6.1. INTRODUCTION

The problem of convective heat transfer in a rectangular porous duct whose vertical walls are maintained at two different temperatures and insulated with horizontal walls received attention by many investigators. Some of these works includes numerical results by a few authors (5-6, 9-10, 13-15, 16).

The investigation of heat transfer in enclosures containing porous media began with the experimental work of Verschoor and Greebler (27). Following Verschoor and Greebler (27) several other investigators studied heat transfer in rectangular enclosures with porous medium (22-23, 24, 25). In particular Bankvall (2-4) has published a great deal of practical work concerning heat transfer by natural convection in rectangular enclosures completely filled with porous media. Burns et. al., (8) analyzed heat transfer flow through porous medium in a rectangular geometry. Cheng et. al. (9) studied the flow and heat transfer rate in a rectangular box with solid walls using Brinkman model. The box is differentially heated in the horizontal direction. Chan et. al., (10) have considered enclosures with aspect ratio greater than or equal to one. Their numerical computations indicate that when Darcy number based on the width of the enclosures is less than $10^{-9}$, Darcy’s law and the Brinkman equation virtually the same results for the heat transfer rate. Joseph et. al., (14) have considered laminar forced convection in rectangular channels with unequal heat addition on adjacent sides. Teoman Ayhan et. al., (26) considered heat transfer and flow structure in a rectangular channel with wing-type Vortex Generator. Ham - Chien Chiu et. al., (12) have discussed mixed convection heat transfer in horizontal rectangular ducts with radiation effects. Chittibabu et. al., (11) discussed convective flow in a porous rectangular duct with differentially heated side wall using Brinkman model.

Literature suggests that the effect of viscous dissipation on heat transfer has been studied for different geometries. Brinkman (7), have studied the viscous dissipation effect on natural convection in horizontal cylinder embedded in porous medium. Their study showed that the viscous dissipation effect on natural convection in a porous cavity and found that the heat transfer rate at hot surface decreases with increase of viscous dissipation parameter. Thermal radiation plays a
significant role in the overall surface heat transfer where convective heat transfer is small. Verschoor et al., (27) have studied the effect of viscous dissipation and radiation on unsteady magneto hydrodynamic free convection flow past vertical plate in porous medium. They found that the temperature profile increases when viscous dissipation increases. A good amount of work has been done to understand natural convection in porous cavity. Inspite of Endeavour efforts to study heat transfer in porous cavity, the combined effect of viscous dissipation and radiation on porous medium filled inside a square cavity has not received attention. Bedridden et al., (1) studied the radiation and viscous dissipation on convective heat transfer in porous cavity. Recently Padmavathi (19) and Nagaradhika (17) have analyzed the convective heat transfer through a porous medium in a rectangular cavity with heat sources and dissipation under varied conditions. Ranga Reddy (20) discussed the natural convective heat and mass transfer in porous rectangular cavity with a differentially heated side walls using Brinkman model. By using Galerkin finite element analysis, the governing equations are solved. Sivaiah et. al., (25) investigated double-diffusive convective heat transfer flow of a viscous fluid through a porous medium with rectangular duct with thermo-diffusion by using finite element technique. Reddaiah et. al., (21) have analyzed the effect of viscous dissipation on convective heat and mass transfer flow of a viscous fluid in a duct of rectangular cross section by employing Galerkin finite element analysis.

In this chapter an attempt has been made to understand the combined influence of radiation and dissipation on the convective heat transfer flow of a viscous fluid through a porous medium in a rectangular cavity using Darcy model. Making use of the incompressibility the governing non-linear coupled equations for the momentum and energy are derived in terms of the non-dimensional stream function and temperature. The Galerkin finite element analysis with linear triangular elements is used to obtain the global stiffness matrices for the values of stream function and temperature. These coupled matrices are solved using iterative procedure and expressions for the stream function and temperature are obtained as linear combinations of the shape functions. The behaviour of temperature, Nusselt number are discussed computationally for different values of the governing parameters Ra, α, N1, and Ec.
Schematic Diagram of the Flow Model
6.2. FORMULATION

We consider the mixed convective heat transfer flow of a viscous incompressible fluid in a saturated porous medium confined in the rectangular duct (Fig. 1) whose base length is a and height b. The heat flux on the base and top walls is maintained as constant. The Cartesian coordinate system \(O(x, y)\) is chosen with origin on the central axis of the duct and its base parallel to \(x\)-axis.

We assume that

i) The convective fluid and the porous medium are everywhere in local thermodynamic equilibrium.

ii) There is no phase change of the fluid in the medium.

iii) The properties of the fluid and of the porous medium are homogeneous and isotropic.

iv) The porous medium is assumed to be closely packed so that Darcy’s momentum law is adequate in the porous medium.

v) The Boussinesq approximation is applicable.

Under these assumptions the governing equations are given by

\[
\frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} = 0 \tag{2.1}
\]

\[
u' = -\frac{k}{\mu} \left( \frac{\partial p'}{\partial x'} \right) \tag{2.2}
\]

\[
v' = -\frac{k}{\mu} \left( \frac{\partial p'}{\partial y'} + \rho' g \right) \tag{2.3}
\]

\[
\rho_c c_p \left( \frac{u'}{\partial x'} + v' \frac{\partial T'}{\partial y'} \right) = K \left( \frac{\partial^2 T'}{\partial x'^2} + \frac{\partial^2 T'}{\partial y'^2} \right) + \rho (T_0 - T) + \left( \frac{\mu}{K} (u'^2 + v'^2) \right) - \frac{\partial (q_c)}{\partial x'} \tag{2.4}
\]

\[
\rho' = \rho_0 \left\{ 1 - \beta_\alpha (T' - T_0)^z - \beta_\lambda (T' - T_0)^z \right\} \tag{2.5}
\]

\[
T_0 = \frac{T_h + T_c}{2}
\]
where \( u' \) and \( v' \) are Darcy velocities along \( O(x, y) \) direction. \( T', p' \) and \( g \) are the temperature, pressure and acceleration due to gravity, \( T_c \) and \( T_h \) is the temperature on the cold and warm side walls respectively. \( \rho' \), \( \mu' \), \( v \), and \( \beta \) are the density, coefficients of viscosity, kinematic viscosity and thermal expansion of the fluid, \( k \) is the permeability of the porous medium, \( K_1 \) is the thermal conductivity, \( C_p \) is the specific heat at constant pressure, \( Q \) is the strength of the heat source, and \( q_r \) is the radiative heat flux.

The boundary conditions are

\[
\begin{align*}
  u' &= v' = 0 \quad \text{on the boundary of the duct} \\
  T' &= T_c \quad \text{on the side wall to the left} \\
  T' &= T_h \quad \text{on the side wall to the right} \\
  \frac{\partial T'}{\partial y} &= 0, \quad \text{on the top (} y = 0 \text{) and bottom}
\end{align*}
\]

\( u = v = 0 \) walls \( (y = 0) \) which are insulated.

Invoking Rosseland approximation for radiation

\[
q_r = \frac{4\sigma^*}{3\beta_R} \frac{\partial T'^4}{\partial y}
\]

Expanding \( T'^4 \) in Taylor’s series about \( T_e \) and neglecting higher order terms

\[
T'^4 \approx 4T_e^3 - 3T_e^4
\]

We now introduce the following non-dimensional variables

\[
\begin{align*}
  x' &= ax; \quad y' = by; \quad c = b/a \\
  u' &= (v/a) u; \quad v' &= (v/a) v; \quad p' = (v^2p/a^2) p \\
  T' &= T_0 + \theta(T_h - T_c)
\end{align*}
\]

The governing equations in the non-dimensional form are

\[
\begin{align*}
  u &= -\left(\frac{K}{a^2}\right) \frac{\partial p}{\partial x} \\
  v &= -\frac{k}{a^2} \frac{\partial p}{\partial y} - \frac{kag}{v^2} + \frac{kag\beta_c(T_h - T_e)\theta}{v^2} + \frac{kag\beta_h(T_h - T_e)^2\theta^2}{v^2}
\end{align*}
\]
In view of the equation of continuity we introduce the stream function \( \psi \) as
\[
u = \frac{\partial \psi}{\partial y}; \quad \psi = -\frac{\partial \psi}{\partial x} \tag{2.12}\]

Eliminating \( p \) from the equation (2.9) and (2.10) and making use of (2.11) the equations in terms of \( \psi \) and \( \theta \) are
\[
\nabla^2 \psi - M_1^2 \frac{\partial \psi}{\partial x} = -Ra(\frac{\partial \theta}{\partial x} + 2\gamma \theta \frac{\partial \theta}{\partial x}) \tag{2.13}
\]
\[
P \left( \frac{\partial \psi}{\partial y} \frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \theta}{\partial y} \right) = \left( 1 + \frac{4}{3N_1} \left( \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right) - \alpha \theta + Ec \left( \frac{\partial \psi}{\partial y} \right)^2 + \left( \frac{\partial \psi}{\partial x} \right)^2 \right) \tag{2.14}
\]

where
\[
G = \frac{g \beta (T_h - T_c) \alpha^3}{v^2} \quad \text{(Grashof number)}
\]
\[
P = \mu \frac{C_p}{K_1} \quad \text{(Prandtl number)}
\]
\[
\alpha = \frac{Q_a c_1}{K_1} \quad \text{(Heat source parameter)}
\]
\[
Ra = \frac{\beta g (T_s - T_c) K a}{v^2} \quad \text{(Rayleigh number)}
\]
\[
\gamma = \frac{\beta_1 (T_2 - T_1)}{\beta_0}
\]
\[
N_1 = \frac{3 \beta - 1}{4 \sigma^* T_r}\]
\[
Ec = \left( \frac{a^4}{\mu K \Delta T} \right) \quad \text{(Eckert number)}
\]

The boundary conditions are
\[
\frac{\partial \psi}{\partial x} = 0, \frac{\partial \psi}{\partial y} = 0 \quad \text{on} \quad x = 0 \& 1 \tag{2.15}
\]
\[
\theta = 1 \quad \text{on} \quad x = 0
\]
\[
\theta = 0 \quad \text{on} \quad x = 1 \tag{2.16}
\]
6.3. **FINITE ELEMENT ANALYSIS AND SOLUTION OF THE PROBLEM**

The region is divided into a finite number of three node triangular elements, in each of which the element equation is derived using Galerkin weighted residual method. In each element \( f_i \), the approximate solution for an unknown \( f \) in the variation formulation is expressed as a linear combination of shape function \( N_k \) which are linear polynomials in \( x \) and \( y \). This approximate solution of the unknown \( f \) coincides with actual values at each node of the element. The variation formulation results in a 3 x 3 matrix equation (stiffness matrix) for the unknown local nodal values of the given element. These stiffness matrices are assembled in terms of global nodal values using inter element continuity and boundary conditions resulting in global matrix equation.

In each case there are \( r \) distinct global nodes in the finite element domain and \( f_p \) (\( p = 1, 2, \ldots, r \)) is the global nodal values of any unknown \( f \) defined over the domain then

\[
f = \sum_{j=1}^{s} \sum_{p=1}^{r} f_p \Phi^i_p,
\]

where the first summation denotes summation over \( s \) elements and the second one represents summation over the independent global nodes and

\[
\Phi^i_p = N^i_N. \text{ If } p \text{ is one of the local nodes say } k \text{ of the element } e_i
\]

\[= 0, \text{ otherwise.}\]

\( f_p \)'s are determined from the global matrix equation. Based on these lines we now make a finite element analysis of the given problem governed by (2.13)-(2.14) subjected to the conditions (2.15) – (2.16).

Let \( \psi^i, \theta^i \) be the approximate values of \( \psi, \theta \) in an element \( e_i \).

\[
\psi^i = N^i_1 \psi^i_1 + N^i_2 \psi^i_2 + N^i_3 \psi^i_3
\]

\[
\theta^i = N^i_1 \theta^i_1 + N^i_2 \theta^i_2 + N^i_3 \theta^i_3
\]
Substituting the approximate value $\psi^i$ and $\theta^i$ for $\psi$ and $\theta$ respectively in (2.13), the error

$$E_i^d = \left(1 + \frac{4}{3N_i} \left( \frac{\partial^2 \theta^i}{\partial x^2} + \frac{\partial^2 \theta^i}{\partial y^2} \right) - P \left( \frac{\partial \psi^i}{\partial y} \frac{\partial \psi^i}{\partial x} - \frac{\partial \psi^i}{\partial x} \frac{\partial \psi^i}{\partial y} \right) - \alpha \theta + E_c \left( \left( \frac{\partial \psi}{\partial y} \right)^2 + \left( \frac{\partial \psi}{\partial x} \right)^2 \right) \right)$$

(3.2)

Inder Galerkin method this error is made orthogonal over the domain of $e_i$ to the respective shape functions (weight functions) where

$$\int_{e_i} E_i^d N_k^i \ d\Omega = 0$$

$$\int_{e_i} N_k^i \left[ \left(1 + \frac{4}{3N_i} \left( \frac{\partial \theta^i}{\partial x} \right) + \frac{\partial \theta^i}{\partial y} \right) - P \left( \frac{\partial \psi^i}{\partial y} \frac{\partial \psi^i}{\partial x} - \frac{\partial \psi^i}{\partial x} \frac{\partial \psi^i}{\partial y} \right) \right]\ d\Omega = 0$$

(3.3)

Using Green’s theorem we reduce the surface integral (3.4) & (3.5) without affecting $\psi$ terms and obtain

$$\int_{e_i} N_k^i \left[ \left(1 + \frac{4}{3N_i} \left( \frac{\partial \theta^i}{\partial x} \right) + \frac{\partial \theta^i}{\partial y} \right) - P \left( \frac{\partial \psi^i}{\partial y} \frac{\partial \psi^i}{\partial x} - \frac{\partial \psi^i}{\partial x} \frac{\partial \psi^i}{\partial y} \right) \right]\ d\Omega = \int_{\Gamma_i} \left( \frac{\partial \psi^i}{\partial n} \right) d\Gamma_i$$

(3.4)

where $\Gamma_i$ is the boundary of $e_i$.

Substituting L.H.S. of (3.1a)- (3.1b) for $\psi^i$, $\theta^i$ in (3.4) we get

$$\sum_{e_i} \int_{e_i} \left(1 + \frac{4N_i}{3} \left( \frac{\partial N_k^i}{\partial x} \frac{\partial N_L^i}{\partial x} + \frac{\partial N_k^i}{\partial y} \frac{\partial N_L^i}{\partial y} \right) - P \sum_{e_j} \psi^i \left( \frac{\partial N_m^j}{\partial y} \frac{\partial N_L^j}{\partial x} - \frac{\partial N_m^j}{\partial x} \frac{\partial N_L^j}{\partial y} \right) \right) d\Omega$$

$$- \alpha \sum_{e_i} \theta^i \int N_k^i d\Omega + E_c \int \left( \left( \frac{\partial \psi}{\partial y} \right)^2 + \left( \frac{\partial \psi}{\partial x} \right)^2 \right) d\Omega$$
where $Q_k^i = Q_{k1}^i + Q_{k2}^i + Q_{k3}^i$, $Q_k^i$'s being the values of $Q_k^i$ on the sides $s = (1, 2, 3)$ of the element $e_i$. The sign of $Q_k^i$'s depends on the direction of the outward normal w.r.t. the element.

Choosing different $N_k^i$'s as weight functions and following the same procedure we obtain matrix equations for three unknowns ($Q_p^i$) viz.,

$$(a_p^i)(\theta_p^i) = (Q_p^i)$$  \hspace{1cm} (3.6)$$

where $(a_p^{ik})$ is a $3 \times 3$ matrix, $(\theta_p^i), (Q_p^i)$ are column matrices.

Repeating the above process with each of $s$ elements, we obtain sets of such matrix equations. Introducing the global coordinates and global values for $\theta_p^i$ and making use of inter element continuity and boundary conditions relevant to the problem the above stiffness matrices are assembled to obtain a global matrix equation. This global matrix is $r \times r$ square matrix with $r$ distinct global nodes in the domain of flow considered.

Similarly substituting $\psi^i, \theta^i$ in (2.12) and defining the error

$$E^i_j = \nabla^2 \psi - M^2 \psi + Ra \left( \frac{\partial \theta}{\partial x} + 2 \gamma \theta \frac{\partial \theta}{\partial x} \right)$$  \hspace{1cm} (3.7)$$

and following the Galerkin method we obtain

$$\int_\Omega E^i_j \psi^j d\Omega = 0$$  \hspace{1cm} (3.8)$$

Using Green’s theorem (3.8) reduces to

$$\int_\Omega \left( \frac{\partial N_k^i}{\partial x} \frac{\partial \psi^i}{\partial x} + \frac{\partial N_k^i}{\partial y} \frac{\partial \psi^i}{\partial y} + Ra \left( \theta^i \frac{\partial N_k^i}{\partial x} + 2 N_k^i (\theta^i)^2 \frac{\partial N_k^i}{\partial x} \right) \right) d\Omega$$

$$= \int_{\Gamma} N_k^i \left( \frac{\partial \psi^i}{\partial x} n_x + \frac{\partial \psi^i}{\partial y} n_y \right) d\Gamma_i + \int_{\Gamma} N_k^i n_x \theta^i d\Gamma_i$$ \hspace{1cm} (3.9)$$
In obtaining (3.9) the Green’s theorem is applied w.r.t derivatives of $\psi$ without affecting $\theta$ terms.

Using (3.1a) and (3.1b) in (3.9) we have

$$
\sum_m \psi_m^i \left\{ \int_\Omega \left( \frac{\partial N_k^i}{\partial x} \frac{\partial N_m^i}{\partial x} + \frac{\partial N_m^i}{\partial y} \frac{\partial N_k^i}{\partial y} \right) d\Omega + \text{Ra} \sum L \left( \theta_L^i \int_\Omega N_k^i \frac{\partial N_L^i}{\partial x} d\Omega + (\theta_L^i)^2 N \int_\Omega N_k^i \frac{\partial N_L^i}{\partial x} d\Omega \right) \right\}
$$

$$
= \int \left( \frac{\partial \psi^i}{\partial x} n_x + \frac{\partial \psi^i}{\partial y} n_y \right) d\Gamma_i + \int N_k^i \theta^i d\Omega_i = \Gamma_k^i \quad (3.10)
$$

In the problem under consideration, for computational purpose, we choose uniform mesh of 10 triangular elements (Fig. ii). The domain has vertices whose global coordinates are (0,0), (1,0) and (1,c) in the non-dimensional form. Let $e_1$, $e_2$......$e_{10}$ be the ten elements and let $\theta_1$, $\theta_2$.....$\theta_{10}$ be the global values of $\theta$ and $\psi_1$, $\psi_2$......$\psi_{10}$ be the global values of $\psi$ at the ten global nodes of the domain (Fig. ii).
6.4. SHAPE FUNCTIONS AND STIFFNESS MATRICES

Range functions in $n : i = \text{element}, j = \text{node}$.

\[
\begin{align*}
  n_{1,1} &= 1 - 3x \\
  n_{1,2} &= 3x - \frac{3y}{C} \\
  n_{2,1} &= 1 - \frac{3y}{C} \\
  n_{2,2} &= -1 + \frac{3y}{C} \\
  n_{2,3} &= 1 - 3x + \frac{3y}{C} \\
  n_{3,1} &= 2 - 3x \\
  n_{3,2} &= -1 + 3x - \frac{3y}{C} \\
  n_{3,3} &= \frac{3y}{C} \\
  n_{4,1} &= 1 - \frac{3y}{C} \\
  n_{4,2} &= -2 + 3x \\
  n_{4,3} &= 2 - 3x \\
  n_{5,1} &= 2 - 3x \\
  n_{5,2} &= -1 + 3x - \frac{3y}{C} \\
  n_{5,3} &= \frac{3y}{C} \\
  n_{6,1} &= 2 - 3x \\
  n_{6,2} &= 3x - \frac{3y}{C} \\
  n_{6,3} &= 1 + \frac{3y}{C} \\
  n_{7,1} &= 2 - \frac{3y}{C} \\
  n_{7,2} &= -2 + 3x \\
  n_{7,3} &= 1 - 3x + \frac{3y}{C} \\
  n_{8,1} &= 3 - 3x \\
  n_{8,2} &= -1 + 3x - \frac{3y}{C} \\
  n_{9,2} &= 3x - \frac{3y}{C} \\
  n_{9,3} &= -1 + \frac{3y}{C}
\end{align*}
\]

Substituting the above shape functions in (3.8),(3.9) and (3.14) w.r.t each element and integrating over the respective triangular domain we obtain the element
in the form (3.8). The 3x3 matrix equations are assembled using connectivity conditions to obtain a 8x8 matrix equations for the global nodes ψ_p, θ_p and ϕ_p.

The global matrix equation for θ is

\[ A_3 X_3 = B_3 \]  

(4.1)

The global matrix equation for φ is

\[ A_4 X_4 = B_4 \]  

(4.2)

The global matrix equation for ψ is

\[ A_5 X_5 = B_5 \]  

(4.3)

where

\[ \begin{pmatrix} -1 & a_{12} & a_{13} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & a_{22} & a_{23} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & a_{32} & a_{33} & a_{34} & a_{35} & 0 & 0 & 0 & 0 \\ 0 & 0 & a_{44} & a_{44} & a_{45} & 0 & 0 & 0 & 0 \\ 0 & 0 & a_{54} & a_{54} & a_{55} & a_{56} & a_{57} & 0 & 0 \\ 0 & 0 & a_{65} & a_{65} & a_{66} & a_{67} & a_{68} & a_{69} & 0 \\ 0 & 0 & a_{75} & a_{76} & a_{77} & a_{78} & a_{79} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & a_{87} & a_{88} & a_{89} & 0 \\ 0 & 0 & 0 & 0 & 0 & a_{97} & a_{98} & a_{99} & a_{910} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{1093} & a_{1094} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{1115} & a_{1116} & -1 \end{pmatrix} \]

\[ \hat{A}_5 = \begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix} \]
The global matrix equations are coupled and are solved following iterative procedures. At the beginning of the first iteration the values of \((\psi_i)\) are taken to be zero and the global equations (4.1) and (4.2) are solved for the nodal values of \(\theta\) and \(\phi\). These nodal values \((\theta_i)\) and \((\phi_i)\) obtained are then used to solve the global equation (4.3) to obtain\((\psi_i)\). In the second iteration these \((\psi_i)\) values are obtained are used in (4.1) and (4.2) to calculate \((\theta_i)\) and \((\phi_i)\) and vice versa. The three equations are thus solved under iteration process until two consecutive iterations differ by a pre-assigned percentage.

The domain consists of three horizontal levels and the solution for \(\Psi\) and \(\theta\) at each level may be expressed in terms of the nodal values as follows,
In the horizontal strip $0 \leq y \leq \frac{c}{3}$

$$\Psi = (\Psi_1 N_1 + \Psi_2 N_2 + \Psi_7 N_7) H(1-\tau_1)$$

$$= \Psi_1 (1-4x) + \Psi_2 4(x - \frac{y}{c}) + \Psi_7 \left( \frac{4y}{c} \right) (1-\tau_1) \quad (0 \leq x \leq \frac{1}{3})$$

$$\Psi = (\Psi_3 N_3 + \Psi_3 N_3 + \Psi_6 N_6) H(1-\tau_2)$$

$$+ (\Psi_2 N_2 + \Psi_7 N_7 + \Psi_6 N_6) H(1-\tau_3) \quad \left( \frac{1}{3} \leq x \leq \frac{1}{3} \right)$$

$$= (\Psi_2 2(1-2x) + \Psi_3 (4x - \frac{4y}{c} - 1) + \Psi_6 \left( \frac{4y}{c} \right)) H(1-\tau_2)$$

$$+ (\Psi_2 2 \left( 1 - \frac{4y}{c} \right) + \Psi_7 \left( 1 + \frac{4y}{c} \right) - 4x) + \Psi_6 (4x - 1)) H(1-\tau_3)$$

$$\Psi = (\Psi_3 N_3 + \Psi_4 N_4 + \Psi_8 N_5) H(1-\tau_3)$$

$$+ (\Psi_3 N_3 + \Psi_5 N_5 + \Psi_6 N_6) H(1-\tau_4) \quad \left( \frac{2}{3} \leq x \leq 1 \right)$$

$$= (\Psi_3 (3-4x) + \Psi_4 2(2x - \frac{2y}{c} - 1) + \Psi_6 \left( \frac{4y}{c} \right) - 4x + 3) H(1-\tau_3)$$

$$+ \Psi_3 (1 - \frac{4y}{c}) + \Psi_5 (4x - 3) + \Psi_6 \left( \frac{4y}{c} \right) H(1-\tau_4)$$

Along the strip $\frac{c}{3} \leq y \leq \frac{2c}{3}$

$$\Psi = (\Psi_7 N_7 + \Psi_6 N_6) H(1-\tau_2) \quad \left( \frac{1}{3} \leq x \leq 1 \right)$$

$$+ (\Psi_6 N_6 + \Psi_9 N_9 + \Psi_6 N_8) H(1-\tau_3) + (\Psi_6 N_6 + \Psi_9 N_9 + \Psi_9 N_9) H(1-\tau_4)$$

$$\Psi = (\Psi_7 2(1-2x) + \Psi_6 (4x - 3) + \Psi_8 \left( \frac{4y}{c} - 1 \right)) H(1-\tau_3)$$

$$+ \Psi_6 (2 \left( 1 - \frac{2y}{c} \right) + \Psi_9 \left( \frac{4y}{c} - 1 \right) + \Psi_8 (1 + \frac{4y}{c} - 4x)) H(1-\tau_4)$$

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+ Ψ₆ (4(1-x) + Ψ₅ (4x - \frac{4y}{c} - 1) + Ψ₂(\frac{2y}{c} - 1))H(1- τ₅)

Along the strip \( \frac{2c}{3} \leq y \leq 1 \)

\[ \Psi = (\Psi₈N₈ + \Psi₉N₉ + \Psi₁₀N₁₀)H(1- τ₆) \] \quad (\frac{2}{3} \leq x \leq 1)

= Ψ₈ (4(1-x)+ Ψ₉4(x - \frac{y}{c})+ Ψ₁₀2(\frac{4y}{c} - 3))H(1- τ₆)

where τ₁ = 4x , τ₂ = 2x , τ₃ = \frac{4x}{3} ,

τ₄ = 4(x - \frac{y}{c}) , τ₅ = 2(x - \frac{y}{c}) , τ₆ = \frac{4}{3} (x - \frac{y}{c})

and H represents the Heaviside function.

The expressions for θ are

In the horizontal strip \( 0 \leq y \leq \frac{c}{3} \)

\[ \theta = (\theta₁(1-4x) + \theta₂4(x - \frac{y}{c}) + \theta₇(\frac{4y}{c}))H(1- τ₁) \] \quad (0 \leq x \leq \frac{1}{3})

\[ \theta = (\theta₂2(1-2x) + \theta₃(4x - \frac{4y}{c} - 1) + \theta₆(\frac{4y}{c}))H(1- τ₂) \]

+ \theta₂(\frac{4y}{c}) + \theta₇(1+ \frac{4y}{c} - 4x) + \theta₆(4x - 1))H(1- τ₃) \quad (\frac{1}{3} \leq x \leq \frac{2}{3})

\[ \theta = \theta₃(3-4x) + 2 \theta₄2(2x - \frac{2y}{c} - 1) + \theta₆(\frac{4y}{c} - 4x + 3)H(1- τ₃) \]

+ (\theta₃(1- \frac{4y}{c}) + \theta₅(4x - 3) + \theta₆(\frac{4y}{c}))H(1- τ₄) \quad (\frac{2}{3} \leq x \leq 1)

Along the strip \( \frac{c}{3} \leq y \leq \frac{2c}{3} \)

\[ \theta = (\theta₂(2(1-2x) + \theta₆(4x - 3) + \theta₈(\frac{4y}{c} - 1))H(1- τ₃) \] \quad (\frac{1}{3} \leq x \leq \frac{2}{3})

138
$$\theta = (\theta_8 4(1-x) + \theta_9 4(x-\frac{y}{c}) + \theta_{10} (\frac{4y}{c} - 1)) H(1-\tau_6) \quad \left(\frac{2}{3} \leq x \leq 1\right)$$

Along the strip $\frac{2c}{3} \leq y \leq 1$

The dimensionless Nusselt numbers (Nu) on the non-insulated boundary walls of the rectangular duct are calculated using the formula

$$\text{Nu} = \left(\frac{\partial \theta}{\partial x}\right)_{x=1}$$

Nusselt number on the side wall $x=1$ in different regions are

$$\text{Nu}_1 = 2 - 4\theta_3 \quad (0 \leq y \leq h/3)$$

$$\text{Nu}_2 = 2 - 4\theta_5 \quad (h/3 \leq y \leq 2h/3)$$

$$\text{Nu}_3 = 2 - 4\theta_7 \quad (2h/3 \leq y \leq h)$$
6.5. DISCUSSION OF THE NUMERICAL RESULTS

In this analysis we investigate the effect of non-linear density temperature variation on convective heat transfer flow of a viscous fluid confined in a rectangular cavity in the presence of heat sources. The equation governing the flow and heat transfer are solved by employing Galerkin finite element analysis with three nodded triangular elements with bilinear functions in x and y as approximate functions.

The temperature ($\theta$) at different horizontal and vertical levels is shown in fig (1-16) for different values of Rayleigh number $Ra$, Radiation parameter $N$ heat source parameter $\alpha$, and Eckert number $Ec$. Figs (1-4) represent the temperature distribution ($\theta$) with Rayleigh number $R$. It is found that at all horizontal levels $y = \frac{h}{3}$ and $y = \frac{2h}{3}$ and at vertical levels $x = \frac{1}{3}$ and $\frac{2}{3}$ the axial temperature enhances with $Ra > 0$ and reduces with $Ra < 0$ at $y = \frac{h}{3}$. At the higher horizontal level $y = \frac{2h}{3}$ the axial temperature depreciates marginally with $|Ra|$. At vertical level $x = \frac{1}{3}$ the axial temperature reduces with $Ra < 2 \times 10^2$ and enhances with higher $Ra < 3 \times 10^2$ and for an increase in $|Ra|$ the axial temperature depreciates in the entire flow region. At the vertical level $x = \frac{2}{3}$ the axial temperature reduces with $Ra \leq 2 \times 10^2$ and enhances with higher $Ra \geq 3 \times 10^2$, while it enhances $|Ra|$. The variation of ‘$\theta$’ with radiation parameter ‘$N$’ is shown in figs 5-8. It is found that the axial temperature depreciates at $y = \frac{h}{3}$ and enhances at $y = \frac{2h}{3}$ with an increase in the radiation parameter ‘$N$’. At the vertical level $x = \frac{1}{3}$ the axial temperature reduces with $N \leq 0.03$ and enhances with $N \geq 0.05$.

The variation of ‘$\theta$’ with heat source parameter ‘$\alpha$’ is shown fig 9-12 at different horizontal and vertical levels. From fig 9 we find that the axial temperature enhances with increase in $|\alpha|$, whereas at higher horizontal level $y = \frac{2h}{3}$ the axial
temperature decreases with $\alpha < 0$ and enhances with $|\alpha|$ Fig-10. At the vertical level $x = \frac{1}{3}$ the axial temperature experiences an enhancement with increase in $|\alpha|$ and at $x = \frac{2}{3}$ level the axial temperature enhances with $\alpha \leq 4$ and reduces $\alpha \geq 6$ and for an increase in $|\alpha|$ the axial temperature enhances in the entire flow region (fig-12). The variation of temperature with Eckert number $Ec$ is shown in fig 13-16. At the horizontal level $y = h/3$ the axial temperature depreciates with increase in Eckert no for $Ec \leq 0.033$ and enhances for $Ec > 0.005$, again depreciates with higher $Ec = 0.007$ and at $y = \frac{2h}{3}$ the axial temperature depreciates with $Ec \leq 0.005$ and enhances with higher $Ec > 0.0007$. At the vertical level $x = \frac{1}{3}$ the axial temperature depreciates with higher $Ec \leq 0.005$ and enhances for $E \geq 0.007$. At the higher vertical level $y = \frac{2}{3}$ the axial temperature depreciates with $Ec \leq 0.003$ and enhances with higher $Ec = 0.005$ and again depreciates $Ec \geq 0.007$. Thus with higher dissipative forces smaller the axial temperature and for further higher dissipative forces larger the axial temperature and for further values of higher dissipative forces smaller the temperature in the entire flow region. It is found that as we move along the vertical direction the axial temperature depreciates.

The Nusselt number at $x = 1$ is shown in tables 1-4 for different values of Ra, N, $\alpha$, Ec and at different positions. Table 1 represents the variation of Nu with increase in Rayleigh number Ra. An increase in the $Ra \leq 2\times10^5$ enhances Nu at all the three quadrants and depreciates for higher $Ra \geq 3\times10^5$. Also the rate of heat transfer enhances in all the three quadrants with increase in $|Ra|$. As we move in vertical direction the rate of heat transfer depreciates. The variation of Nu with radiation parameter ‘N’ is shown in table 2. It depreciates Nusselt number in first quadrant, while in the second and third quadrants the Nusselt number enhances with $N \leq 0.03$, depreciates at $N = 0.05$ and again enhances at $N = 0.07$. The variation of Nu with heat source parameter ‘$\alpha$’ is shown in table 4. It is found that Nusselt
number sat first and middle quadrants depreciates with $\alpha \geq 6$. At the highest quadrant the rate of heat transfer enhances with ‘$\alpha$’. An increase in $|\alpha|$ enhances Nu at the first and middle quadrants and reduces at the highest quadrant. The variation of Nu, Eckert number Ec is shown in table 3. An increase in $Ec \leq 0.003$ enhances Nu at the first quadrant and reduces at the middle and third quadrant and depreciates at all the three quadrants at $Ec = 0.005$ and again enhances at all the three quadrants at higher $Ec = 0.007$. 
6.6. APPENDIX

\[
\begin{align*}
\sin^2 \theta &= \frac{8 \alpha \beta \gamma + 2 \gamma \beta + 2 \alpha \beta}{2} \\
\cos \theta &= \frac{2 \alpha \beta \gamma + 2 \gamma \beta + 2 \alpha \beta}{2} \\
\tan \theta &= \frac{2 \alpha \beta \gamma + 2 \gamma \beta + 2 \alpha \beta}{2}
\end{align*}
\]
\[ a_{70} = \frac{20}{3s} + \frac{4 P f}{5} + \frac{4 P f}{15} - \frac{2 P a g}{5} - \frac{4 P a g}{15} \]

\[ a_{80} = \frac{20}{15s} - \frac{4 P f}{8} - \frac{2 P f}{9} + \frac{4 P a g}{8} + \frac{2 P a g}{5} \]

\[ a_{70} = \frac{10}{3s} - \frac{P f}{15} - \frac{P f}{15} - \frac{P f}{30} + \frac{1}{15} P a g + \frac{1}{15} P a g + \frac{1}{30} P a g \]

\[ 200 + P s \left( -f + 0 f + 3 f + a \left( g + 6 g - 3 g \right) \right) \]

\[ a_{90} = \frac{15s}{15s} \]

\[ a_{90} = \frac{20}{3s} - \frac{P f}{30} - \frac{P f}{5} - \frac{1}{30} P a g - \frac{1}{15} P a g + \frac{1}{5} P a g \]

\[ 200 + P s \left( -3 f - 4 f + f + 3 a g + 8 a g \right) \]

\[ a_{100} = \frac{15s}{15s} \]

\[ a_{110} = \frac{10}{3s} + \frac{P f}{30} + \frac{P f}{15} + \frac{1}{30} P a g - \frac{1}{15} P a g \]

\[ a_{1010} = -\frac{20}{3s} + \frac{2 P f}{15} + \frac{4 P f}{15} - \frac{2 P a g}{5} - \frac{4 P a g}{15} \]

\[ 4 \left( -100 + P s \left( f - f - a g \right) \right) \]

\[ a_{1210} = -\frac{15s}{15s} \]

\[ a_{1310} = \frac{20}{3s} - \frac{4 P f}{10} - \frac{2 P f}{5} + \frac{4 P a g}{15} \]

\[ b_{1, 2} = -\frac{2}{3} \frac{40}{3s} \]

\[ b_{1, 2} = \frac{1}{6} + \frac{5}{3s} \]

\[ b_{1, 2} = \frac{8}{3s} + \frac{80}{3s} \]

\[ b_{1, 2} = \frac{14}{3s} + \frac{70}{3s} \]

\[ b_{1, 2} = \frac{10}{3} - \frac{40}{3} \]

\[ b_{1, 2} = \frac{1}{2} + \frac{5}{3s} \]

\[ b_{1, 2} = -\frac{10}{3} - \frac{40}{3s} \]
\[
\begin{align*}
  b &= 8 + \frac{80}{3} \\
  b &= 14 + \frac{40}{3} \\
  b &= 5 + \frac{5}{3} \\
  b &= 14 + \frac{40}{3} \\
  b &= 28 + \frac{70}{3} \\
  b &= -6 + \frac{40}{3} \\
  b &= 5 + \frac{5}{3} \\
  b &= -6 + \frac{40}{3} \\
  b &= 40 + \frac{80}{3} \\
  b &= 22 + \frac{40}{3} \\
  b &= 5 + \frac{5}{3} \\
  b &= 14 + \frac{70}{3} \\
  b &= -6 + \frac{40}{3} \\
  b &= 5 + \frac{5}{3} \\
  b &= 26 + \frac{40}{3} \\
  b &= 26 + \frac{40}{3} \\
  b &= 56 + \frac{80}{3} \\
  b &= -10 + \frac{40}{3} \\
  b &= 7 + \frac{5}{3} \\
  b &= -10 + \frac{40}{3} \\
  b &= 56 + \frac{70}{3} \\
  b &= -34 + \frac{40}{3} \\
  b &= -34 + \frac{40}{3} \\
  b &= -34 + \frac{40}{3} \\
  b &= 38 + \frac{40}{3} \\
  b &= -1 - \frac{1}{3}
\end{align*}
\]
Chapter – VI

\[ \begin{align*}
\text{Section} & \quad (13.3333333333333333^2 \cdot 4.666666666666666^2 \cdot n_1 \cdot (-2.3333333333333333^2 \cdot 4.666666666666666^2 \cdot 83.33333333333333 \cdot \bar{n})^2 + 33.33333333333333 \cdot \bar{n}^2 +
\end{align*} \]

\[ \begin{align*}
\text{Section} & \quad (13.3333333333333333^2 \cdot 83.33333333333333 \cdot \bar{n})^2 + 33.33333333333333 \cdot \bar{n}^2 +
\end{align*} \]

\[ \begin{align*}
\text{Section} & \quad (13.3333333333333333^2 \cdot 83.33333333333333 \cdot \bar{n})^2 + 33.33333333333333 \cdot \bar{n}^2 +
\end{align*} \]

\[ \begin{align*}
\text{Section} & \quad (13.3333333333333333^2 \cdot 83.33333333333333 \cdot \bar{n})^2 + 33.33333333333333 \cdot \bar{n}^2 +
\end{align*} \]

\[ \begin{align*}
\text{Section} & \quad (13.3333333333333333^2 \cdot 83.33333333333333 \cdot \bar{n})^2 + 33.33333333333333 \cdot \bar{n}^2 +
\end{align*} \]

\[ \begin{align*}
\text{Section} & \quad (13.3333333333333333^2 \cdot 83.33333333333333 \cdot \bar{n})^2 + 33.33333333333333 \cdot \bar{n}^2 +
\end{align*} \]

\[ \begin{align*}
\text{Section} & \quad (13.3333333333333333^2 \cdot 83.33333333333333 \cdot \bar{n})^2 + 33.33333333333333 \cdot \bar{n}^2 +
\end{align*} \]

\[ \begin{align*}
\text{Section} & \quad (13.3333333333333333^2 \cdot 83.33333333333333 \cdot \bar{n})^2 + 33.33333333333333 \cdot \bar{n}^2 +
\end{align*} \]


\[
\Psi = \Phi - \frac{1}{H} \left( H - \frac{1}{2} \sum_{n=1}^{N} (1 - \theta_{n}) \right)
\]

Where \( H \) is the Hamiltonian, \( \Phi \) is the total energy, and \( \theta_{n} \) are the coefficients related to the states.

\[
H = \sum_{n=1}^{N} \sum_{j=1}^{3} \left( -\frac{1}{2} m \omega^2 \right) x_{j}^2 + \sum_{n=1}^{N} \sum_{j=1}^{3} \sum_{k=1}^{3} J_{jk} x_{j} x_{k} + \sum_{n=1}^{N} \sum_{j=1}^{3} \sum_{k=1}^{3} \sum_{l=1}^{3} \sum_{m=1}^{3} C_{ijkl} x_{j} x_{k} x_{l} x_{m}
\]

\( \theta_{n} \) are determined by the boundary conditions and the interaction terms in the Hamiltonian.
6.7. REFERENCES


17) Nagaradhika, V.: Convective heat transfer in a porous medium in a rectangular cavity under the influence of radiation, viscous dissipation and temperature gradient dependent heat source, IJEECT,vol. 2, No. 4, (2011)


Fig. 1: Variation of $\theta$ with $Ra$ at $y = \frac{h}{3}$ level

<table>
<thead>
<tr>
<th>$Ra$</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
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<tbody>
<tr>
<td></td>
<td>$10^2$</td>
<td>$2 \times 10^2$</td>
<td>$3 \times 10^2$</td>
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<td>$-2 \times 10^2$</td>
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</table>

Fig. 2: Variation of $\theta$ with $Ra$ at $y = \frac{2h}{3}$ level

<table>
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<th>III</th>
<th>IV</th>
<th>V</th>
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<td>$2 \times 10^2$</td>
<td>$3 \times 10^2$</td>
<td>$-10^2$</td>
<td>$-2 \times 10^2$</td>
</tr>
</tbody>
</table>
Fig. 3: Variation of $\theta$ with $Ra$ at $x = \frac{1}{3}$ level

<table>
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<th>III</th>
<th>IV</th>
<th>V</th>
</tr>
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<tr>
<td>$2x10^2$</td>
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<tr>
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Fig. 4: Variation of $\theta$ with $Ra$ at $x = \frac{2}{3}$ level

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<tr>
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<tr>
<td>$-2x10^2$</td>
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</tbody>
</table>
Fig. 5: Variation of $\theta$ with $N$ at $y = \frac{h}{3}$ level

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<td>N</td>
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<td>0.02</td>
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Fig. 6: Variation of $\theta$ with $N$ at $y = \frac{2h}{3}$ level

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Fig. 7: Variation of $\theta$ with $N$ at $x = \frac{1}{3}$ level

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<th>IV</th>
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Fig. 8: Variation of $\theta$ with $N$ at $x = \frac{2}{3}$ level

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<td>0.02</td>
<td>0.03</td>
<td>0.05</td>
<td>0.07</td>
</tr>
</tbody>
</table>
Fig. 9 : Variation of $\theta$ with $\alpha$ at $y = \frac{h}{3}$ level

$\alpha$ | I | II | III | IV | V
--- |---|---|---|---|---
$\alpha$ | 2 | 4 | 6 | -2 | -4

Fig. 10 : Variation of $\theta$ with $\alpha$ at $y = \frac{2h}{3}$ level

$\alpha$ | I | II | III | IV | V
--- |---|---|---|---|---
$\alpha$ | 2 | 4 | 6 | -2 | -4
Fig. 11 : Variation of $\theta$ with $\alpha$ at $x = \frac{1}{3}$ level

<table>
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<th>III</th>
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<th>V</th>
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<td>6</td>
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<td>-4</td>
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</table>

Fig. 12 : Variation of $\theta$ with $\alpha$ at $x = \frac{2}{3}$ level

<table>
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<th>II</th>
<th>III</th>
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Fig. 13: Variation of $\theta$ with $Ec$ at $y = \frac{h}{3}$ level

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<tr>
<td>Ec</td>
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</table>

Fig. 14: Variation of $\theta$ with $Ec$ at $y = \frac{2h}{3}$ level

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<tbody>
<tr>
<td>Ec</td>
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<td>0.003</td>
<td>0.005</td>
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</table>
Fig. 15: Variation of $\theta$ with $Ec$ at $x = \frac{1}{3}$ level

<table>
<thead>
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<th>IV</th>
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<td>0.003</td>
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</table>

Fig. 16: Variation of $\theta$ with $Ec$ at $x = \frac{2}{3}$ level

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<th>III</th>
<th>IV</th>
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</thead>
<tbody>
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<td>0.001</td>
<td>0.003</td>
<td>0.005</td>
<td>0.007</td>
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</table>
**Table-1**  
Nusselt Number (Nu) at y = 1

<table>
<thead>
<tr>
<th></th>
<th>Nu1</th>
<th>Nu2</th>
<th>Nu3</th>
<th>Ra</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nu1</td>
<td>2.084</td>
<td>3.728</td>
<td>2.956</td>
<td>2.584</td>
</tr>
<tr>
<td>Nu2</td>
<td>2.040</td>
<td>3.480</td>
<td>2.228</td>
<td>2.488</td>
</tr>
<tr>
<td>Nu3</td>
<td>2.012</td>
<td>3.316</td>
<td>1.980</td>
<td>2.424</td>
</tr>
<tr>
<td>Ra</td>
<td>100</td>
<td>200</td>
<td>300</td>
<td>-100</td>
</tr>
</tbody>
</table>

**Table-2**  
Nusselt Number (Nu) at y = 1

<table>
<thead>
<tr>
<th></th>
<th>Nu1</th>
<th>Nu2</th>
<th>Nu3</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nu1</td>
<td>1.956</td>
<td>1.860</td>
<td>1.680</td>
<td>0.01</td>
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<tr>
<td>Nu2</td>
<td>1.948</td>
<td>2.020</td>
<td>1.940</td>
<td>0.03</td>
</tr>
<tr>
<td>Nu3</td>
<td>1.940</td>
<td>2.128</td>
<td>2.112</td>
<td>0.05</td>
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<tr>
<td>N</td>
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<td>0.03</td>
<td>0.05</td>
<td>.07</td>
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</tbody>
</table>

**Table-3**  
Nusselt Number (Nu) at y = -1

<table>
<thead>
<tr>
<th></th>
<th>Nu1</th>
<th>Nu2</th>
<th>Nu3</th>
<th>Ec</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nu1</td>
<td>1.956</td>
<td>2.472</td>
<td>-0.40</td>
<td>0.001</td>
</tr>
<tr>
<td>Nu2</td>
<td>2.052</td>
<td>2.004</td>
<td>-0.12</td>
<td>0.003</td>
</tr>
<tr>
<td>Nu3</td>
<td>1.940</td>
<td>1.934</td>
<td>-0.60</td>
<td>0.005</td>
</tr>
<tr>
<td>Ec</td>
<td>0.001</td>
<td>0.003</td>
<td>0.005</td>
<td>0.007</td>
</tr>
</tbody>
</table>

**Table-4**  
Nusselt Number (Nu) at y = -1

<table>
<thead>
<tr>
<th></th>
<th>Nu1</th>
<th>Nu2</th>
<th>Nu3</th>
<th>α</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nu1</td>
<td>1.988</td>
<td>1.704</td>
<td>3.896</td>
<td>2</td>
</tr>
<tr>
<td>Nu2</td>
<td>1.948</td>
<td>1.872</td>
<td>3.860</td>
<td>4</td>
</tr>
<tr>
<td>Nu3</td>
<td>1.940</td>
<td>1.984</td>
<td>3.836</td>
<td>6</td>
</tr>
<tr>
<td>α</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>-2</td>
</tr>
</tbody>
</table>

α: 2; 4; 6; -2; -4