CHAPTER 1

INTRODUCTION

1.1 SYSTEM IDENTIFICATION

The complete knowledge about a system and its environment, which is required to design a control system, is rarely available \textit{a-priori} for many practical systems, even if the structure of the system is known. The engineer often meets a situation wherein he has a reasonable confidence in the functional form of the mathematical model of a certain phenomena, but he is not sure about the numerical values of the various parameters of the model. The problem of estimating the numerical values of these parameters, from input-output data obtained by observation of the phenomenon, is known as parameter identification. The problem of system identification is generally referred to as the determination of a mathematical model for a system or a process, by observing its input-output relationships. Historically, system identification has been motivated by the need to design efficient control strategy. Very frequently, there are situations faced with the necessity of experimentally determining some important physical parameters such as heat transfer coefficient, chemical reaction rate, damping factor, mass, and so on. The need for highly accurate system models has been intensified by the development of optimal and adaptive control theories. In adaptive systems' design it is often necessary to update the values of some time varying parameters of a plant and its environment in order to maintain optimal system performance at all times. Other engineering applications of parameter estimation of dynamic systems include communication systems and fault diagnosis of physical systems.

The need for modelling arises in many other disciplines too. Workers in the field of econometrics have long sought to establish mathematical relationship between endogenous variables and exogenous variables. There has been very
significant progress in the application of system identification techniques to physiological and biomedical problems. Very successful models have been obtained for human performance in a man-machine environment, control functions of the pupil and the muscle, metabolism, brain waves, and so on. Thus the subject of system identification is attracting increasing attention from maintenance engineers, military personnel, medical and life scientists. Similar modelling applications can be found in such areas as ecology, transportation, and sociology. In addition, the availability of modern estimation theory and sophisticated computational algorithms has contributed to the rapid growth of the system identification technology.

The block diagram representation of the system identification problem is shown in Figure 1.1 with its input and output. The procedure for carrying out system identification can be divided into the following steps.

1. Specify and parameterise a class of mathematical models that represents the system to be identified.
2. Apply an appropriately chosen test signal to the system and record the input-output data.
3. Perform the parameter identification to select the model in the specified class that best fits the statistical data.
4. Perform a validation test to see if the model chosen adequately represents the system with respect to final identification objectives.
5. If the validation test is passed, the procedure ends. Otherwise, another class of models must be selected and steps 2 through 4 performed until a satisfactory model is obtained.

The consideration given in step 1 essentially concern the problem of representation. Dynamic system models can be represented in continuous-time or discrete-time. The essential difference between the two is that the signals in the system are continuous in one case and discrete in the other. The majority of system identification techniques are digitally oriented as a result of the deployment
Figure 1.1 Block diagram of system identification problem
of digital computers. Therefore, discrete system models are more convenient to deal with. In discrete domain representation the choice is between difference equations or waveform method. The algorithms dealt with in this thesis are also of discrete type. The choice depends on the identification objective and its associated input-output data.

Identification can be accomplished either off-line or on-line. In the off-line method, a record of input-output data is first obtained and then the parameters are estimated separately. In the on-line identification method, the parameters are estimated when the system is functioning or on stream or in-flight, after every measurement recursively by using some form of updating the last estimate. If the updating process is very fast, then it is possible to obtain parameter estimates of time varying systems with reasonable accuracy. On-line methods are more important because they are needed for optimal control of processes and for the analysis of biological processes. The term adaptive identification is used for on-line identification interchangeably in this thesis. On-line identifiers can be further classified into Expanding window (Expanding Memory) type identifiers and Finite data window (Fixed Memory) type identifiers. Expanding memory identification methods use all data sets right from the starting up to the present instant for computation purposes. As the new set arrives, the size of the window also increases as shown in Figure 1.2, whereas finite data window identification methods use only a limited number of data sets at any time. As the new data set arrives, the window also keeps moving forward so as to keep the window width constant, as shown in Figure 1.3, thereby including the new data and excluding the oldest one.

Important aspects to be considered when deciding on the type of Identification technique to be used, are identifier stability properties and convergence rate. However, when employed within a real time adaptive controller, the amount of computation required in order to carry out an identification (or estimation) update can be a prime consideration. Within an adaptive controller the computations necessary for the parameter identification alone can often contribute
At $k = 5$
No. of measurement = 5

At $k = 10$
No. of measurement = 10

Figure 1.2 Expanding memory type identifier
Figure 1.3  Fixed memory type identifier
the majority of the total time required, especially if an implicit control objective is being used or if the system model is of higher order. Problems can therefore be encountered when attempting to implement an adaptive controller simply because too much time is taken for the calculation to be made, with the blame lying firmly on the shoulders of the parameter identifier (estimator). This is especially true where a low cost scheme is required and/or where the available computing power is fairly restricted.

There are a number of well known parameter estimation techniques that have been successfully applied to the identification problem. These are, the methods of maximum likelihood, least squares, cross correlation, instrumental variable, stochastic approximation, minimum variance, projection technique and a variety of other optimization methods (Isermann, 1981; Mendel, 1973; Eykoff, 1974; Hsia, 1977).

Two popular identification algorithms suggested in literature (Goodwin and Sin 1984, Wen Teng Wu et al. 1988) are: (i) Projection algorithm (ii) Least squares algorithm. The most popular technique is the method of least squares (LS) for a number of important reasons (Hsia, 1977; Gustavsson et al., 1977; Astrom and Wittenmark, 1980; Young, 1984). (1) This method is classical and scientific workers in many disciplines are familiar with it. (2) It offers conceptual simplicity and applicability to a wide range of situations in which other statistical estimation theories may be difficult to apply, yet exhibits statistical properties that are as good as those of the maximum likelihood method for most practical situations. (3) This identification algorithm can be related easily to many other identification algorithms, making possible a unified treatment of the system identification problem. (4) This algorithm generally has much faster convergence than the projection algorithm. (5) The least squares algorithm can be used for noisy signals.
1.2 IDENTIFICATION REPRESENTATION

A structurally known multivariable linear discrete-data system shown in Figure 1.4 can be represented by

\[ x(k+1) = A x(k) + B u(k) \]  \hspace{3cm} (1.1)

where

- \( x(k) \) is n×1 output vector
- \( u(k) \) is r×1 input vector
- \( A \) is nxn state transition matrix
- \( B \) is the nxr control input matrix
- \( k \) is the sampling instant

It is assumed that all states are accessible for measurement. As far as the problem of identification is concerned the equation (1.1) can be rewritten as

\[ x(k+1) = S y(k) \]  \hspace{3cm} (1.2)

where

- \( S = [ A : B ] \) is a nxN matrix
- \( y^T(k) = [x^T(k):u^T(k)] \) is a composite vector consisting of n states and r inputs
- \( N = n + r \), the number of parameters in one row of \( S \) to be estimated

If the measurements are corrupted with noise, equation (1.2) can be rewritten as

\[ x_c(k+1) = S y_c(k) - S \epsilon_1(k) + \epsilon_2(k) \]  \hspace{3cm} (1.3)
Figure 1.4 Discrete data system

Figure 1.5 Identification Representation
where
\[ x_{(k+1)} \] is the actual measured output,
\[ y_n(k) \] is the measured vector,
\[ e_1(k) \] and \[ e_2(k) \] are the noise vectors.

Figure 1.5 explains the above cases.

With reference to equation (1.2) the problem of identification can be briefly stated as the estimation of the elements of \( S \) from the measurement vectors \( x \) and \( y \). Let us assume that the estimated output of the system at the next instant would be

\[ \hat{x}(k+1) = S \ y(k) \] \hspace{1cm} (1.4)

The actual output is however \( x(k+1) \), which is measured at \( (k+1)_{th} \) instant. Now the error in the estimated output would be

\[ e(k+1) = x(k+1) - \hat{x}(k+1) \] \hspace{1cm} (1.5)

Had the estimates \( S(k) \) been equal to \( S \), then \( e(k+1) \) would have been zero. In the literature (Mendel, 1973; Eykoff, 1974) this error is known as the equation error. The core function of all estimation techniques is to minimise certain appropriately defined error criterion as a means to optimally fit the model to the system data. Minimisation of equation error is the most commonly used error criteria. Other types of errors which are also used in objective functions include output error, input error and generalised error.

In general, the schemes of identification using the equation error formulation can be pictorially represented as shown in Figure 1.6. It is only the algorithm that varies from scheme to scheme.
Figure 1.6  Equation error identifier
1.3 EXPANDING MEMORY TYPE IDENTIFIERS

With reference to equation (1.2) the problem of identification can be briefly stated as the estimation of the elements of $S$ from the measurement vectors $x$ and $y$. The estimation of $S$ can be obtained by taking the least squares solution for equation (1.2). The estimation of $S$ can be studied from two points of view (i) classical batch processing approach and (ii) modern sequential processing approach. As the interest in this thesis is only in adaptive techniques, the sequential approach considered here, can be either expanding memory type or fixed memory type approach described in the literature (Mendel, 1973).

For the representation shown in Figure 1.5 the equation set at $k^{th}$ instant, is

$$
\begin{align*}
\mathbf{x}(1) &= S \mathbf{y}(0) \\
\mathbf{x}(2) &= S \mathbf{y}(1) \\
\vdots & \hspace{1cm} \vdots \\
\vdots & \hspace{1cm} \vdots \\
\mathbf{x}(k) &= S \mathbf{y}(k-1)
\end{align*}
$$

Equation (1.6) is written in matrix form as

$$
\mathbf{X}(k) = S \mathbf{G}(k)
$$

where

$$
\begin{align*}
\mathbf{X}(k) &= [\mathbf{x}(1):\mathbf{x}(2):...:\mathbf{x}(k)] \\
(\mathbf{n} \times \mathbf{k}) \\
\mathbf{G}(k) &= [\mathbf{y}(0):\mathbf{y}(1):...:\mathbf{y}(k-1)] \\
(\mathbf{N} \times \mathbf{k})
\end{align*}
$$
In the least squares method any estimate can be attempted only after $k$ exceeds the value $N$. The least squares solution of equation (1.7) is

$$S(k) = X(k) G^T(k) [G(k) G^T(k)]^{-1}$$

(1.10)

At the next iteration,

$$S(k+1) = X(k+1) G^T(k+1)[G(k+1) G^T(k+1)]^{-1}$$

(1.11)

where

$$G(k+1) = [G(k);y(k)]$$

(1.12)

$$X(k+1) = [X(k);x(k+1)]$$

(1.13)

It is clear that the dimension of $G(k+1)$ has increased.

If

$$P(k) = [G(k) G^T(k)]^{-1}$$

(1.14)

and

$$P(k+1) = [G(k+1) G^T(k+1)]^{-1}$$

(1.15)

then, the following recursive relationship could be derived

$$S(k+1) = S(k) + e(k+1) y^T(k) P(k)$$

$$1 + y^T(k) P(k) y(k)$$

(1.16)

$$= S(k) + e(k+1) y^T(k) P(k+1)$$

(1.17)

$$P(k+1) = P(k) - \frac{P(k) y(k) y^T(k) P(k)}{1 + y^T(k) P(k) y(k)}$$

(1.18)
where
\[ x(k+1) = S(k) y(k) \]  \hspace{1cm} (1.19)
and
\[ e(k+1) = x(k+1) - \hat{x}(k+1) \]  \hspace{1cm} (1.20)

The new estimate is given by the old estimate plus a correction term based on the equation error, the updated P(k+1) and the observation vector y(k).

Equation (1.10) is the least squares solution for the system represented by equation (1.7). Least Squares theory was first proposed by Karl Gauss (1809) for carrying out his work in orbit prediction of planets. Least squares theory has since become a major tool for parameter estimation from experimental data. Equation (1.10) is known as the batch least squares (off-line) algorithm according to Ljung and Soderstron (1983). The need for a recursive solution arises when fresh experimental data are continuously in supply and we wish to improve parameter estimates by making use of new information. With a recursive formula, the estimates can be updated step by step without repeatedly computing matrix solution of equation (1.7), in which the matrix inversion is quite time consuming. This recursive solution procedure is often referred to as Sequential or On-line estimation or Recursive Least Squares (RLS) estimation. The RLS algorithm (1.16) - (1.18) appears to have been derived by Plackett (1950).

The RLS algorithm initially has a much faster convergence. The basic difficulty with the RLS method is that the covariance matrix P gradually decays to a small value and therefore the algorithm does not retain its alertness or adaptivity. This is easily seen from the correction term on the right hand side of equation (1.18). This term is always positive or zero. If there is sufficient new information it is positive and consequently P(k+1) becomes smaller, and if there is no new information P(k) y(k) = 0. Hence, the RLS in its simplest form cannot track time varying parameters. Many modifications have been suggested in the RLS algorithm as listed in Warwick (1988) to reduce the effect of past data, to improve its alertness and to track the time varying parameters.
The RLS method and its various modifications use the influence of past measurements right from the starting up to the present instant either completely or partially. Hence they are termed as Expanding memory type identifiers (Mendel, 1973). An alternative way to deal with the time varying systems is to use the finite data type identifiers as suggested by Goodwin and Payne (1977), and Goodwin and Sin (1984). Finite Data Window Identification Algorithms which are of more interest in this thesis, are discussed in the following sections.

1.4 FINITE DATA WINDOW IDENTIFICATION ALGORITHMS

The philosophy in finite data window identification technique is to discard the old data as the new measurement comes in and at the same time keep the algorithm alive. There are several alternative ways of achieving this as reported in Goodwin and Sin (1984). They are:

a) The data prior to some point can be periodically discarded.

b) A moving window of fixed length can be used by throwing away an old data point each time a new one is added.

c) The memory length can be varied by adding and discarding data in some pattern.

The method proposed by Renganathan (1981), is known as Rectangular window type Fixed Memory (FM) Identifier. In FM identifier, as the new set data arrives the oldest set is deleted keeping only the 'm' number of data set which is greater than the number of parameters 'N' to be estimated. All the measurements within the window are given equal weightage. FM identification algorithm is given by

\[
R(k) = P(k) - \frac{P(k) y(k) y^T(k) P(k)}{1 + y^T(k) P(k)y(k)}
\]

(1.21)
P(k+1) = R(k) + \frac{R(k) y(k-m) y^T(k-m) R(k)}{1 - y^T(k-m) R(k) y(k-m)} \tag{1.22}

S(k+1) = S(k) + e(k) y^T(k) P(k+1) - e_m(k) y^T(k-m) P(k+1) \tag{1.23}

where

\begin{align*}
e(k+1) & = x(k+1) - S(k)y(k) = x(k+1) - \hat{x}(k+1) = e(k) & \tag{1.24} \\
e(k-m+1) & = x(k-m+1) - S(k)y(k-m) \\
 & = x(k-m+1) - \hat{x}(k-m+1) = e_m(k) & \tag{1.25}
\end{align*}

The procedure suggested by Goodwin and Payne (1974) is to add the new data and then delete the old data so as to keep the length of the window constant. This involves two stages.

In the first stage, the latest observation is added and an estimate based on observation between k and k+m is obtained as

\begin{align*}
P'(k+m) & = P'(k+m-1) - \frac{P'(k+m-1)y(k+m)y^T(k+m)P'(k+m-1)}{1 + y^T(k+m)P'(k+m-1)y(k+m)} & \tag{1.26} \\
S'(k+m) & = S(k+m-1) + (x(k+m)-S(k+m-1)y(k+m-1))y^T(k+m-1)P(k+m-1) & \tag{1.27}
\end{align*}

In the second stage, the data at the kth instant is discarded and the estimation of the parameters is based on observation between k+1 and k+m

\begin{align*}
P(k+m) & = P'(k+m) - \frac{P'(k+m)y(k)y^T(k)P'(k+m)}{1 + y^T(k)P(k+m)y(k)} & \tag{1.28} \\
S(k+m) & = S'(k+m) + (x(k+1)-S'(k+m)y(k))y^T(k)P(k+m) & \tag{1.29}
\end{align*}
The above method requires that the last 'm' data points be stored. For large window sizes, it may be preferable to use an oscillating memory estimator, since this will usually result in a considerable decrease in the storage requirements as reported by Jazurnski (1970). The oscillating memory estimator discards old data in blocks of 'm'. The effective memory varies between 'm' and '2m'. The data sets are added and when the window length of '2m' is reached, the first 'm' data are simply discarded. Thus to discard 'm' points all that is needed, is to store the estimate \( S(k, k+m) \) and the covariance \( P(k, k+m) \), rather than the points themselves.

The Weighted Moving (WM) identification algorithm proposed by Wen Teng Wu et al. (1987 & 1988) is similar to the one reported by Goodwin & Payne (1974). Here, also two steps are involved. In the first step the old data set is discarded and in the second step the new measurement is added. But, while discarding the old data set, a selective weight is given for the discarded data set and an exponential weight is given to the new data set that is added.

For the first step, the intermediate estimated parameter vector \( S'(k+1) \) is obtained as

\[
S'(k+1) = S(k) - \frac{\alpha(k)P(k)y(k-m)[x(k-m+1)-y^T(k-m)S(k)]}{1 - \alpha(k)y^T(k-m)P(k)y(k-m)} \tag{1.30}
\]

where \( \alpha(k) \) is a selective data weighting factor for the old data.

\[
P'(k+1) = P(k) + \frac{\alpha(k)P(k)y(k-m)y^T(k-m)P(k)}{1 - \alpha(k)y^T(k-m)P(k)y(k-m)} \tag{1.31}
\]
The second step gives the desired parameters $S(k+1)$ as

$$S(k+1) = S'(k+1) + \frac{P'(k+1)y(k)[x(k+1) - y^T(k)S'(k+1)]}{\beta(k) + y^T(k)P'(k+1)y(k)}$$

(1.32)

where $\beta(k)$ is the exponential data weighting factor that gives more weight to the more recent data.

$$P(k+1) = \frac{1}{\beta(k)} \left\{ P'(k+1) - \frac{P'(k+1)y(k)y^T(k)P'(k+1)}{\beta(k) + y^T(k)P'(k+1)y(k)} \right\}$$

(1.33)

1.5 AIM OF THE THESIS

The objectives of this thesis are:

i) to study FM and WM identification algorithms in depth.

ii) to improve their numerical accuracies through the UD factorisation technique.

iii) to improve their transient characteristics by introducing a new by-pass scheme in estimates.

iv) to apply them for identifying parameters of a passive oscillatory system and to determine the weight of materials transported on a conveyor belt system.

vi) to apply them for the design of adaptive observers both for SISO & MIMO systems.

vii) to apply them for the design of adaptive controllers based on

a) explicit type (indirect) closed loop pole assignment law

b) implicit type (direct) minimum variance law

c) state variable feedback pole placement law
1.6 OUTLINE OF THE THESIS

The derivation of FM and WM techniques (algorithms) with convergence analysis for FM technique are made in chapter 2. In chapter 3, FM and WM algorithms' convergence and tracking properties are studied on a simulated system. UD factorisation technique is applied to update the covariance matrix $P$ to improve the algorithms' numerical accuracies. The necessary proofs for the proposed algorithms are also included in chapter 3 with the computer simulation results. A new technique to reduce overshoots and undershoots in estimates so as to improve the transient behaviour of algorithms is suggested in chapter 4 with necessary proof and computer simulation results.

The improved FM and WM identification algorithms are applied for the estimation of parameters of a passive oscillatory system and to determine the weight of materials transported on a conveyor belt system which are explained along with the results in chapter 5.

In chapter 6, the application of the improved algorithms to the design of an adaptive observer for a SISO system is described. The state variable filter structure required to reconstruct the states of the system is also analysed in that chapter with computer simulation results. In chapter 7, the application to the design of an adaptive observer is extended to MIMO system.

In chapter 8, the adaptive observer developed in chapter 6 is applied to design a pole placement adaptive controller based on state variable feedback approach. The design of an explicit type (indirect) self tuning controller based on pole placement law and an implicit self tuning controller based on minimum variance law, are described in the same chapter with the computer simulation results.

Finally the conclusion is given in chapter 9 with suggestions for scope for future research.