APPENDIX 1

COMPUTATION OF STATE TRANSITION AND CONTROL TRANSITION MATRICES

Let us consider the continuous-time state space model

$$
\dot{X}(t) = AX(t) + BU(t); \ X(0) = X_0
$$

(A1.1)

Equation (A1.1) is solved and its discrete-time equivalent is obtained (Nagrath and Gopal 1991, Katsuhiko Ogata 1991 and Benjamin C. Kuo 1981) as given below:

$$
X(k) = \phi X(k-1) + \Gamma U(k-1); \ X(0) = X_0
$$

(A1.2)

where $\phi$ is the State Transition matrix and $\Gamma$ is the Control Transition matrix. One of the familiar methods of computing $\phi$ and $\Gamma$ is to evaluate the following:

$$
\phi = L^{-1}[(sI - A)^{-1}]_{t=\tau}
$$

(A1.3)

$$
\Gamma = \int_{0}^{\tau} e^{\lambda(t-s)}Bd\lambda
$$

(A1.4)

where $L^{-1}$ denotes the Inverse Laplace Transform operator and $\tau$ is the sampling period. We shall apply this method for the computation of $\phi$ and $\Gamma$ in the Boiler subsystem models.
A 1.1 COMPUTATION OF STATE TRANSITION MATRIX

Let us consider the linearized state space equation (3.19) for the boiler furnace where the dimension of matrix $A$ is 2x2 and that of $B$ is 2x5 and are represented as follows for convenience

$$
A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} B_{11} & B_{12} & B_{13} & B_{14} & B_{15} \\ B_{21} & B_{22} & B_{23} & B_{24} & B_{25} \end{bmatrix}
$$

Now, $(sI-A)^{-1} = \frac{1}{(s-\beta_1)(s-\beta_2)} \begin{bmatrix} s-A_{22} & A_{12} \\ A_{21} & s-A_{11} \end{bmatrix}$ \hspace{1cm} (A1.5)

where $\beta_1 = \frac{1}{2} \left[ (A_{11} + A_{22}) + \sqrt{(A_{11} + A_{22})^2 - 4(A_{11}A_{22} - A_{12}A_{21})} \right]$ and $\beta_2 = \frac{1}{2} \left[ (A_{11} + A_{22}) - \sqrt{(A_{11} + A_{22})^2 - 4(A_{11}A_{22} - A_{12}A_{21})} \right]$

Equation (A1.3) in the light of equation (A1.5) yields

$$
\phi = \frac{1}{\beta_1 - \beta_2} \begin{bmatrix} (\beta_1 - A_{22})e^{\beta_1 \tau} - (\beta_2 - A_{22})e^{\beta_2 \tau} & A_{12} (e^{\beta_1 \tau} - e^{\beta_2 \tau}) \\ A_{21} (e^{\beta_1 \tau} - e^{\beta_2 \tau}) & (\beta_1 - A_{11})e^{\beta_1 \tau} - (\beta_2 - A_{11})e^{\beta_2 \tau} \end{bmatrix}
$$

(A1.6)

A 1.2 COMPUTATION OF CONTROL TRANSITION MATRIX

We know that $e^{A\lambda} = L^{-1}[(sI-A)^{-1}]_{\lambda}$ \hspace{1cm} (A1.7)

Let $a_1 = \frac{1 - e^{\beta_1 \tau}}{\beta_1}$ and $a_2 = \frac{1 - e^{\beta_2 \tau}}{\beta_2}$
Using equation (A1.7), the integral equation (A1.4) is evaluated and \( \Gamma \) is obtained in the form

\[
\Gamma = \begin{bmatrix}
\Gamma_{11} & \Gamma_{12} & \Gamma_{13} & \Gamma_{14} & \Gamma_{15} \\
\Gamma_{21} & \Gamma_{22} & \Gamma_{23} & \Gamma_{24} & \Gamma_{25}
\end{bmatrix}
\]

where

\[
\Gamma_{11} = \frac{1}{\beta_1 - \beta_2} \left\{ [B_{21}A_{12} + (\beta_2 - A_{22}) B_{121}]a_2 - [B_{21}A_{12} + (\beta_1 - A_{22}) B_{111}]a_1 \right\}
\]

\[
\Gamma_{12} = \frac{1}{\beta_1 - \beta_2} \left\{ [B_{22}A_{12} + (\beta_2 - A_{22}) B_{122}]a_2 - [B_{22}A_{12} + (\beta_1 - A_{22}) B_{122}]a_1 \right\}
\]

\[
\Gamma_{13} = \frac{1}{\beta_1 - \beta_2} \left\{ [B_{23}A_{12} + (\beta_2 - A_{22}) B_{123}]a_2 - [B_{23}A_{12} + (\beta_1 - A_{22}) B_{123}]a_1 \right\}
\]

\[
\Gamma_{14} = \frac{1}{\beta_1 - \beta_2} \left\{ [B_{24}A_{12} + (\beta_2 - A_{22}) B_{124}]a_2 - [B_{24}A_{12} + (\beta_1 - A_{22}) B_{124}]a_1 \right\}
\]

\[
\Gamma_{15} = \frac{1}{\beta_1 - \beta_2} \left\{ [B_{25}A_{12} + (\beta_2 - A_{22}) B_{125}]a_2 - [B_{25}A_{12} + (\beta_1 - A_{22}) B_{125}]a_1 \right\}
\]

\[
\Gamma_{21} = \frac{1}{\beta_1 - \beta_2} \left\{ [B_{11}A_{21} + (\beta_2 - A_{11}) B_{211}]a_2 - [B_{11}A_{21} + (\beta_1 - A_{11}) B_{211}]a_1 \right\}
\]

\[
\Gamma_{22} = \frac{1}{\beta_1 - \beta_2} \left\{ [B_{12}A_{21} + (\beta_2 - A_{11}) B_{221}]a_2 - [B_{12}A_{21} + (\beta_1 - A_{11}) B_{221}]a_1 \right\}
\]

\[
\Gamma_{23} = \frac{1}{\beta_1 - \beta_2} \left\{ [B_{13}A_{21} + (\beta_2 - A_{11}) B_{231}]a_2 - [B_{13}A_{21} + (\beta_1 - A_{11}) B_{231}]a_1 \right\}
\]

\[
\Gamma_{24} = \frac{1}{\beta_1 - \beta_2} \left\{ [B_{14}A_{21} + (\beta_2 - A_{11}) B_{241}]a_2 - [B_{14}A_{21} + (\beta_1 - A_{11}) B_{241}]a_1 \right\}
\]

\[
\Gamma_{25} = \frac{1}{\beta_1 - \beta_2} \left\{ [B_{15}A_{21} + (\beta_2 - A_{11}) B_{251}]a_2 - [B_{15}A_{21} + (\beta_1 - A_{11}) B_{251}]a_1 \right\}
\]