5.1 INTRODUCTION

The application of identification procedure for the estimation of parameters and determination of mass in a passive oscillatory system is gaining importance due to the increased use of automatic weight measurements in packaging, food processing, mining, power and other process industries. In oscillatory second order systems, steady state is reached after a long time, only after which the parameters of the system can be measured. If the parameter determination is to be done faster, then the measurements are to be taken at a predetermined number of closely spaced instants of time and the mass, one of the parameters of such a system is estimated even while the system is oscillating as suggested by Jost (1980). Usually, the measurements are taken and arranged in a rectangular matrix and the pseudo inverse is computed. By further processing, it is possible to arrive at the best estimate of the parameters and thus the weight, in the least squares sense. The best estimate will be equal to the true value, if the measurements are not corrupted by noise and also if numerical errors do not creep into the computation. This conventional procedure, though quite ingenious, is off-line in character. If the computation time is not sufficiently longer than the period of oscillations, this method may yield satisfactory results. Further a drawback of this conventional procedure is the necessity of including a matrix inversion subroutine in the software which may demand excessive memory space in certain cases.
Using FM and WM algorithms most of the above limitations can be removed. In the above procedures the data can be processed as and when they are supplied to the computer by the measuring device, without waiting to collect the entire data set. The usual RLS schemes do not track suddenly varying parameters efficiently because of the persistence of the effect of past measurements and initial estimates. Modified form of RLS schemes also do not completely wipe out the past measurements. To completely wipe out the past effect in systems with abrupt changes in parameters, the FM and WM algorithms can be utilised. These identification schemes have a good convergent property. Owing to the computer round off error, their efficiencies are degraded causing numerical instability particularly when the measurements are corrupted with noise. Hence, UD factorisation technique known to have less computational error is combined with FM and WM identification algorithms to update the covariance matrix which minimises the numerical instability, thereby improving the convergence properties of the algorithms particularly when the parameters change after the initial convergence takes place. In this thesis, simulation results obtained for a passive oscillatory system, modelled as a second order Mass-Damper-Spring system using FM and WM Identification algorithms are presented. The experimental results obtained on a real 'Conveyor belt system' for determining the total weight of the solid material transported on it using FM and WM algorithms are also presented in this chapter.

5.2 THE PASSIVE OSCILLATORY SYSTEM

The passive oscillatory system considered here is a Mass - Damper-Spring system. Mass - Damper - Spring system is modelled as a second order oscillatory system. The FM and WM adaptive identification algorithms combined with UD factorisation (UDF) technique are applied to identify the parameters and then to determine the mass (weight).
Figure 5.1 Second order oscillatory system with computer connection

The second order oscillatory system with computer connection shown in the Figure 5.1, is considered for mass measurement. This system is described by the differential equation as

\[ (M \dddot{x} + B \dot{x} + K) x(t) = M a u(t) \]
where

\( x(t) \) is the displacement of the mass
\( u(t) \) is the unit step function
\( a_g \) is the acceleration due to gravity
\( D \) is the differential operator \( (d/dt) \)
\( M \) is the mass
\( B \) is the damping coefficient
\( K \) is the spring constant

Since the strain gauge is used to measure the displacement, the force acting on the strain gauge is

\[ f(t) = R_s x(t) \]  \( (5.2) \)

where \( R_s \) is the strain gauge system constant.

The modified differential equation in terms of the force acting on the strain gauge is

\[ (M_sD^2 + B_sD + K_s)f(t) = R_sM_s a_g u(t) \]  \( (5.3) \)

In the Laplace domain, the above equation can be written as

\[ (M_s s^2 + B_s s + K_s)F(s) = R_s M_s a_g U(s) \]

\[ H(s) = \frac{F(s)}{U(s)} = \frac{R_s M_s a_g}{M_s s^2 + B_s s + K_s} \]

\[ = \frac{R_s a_g}{s^2 + B_s s/M_s + K_s/M_s} \]

\[ = \frac{R_s a_g}{s^2 + 2c_s s + c_2} \]  \( (5.4) \)
where
\[ c_1 = \frac{B_o}{2M_s} \]
\[ c_2 = \frac{K_s}{M_s} \]

The Z-Transform of the above system with sampler and zero order holding device can be written as

\[ H(z) = \frac{A_1z^{-1} + A_2z^{-2}}{1-A_2z^{-1} - A_4z^{-2}} \] (5.5)

where
\[ A_1 = -e^{-\omega T} \]
\[ A_2 = -2e^{-\omega T} \cos \omega T \]
\[ A_3 = M_a(N_1e^{-\omega T}) \]
\[ -e^{-\omega T} \{ \cos \omega T \cdot (c_1/\omega) \sin \omega T \} \] (5.6)
\[ A_4 = M_a[N_1(1-e^{-\omega T}) \cos \omega T + (c_1/\omega) \sin \omega T] \]
\[ \omega^2 = c_2 - c_1^2 \]
\[ T = \text{sampling time} \]

In the discrete time domain, the system obeys the following ARMA model.

\[ f(k) = A_2 f(k-1) + A_1 f(k-2) + A_4 u(k-1) + A_3 u(k-2) \] (5.7)

The system is subjected to a constant force due to gravity. This fact gets translated in the Z-domain as

\[ u(k-2) = u(k-1) = 1.0 \text{ for all } k. \]

Hence, the equation (5.7) can be written as

\[ f(k) = A_2 f(k-1) + A_1 f(k-2) + A_3 \] (5.8)
where

\[ A_{34} = A_3 + A_4 \]

The output at the \( k \)th instant is

\[ y(k) = f(k) \quad (5.9) \]

The measurement vector at the \( k \)th instant is

\[
g(k) = \begin{bmatrix} f(k-2) \\ f(k-1) \\ 1.0 \end{bmatrix} \quad (5.10)
\]

The parameter vector is

\[
s = \begin{bmatrix} A_1 \\ A_2 \\ A_{34} \end{bmatrix}
\]

This problem can be put into the matrix notation as

\[ y(k) = s^T g(k) \quad (5.11) \]

To solve the equation (5.11), it is sufficient if 3 measurements are taken when there is no noise in measurement. When measurements are corrupted by zero mean random noise, more measurements than the minimum 3 would be necessary. Therefore, the algorithmic equations of FM and WM identification schemes updated through UD factorisation technique using \( m \) measurements where \( m > N \) are used.
5.3 ALGORITHM

Let

\[ g_1 = g(k-m+1) = \begin{bmatrix} f(k-m+1) \\ f(k-m) \\ 1.0 \end{bmatrix} \]  

(5.12)

is the measurement at the \((k-m)\)th instant and

\[ g_2 = g(k+1) = \begin{bmatrix} f(k-1) \\ f(k) \\ 1.0 \end{bmatrix} \]  

(5.13)

is the measurement at the \(k\)th instant

\[ G(k) = \begin{bmatrix} g(k-m+1); g(k-m+2); \ldots; g(k) \end{bmatrix} \]  

(5.14)

\[ P(k) = G(k)G^T(k)^{-1} \]  

(5.15)

5.3.1 FM algorithm

The FM identification algorithmic steps can be written down for this problem in the following manner:

\[ R(k) = P(k) - \frac{P(k)g_2g_2^T P(k)}{1 + g_2^T P(k)g_2} \]  

(5.16)

\[ P(k+1) = R(k) + \frac{R(k)g_1g_1^T R(k)}{1 - g_1^T R(k)g_1} \]  

(5.17)

\[ s(k+1) = s(k) + P(k+1) \{ g_2 e(k+1) - g_1 e(k-m+1) \} \]  

(5.18)
where
\[ e(k+1) = y(k+1) - s^T(k)g_2 \]
\[ e(k-m+1) = y(k-m+1) - s^T(k)g_1 \]

5.3.2 WM algorithm

The WM identification algorithmic steps can be written down for this problem in the following manner.

For the first step, the intermediate estimate is obtained as

\[ s^*(k+1) = s(k) - \frac{\alpha(k)P(k)g_1 e(k-m+1)}{(1 - \alpha(k)g_i^T P(k)g_i)} \]  \hspace{1cm} (5.19)

Where \( \alpha(k) \) is a selective data weightage factor for the old data. The choice of selection \( \alpha(k) \) is same as that discussed in chapter 2.

\[ P^*(k+1) = P(k) + \frac{\alpha(k)P(k)g_1 g_i^T P(k)}{(1 - g_i^T P(k)g_i)} \]  \hspace{1cm} (5.20)

The second step gives the desired parameters \( s(k+1) \) as

\[ s(k+1) = s^*(k+1) + \frac{P^*(k+1)g_2 e^*(k-1)}{\{\beta(k) + g_2^T P^*(k+1)g_2\}} \]  \hspace{1cm} (5.21)

where \( \beta(k) \) is the exponential data weighing factor that gives more weight to more recent data. The choice of selection of \( \beta(k) \) is same as that discussed in chapter 2. and
\text{e}^{T}(k+1) = y(k+1) - s^{T}(k+1)g_{2}

\begin{equation}
\begin{split}
P(k+1) &= \frac{1}{\beta(k)} \left[ P'(k+1) - \frac{P'(k+1)g_{2}g_{2}^{T}P'(k+1)}{\beta(k) + g_{2}^{T}P'(k+1)g_{2}} \right] \tag{5.22}
\end{split}
\end{equation}

\textbf{5.4 SIMULATION RESULTS}

\textbf{5.4.1 Simulation}

The oscillatory system with
mass \(M\), variable in steps: 5, 10 and 15 Kg.
Spring constant \(K\): 400 N/m
Damping coefficient \(B\): 5 N - s/m

was simulated on a PC/AT. FM and WM identification algorithms were used for
each arrival of the measurement vector \(g\), whose form is given equation (5.10) (see
Figure 5.2). Windows of width 7 was used for simulation. \(\beta_{\text{wm}}\) in WM chosen was
0.65.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure5_2.png}
\caption{Measuring Realisation}
\end{figure}
The sampling interval of 0.15 second was tried with noise free case and also for measurements corrupted by random noise (2% of zero mean random noise of magnitude ±0.5).

The mass calculation was done after estimating the parameters \( A_1, A_2 \) and \( A_{34} \) using the relation

\[
M_a = \frac{1}{a_k} \frac{A_{34}}{1 - A_2 - A_1}
\]  

(5.23)

Sudden step change in weights were given in all cases after the parameters converged to the new true values.

5.4.2 Scaling

The output of the strain gauge was scaled down by a suitable factor, so that the magnitude of the signal \( f(k) \) used for computation was comparable to unity. Correspondingly at the end of the computation, descaling was applied to the coefficient \( A_{34} \) so that the true values of the mass, spring constant and damping coefficient could be obtained.

5.4.3 Result

The results are shown on the graphs. Figure 5.3 show the output of the simulated system. Figures 5.4a&5.4b show the parameter estimates and mass using FM algorithm under noise free case. Figures 5.5a&5.5b show the estimate of parameters and mass using FM under noisy condition (2% of random noise of magnitude ±0.5 was added to the output). Figures 5.6a and 5.6b show the estimate of parameters and mass under noise free case using FM with by-pass scheme. Figures 5.7a&5.7b show the estimate of parameters and mass under noise free condition using WM algorithm. Figures 5.8a&5.8b show the estimate of parameters and mass using WM under noisy condition. Figures 5.9a&5.9b show the estimate of parameters and mass using WM under noise free condition with by-pass scheme. Results of the by-pass scheme are better than original ones.
Figure 5.3  Response of the simulated system (SG output)  $T = 0.15$ sec
Mass changed at 50th and 100th sampling instants
Figure 5.4 Estimates of the simulated system using FM (T = 0.15 sec) a) Parameters b) Mass
Mass changed at 50th and 100th sampling instants
Figure 5.5 Estimates of the simulated system under 2% noise condition using FM (T = 0.15 sec) a) Parameters b) Mass
Mass changed at 50th and 100th sampling instants
Figure 5.6 Estimates of the simulated system using FM with by-pass scheme
(T = 0.15 sec) a) Parameters b) Mass
Mass changed at 50th and 100th sampling instants
Figure 5.7  Estimates of Simulated system using WM (T = 0.15 sec)  
Mass changed at 50th and 100th sampling instants
Figure 5.8  Estimates of Simulated system under 2% noise condition using WM (T = 0.15 sec) Mass changed at 50th and 100th sampling instants
Figure 5.9 Estimates of simulated system using WM with by-pass scheme
(T = 0.15 sec) Mass changed at 50\textsuperscript{th} and 100\textsuperscript{th} sampling instants
5.5 CONVEYOR BELT SYSTEM

To substantiate the practical applicability of FM and WM algorithms, the above procedures were applied to a practical system. A conveyor belt system in Figure 5.10 was used. A strain gauge (SG) type load cell was used to measure the weight (mass) of the material transported on it. In our case, this system was used to verify the algorithms' convergence property. The load cell (strain gauge) output proportional to load applied on the weighing platform of the system was taken and amplified by using an instrumentation amplifier. The amplified signal was then interfaced to a PC through an ADC card (Burr-Brown PCI-2000M Personal Computer intelligent instrumentation system) which contains a 16 channel ADC card and an analog terminal block.

5.5.1 Experiment

For the belt at rest initially a 2 Kg weight was dropped on the weighing platform of the system 3 seconds after starting the system. The strain gauge output corresponding to the force acting on the load cell was amplified. The discretised data acquired by LABTECH NOTEBOOK, a menu driven software, was used in FM and WM algorithms to estimate parameters. The sampling time chosen was 0.1 second. At the 19th second, a 1 Kg weight was added. Figure 5.11 shows the output of the strain gauge load cell and the estimate of mass using FM algorithm. Figure 5.12 shows the output of the strain gauge load cell and the estimate of mass using WM algorithm.

For the determination of mass flow rate for grains (wheat) transported on the conveyor belt, the belt speed was maintained constant at 46 cm/min. Also, constant mass flow was maintained by keeping the head constant at 50 cm. The mass flow rate was determined by using the formula $Q = MS/L$ Kg/min, where, $M$ is the mass (weight) of material on section of length $L$ and $S$ is the speed of the belt. Also, the total amount of material transported along the conveyor belt was determined. 3000 data at a sampling time of 0.1 second were used. The grain flow
started increasing at 71st sampling instant, the flow dropped at 1832nd sampling instant and then increased at 2300th sampling instant. Figure 5.13 shows the strain gauge (SG) output and the estimated mass flow rate using FM and WM algorithms. Figure 5.14 shows the total mass transported over 300 seconds.

The total mass transported was 8.20 Kg, and the estimated one was 8.14 Kg.

5.6 CONCLUSION

In this chapter, a second order passive oscillatory system has been simulated. Its unknown parameters have been estimated using FM and WM algorithms. These algorithms have been applied to determine the weight of the material transported on a real conveyor belt system available in the laboratory. The results are given and they are found to be satisfactory.
Figure 5.10 Conveyor belt system
Figure 5.11 Strain gauge (SG) output and estimate of mass in conveyor belt system at rest using FM ($T = 0.1$ sec)

Mass changed at 190th sampling instant
Figure 5.12 Strain gauge (SG) output and estimate of mass in conveyer belt system at rest using WM (T = 0.1 sec)
Mass changed at 190th sampling instant
Figure 5.13 Strain gauge (SG) output and estimate of mass flow rate of grain transported by the conveyor belt system using FM and WM ($T = 0.1$ sec).

Mass flow rate changed at about 71st, 1832nd, and 2300th sampling instants.
Figure 5.14 Total mass transported by the conveyor belt system using FM and WM (T = 0.1 sec)