languages.

In chapter 3, we have made an attempt to extend Myhill-Nerode theorem of finite automata to fuzzy automata (under max-min composition). With a fuzzy automaton M and $\alpha \in [0,1]$, we associate a non deterministic automaton $D_\alpha (M)$. It so happens that $L_\alpha = L (D_\alpha (M))$ so that we can apply Myhill-Nerode theorem of finite automaton. But unlike in the finite automata case, this has not helped us in reducing the number of states in the fuzzy automata. Hence we have provided an alternate method to reduce the number of states in a fuzzy automaton. We have provided examples to illustrate the techniques used. An algorithm also has been developed to reduce the number of states.

Chapter 4 deals with max-product composition. Unlike max-min composition, in this case $L_\alpha$ is only subset of $L (D_\alpha (M))$. This lend to challenges. Nevertheless, we could prove the analogue of Myhill-Nerode theorem. We applied Myhill-Nerode theorem of finite automata to $L (D_\alpha (M))$ and considered its restriction.
In chapter 5, we have considered max-average composition. Examples show that in this case, even \( L_a \) is not contained in \( L \left( D_a \left( M \right) \right) \) as it happens in max-product composition. This lends to splitting of \( L_a \) and we could extend Myhill-Nerode theorem to this case also.

Chapter 6 considers some of the applications of a given fuzzy automata. In the first application, we have given a procedure to convert a given finite automaton to fuzzy automaton. Fuzzy automaton allows us to define individual levels of similarity for pairs of symbols or sequence of symbols and hence can be used as a base for providing a better string search operation. Fuzzy automata are more useful in performing comparison operations between two strings. Using finite automata, it is not possible to decide how close two given strings are. Finite automata, can help us in determining whether a given string is accepted or not whereas fuzzy automata can tell us the extent to which the given string is accepted.

The main result that A*-algorithm points to is that an informed search strategy which we have used is clearly superior to any uninformed strategy while searching approximate match for a word in dictionary of words. In particular, the heuristic of the strategy we have used is fast to compute ahead of time taking only linear time and space in the size of the automaton. This often results in a dramatic reduction of the search space that needs to be explored in order to find approximate matches.

In the last application, we have proposed a method for selecting the parameters of a similarity measurement system based on fuzzy automaton which models the typical edit operations used to transform an observed string into a pattern string. The similarity value is the membership value of the observed string to the pattern string via the fuzzy automaton.

We have not been able to carry out the minimization of fuzzy automata under max-product and max-average compositions. It will also be interesting to study fuzzy automata under other compositions such as min-max, max-max and min-min compositions.