CHAPTER III

ON THE ANALYSIS OF THE CRUSHING SYSTEMS OF A CRUSHING PLANT OF THE MINING AREA

3.1 INTRODUCTION

During the last three decades, reliability technology has been developed in order to use it in technological fields. It is most frequently used in the development of electrical and electronic equipments. Further now a days most of the problems of transportation and industries are also studied with the help of this technology. For bibliographies see references [22,26]. Recently Kumar et al [129] have studied a feeding system of a sugar mill and evaluated mean time to system failure and availability of the system by using supplementary variable technique. Kochar et al [117] have developed a systematic method for investment decision on an addional equipment to form a standby or redundant system in a production system. They [118] have also analysed the electric motor system for irrigation and obtained reliability of the system. Dhillon et al [27,28] and Natesan et al [163] have studied different pulverizer systems by using supplementary variable technique and obtained several characteristics of the system under study. However very few attentions have been given for the mathematical formulations and analysis of the industrial systems. Since industries are very helpful for the fast development of the nation, so its problems should be studied on the top priority basis. This chapter is devoted to the study of two models C and D relating to the crushing plant of Bailadila iron ore mining area which is situated in Kandul (M.P.). Model C describes a crushing plant having one appron feeder, one grizzly and one primary Gyratory crusher. Model D deals with the crushing plant having two
identical parallel crushing subsystems.

3.2 MODEL C

The purpose of the present model C is to study a system of the mining area which is used to construct Iron ores. Although there are many functional units in iron ore system, we have taken the main part containing the Appron feeder, Grizzly and one Primary Gyratory Crusher. Appron feeder holds the quarried stones. It is just installed before the crusher as a quantity controller. It is a high efficient feeder capable of feeding the Primary crusher with feed material in a constant quantity as much as possible. This promotes the efficiency of crusher and at the same time raising up fully the over all working ratio of the crushing plant. Grizzly works as a passage medium for the quarried stones from the feeder to the crusher. Primary crusher crushes the quarried stones into different sizes. In an iron ore system, iron ore are produced through some processes. In this system first of all stones in a specific amount are sent to crushing machines with the help of Grizzly. After crushing process is over, crushed stones are washed to get pure ores. The production is adversely affected if the feeding system fails. In practice, only crushing machine fails most frequently so same one is taken into standby in order to make the system up to a maximum time. Since functioning of these systems affects the production considerably, so they require more attention for study. Using regeneration point technique, several measures of system effectiveness are obtained.

(a) SYSTEM DESCRIPTION AND ASSUMPTIONS

(i) The crushing system consists of one Appron feeder, one Grizzly and two Primary Gyratory Crusher in cold standby configuration.

(ii) Whenever Appron feeder or Grizzly fails, system fails completely because failure of any one of them, stops the supply
of stones to crusher.

(iii) After a random time (maximum operation time) system is stopped for preventive maintenance.

(iv) Failure and repair time of the units are taken to be negative exponential while as time to preventive maintenance is taken to be arbitrary.

(v) Switch is perfect and instantaneous.

(vi) There is a single repair facility which is used to repair the failed units. Priority in repair is given to Appron feeder and Grizzly over crushing unit.

(vii) After repair unit works as new.

(b) NOTATION

\[ c_1, r_1 \quad \text{constant failure/repair rate of } A \]

\[ c_2, r_2 \quad \text{constant failure/repair rate of } G \]

\[ c_3, r_3 \quad \text{constant failure/repair rate of } C \]

\[ a(t), A(t) \quad \text{pdf and cdf time until preventive maintenance} \]

\[ b(t), B(t) \quad \text{pdf and cdf of time to accomplish preventive maintenance} \]

\[ A_{ij} = (1-p_{ij}), q_{ij}q_{jk} = q_{ijk}, q_{ij}q_{ji} = \bar{q}_{ij}, (1-q_{ij}) = \bar{B}_{ij} \]

(c) SYMBOLS USED FOR THE STATES OF THE SYSTEM

\[ A_0, A_r \quad \text{Appron feeder is under operation/good and non-operative/under repair} \]

\[ G_0, G_r \quad \text{Grizzly is under operation/good and non-operative/under repair} \]

\[ C_0, C_s, C_g \quad \text{Primary Gyratory crusher is under operation/standby/good and non-operative} \]

\[ C_r, C_{wr} \quad \text{Primary Gyratory Crusher is under repair/waiting for repair} \]

SUPM \quad \text{System under preventive maintenance}
Considering these symbols the system may be in any one of the following states.

(d) Up states

\[ S_0 = \begin{pmatrix} A_0 & G_0 \\ C_0 & C_S \end{pmatrix}, \quad S_1 = \begin{pmatrix} A_0 & G_0 \\ C_r & C_0 \end{pmatrix} \]

(e) Failed States

\[ S_2 = \begin{pmatrix} A_r & G_g \\ C_g & C_S \end{pmatrix}, \quad S_3 = \begin{pmatrix} A_g & G_r \\ C_g & C_S \end{pmatrix}, \quad S_4 = \text{SUPM}, \quad S_5 = \begin{pmatrix} A_r & G_g \\ C_{wr} & C_g \end{pmatrix}, \quad S_6 = \begin{pmatrix} A_g & G_r \\ C_r & C_{wr} \end{pmatrix}, \quad S_7 = \begin{pmatrix} A_r & G_g \\ C_{wr} & C_g \end{pmatrix} \]

Transition between states are shown in Fig. C.1.

3.3 TRANSITION PROBABILITIES AND SOJOURN TIMES

Simple probabilistic consideration yields the following expression for distribution functions of transition times, the non-zero elements of \( Q = Q_{ij}(t) \) are:

\[
Q_{01}(t) = \int_0^t \alpha_3 e^{-A_{14} t} A(t) dt, \quad Q_{02}(t) = \int_0^t \alpha_1 e^{-A_{14} t} A(t) dt \\
Q_{03}(t) = \int_0^t \alpha_2 e^{-A_{14} t} A(t) dt, \quad Q_{04}(t) = \int_0^t a(t) e^{-A_{14} t} dt \\
Q_{10}(t) = \int_0^t e^{-A_{15} t} r_3 A(t) dt, \quad Q_{15}(t) = \int_0^t e^{-A_{15} t} \alpha_2 A(t) dt \\
Q_{16}(t) = \int_0^t \alpha_3 e^{-A_{15} t} A(t) dt, \quad Q_{17}(t) = \int_0^t \alpha_1 e^{-A_{15} t} A(t) dt
\]
\[ Q_{18}(t) = \int_0^t a(t) e^{-A_{15}t} \, dt, \quad Q_{20}(t) = Q_{71}(t) = \int_0^t r_1 e^{-r_1 t} \, dt, \]
\[ Q_{30}(t) = Q_{51}(t) = \int_0^t r_2 e^{-r_2 t} \, dt, \quad Q_{40}(t) = Q_{81}(t) = \int_0^t b(t) \, dt, \]
\[ Q_{61}(t) = \int_0^t r_3 e^{-r_3 t} \, dt \quad [1-16] \]

Letting \( t \to \infty \), expressions [1-16] give non-zero transition probabilities \( (p_{ij}) \)
\[ p_{01} = a_3 \int_0^\infty e^{-A_{14}t} A(t) \, dt = a_1 A_{14}^{-1} \left[1-a A_{14}\right] \]
similarly
\[ p_{02} = a_1 A_{14}^{-1} \quad p_{03} = a_2 A_{14}^{-1} \quad p_{04} = a A_{14}^{-1} \quad p_{10} = r_3 A_{15}^{-1} \]
\[ p_{15} = a_2 A_{15}^{-1} \quad p_{16} = a_3 A_{15}^{-1} \quad p_{17} = a_1 A_{15}^{-1} \quad p_{18} = a A_{15}^{-1} \]
\[ p_{20} = p_{30} = p_{40} = p_{51} = p_{61} = p_{71} = p_{81} = 1 \quad [17-26] \]

It can be easily verify that
\[ p_{01} + p_{02} + p_{03} + p_{04} = 1, \quad p_{10} + p_{15} + p_{16} + p_{17} + p_{18} = 1 \]

The mean sojourn time \( \mu_i \) in state \( S_i \) based on the similar arguments as in chapter 2 section 2.3 are:
\[ \mu_0 = A_{14}^{-1} \left[1-a A_{14}\right], \quad \mu_1 = A_{15}^{-1} \left[1-a A_{15}\right], \quad \mu_2 = \mu_7 = 1/r_1 \]
\[ \mu_3 = \mu_5 = 1/r_2, \quad \mu_6 = 1/r_3, \quad \mu_4 = \mu_8 = \int_0^\infty B(t) \, dt \quad [27-32] \]

where
\[ A_{14} = (\alpha_1 + \alpha_2 + \alpha_3), \quad A_{15} = (\alpha_1 + \alpha_2 + \alpha_3 + r_3) \]

To calculate \( m_{ij} \), we note that the Laplace Stieljes transform of
State Transition diagram
Fig C-1

○ : up State
□ : Down state
• : Regenerative point
\( Q_{ij} \) is equal to Laplace transform of \( q_{ij}(t) \), i.e. \( \tilde{Q}_{ij}(s) = q^*_{ij}(s) \) so that

\[
\tilde{Q}_{ij}(0) = q^*_{ij}(0) = p_{ij}
\]  

[33]

In terms of Laplace-Stieltjes transform of \( Q_{ij}(t) \), we have

Using [1-16] the Laplace Stieltjes transform of \( Q_{ij}(t) \) are:

\[
\tilde{Q}_{01}(s) = \int_0^\infty e^{-(s+A_{14})t} A(t)dt - \frac{\alpha_3[1-a^*(s+A_{14})]}{(s+A_{14})}
\]

\[
\tilde{Q}_{02}(s) = \int_0^\infty e^{-(s+A_{14})t} A(t)dt - \frac{\alpha_1[1-a^*(s+A_{14})]}{(s+A_{14})}
\]

\[
\tilde{Q}_{03}(s) = \int_0^\infty e^{-(s+A_{14})t} A(t)dt - \frac{\alpha_2[1-a^*(s+A_{14})]}{(s+A_{14})}
\]

\[
\tilde{Q}_{04}(s) = \int_0^\infty a(t)e^{-(s+A_{14})t} = a^*(s+A_{14})
\]

\[
\tilde{Q}_{10}(s) = \int_0^\infty r_3 e^{-(s+A_{15})t} A(t)dt - \frac{\alpha_3[1-a^*(s+A_{15})]}{(s+A_{15})}
\]

\[
\tilde{Q}_{15}(s) = \int_0^\infty e^{-(s+A_{15})t} A(t)dt = \frac{\alpha_2[1-a^*(s+A_{12})]}{(s+A_{12})}
\]

\[
\tilde{Q}_{16}(s) = \int_0^\infty e^{-(s+A_{15})t} A(t)dt = \frac{\alpha_3[1-a^*(s+A_{15})]}{(s+A_{15})}
\]

\[
\tilde{Q}_{17}(s) = \int_0^\infty e^{-(s+A_{15})t} A(t)dt = \frac{\alpha_1[1-a^*(s+A_{15})]}{(s+A_{15})}
\]
\[ \tilde{Q}_{18}(s) = \int_{0}^{\infty} a(t) e^{-(s+15) t} = a^*(s+15) \]

\[ \tilde{Q}_{20}(s) = \tilde{Q}_{71}(s) = \int_{0}^{\infty} r_{1} e^{-(s+r_{1}) t} dt = r_{1}(s+r_{1})^{-1} \]

\[ \tilde{Q}_{30}(s) = \tilde{Q}_{51}(s) = \int_{0}^{\infty} r_{2} e^{-(s+r_{2}) t} dt = r_{2}(s+r_{2})^{-1} \]

\[ \tilde{Q}_{61}(s) = \int_{0}^{\infty} r_{3} e^{-(s+r_{3}) t} dt = r_{3}(s+r_{3})^{-1} \]

\[ \tilde{Q}_{40}(s) = \tilde{Q}_{81}(s) = \int_{0}^{\infty} b(t) e^{-st} dt = b^*(t) \]

In the view of the relation [34-46], we have

\[ m_{01} = -\frac{d}{ds} \tilde{Q}_{01}(s) \bigg|_{s=0} = -\tilde{Q}_{01}(0) = \alpha_{3} \int_{0}^{\infty} t e^{-A_{14} t} A(t) dt \]

\[ m_{02} = \alpha_{1} \int_{0}^{\infty} t e^{-A_{14} t} A(t) dt, \quad m_{03} = \alpha_{2} \int_{0}^{\infty} t e^{-A_{14} t} A(t) dt \]

\[ m_{04} = \int_{0}^{\infty} t e^{-A_{14} t} a(t) dt, \quad m_{10} = r_{3} \int_{0}^{\infty} t A(t) e^{-15 t} dt, \]

\[ m_{15} = \alpha_{2} \int_{0}^{\infty} t e^{-A_{15} t} A(t) dt, \quad m_{16} = \alpha_{3} \int_{0}^{\infty} t e^{-A_{15} t} A(t) dt \]

\[ m_{17} = \alpha_{1} \int_{0}^{\infty} t e^{-A_{15} t} A(t) dt, \quad m_{18} = \int_{0}^{\infty} t a(t) e^{-15 t} dt \]

\[ m_{20} = m_{71} = \int_{0}^{\infty} t r_{1} e^{-r_{1} t} dt, \quad m_{30} = m_{51} = \int_{0}^{\infty} t r_{2} e^{-r_{2} t} dt \]

\[ m_{40} = m_{81} = \int_{0}^{\infty} t b(t) dt, \quad m_{61} = \int_{0}^{\infty} t r_{3} e^{-3 t} dt \]

[34-46]
It can be easily seen that
\[
\begin{align*}
& m_{01} + m_{02} + m_{03} + m_{04} = \mu_0, \quad m_{10} + m_{15} + m_{16} + m_{17} + m_{18} = \mu_1, \quad m_{20} = \mu_2 \\
& m_{30} = \mu_3, \quad m_{40} = \mu_4, \quad m_{51} = \mu_5, \quad m_{61} = \mu_6, \quad m_{71} = \mu_7, \quad m_{81} = \mu_8
\end{align*}
\]

[60-68]

3.4 TIME TO SYSTEM FAILURE

Employing the arguments as in the section 2.4 of chapter 2 we obtain the following recursive relations for \( \pi_i(t) \):

\[
\begin{align*}
\pi_0(t) &= Q_{01}(t) \pi_1(t) + \sum_{j=2}^{4} Q_{0j}(t) \\
\pi_1(t) &= Q_{10}(t) \pi_0(t) + \sum_{j=5}^{8} Q_{1j}(t)
\end{align*}
\]

[1-2]

Taking Laplace-Stieltjes transforms of equations [1-2], the solution for \( \tilde{\pi}_i(s) \) can be written in the following matrix form:

\[
(\tilde{\pi}_0 : \tilde{\pi}_1) = \tilde{Q}^{-1} \left( \tilde{Q}_{02} + \tilde{Q}_{03} + \tilde{Q}_{04}, \tilde{Q}_{10} + \tilde{Q}_{15} + \tilde{Q}_{16} + \tilde{Q}_{17} + \tilde{Q}_{18} \right)
\]

[3]

\[
\tilde{Q} = \left[ \begin{array}{ccc}
-1 & \tilde{Q}_{01} \\
\tilde{Q}_{10} & -1
\end{array} \right]
\]

Computing the relevant elements of \( \tilde{Q}^{-1} \), we have

\[
\tilde{\pi}_0(s) = \frac{N_1(s)}{D_1(s)}
\]

[4]

Where

\[
N_1(s) = \tilde{Q}_{02} + \tilde{Q}_{03} + \tilde{Q}_{04} + \tilde{Q}_{01} (\tilde{Q}_{15} + \tilde{Q}_{16} + \tilde{Q}_{17} + \tilde{Q}_{18})
\]

[5-6]

\[
D_1(s) = 1 - \tilde{Q}_{01} \tilde{Q}_{10}
\]

Using the equations [17-26] of section 3.3 and \( \lim_{s \to 0} \tilde{\pi}_{ij}(s) = p_{ij} \)

we have

\[
N_1(0) = p_{02} + p_{03} + p_{04} + p_{01} (p_{15} + p_{16} + p_{17} + p_{18}) = 1 - p_{01} p_{10} = D_1(0)
\]

[7]
Thus $\pi_0(0) = 1$.

This shows that $\pi_0(0)$ is a proper cdf. Therefore, mean time to system failure when the system starts from $S_0$ is

$$E(T) = -\frac{d}{ds} \pi_0(s) \bigg|_{s=0} = \frac{D_1'(0) - N_1'(0)}{D_1(0)} \quad [8]$$

To obtain the numerator of [10] we collect the coefficient of relevant $m_{ij}$ in $D_1'(0) - N_1'(0)$ in the same way as in the section 2.4 of chapter 2.

Coefficient of $m_{01} = -\pi_{01}(0) = p_{10} + p_{15} + p_{16} + p_{17} + p_{18} = 1$

Coefficient of $m_{02} = \text{Coefficient of } m_{03} = \text{Coefficient of } m_{04} = 1$

Coefficient of $m_{10} = \text{Coefficient of } m_{15} = p_{01}$

Coefficient of $m_{16} = \text{Coefficient of } m_{17} = \text{Coefficient of } m_{18} = p_{01}$

using the relation [17-26] & [27-32] of Section 3.3 we have

$$MTSF = E(T) = \frac{\mu_0 + \mu_1 p_{01}}{1 - p_{01} p_{10}} \quad [9]$$

3.5 AVAILABILITY ANALYSIS

As defined earlier $M_i(t)$ is the probability that the system is up in state $S_i \equiv E$ has no transition till time $t$, then

$$M_0(t) = \tilde{A}(t) e^{-A_{14} t}, \quad M_1(t) = \tilde{A}(t) e^{-A_{15} t} \quad [1-2]$$

Similar arguments as in section 2.5 of chapter 2 yield for $A_i(t)$ the following recursive relations:

$$A_0(t) = M_0(t) + \sum_{j=1-4} q_{0j}(t) A_j(t)$$

$$A_1(t) = M_1(t) + \sum_{j=0,5-8} q_{1j}(t) A_j(t)$$
\[ A_i(t) = q_{ij}(t) \quad A_j(t), \quad (i,j) = (2,0), (3,0), (4,0), (5,1), (6,1), (7,1), (8,1) \] 

Taking the Laplace-transform of \([4-6]\) the solution for \(A_i^*(s)\) \((i=1-8)\) can be written in the following matrix form:

\[
(A_0^*, A_1^*, A_2^*, A_3^*, A_4^*, A_5^*, A_6^*, A_7^*, A_8^*) = q^{-1} (M_0^*, M_1^*, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)^T
\]

Here we have omitted the argument 's' for brevity. Computing the relevant elements of the inverse matrix, the Laplace transform of the pointwise availability is seen to be

\[
\frac{A_0^*(s)}{A_0^*(s)} = \frac{N_2(s)}{D_2(s)}
\]

Where

\[
N_2(s) = M_0(s)(R_{18}^* - q_{15}^* - q_{16}^* - q_{17}^*) + M_1(s)q_{01}^*
\]

\[
D_2(s) = (R_{02}^* - q_{03}^* - q_{04}^*) (R_{18}^* - q_{15}^* - q_{16}^* - q_{17}^*) - q_{01}^*
\]
To find steady state availability we first calculate

\[ M^*_0(0) = \int_{0}^{\infty} e^{-A_{14}t} A(t) \, dt = A_{14}^{-1} [1-a^*(A_{14})] = \mu_0 \]

\[ M^*_1(0) = \int_{0}^{\infty} e^{-A_{15}t} \, dt = A_{15}^{-1} [1-a^*(A_{15})] = \mu_1 \]  \[11-12\]

Using relations \( q^*_i(0) = \int_{0}^{\infty} q_{ij}(t) \, dt = p_{ij} \) and \([17-26]\) of section 3.3 we have

\[ D_2(0) = (\bar{A}_{02} - \bar{p}_{03} - \bar{p}_{04})[\bar{A}_{18} - \bar{p}_{15} - \bar{p}_{16} - \bar{p}_{17}] - \bar{p}_{01} = 0 \text{ as it should.} \]

Substituting \([11-12]\) in \([9]\) and using \([17-26]\) of section 3.3 we get

\[ N_2(0) = \mu_0 \bar{p}_{10} + \mu_1 \bar{p}_{01} \]  \[13\]

Thus the steady state availability of the system when the system starts from \( S_i \in E \) is obtained as follows.

\[ A_0(\omega) = \lim_{s \to 0} A_0^*(s) = \frac{N_2(0)}{D'_2(0)} \text{ for } \left. \frac{s}{D(s)} \right|_{s=0} \]  \[14\]

To obtain the value of \( D'_2(0) \), we collect the coefficients of relevant \( m_{ij} \) in \( D'_2(0) \)

Coefficient of \( m_{01} = \bar{p}_{10} \), Coefficient of \( m_{02} = 1-p_{18} - p_{15} - p_{16} - p_{17} = \bar{p}_{10} \)

Coefficient of \( m_{03} = \text{Coefficient of } m_{04} = 1-p_{18} - p_{15} - p_{16} - p_{17} = \bar{p}_{10} \)

Coefficient of \( m_{10} = \bar{p}_{01} \), Coefficient of \( m_{15} = 1-p_{02} - p_{03} - p_{04} = \bar{p}_{01} \)

Coefficient of \( m_{16} = \text{Coefficient of } m_{17} = 1-p_{02} - p_{03} - p_{04} = \bar{p}_{01} \)

Coefficient of \( m_{18} = 1-p_{02} - p_{03} - p_{04} = \bar{p}_{01} \)

Coefficient of \( m_{20} = \bar{p}_{02} (1-p_{15} - p_{16} - p_{17} - p_{18}) = \bar{p}_{02} \bar{p}_{10} \)
Coefficient of \( m_{30} = p_{03}(1-p_{15}p_{16}p_{17}p_{18}) = p_{03}p_{10} \)

Coefficient of \( m_{40} = p_{04}(1-p_{15}p_{16}p_{17}p_{18}) = p_{04}p_{10} \)

Coefficient of \( m_{51} = p_{15}(1-p_{02}p_{03}p_{04}) = p_{01}p_{15} \)

Coefficient of \( m_{61} = p_{16}(1-p_{02}p_{03}p_{04}) = p_{01}p_{16} \)

Coefficient of \( m_{71} = p_{17}(1-p_{02}p_{03}p_{04}) = p_{01}p_{17} \)

Coefficient of \( m_{81} = p_{18}(1-p_{02}p_{03}p_{04}) = p_{01}p_{18} \)

Using \([17-26]\) of section 3.3 and above computed results, we have

\[
D(0) = \sum_{t=0}^{\infty} D(t) = p_{10}(1+p_{02}p_{03}p_{04}) + p_{01}(p_{15}p_{16}p_{17}p_{18})
\]

Using \([13]\) and \([15]\) in \([14]\) we get the expression for \( A_0(\omega) \).

### 3.6 BUSY PERIOD ANALYSIS

(a) Busy period analysis of the repairman in \((0,t)\)

Let \( W_i(t) \) denote the probability that the repairman is busy initially in regenerative state \( S_i \) and remains busy at epoch \( t \) without transiting to any other state or returning to itself through one or more regenerative states. By probabilistic arguments we have

\[
W_1(t) = M_1(t), \quad W_2(t) = W_7(t) = e^{-r_1 t}, \quad W_3(t) = W_5(t) = e^{-r_2 t}
\]

\[
W_4(t) = W_6(t) = R(t), \quad W_8(t) = e^{-r_3 t}
\]

We define \( R_i(t) \), the probability that repairman is busy at epoch \( t \) starting from state \( S_i \in E \). By probabilistic arguments similar to those in section 2.14 of chapter 2, \( R_i(t) \) are seen to satisfy the following recursive relations:

\[
R_i(t) = \sum_{j=1}^{\infty} a_{ij}(t) c(t) R_j(t)
\]
\[ B^1_1(t) = W_1(t) + \sum_{j=0,5-8} q_{ij} B^1_j(t) \]
\[ B^1_1(t) = W_1(t) + q_{ij} B^1_j(t), \quad (i,j) = (2,0),(3,0),(4,0),(5,1),(6,1) \]
\[ (7,1),(8,1) \quad [6-8] \]

Taking Laplace transform of [6-8] and computing the value of \( B^1_0(s) \), we have

\[ B^1_0(s) = \frac{N_3(s)}{D_2(s)} \quad [9] \]

Where \( D_2(s) \) is obtained earlier as the expression of \( A_0^*(s) \) and

\[ \begin{align*}
N_3(s) &= B^*_{18} \left[ W^*(s)q_0^2 + W^*(s)q_0^3 + W^*(s)q_0^4 + q_0^1 \left( W^*_1(s) + W^*_5(s)q_1^5 + W^*_6(s)q_1^6 \right) \\
&\quad + W^*_7(s)q_1^7 + W^*_8(s)q_1^8 \right] - (q_1^5 + q_1^6 + q_1^7) \left( W^*_2(s)q_0^2 + W^*_3(s)q_0^3 + W^*_4(s)q_0^4 \right)
\end{align*} \]

[10]

To find, in steady state, the fraction of time for which the system is under repair, we first calculate

\[ \begin{align*}
W^*_1(0) &= \int_0^\infty A(t) e^{-A_{15} t} dt = \mu_1, \quad W^*_2(0) = W^*_7(0) = \int_0^\infty e^{-A_{15} t} dt = \mu_2 = \mu_7 \\
W^*_3(0) = W^*_5(0) &= \int_0^\infty e^{-A_{12} t} dt = \mu_3 = \mu_5, \quad W^*_6(0) = \int_0^\infty e^{-A_{16} t} dt = \mu_6 \\
W^*_4(0) = W^*_8(0) &= \int_0^\infty B(t) dt = \mu_4 = \mu_8 \quad [11-15]
\end{align*} \]

Using above [11-15] and relation [17-26] of section 3.3 we get

\[ \begin{align*}
N_3(0) &= A_{18} (\mu_2 p_0^2 + \mu_3 p_0^3 + \mu_4 p_0^4 + p_0^1 (\mu_1 + \mu_5 + \mu_6 + \mu_7 + \mu_8) p_1^0) \\
&\quad - (A_{10} + p_{18}) (\mu_2 p_0^2 + \mu_3 p_0^3 + \mu_4 p_0^4) \\
\end{align*} \]

[16]

Therefore, in long run, the fraction of time for which the system
is under repair by the repairman is given by

$$B_0^1(\infty) = \lim_{t \to \infty} B_0^1(t) = \lim_{s \to 0} s B_0^1(s) = \frac{N_4(0)}{D_2'(0)} \quad [17]$$

(b) Busy period of the repairman under preventive maintenance in \((0, t)\)

As the case mentioned above the probabilistic equations that the repairman is busy in preventive maintenance are:

$$B_0^2(t) = \sum_{j=1-4} q_{0j}(t) B_j^2(t)$$

$$B_1^2(t) = \sum_{j=0,5-8} q_{1j}(t) B_j^2(t)$$

$$B_2^2(t) = q_{ij}(t) B_j^2(t), (i,j) = (2,0), (3,0), (5,1), (6,1), (7,1)$$

$$B_3^2(t) = W_i(t) + q_{ij}(t) B_j^2(t), (i,j) = (4,0), (8,1) \quad [18-20]$$

Taking Laplace transform of [18-20] and computing the value of $B_0^2(s)$, we have

$$B_0^2(s) = \frac{N_4(s)}{D_2(s)} \quad [21]$$

Where $D_2(s)$ is obtained earlier as the expression of $A_0^*(s)$ and

$$N_4(s) = W_4^*(s)q_{104}^* + W_8^*(s)q_{018}^* \quad [22]$$

To find the fraction of time for which the system is under service of the repairman in preventive maintenance, we first calculate

$$W_4^*(0) = W_8^*(0) = \int_0^\infty B(t) \, dt = \mu_4 = \mu_8 \quad [23]$$

Using above [23] and relation [17-26] of section 3.3 we get

$$N_4(0) = \mu_4 p_{04} p_{401} + \mu_8 p_{01} p_{18} \quad [24]$$

Therefore in the long run, the fraction of time for which the system is under repair of the repairman is given by
3.7 PARTICULAR CASES

When all time distributions are taken to be negative exponential, i.e. \( a(t) = \theta \exp(-\theta t) \) and \( b(t) = \delta \exp(-\delta t) \) then the steady state equations become:

\[
E(T) = \frac{K_0}{K_1}, \quad A_0 = \frac{(r_3 + \alpha_3)\delta r_1 r_2 r_3}{K_{12}}
\]

\[
B_0^1 = \frac{[K_{13} - K_{11} \delta r_3 (\alpha_1 r_2 + \alpha_2 r_1)]}{[K_{12} (r_3 + \alpha_1 + \alpha_2 + \alpha_3 + \Theta)(r_3 + \alpha_3)]}
\]

\[
B_0^2 = \frac{\Theta r_1 r_2 r_3 (r_3 + \alpha_3)}{K_{12}}
\]

where

\[
K_0 = r_3 + \alpha_1 + \alpha_2 + 2\alpha_3 + \Theta , \quad K_1 = K_{11} - \alpha_3 r_3
\]

\[
K_{11} = (\alpha_1 + \alpha_2 + \alpha_3 + \Theta)(r_3 + \alpha_1 + \alpha_2 + \alpha_3 + \Theta)
\]

\[
K_{12} = (r_3 + \alpha_3)[\delta r_3 (r_1 r_2 + \alpha_1 r_2 + \alpha_2 r_1 + \delta r_1 r_2 r_3) + \delta r_3 r_1 r_2]
\]

\[
K_{13} = (r_3 + \alpha_1 + \alpha_2 + \alpha_3 + \Theta)[(\delta r_1 r_2 + \delta r_1 r_2)(r_3 + \alpha_1 + \alpha_2 + 2\alpha_3) r_3 + \delta r_1 r_2 r_3 (r_3 + \alpha_3)]
\]

3.8. COST ANALYSIS

The cost benefit analysis of the system can be carried out by considering the expected busy period of the repairman in repair and PM of the unit in \((0,t]\). Therefore

\[
G(t) = \text{expected total revenue earned by the system in (0,t]} - \text{expected repair cost of the repairman in repair in (0,t]} - \text{expected preventive maintenance of the repairman in (0,t].}
\]

\[
= C_{1}^{\mu_1} p(t) - C_{2}^{\mu_1} b(t) - C_{3}^{\mu_2} b(t)
\]
Where

\[ \mu_{up}(t) = \int_{0}^{t} A_0(t) \, dt, \quad \mu_b^1(t) = \int_{0}^{t} B_0^1(t) \, dt, \quad \mu_b^2(t) = \int_{0}^{t} B_0^2(t) \, dt \]

The expected total profit per unit of time in steady state is

\[ G = \lim_{t \to \infty} \frac{G(t)}{t} = \lim_{s \to 0} s^2 G^*(s) = C_1 \mu_{up} - C_2 \mu_b^1 - C_3 \mu_b^2 \]

where \( C_1 \) is the revenue per unit up time and \( C_2 \) & \( C_3 \) are the repair and preventive maintenance cost per unit of time respectively.

3.9 GRAPHICAL REPRESENTATION

To make an accurate study of system behaviour, we plot the graphs for MTSF, Availability and Expected profit. Fig.C.2 shows the curve for MTSF with respect to \( \alpha_3 \) for varying values of \( \alpha_1 \) when other parametric values are taken to be constant. From the graph we can predict that MTSF decreases as \( \alpha_1 \) increases. Fig.C.3 and Fig.C.4 shows the curves for Availability and Expected profit with respect to \( \alpha_3 \) for varying values of \( \alpha_1 \). It is evident from the graphs that as the values of \( \alpha_1 \) increases, Availability and Expected profit decreases.
EFFECT OF FAILURE RATE OF CRUSHER ON MTSF FOR VARYING VALUES OF $\alpha_1$

EFFECT OF $\alpha_3$ ON AVAILABILITY FOR VARYING VALUES OF $\alpha_1$
EFFECT OF ON EXPECTED PROFIT
FOR VARYING VALUES OF a

EXPECTED PROFIT (Thousands)

Failure rate of crusher ( )

FIG C.4
3.10 MODEL D

This model studies a system of a crushing plant. This system consists of two identical parallel crushing sub-systems. Each sub-system consists of one Appron feeder installed just before the Primary Gyratory Crusher and is capable of feeding the crusher with materials in a constant quantity. However, if quarried stones are fed unlimitedly to the crusher, without the help of Appron feeder then the Primary Gyratory crusher does not crush the stones into a specific size and the over feed of the stones to it, causes damage to the crushing unit and creates loss in the production. So to avoid this over feeding of stones to Primary Gyratory Crusher, Appron feeder works as a good quantity controller for the crushing machine. This unit certainly promotes the efficiency of the crusher and at the same time raises up the overall working ratio of the crushing plant. Further, the main function of the Primary Gyratory Crusher is to grind the fed stones. This unit crushes the fed stones into minimum sizes. After the completion of the stone crushing process, stones are washed to get pure ores. Using regeneration point technique various measures of system effectiveness are obtained.

(a) SYSTEM DESCRIPTION AND ASSUMPTIONS

1. The whole system consists of two sub-systems of crushing units in parallel configuration. Each sub-system consists two units - one Appron feeder and other Primary Gyratory Crusher.

2. System fails completely if either both Appron feeders fail or both Primary Gyratory Crushers fail.

3. Since both sub-systems are working parallel to each other so failure of Appron feeder or Primary Gyratory crusher of either of the sub-system do not affect the working of the other.

4. After a random time, one sub-system completely undergoes to preventive maintenance if both sub-systems are under operation.
5. There is a single repair facility to repair the failed units on first come first serve basis. During preventive maintenance of one sub-system if other fails due to failure of any of its unit, then it waits for repair. i.e. Priority is given to preventive maintenance over repair.

6. Failure and repair time distributions of all the units are taken to be negative exponential whereas preventive maintenance time is taken to be arbitrary.

7. Switching device is perfect and instantaneous.

8. After repair unit works as new.

(b) NOTATION

\begin{align*}
A & \quad \text{apron feeder} \\
C & \quad \text{Primary gyratory crusher} \\
\text{SUPM}_C & \quad \text{Preventive maintenance continues from the earlier state} \\
\text{RP}_R & \quad \text{repair continues from earlier state} \\
\lambda_1 & \quad \text{constant failure rate of Appron feeder} \\
\lambda_2 & \quad \text{constant failure rate of Crusher} \\
g_1(t),G_1(t) & \quad \text{pdf and cdf of repair time of the failed apron feeder} \\
g_2(t),G_2(t) & \quad \text{pdf and cdf of repair time of the failed crusher} \\
a(t),A(t) & \quad \text{pdf and cdf time until preventive maintenance} \\
b(t),B(t) & \quad \text{pdf and cdf of time to accomplish preventive maintenance}
\end{align*}

(c) SYMBOLS USED FOR THE STATES OF THE SYSTEM

\begin{align*}
A_0,A_r,A_g & \quad \text{Appron feeder is under operation/under repair/good and non-operative} \\
A_{wr},A_R & \quad \text{Appron feeder is waiting for repair/repair continues from previous state.} \\
C_0,C_r,C_g & \quad \text{Primary gyratory crusher is under operation/under repair/good and non-operative.}
\end{align*}
Primary gyratory crusher is waiting for repair/repair continues from previous state.

Considering these symbols the system may be in any one of the following states.

(d) Up states

\[ S_0 = \begin{bmatrix} A_0^r & C_0^r \\ A_0 & C_0 \end{bmatrix}, \quad S_1 = \begin{bmatrix} A_0^r & C_0^r \\ A_0 & C_0 \end{bmatrix}, \quad S_2 = \begin{bmatrix} A_0^g & C_0^g \\ A_0 & C_0 \end{bmatrix}, \quad S_3 = \begin{bmatrix} \text{SUPM} & C_0^r \\ A_0 & C_0 \end{bmatrix} \]

(e) Failed States

\[ S_4 = \begin{bmatrix} A_0^r & C_0^r \\ A_{wr} & C_{wr} \end{bmatrix}, \quad S_5 = \begin{bmatrix} A_0^r & C_0^r \\ A_{wr} & C_{wr} \end{bmatrix}, \quad S_6 = \begin{bmatrix} A_0^g & C_0^g \\ A_{wr} & C_{wr} \end{bmatrix}, \quad S_7 = \begin{bmatrix} A_0^g & C_0^g \\ A_{wr} & C_{wr} \end{bmatrix} \]

\[ S_8 = \begin{bmatrix} \text{SUPM} & C_0 \\ A_{wr} & C_{wr} \end{bmatrix}, \quad S_9 = \begin{bmatrix} \text{SUPM} & C_0 \\ A_{wr} & C_{wr} \end{bmatrix} \]

Transition between states are shown in Fig. B.1i.

3.11 TRANSITION PROBABILITIES AND SOJOURN TIMES

Simple probabilistic arguments yield the following equations:

\[ Q_{01}(t) = \int_0^t 2\alpha_1 e^{-2A_{16}t} A(t) \, dt, \quad Q_{02}(t) = \int_0^t 2\alpha_2 e^{-2A_{16}t} A(t) \, dt \]

\[ Q_{03}(t) = \int_0^t a(t) e^{-2A_{16}t} \, dt, \quad Q_{10}(t) = \int_0^t g_1(t) e^{-A_{16}t} \, dt \]

\[ Q_{11}^{(4)}(t) = A_{16}^{-1} \int_0^t g_1(t)[1 - e^{-A_{16}t}] \, dt, \]

\[ Q_{12}^{(5)}(t) = A_{16}^{-1} \int_0^t g_1(t)[1 - e^{-A_{16}t}] \, dt, \quad Q_{20}(t) = \int_0^t g_2(t) e^{-A_{16}t} \, dt \]

\[ Q_{21}^{(6)}(t) = A_{16}^{-1} \int_0^t g_2(t)[1 - e^{-A_{16}t}] \, dt, \]
\[ Q_{22}(t) = \alpha_2 A_{16}^{-1} \int_0^t g_2(t)[1 - e^{-A_{16}t}] \, dt, \quad Q_{30}(t) = \int_0^t b(t) e^{-A_{16}t} \, dt \]

\[ Q_{31}(t) = \alpha_1 A_{16}^{-1} \int_0^t b(t)[1 - e^{-A_{16}t}] \, dt, \]

\[ Q_{32}(t) = \alpha_2 A_{16}^{-1} \int_0^t b(t)[1 - e^{-A_{16}t}] \, dt, \] [1-12]

Letting \( t \to \infty \), expressions [1-12] give non-zero transition probabilities \( (p_{ij}) \)

\[ p_{01} = 2\alpha_1 \int_0^\infty e^{-A_{16}t} A(t) \, dt = 2\alpha_1 A_{16}^{-1} [1 - * A_{16}] \]

\[ p_{02} = 2\alpha_2 A_{01} ; \quad p_{03} = * A_{16} ; \quad p_{10} = g_1 A_{16} ; \quad p_{11} = \alpha_1 A_{02} \]

\[ p_{11} = \alpha_2 A_{02} ; \quad p_{20} = g_2 A_{16} ; \quad p_{21} = * A_{03} ; \quad p_{22} = \alpha_2 A_{03} \]

\[ p_{30} = b A_{16} ; \quad p_{31} = \alpha_1 A_{04} ; \quad p_{32} = \alpha_2 A_{04} \]

\[ n_1 = \int_0^\infty \tilde{G}_1(t) \, dt ; \quad n_2 = \int_0^\infty \tilde{G}_2(t) \, dt , \quad n_3 = \int_0^\infty \tilde{B}(t) \, dt \] [13-27]

Where

\[ A_{16} = (\alpha_1 + \alpha_2) , \quad A_{01} = [1 - * 2A_{16}]/2A_{16} ; \quad A_{02} = [1 - g_1 A_{16}]/A_{16} \]

\[ A_{03} = [1 - * A_{16}]/A_{16} ; \quad A_{04} = [1 - b A_{16}]/A_{16} \]

It can be easily verify that

\[ p_{01} + p_{02} + p_{03} + p_{04} + p_{05} = 1 ; \quad p_{60} + p_{8,10} = 1 ; \quad p_{70} + p_{7,11} = 1 \]

\[ p_{80} + p_{8,12} = 1 ; \quad p_{90} + p_{9,13} = 1 \]

Mean sojourn time \( \mu_i \) in state \( S_i \) is based on the similar arguments as in section 2.3 of chapter 2 are:

\[ \mu_0 = A_{01} ; \quad \mu_1 = A_{02} ; \quad \mu_2 = A_{03} ; \quad \mu_3 = A_{04} \] [28-31]
State transition diagram

Fig. 0-1

Key word: □ Up state, ◯ Down state, • Regenerative point
To calculate $m_{ij}$, we note that the Laplace Stieljes transform of $Q_{ij}$ is equal to Laplace transform of $q_{ij}(t)$, i.e. $\tilde{Q}_{ij}(s) = \tilde{q}_{ij}^*(s)$ so that

$$\tilde{Q}_{ij}(0) = \tilde{q}_{ij}^*(0) = p_{ij}$$

[32]

In terms of Laplace-Stieltjes transform of $Q_{ij}(t)$, we define $m_{ij}$ as follows

$$m_{ij} = -\tilde{Q}_{ij}(0)$$

[33]

Using [1-12] the Laplace Stieltjes transform of $Q_{ij}(t)$ are:

$$\tilde{Q}_{01}(s) = 2\alpha_1 \int_0^\infty e^{-(s+2A_{16})t} A(t) \, dt = 2\alpha_1 (s+2A_{16})^{-1} [1 - a^*(s+2A_{16})]$$

$$\tilde{Q}_{02}(s) = 2\alpha_2 \int_0^\infty e^{-(s+2A_{16})t} A(t) \, dt = 2\alpha_2 (s+2A_{16})^{-1} [1 - a^*(s+2A_{16})]$$

$$\tilde{Q}_{03}(s) = \int_0^\infty e^{-(s+2A_{16})t} a(t) \, dt = a^*(s+2A_{16})$$

$$\tilde{Q}_{10}(s) = \int_0^\infty e^{-(s+2A_{16})t} g_1(t) \, dt = g_1^*(s+2A_{16})$$

$$\tilde{Q}_{11}(s) = \alpha_1 A_{16}^{-1} \int_0^\infty e^{-st} g_1(t) [1 - e^{-A_{16} t}] \, dt = \alpha_1 A_{16}^{-1} [g_1^*(s) - g_1^*(s+A_{16})]$$

$$\tilde{Q}_{12}(s) = \alpha_2 A_{16}^{-1} \int_0^\infty e^{-st} g_1(t) [1 - e^{-A_{16} t}] \, dt = \alpha_2 A_{16}^{-1} [g_1^*(s) - g_1^*(s+A_{16})]$$

$$\tilde{Q}_{20}(s) = \int_0^\infty e^{-(s+2A_{16})t} g_2(t) \, dt = g_2^*(s+2A_{16})$$

$$\tilde{Q}_{21}(s) = \alpha_1 A_{16}^{-1} \int_0^\infty e^{-st} g_2(t) [1 - e^{-A_{16} t}] \, dt = \alpha_1 A_{16}^{-1} [g_2^*(s) - g_2^*(s+A_{16})]$$

$$\tilde{Q}_{22}(s) = \alpha_2 A_{16}^{-1} \int_0^\infty e^{-st} g_2(t) [1 - e^{-A_{16} t}] \, dt = \alpha_2 A_{16}^{-1} [g_2^*(s) - g_2^*(s+A_{16})]$$
\[ Q_{30}(s) = \int_0^\infty e^{-(s+A_{16})t} b(t) \, dt = b^*(s+A_{16}) \]

\[ \tilde{Q}_{31}(s) = \alpha_{16}^{-1} \int_0^\infty e^{-st} b(t) [1 - e^{-A_{16}t}] \, dt = \alpha_{16}^{-1} [b^*(s) - b^*(s+A_{16})] \]

\[ \tilde{Q}_{32}(s) = \alpha_{16}^{-1} \int_0^\infty e^{-st} b(t) [1 - e^{-A_{16}t}] \, dt = \alpha_{16}^{-1} [b^*(s) - b^*(s+A_{16})] \]

In the view of the relation [33], we have

\[ m_{01} = -\frac{d}{ds} \tilde{Q}_{01}(s) \bigg|_{s=0} = -\tilde{Q}'_{01}(0) = 2a_1 \int_0^\infty t e^{-2A_{16}t} A(t) \, dt \]

\[ m_{02} = 2a_2 \int_0^\infty t e^{-2A_{16}t} A(t) \, dt, \quad m_{03} = \int_0^\infty t a(t) e^{-2A_{16}t} \, dt \]

\[ m_{10} = \int_0^\infty t g_1(t) e^{-A_{16}t} \, dt, \quad m_{11} = \alpha_{16}^{-1} \int_0^\infty t g_1(t) [1 - e^{-A_{16}t}] \, dt \]

\[ m_{12} = \alpha_{16}^{-1} \int_0^\infty t g_1(t) [1 - e^{-A_{16}t}] \, dt, \quad m_{20} = \int_0^\infty t g_2(t) e^{-A_{16}t} \, dt \]

\[ m_{21} = \alpha_{16}^{-1} \int_0^\infty t g_2(t) [1 - e^{-A_{16}t}] \, dt, \quad m_{20} = \int_0^\infty t g_2(t) e^{-A_{16}t} \, dt \]

\[ m_{22} = \alpha_{16}^{-1} \int_0^\infty t g_2(t) [1 - e^{-A_{16}t}] \, dt, \quad m_{30} = \int_0^\infty t b(t) e^{-A_{16}t} \, dt \]

\[ m_{31} = \alpha_{16}^{-1} \int_0^\infty t b(t) [1 - e^{-A_{16}t}] \, dt \]

\[ m_{32} = \alpha_{16}^{-1} \int_0^\infty t b(t) [1 - e^{-A_{16}t}] \, dt \]
It can be easily seen that
\begin{align*}
m_{01} + m_{02} + m_{03} &= \mu_0, \\
m_{10} + m_{11} + m_{12} &= \int_0^\infty t g_1(t) \, dt = \int_0^\infty \tilde{G}_1(t) \, dt = n_1
\end{align*}
\begin{align*}
m_{20} + m_{21} + m_{22} &= \int_0^\infty t g_2(t) \, dt = \int_0^\infty \tilde{G}_2(t) \, dt = n_2
\end{align*}
\begin{align*}
m_{30} + m_{31} + m_{32} &= \int_0^\infty t b(t) \, dt = \int_0^\infty \tilde{b}(t) \, dt = n_3
\end{align*}

3.12 TIME TO SYSTEM FAILURE

Using the arguments as discussed in section 2.4 of chapter 2, we have the following recursive relations for \( \pi_i(t) \):
\begin{align*}
\pi_0(t) &= \sum_{j=1}^{3} Q_{0j}(t) \pi_j(t), \\
\pi_1(t) &= Q_{10}(t) \pi_0(t) + \sum_{j=4}^{5} Q_{1j}(t) \\
\pi_2(t) &= Q_{20}(t) \pi_0(t) + \sum_{j=6}^{7} Q_{2j}(t) \\
\pi_3(t) &= Q_{30}(t) \pi_0(t) + \sum_{j=8}^{9} Q_{3j}(t) \\
\end{align*}
[1-4]

Taking Laplace-Stieltjes transforms of equations [1-4], the solution for \( \tilde{\pi}_i(s) \) can be written in the following matrix form:
\begin{align*}
(\tilde{\pi}_0, \tilde{\pi}_1, \tilde{\pi}_2, \tilde{\pi}_3)' &= Q^{-1}(0, \tilde{Q}_{14} + \tilde{Q}_{15}, \tilde{Q}_{26} + \tilde{Q}_{27}, \tilde{Q}_{38} + \tilde{Q}_{39})' \\
\end{align*}
[5]
\begin{align*}
Q &= \begin{bmatrix}
-1 & Q_{01} & Q_{02} & Q_{03} \\
Q_{10} & -1 & 0 & 0 \\
Q_{20} & 0 & -1 & 0 \\
Q_{30} & 0 & 0 & -1 \\
\end{bmatrix}
\end{align*}

For brevity we have omitted the argument 's' from \( \tilde{Q}_{ij}(s) \) and
Computing the relevant elements of $Q^{-1}$, we have

$$\pi_0(s) = \frac{N_1(s)}{D_1(s)} \quad [6]$$

Where

$$N_1(s) = \sum (Q_{14} + Q_{15})Q_{01} + (Q_{26} + Q_{27})Q_{02} + (Q_{38} + Q_{39})Q_{03}$$

$$D_1(s) = 1 - Q_{01}Q_{10} - Q_{02}Q_{20} - Q_{03}Q_{30} \quad [7-8]$$

Using the equations [13-27] of section 3.11 and $\lim \tilde{Q}_{ij}(s) = p_{ij}$ as $s \to 0$, we get:

$$N_1(0) = (p_{14} + p_{15})p_{01} + (p_{26} + p_{27})p_{02} + (p_{38} + p_{39})p_{03} = p_{01}p_{10} + p_{02}$$

Thus $\pi_0(0) = 1$.

This shows that $\pi_0(0)$ is a proper cdf. Therefore, mean time to system failure when the system starts from $S_0$ is

$$E(T) = - \frac{d}{ds} \pi_0(s) \bigg|_{s=0} = \frac{D_1'(0) - N_1'(0)}{D_1(0)} \quad [9]$$

To obtain the numerator of [9] we collect the coefficient of relevant $m_{ij}$ in $D_1'(0) - N_1'(0)$ in the same way as in the section 2.4 of chapter 2.

Coefficient of $m_{01} = \tilde{Q}_{01}'(0) = p_{10} + p_{14} + p_{15} = 1$

Coefficient of $m_{02} = $ Coefficient of $m_{03} = 1$

Coefficient of $m_{10} = $ Coefficient of $m_{14} = $ Coefficient of $m_{15} = p_{01}$

Coefficient of $m_{20} = $ Coefficient of $m_{26} = $ Coefficient of $m_{27} = p_{02}$

Coefficient of $m_{38} = $ Coefficient of $m_{39} = p_{03}$

\[
\text{MTSF} = E(T) = \frac{\mu_0 + \mu_1 p_{01} + \mu_2 p_{02} + \mu_3 p_{03}}{1 - p_{01} p_{10} - p_{02} p_{20} - p_{03} p_{30}}
\]

[10]

### 3.13 Availability Analysis

As defined earlier, \( M_i(t) \) is the probability that the system is up in state \( S_i \in E \) has no transition till time \( t \), then

\[
M_0(t) = A(t) e^{-2(\alpha_1 + \alpha_2)t}, \quad M_1(t) = G_1(t) e^{-(\alpha_1 + \alpha_2)t}
\]

\[
M_2(t) = G_2(t) e^{-(\alpha_1 + \alpha_2)t}, \quad M_3(t) = B(t) e^{-(\alpha_1 + \alpha_2)t}
\]

[1-4]

similar arguments as in section 2.5 of chapter 2 yield for \( A_i(t) \) the following recursive relations:

\[
A_0(t) = M_0(t) + \sum_{j=1}^{3} q_{0j}(t) A_j(t)
\]

\[
A_1(t) = M_1(t) + q_{10}(t) A_0(t) + q_{11}(t) A_1(t) + q_{12}(t) A_2(t)
\]

\[
A_2(t) = M_2(t) + q_{20}(t) A_0(t) + q_{21}(t) A_1(t) + q_{22}(t) A_2(t)
\]

\[
A_3(t) = M_3(t) + q_{30}(t) A_0(t) + q_{31}(t) A_1(t) + q_{32}(t) A_2(t)
\]

[5-8]

Taking the Laplace-transform of [5-8] the solution for \( A_i^*(s) \) (\( i=0-3 \)) can be written in the following matrix form:

\[
(A_0^*, A_1^*, A_2^*, A_3^*) = q^{-1}(M_0^*, M_1^*, M_2^*, M_3^*)
\]

[9]

\[
q^{-1} = -1
\]

\[
\begin{bmatrix}
-1 & q_01 & q_02 & q_03 \\
q_10 & -(1-q_{11}^*(4)) & q_{12}^*(5) & 0 \\
q_20 & q_{21}^*(6) & -(1-q_{22}^*(7)) & 0 \\
q_30 & q_{31}^*(8) & q_{32}^*(9) & -1
\end{bmatrix}
\]
Here we have omitted the argument 's' for brevity. Computing the relevant elements of the inverse matrix, the Laplace transform of the pointwise availability is seen to be

\[ A_0^*(s) = \frac{N_2^*(s)}{D_2^*(s)} \]  \[10\]

Where

\[ N_2^*(s) = M_0^*(s)[(1-q_{14}^*(4))(1-q_{22}^*(7))-q_{12}^*(q_{21}^*(5)*q_{21}^*(6))] + M_1^*(s)[q_{01}^*(1-q_{22}^*(7)) + q_{02}^*(6)] + M_2^*(s)[q_{03}^*(4) + q_{03}^*(9) + q_{03}^*(8)] + M_3^*(s)[q_{03}^*(1-q_{11}^*(4))]

\[ D_2^*(s) = [1-q_{14}^*(4)(1-q_{22}^*(7))-q_{12}^*(q_{21}^*(5)*q_{21}^*(6))] + q_{10}^*[q_{01}^*(1-q_{22}^*(7)) - q_{02}^*(6)]

To find steady state availability we first calculate

\[ M_0^*(0) = \int_{0}^{\infty} A(t) e^{-A_{16}t} dt = \mu_0, \quad M_1^*(0) = \int_{0}^{\infty} G_1(t) e^{-A_{16}t} dt = \mu_1 \]

\[ M_2^*(0) = \int_{0}^{\infty} G_2(t) e^{-A_{16}t} dt = \mu_2, \quad M_3^*(0) = \int_{0}^{\infty} B(t) e^{-A_{16}t} dt = \mu_3 \]  \[13-16\]

Using relations \[ q_{ij}^*(0) = \int_{0}^{\infty} q_{ij}(t) dt = p_{ij} \] and substituting in [11], we have
Thus the steady state availability of the system when the system starts from $S_i \in E$ is obtained as follows.

$$A_0(\omega) = \lim_{s \to 0} s A_0^*(s) = \frac{N_2(0)}{D_2'(0)} \text{ for } \frac{s}{D(s)} \bigg|_{s=0}$$  \[18\]

To obtain the value of $D_2'(0)$, we collect the coefficients of relevant $m_{ij}$ in $D_2'(0)$

Coefficient of $m_{01} = p_{10}^1(1-p_{22}^7) + p_{12}^1 p_{20}^1 = p_{10}^1(p_{20}^1 + p_{21}^1) + p_{12}^1 p_{20}^1$

$$= p_{10}^6 p_{21} + p_{20}^1 (1-p_{11}^4)$$

Coefficient of $m_{02} = \text{Coefficient of } m_{03} = p_{10}^8 p_{21} + p_{20}^1 (1-p_{11}^4)$

Coefficient of $m_{10} = p_{01}^1(1-p_{22}^7) + p_{02}^1 p_{21} + p_{03}^1(p_{21}^6 p_{32} + p_{31}^6 (1-p_{22}^7))$

$$= p_{01}^4 (p_{20}^1 + p_{21}^1) + p_{02}^6 p_{21}^1 + p_{03}^6 [p_{21}^6 p_{32} + p_{20}^1 p_{31}^6 + p_{21}^1 p_{31}^6]$$

$$= p_{01}^6 p_{20}^1 + p_{03}^6 p_{21}^1 p_{31}^6 - p_{03}^6 p_{30}^1 p_{21}^1$$

Coefficient of $m_{11} = \text{Coefficient of } m_{12}$

$$= p_{01}^6 p_{20}^1 + p_{03}^6 p_{21}^1 p_{31}^6 - p_{03}^6 p_{30}^1 p_{21}^1$$

Coefficient of $m_{20} = p_{01}^5 p_{12} + p_{02}^5 (1-p_{11}^4) + [(1-p_{11}^4) p_{32} + p_{12}^5 p_{31}^1] p_{03}^5$

$$= p_{01}^5 p_{12} + p_{02}^5 (p_{10}^5 p_{12}^1) + p_{03}^5 (p_{10}^5 p_{12}^1) + p_{03}^5 p_{12}^5 p_{31}^1$$

$$= p_{12}^5 p_{10}^5 p_{12}^5 p_{10}^5 p_{32}^5 - p_{30}^5 p_{12}^5 p_{30}^5$$

Coefficient of $m_{22} = p_{12}^5 p_{02}^5 p_{10}^5 p_{03}^5 p_{10}^5 p_{32}^5 - p_{30}^5 p_{12}^5 p_{30}^5$
Coefficient of $m_{21}^{(6)} = p_{12}^{(5)}+p_{02}^{(5)}+p_{03}^{(9)}-p_{03}^{(12)}p_{30}^{(5)}$

Coefficient of $m_{30}^{(6)} = p_{03}^{(10)}p_{21}^{(6)}+p_{03}^{(20)}(1-p_{11})^{(4)}$

Coefficient of $m_{31}^{(8)} = \text{Coefficient of } m_{32}^{(9)} = p_{03}^{(10)}p_{21}^{(6)}+p_{03}^{(20)}(1-p_{11})^{(4)}$

Using the equations [13-27] and [28-31] of section 3.12 we have

\[ D_2^{(0)} = \mu_0[p_{10}^{(21)}+p_{20}(1-p_{11})]+n_1[p_{21}^{(9)}+p_{201}^{(20)}+p_{203}^{(31)}-p_{03}^{(21)}]+n_2[p_{12}^{(6)}+p_{102}^{(103)}-p_{03}^{(12)}]+n_3[p_{103}^{(21)}+p_{203}(1-p_{11})] \] [19]

Using [17] and [19] in [18] we get the expression for $A_0^{(m)}$.

3.14 BUSY PERIOD ANALYSIS

(a) Busy period analysis of the repairman for repair in (0,t)

Let $W_1(t)$ denote the probability that the repairman busy initially in regenerative state $S_i$ and remains busy at epoch $t$ without transiting to any other state or returning to itself through one or more regenerative states. By probabilistic arguments, we have

$W_1(t) = G_1(t), \quad W_2(t) = G_2(t)$ [1-2]

By similar arguments as in section 2.14 of chapter 2, $B_1(t)$ are seen to satisfy the following recursive relations:

\[ B_0^1(t) = \sum_{j=1}^{3} q_{0j}(t) B_j(t) \]

\[ B_1^1(t) = W_1(t)+q_{10}(t) B_0^1(t)+q_{11}(t) B_1^1(t)+q_{12}(t) B_2^1(t) \]

\[ B_2^1(t) = W_2(t)+q_{20}(t) B_0^1(t)+q_{21}(t) B_1^1(t)+q_{22}(t) B_2^1(t) \]

\[ B_3^1(t) = q_{30}(t) B_0^1(t)+q_{31}(t) B_1^1(t)+q_{32}(t) B_2^1(t) \] [3-6]
Taking Laplace transform of [3-6] and computing the value of \( B_0^1(s) \), we have

\[
B_0^1(s) = \frac{N_3(s)}{D_2(s)} \quad [7]
\]

Where \( D_2(s) \) is obtained earlier as the expression of \( A_0^*(s) \) and

\[
N_3(s) = \sum_{i=1}^{3} \sum_{j=1}^{2} N_3(i,j) \quad [8]
\]

To find the steady state, the fraction of time for which system is under repair, we first calculate

\[
W_0^*(0) = \lim_{t \to \infty} -G_1(t) \, dt = n_1, \quad W_1^*(0) = \lim_{t \to \infty} -G_2(t) \, dt = n_2 \quad [9-10]
\]

Using above [9-10] and relation [13-27] of section 3.12 we get

\[
N_3(0) = n_1[p_02(1-p_{22})+p_2(6)+p_3(5)(9)+p_3(8)(9)(1-p_{22})]+n_2[p_01
\]

\[
p_12+p_2(1-p_{11})+p_3[(1-p_{11})p_{32}+p_{31}] \quad [11]
\]

Therefore in the long run, the fraction of time for which the system is under repair of the repairman is given by

\[
B_0^1(\omega) = \lim_{t \to \infty} B_0^1(t) = \lim_{s \to 0} s B_0^1(s) = \frac{N_3(0)}{D_2(0)} \quad [12]
\]

(b) Busy period of the repairman in preventive maintenance in

\((0, t)\)

As the case mentioned above the probabilistic equations that the repairman is busy in preventive maintenance are:
\[ B(t) = \sum_{j=1}^{3} q_{0j}(t) B_j(t) \]
\[ B_0^2(t) = q_{11}(t) B_0^2(t) + q_{12}(t) B_1^2(t) + q_{13}(t) B_2^2(t) \]
\[ B_1^2(t) = q_{10}(t) B_0^2(t) + q_{11}(t) B_1^2(t) + q_{12}(t) B_2^2(t) \]
\[ B_2^2(t) = q_{10}(t) B_0^2(t) + q_{11}(t) B_1^2(t) + q_{12}(t) B_2^2(t) \]
\[ B_3^2(t) = W_3(t) + q_{30}(t) B_0^2(t) + q_{31}(t) B_1^2(t) + q_{32}(t) B_2^2(t) \]

Taking Laplace transform of [1-4] and computing the value of \( B_0^2(s) \), we have
\[ B_0^2(s) = \frac{N_4(s)}{D_2(s)} \]  
Where \( D_2(s) \) is obtained earlier as the expression of \( A_0^*(s) \) and 
\[ N_4(s) = W_3(s) q_{11}^*(4) (1-q_{11}^*(7)) q_{12}^*(5) q_{13}^*(6) \]  
To find the steady state, the fraction of time for which system is under preventive maintenance, we first calculate 
\[ W_3^*(0) = \int_0^\infty B(t) \, dt = n_3 \]  
\[ N_4(0) = n_3 p_{03} [(1-p_{11}^4) (1-p_{22}^7) - p_{12}^5 p_{21}^6] \]  
Therefore in the long run, the fraction of time for which the system is under preventive maintenance of the repairman is given by
\[ B_0^2(x) = \lim_{t \to \infty} B_0^2(t) = \lim_{s \to 0} s B_0^2(s) = \frac{N_4(0)}{D_2'(0)} \]

3.15 PARTICULAR CASES

When all time distributions are taken to be negative
exponential, i.e. \( g_1(t) = r_1 \exp(-r_1 t) \), \( g_2(t) = r_2 \exp(-r_2 t) \). 

\[ b(t) = \eta \exp(-\eta t) \]

then the steady state equations become:

MTSF = \( \frac{L_{00}}{L_{01}} \), \( A_0(t) = \frac{L_{02}}{L_{03}} \), \( B_0(t) = \frac{L_{03}}{L_{22}} \)

\[ B_0(t) = r_1 r_2 [2r_1 \eta L_5 + 2\eta r_1 \alpha (L_3 + \alpha_2) + 2\eta r_1 \alpha_2 L_5] \]

\[ L_{00} = [2L_2^2 L_3 (L_4 + \eta) + 4L_4 (\alpha_1 L_3 + \alpha_2 L_2)] \]

\[ L_{01} = [L_2 L_3 (L_4 - 2\eta) - 4L_4 (\alpha_1 L_3 + \alpha_2 L_2) + 2\eta L_5] \]

\[ L_{02} = \nu r_1 r_2 [2r_1 \eta L_5 + 2\eta \alpha_1 (4\alpha_1 L_4 + 2\eta) + 2\eta \alpha_2 (2L_4 + \eta) + 2\eta r_1 L_5] \]

\[ L_{03} = \nu [r_1 r_2 [(2L_4 + \eta) 2\alpha_1 (L_5 + \alpha_2)] + 2\alpha_2 r_1 r_2 (2L_4 + \eta)] + 4r_2 \alpha_1 (\alpha_1 \alpha_2 (2L_4 + \eta)] + 2r_1 \alpha_2 L_5 + 2\alpha_2 (2L_4 + \eta) + r_1 \alpha_2 (2L_4 + \eta)] \]

\[ L_1 = (2\alpha_1 + \alpha_2 + \eta), \quad L_2 = (r_1 + \alpha_1 + \alpha_2), \quad L_3 = (r_2 + \alpha_1 + \alpha_2) \]

3.16 COST ANALYSIS

The cost benefit analysis of the system can be carried out by considering the expected busy period of the repairman in repair and PM of the unit in \((0, t]\). Therefore

\[ G(t) = \text{expected total revenue earned by the system in } (0, t] \]

-expected repair cost of the repairman in repair and preventive maintenance in \((0, t] \).

\[ = C_1 \mu_{up}(t) - C_2 \mu_b(t) - C_3 \mu_b^2(t) \]
\[
\mu_{up}(t) = \int_0^t A_0(t) \, dt, \quad \mu_b^1(t) = \int_0^t B_0^1(t) \, dt, \quad \mu_b^2(t) = \int_0^t B_0^2(t) \, dt
\]

The expected total profit per unit of time in steady state is

\[
G = \lim_{t \to \infty} \frac{G(t)}{t} = \lim_{s \to 0} s^2 G(s) = C_1 \mu_{up} - C_2 \mu_b^1 - C_3 \mu_b^2 \tag{2}
\]

where \(C_1\) is the revenue per unit up time and \(C_2 \, C_3\) are the repair and preventive maintenance cost per unit of time respectively.

3.17 GRAPHICAL REPRESENTATION

To make an accurate study of system behavior, we plot the graphs for MTSF, Availability and Expected profit. Fig. D.2 shows the curve for MTSF with respect to \(\alpha_2\) for varying values of \(\alpha_1\). It is observed that MTSF decreases as \(\alpha_1\) increases. Also Fig. D.3 and Fig. D.4 represents the curves for Availability and Expected profit of the system with respect to \(\alpha_2\) for different values of \(\alpha_1\). The steady state availability decreases as \(\alpha_1\) increases. The same is the result of the curve of Fig. D.4.
EFFECT OF $\alpha_1$ ON MTSF FOR VARYING VALUES OF $\alpha_1$

EFFECT OF $\alpha_2$ ON AVAILABILITY FOR VARYING VALUES OF $\alpha_1$
EFFECT OF $\alpha_2$ ON EXPECTED PROFIT FOR VARYING VALUES OF $\alpha_1$

FIG D-4

$\alpha_1 = 0.01, 0.006, 0.002$

Failure rate of crusher ($\alpha_2$)

Expected profit