CHAPTER 2

DESIGN OF PLANAR MICROSTRIP ANTENNA ARRAYS AND MUTUAL COUPLING EFFECTS

In patch antennas, characteristics such as high gain, beam scanning, or steering capability are possible only when discrete patch elements are combined to form arrays. The elements of an array may be spatially distributed to form a linear, planar, or volume array. Planar arrays have elements distributed on a plane. Planar array configurations are extensively used in both communication and radar systems where a narrow pencil beam is required [51]. Individual elements can be positioned along a rectangular grid to form a planar array for better control of beam shape and position in space. Planar arrays of printed radiating elements are potentially good candidates for low cost scanning array applications [52]. Microstrip patch arrays are versatile as they can be used to synthesize a required pattern that cannot be achieved with a single element. In addition, they are used to increase the directivity and perform various other functions, which would be difficult with any one single element [53].

The chapter presents the basic characteristics and structure of a microstrip antenna, modeling and analyzing the behavior of a four and a six element rectangular microstrip patch antenna array. Configurations of a symmetrical 2×2 and asymmetrical 2×3 patch array have been analyzed and an optimum frequency range of the patch antenna array has been arrived at. It is inferred that for any planar array configuration, optimized antenna characteristics can be obtained, depending upon element spacing. The effects of surface waves and mutual coupling can be minimized by optimizing the inter element spacing in both the planes. The antenna provides frequency close to the designed operating frequency with an acceptable Directivity and Gain. When antenna structure is closely spaced, the return loss improves in the E plane. It has been shown that with the increasing array spacing, the gain of the antenna gets reduced significantly.

2.1 Microstrip Antennas

As discussed in Chapter1, conventional antennas and microstrip antennas are crucial for application in modern communication and navigation systems. Unlike
conventional microwave antennas, microstrip antennas can conform to both planar and non-planar surfaces. Military or civilian applications such as space and weight restricted aerospace vehicle structure microstrip antennas are better suited in comparison to conventional antennas.

2.1.1 Applications of Microstrip Antennas

The design and analysis of microstrip antenna to suit in applications related to aerospace viz. high performance aeroplane, satellite for military purpose, spacecraft and missile may necessitate critical examination of the following aspects.

- Antenna radiation pattern i.e. radiated field.
- Related to antenna radiation pattern are important parameters such as directivity, gain, radiation efficiency and power output.
- Return loss which ensures that there is impedance match between the feeding network and the microstrip patch antenna.
- Pattern and impedance bandwidth. Low impedance bandwidth results due to use of thin dielectric substrate, alternatively use thick dielectric but it will enhance surface wave and deteriorate radiation pattern and efficiency.

2.1.2 Advantages and Limitations of Microstrip Antennas

It is worth mentioning the advantages as well as the limitations of microstrip antennas. Well known facts which highlight the microstrip antenna as compared to conventional microwave antenna are:

- Because of its size, light weight, ease of installation and thin profile configurations it can be made conformal.
- Integration with microwave integrated circuits is relatively easier.
- It is versatile as far as resonance frequency, polarization, radiation pattern and impedance bandwidth is concerned.
- Along with the microstrip antennas structure, associated feed lines and matching network can be simultaneously fabricated.
- Can be made mechanically robust when conformed on rigid surfaces.

Limitations or disadvantages of microstrip antenna as compared to conventional antenna are:

- High Q resulting in narrow bandwidth, generally a few percent.
• Undesired radiation from the feed.
• Low gain and efficiency, in addition to high levels of cross polarization.
• Can handle low power of the order of tens of watts.
• It is difficult to achieve polarization purity.
• Thicker substrate results in excitation of surface waves.
• Most microstrip antennas radiate into half space.

2.1.3 Why Microstrip Antennas?

Despite the above stated limitations of the microstrip antennas, it is these properties of low profile, light in weight, conformable to both planar and non planar structure, easy to fabricate and integration to MIC that makes it superior to conventional flush mounted antennas. Hence in practice, microstrip antennas are extensively used in several applications meet the challenging system requirement due to the above stated facts. In addition effort to develop microstrip antennas configuration with accurate and rugged analytical model needs understanding of its limitations as well as improve design and optimize its performance [52].

2.1.4 Performance Improvement Techniques

There are techniques to overcome some of these limitation/disadvantages. Techniques exists to improve Q factor and hence enhancement of bandwidth. Microstrip antenna arrays can overcome the lower gain and lower power handling capacity. Use of photonic gap structure can overcome limitations such as poor efficiency, degraded radiation pattern and lower gain primarily by reducing the surface waves [54].

2.2 Construction of Microstrip Antenna

Microstrip antenna, shown in Figure 2.1 consists of a metallic patch, placed above a ground plane with a dielectric substrate sandwich between them. The thickness $t$ of the metallic patch is very small compared to free space wavelength $\lambda_0$, i.e. $t << \lambda_0$. 

![Figure 2.1: Construction of microstrip patch antenna.](image)
The height $h$ of the substrate above the ground plane on which patch is placed is very much less than $\lambda_0$. Generally range of $h$ lies within $0.003\lambda_0 \leq h \leq 0.05\lambda_0$.

### 2.2.1 The Permittivity Limitations of Microstrip Antennas

In the design of a microstrip patch antenna, $\varepsilon_r$, the relative permittivity of the dielectric substrate, which separates the patch from the ground plane, is generally taken to be in the range of 2.2 and 12 i.e. $2.2 \leq \varepsilon_r \leq 12$. To achieve better efficiency and larger bandwidth one must use low permittivity and thick substrate. However, if smaller patch elements are required it may be necessary to use thin substrate with high permittivity that will result in low efficiency and smaller bandwidth. The impedance bandwidth may lie between 1% to few tens of percent for substrate satisfying the criteria $h/\lambda_0< 0.023$ for $\varepsilon_r=10.2$ to $h/\lambda_0< 0.07$ for $\varepsilon_r= 2.3$ [52].

### 2.2.2 Impact of Patch Dimension on Microstrip Antennas

As explained above in Section 2.1, suitability of microstrip antenna in aerospace related applications is primarily due to limitation of space available in the parent structure. It is well known that larger patch width results in generation of grating lobes in addition to space requirements. Cross polarization is also an important characteristic related to the patch width. Therefore in addition to achieving good radiation efficiency, the patch width selection should be based on the space requirement, suppression of grating lobes and avoidance of cross polarization.

In the following section and subsections the theory related to the characteristics, design considerations of microstrip antennas on the basis of bandwidth, radiation characteristics and antenna pattern are discussed.

### 2.3 Characteristics and Design Aspects of Microstrip Antennas

#### 2.3.1 Theoretical Model of a Microstrip Antenna

Theoretical model of a microstrip antenna's can be explained as in Figure 2.2 [54]. At the resonant frequency significant radiation is produced due to strong field inside the cavity and strong current on the bottom surface of the patch.

$$\vec{E}_t = 0 \text{ and } \vec{E} = \hat{a}_x E_x(x,y)$$

Tangential component of the electric field on the patch and ground plane is zero and the electric field is expressed as given above. Separation $h$ between microstrip patch
antenna and the ground plane results in close proximity between them thus $\vec{E}$ has $z$ component.

$$\vec{V} \times \vec{E} = -j \omega \mu \vec{H}$$

$$\vec{E} = -\frac{1}{j \omega \mu} \vec{V} \times \vec{E} = -\frac{1}{j \omega \mu} \vec{V} \times (\hat{a}_z E_z(x, y)) = -\frac{1}{j \omega \mu} (-\hat{a}_z \times \vec{V} E_z(x, y))$$

$$\vec{H}(x, y) = \frac{1}{j \omega \mu} (\hat{a}_z \times \vec{V} E_z(x, y))$$

Using Maxwell’s equation, $\vec{H}$ is determined as shown above.

The mode is TM$_z$ and the magnetic field is purely horizontal. As can been seen above the vector field $\vec{H}$ has only x y components in the region bound by the patch and the ground plane. For the regions mentioned above, field is independent of $z$-coordinates.

$$\vec{J}_s : \hat{a}_n = 0$$

$\vec{J}_s$ is the surface current and on the edges of the patch we have

$$\vec{J}_s = (\hat{a}_z \times \vec{H}) ; \therefore \vec{H}_z = 0$$

$$\hat{a}_n \times \vec{H}(x, y) = 0$$

$$\hat{a}_n \times (\hat{a}_z \times \vec{V} E_z(x, y)) = 0$$

$$\hat{a}_z (\hat{a}_n \cdot \vec{V} E_z(x, y)) = 0 \text{ or } \frac{\partial E_z}{\partial n} = 0$$
On lower surface of the patch there is no electric current component normal to the edges of the patch; hence tangential $\vec{H}$ component does not exist, i.e. Perfect Magnetic Conductor (PMC) [14].

2.3.2 Design Considerations

Microstrip antenna can be called a resonant antenna [55]. System requirement specifies the operating frequency of antenna, and to meet the goal, appropriate antenna geometry is decided upon considering a rectangular patch, either used as a single element or number of elements forming an array. Design procedure is based on specific system requirements. The following design considerations are worth mentioning:

- Geometry of the patch antenna consists of a dielectric substrate separating the patch and the ground plane. Suitable dielectric material of height $h$, having a relative permittivity $\varepsilon_r$ and a specified loss tangent needs to be selected. Selected substrate having higher loss tangent effect the antenna efficiency.

$$f_r = \frac{c}{2L\sqrt{\varepsilon_r}} \quad \ldots \ (2.1)$$

- Dimension of the rectangular patch of width $W$ and length $L$ has effect on performance of the antenna. Resonant frequency $f_r$ of the rectangular patch antenna is dependent on the length $L$ and relative permittivity $\varepsilon_r$. For $\text{TM}_{10}$ mode, $f_r$ is given by equation (2.1).

$$\varepsilon_{re} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2} \left[ 1 + \frac{12h}{W} \right]^{-1/2} \quad \ldots \ (2.2)$$

Since the fringing field that results in increase of effective length of the patch, it can be accounted for by considering effective dielectric constant $\varepsilon_{re}$ given by the equation (2.2) [52].

$$\Delta L = 0.412h \frac{(\varepsilon_{re} + 0.3) \left[ \frac{W}{h} + 0.264 \right]}{(\varepsilon_{re} - 0.258) \left[ \frac{W}{h} + 0.813 \right]} \quad \ldots \ (2.3)$$

The increase in length of the patch $\Delta L$ due to fringing field is given by equation (2.3) [52].
\[ L = L_{\text{eff}} - 2\Delta L = \frac{c}{2f_r \sqrt{\varepsilon_{\text{re}}}} - 2\Delta L \quad \ldots (2.4) \]

The actual length of the patch is given by equation (2.4) [52], where \( L_{\text{eff}} = \frac{c}{2f_r \sqrt{\varepsilon_{\text{re}}}} \) is the effective length of the patch.

It is important to note that width \( W \) has insignificant effect on the resonant frequency \( f_r \) and the radiation pattern of the antenna. Nevertheless, input impedance and hence the bandwidth is affected by the width of the patch.

### 2.3.3 The Bandwidth of Microstrip Antennas

The bandwidth of microstrip antennas can be improved by increasing patch width \( W \). Due to increase in \( W \) the resonant resistance is reduced and as a result power output increases. Further, with proper excitation, \( W > L \) is resorted to in order to achieve the desired mode and suppression of undesired modes. However, larger \( W \) in antenna arrays, unlike a single patch antenna, may not suit the system requirements and in addition result in production of grating lobes. It has been suggested [56], [57] that for the limitation mentioned above, one must choose \( 1 < W/L < 2 \).

\[ W = \sqrt{\frac{h \lambda_g}{\ln \left( \frac{\lambda_g}{h} \right) - 1}} \quad \ldots (2.5) \]

To obtain \( 50\Omega \) input impedance patch width \( W \) can be found using the equation (2.5) [58], where \( \lambda_g = \frac{\lambda_0}{\sqrt{\varepsilon_{\text{re}}}} \). It is significant to mention that the factor \( \sqrt{\varepsilon_{\text{re}}} \) is due to loading by the substrate and strictly valid for a very wide patch for TM_{10} mode [52].

### 2.3.4 The Radiation Characteristics of Microstrip Antennas

Radiation characteristics determine the radiation pattern of the antenna. Further, radiation characteristics not only includes beam pattern i.e. beamwidth, sidelobe level, but also includes directivity, power output and polarization characteristics. The potential function may be used to derive the radiation pattern [52].

\[ \vec{E} = \frac{\varepsilon}{4\pi} \int_\mathcal{S} \vec{M} \left( \vec{r}' \right) e^{-j\vec{k}_* \cdot \left( \vec{r} - \vec{r}' \right)} d\mathcal{S}' \quad \ldots (2.6) \]
Let $\mathbf{F}$ be the vector electric potential which is expressed as shown in equation (2.6), where $\mathbf{M}(\mathbf{r})$ is the surface magnetic current density vector at a point $\mathbf{r}'$ from the origin and $dS'$ is the element surface shown in Figure 2.3.

![Figure 2.3: Determination of radiation characteristic of a rectangular patch](image)

Equation (2.6) is:

$$\mathbf{E} = \frac{\mu}{4\pi} \left[ \int \mathbf{J}(\mathbf{r}) \cdot \mathbf{E} - \frac{1}{\varepsilon} \mathbf{A} \times \nabla \times \mathbf{A} \right] dS$$

Similarly, the vector magnetic potential is given by equation (2.7), where $\mathbf{J}(\mathbf{r})$ is the surface electric current vector. Since the radiation characteristics of microstrip antennas may be studied by considering the expressions for electric and magnetic field components, it is necessary to consider the expressions for these components.

$$\mathbf{E} = \frac{1}{j\omega\mu\varepsilon} \nabla(\mathbf{A} \cdot \mathbf{E}) - j\omega\mathbf{A} - \frac{1}{\varepsilon} (\nabla \times \mathbf{A})$$

$$\mathbf{H} = \frac{1}{j\omega\mu\varepsilon} \nabla(\mathbf{A} \times \mathbf{E}) - j\omega\mathbf{F} - \frac{1}{\mu} (\nabla \times \mathbf{A})$$

The total electric and magnetic fields due to electric and magnetic current sources are given by equation (2.8) and (2.9) respectively [52].

$$H_\theta = -j\omega F_\theta \quad \text{and} \quad H_\phi = -j\omega F_\phi$$

Far field components that are transverse to the direction of propagation are $\theta$ and $\phi$ components. Considering magnetic field components which are function of electric vector potential is given in the equation (2.10) [54].

$$\mathbf{E} = -\eta_0 (\mathbf{r} \times \mathbf{H}) = -\eta_0 (\hat{\phi} H_\phi - \hat{\theta} H_\theta)$$

$$\mathbf{E} = j\omega\eta_0 (\hat{\phi} F_\phi - \hat{\theta} F_\theta)$$

$E_\theta = -j\omega A_\theta$ and $E_\phi = -j\omega A_\phi$
The electric field in the free space thus can be obtained as shown in equation (2.11) and (2.11(a)), where η₀ is the free space impedance, the electric field component \( E_\theta \) and \( E_\varphi \) may be expressed as given in equation (2.12).

\[
\vec{H} = r \times \vec{E} / \eta_0
\]  
... (2.13)

\[
\vec{F} = \frac{\varepsilon}{4\pi} e^{-jk_0r} \int \int \vec{M}(r') e^{jk_0r' \cos \varphi} \, ds'
\]  
... (2.14)

\[
\vec{A} = \frac{\mu}{4\pi} e^{-jk_0r} \int \int \vec{J}(r') e^{jk_0r' \cos \varphi} \, ds'
\]  
... (2.15)

Since \( r \cos \varphi = x \sin \theta \cos \varphi + y \sin \theta \sin \varphi \)  
... (2.16)

The \( \vec{H} \) in free space may be expressed as in equation (2.13). For the far field condition \( r >> r' \) and \( |\vec{r} - \vec{r}'| = r - r' \cos \varphi \) in the numerator and \( \vec{r} - \vec{r}' = r \) in the denominator, we have expression for vector electric and magnetic potential as given by equation (2.14) and (2.15) respectively [52].

\[
\vec{F} = \varepsilon_0 \left( e^{-jk_0r} / r \right) \int_{-L/2}^{L/2} \int_{-W/2}^{W/2} \vec{M}(x', y') e^{jk_0(x' \sin \theta \cos \varphi + y' \sin \theta \sin \varphi)} \, dx' \, dy'.
\]  
... (2.17)

\[
\therefore \vec{M}(x', y') = (M_x(x', y') \hat{x} + M_y(x', y') \hat{y})
\]  
... (2.18)

\[
\therefore F_x = \varepsilon_0 \left( e^{-jk_0r} / r \right) \int_{-L/2}^{L/2} \int_{-W/2}^{W/2} M_x(x', y') e^{jk_0(x' \sin \theta \cos \varphi + y' \sin \theta \sin \varphi)} \, dx' \, dy'
\]  
... 2.19(a)

\[
F_y = \varepsilon_0 \left( e^{-jk_0r} / r \right) \int_{-L/2}^{L/2} \int_{-W/2}^{W/2} M_y(x', y') e^{jk_0(x' \sin \theta \cos \varphi + y' \sin \theta \sin \varphi)} \, dx' \, dy'
\]  
... 2.19(b)

\[
F_z = 0
\]  
... (2.19(c))

From equation (2.14) and (2.16), integrating over two dimensional surface area between \(-L/2 \) to \( L/2 \) and \(-W/2 \) to \( W/2 \), far fields of a rectangular magnetic current source is obtained in equation (2.17). The surface magnetic current density defined in equation (2.18) and the corresponding components of electric potential given in equation (2.19(a)) to equation (2.19(c)) [52].

\[
E_\theta = j \omega \eta_0 (F_x \sin \varphi - F_y \cos \varphi)
\]  
... (2.20(a))

\[
E_\varphi = j \omega \eta_0 (F_x \cos \theta \cos \varphi - F_y \cos \theta \sin \varphi)
\]  
... (2.20(b))
Corresponding electric field components in polar coordinates obtained in equation (2.20(a)) and (20(b)) [52].

2.3.5 Radiation pattern based on two slots model:

Similarly using two slots-model radiation patterns for the TM$_{10}$ mode can be determined. However radiation pattern gets affected due to ground plane and the substrate.

$$F_3(\theta) = \frac{2 \cos \theta \sqrt{\varepsilon_r - \sin^2 \theta}}{\sqrt{\varepsilon_r - \sin^2 \theta - j \varepsilon_r \cos \theta \cot(k_0 h \sqrt{\varepsilon_r - \sin^2 \theta})}} \quad ... (2.21)$$

Applying reciprocity theorem to two infinitesimal dipoles, one located on the surface of the substrate and the other in free space at a far distance results in a factor $F_3(\theta)$ which needs to be included is [52].

$$F_4(\theta) = \frac{2 \cos \theta}{\cos \theta - j \sqrt{\varepsilon_r - \sin^2 \theta} - \cot(k_0 h \sqrt{\varepsilon_r - \sin^2 \theta})} \quad ... (2.22)$$

Similarly, for the $E$ plane pattern ($\varphi = 0^\circ$). For $H$-plane pattern $\varphi = 90^\circ$

$$E_{\varphi}(\theta, \varphi) = \sin \varphi \left(\frac{j \omega \mu_0}{4 \pi r}\right) e^{(-j \beta r)} J(\theta, \varphi) F_3(\theta) \quad ... (2.23)$$

$$E_{\theta}(\theta, \varphi) = -\cos \varphi \left(\frac{j \omega \mu_0}{4 \pi r}\right) e^{(-j \beta r)} J(\theta, \varphi) F_3(\theta) \quad ... (2.24)$$

$$\vec{J}(x, y) = \hat{x} \frac{V_0}{Z_0 W} \sin(\beta x) \quad \text{for } 0 < x < L \ , \ 0 < y < W \quad ... (2.25)$$

$$\vec{J}(\theta, \varphi) = \frac{V_0}{Z_0} \frac{2 \sqrt{\varepsilon_{re}}}{k_0} \frac{\cos \left(\pi \sin \theta \cos \frac{\varphi}{2} \frac{\sqrt{\varepsilon_{re}}}{2}\right)}{(\sin^2 \theta \cos^2 \varphi - \varepsilon_{re})} \times \frac{\varepsilon_{re} \sin^2 (0.5k_0 W \sin \theta \sin \varphi)}{Z_0} \quad ... (2.26)$$

Radiation pattern based on the model where surface current replaces the metallic patch and taking into account grounded substrate as given in equation (2.23) and equation(2.24) [52], where $\vec{J}(\theta, \varphi)$ is the Fourier transform of the patch current. Patch current for the TM$_{10}$ mode is given by equation (2.25). Where $\beta = \sqrt{\varepsilon_{re} k_0}$ and $Z_0$ is the characteristic impedance corresponding to width $W$. $\vec{J}(\theta, \varphi)$ is obtained as shown in equation (2.26). $V_0$ is the voltage across either radiating slot.

Patch length $L$ for the TM$_{10}$ mode is given by $L = \lambda_0/2$
\[ k_o L = \frac{\beta}{\sqrt{\varepsilon_r}} \times \frac{\lambda_g}{2} = \frac{2\pi}{\lambda_g} \times \frac{1}{\sqrt{\varepsilon_r}} \times \frac{\lambda_g}{2} = \frac{\pi}{\sqrt{\varepsilon_r}} \]

\[ |E_\phi(\theta)|^2 = \varepsilon_r \left[ 1 + \varepsilon_r \cot^2 \left( k_o h \sqrt{\varepsilon_r} \right) \right] \times \frac{\cos^2 \left( k_o L \sin \frac{\theta}{2} \right) \left( \varepsilon_r - \sin^2 \theta \right) \cos^2 \theta}{\left( \varepsilon_r - \sin^2 \theta \right)^3 + \varepsilon_r^2 \cos^2 \theta \cot^2 \left( k_o h \sqrt{\varepsilon_r - \sin^2 \theta} \right) \cdots} \]

(2.27)

\[ |E_\phi(\theta)|^2 = \left[ 1 + \varepsilon_r \cot^2 \left( k_o h \sqrt{\varepsilon_r} \right) \right] \times \frac{\cos^2 \theta \sin \left( k_o W \sin \frac{\theta}{2} \right)}{\left( \varepsilon_r - \sin^2 \theta \right) \cot^2 \left( k_o h \sqrt{\varepsilon_r - \sin^2 \theta} \right) + \cos^2 \theta \cdots} \]

(2.28)

\[ E\text{-plane pattern corresponding to } \varphi = 0^\circ \text{ plane } E_\varphi = 0 \text{ is obtained in equation (2.27)} \]

[52], and \( H\text{-plane which corresponds to } \varphi = 90^\circ \text{ plane } E_\theta = 0 \) is given in equation (2.28) [52].

\[ P_r = \frac{1}{2} \int_0^{2\pi} \int_0^{\pi/2} \left[ |E_\theta|^2 + |E_\phi|^2 \right] r^2 \sin \theta d\theta d\varphi \quad \cdots (2.29) \]

\[ P_r = \frac{V_o^2}{Z_0} \frac{60\varepsilon_r \pi^2}{\varepsilon_r} \int_0^{\pi/2} \cos^2 \left( \pi \sin \theta \cos \frac{\varphi}{\sqrt{\varepsilon_r}} \right) \sin^2 \left( k_o W \sin \frac{\varphi}{2} \sin \theta \right) \sin \theta d\theta d\varphi \times \]

\[ \left[ \frac{\varepsilon_r - \sin^2 \theta}{\cos^2 \theta \cot^2 \left( k_o h \sqrt{\varepsilon_r - \sin^2 \theta} \right) + \cos^2 \theta \varepsilon_r^2 \cos^2 \theta \cot^2 \left( k_o h \sqrt{\varepsilon_r - \sin^2 \theta} \right) \cdots} \right] \]

Hence the radiated power \( P_r \) can be obtain substituting equation (2.27) and equation (2.28) in equation (2.29). Hence we have the radiated power.

\[ P_r = \frac{(E_o h)^2 A^4}{23040} \left[ 1 - B \left( 1 - \frac{A}{15} + \frac{A^2}{420} \right) + \frac{B^2}{5} \left( 2 - \frac{A}{7} + \frac{A^2}{189} \right) \right] \]

where \( A = \left( \frac{\pi W}{\lambda_0} \right)^2 \) and \( B = \left( \frac{2L}{\lambda_0} \right)^2 \)

Simpler expression is obtained by neglecting the effect of the substrate [59].

\[ R_r = \frac{V_o^2}{2P_r} = \varepsilon_r Z_0^2 \frac{Z_0^2}{120I_2} \] is valid for \( h \leq 0.03\lambda_0 \) and \( \varepsilon_r \leq 0 \) for an accuracy of 10% [60].

where, \( I_2 = (k_o h)^2 \left\{ 0.53 - 0.03795 \left( \frac{k_o W}{2} \right)^2 - \frac{0.03553}{\varepsilon_r} \right\} \) for \( \varepsilon_r \leq 5 \)

\[ I_2 = I_L / I_1 \text{ for } 5 \leq \varepsilon_r \leq 10 \]
and $I_L = (k_0 h)^2 \left[ 1.3 - \frac{4}{3 \varepsilon_r} + \frac{0.53}{\varepsilon_r^2} - \left( \frac{k_0 W}{2} \right)^2 \left\{ 0.08856 - \frac{0.08856}{\varepsilon_r} + \frac{0.03795}{\varepsilon_r^2} \right\} \right]$

and $I_i = (\varepsilon_r - 1) \left( 1.29 - \frac{3.57 h}{\lambda_0} \right) / 9$

Radiation resistance $R_r$ is as given above.

$R_m = R_r \cos^2 \left( \frac{\pi x_f}{L} \right)$

If the patch is fed at a distance $x_f$ from one of the radiating edge, the input resistance is obtained as given above. The factor $\cos \left( \frac{\pi x_f}{L} \right)$ is due to field variation for the dominant mode.

$$D = \frac{\frac{r^2}{2 \eta_0} \left( |E_\theta|^2 + |E_\phi|^2 \right)_{\theta=0}}{\frac{P_r}{4\pi}} \quad \ldots \ (2.30)$$

Now we obtain the Directivity of the patch antenna is given by equation (2.30)

$$D \approx \frac{4(k_0 W)^2}{\pi \eta_0 G_r} \cdot$$

For a rectangular patch antenna, the directivity can be approximated as [52], where $G_r = 1/R_r$ is the radiation conductance of the patch.

$G = kD$

Gain of an antenna is defined as $G$, where $k = \text{radiation efficiency of the antenna}$

$0 < k < 1$

$$\theta_E = 2 \sin^{-1} \left( \frac{7.03}{3k_0^2 L^2 + k_0^2 h^2} \right)^{1/2} \quad \ldots \ (2.31(a))$$

$$\theta_H = 2 \sin^{-1} \left( \frac{1}{2 + k_0^2 W} \right)^{1/2} \quad \ldots \ (2.31(b))$$

The half power beamwidth $\theta_E$ and $\theta_H$ for $E$ and $H$ plane may be expressed respectively by the empirical relationship in terms of equations (2.31(a)) and (2.31(b)) [52].
2.4 Microstrip Antenna Arrays

Single microstrip patch antenna may not be suitable for application, which needs high gain, beam scanning or enhance bandwidth. In order to enhance gain and to achieve beam steering capability the arrays formation of microwave antenna is resorted to. The same concept of array formation may therefore be used with the microstrip patch antenna as well. Antenna arrays may be linear, planar or conformal. In applications related to radar and communication systems narrow beam is desired and hence planar array configuration may be used for such a requirement.

2.4.1 Microstrip Planar Arrays

Considering that edge effects are subjected to all the elements, the design of finite arrays necessitates grouping of the patches in a symmetrical pattern so that radiation in the desired direction is obtained. This can be achieved only when fields due to individual patches get combined in phase in the desired direction and cancel each other in all other directions. In other words each patch output is combined to obtain the fields radiated by the array. It is important to note that radiation pattern of individual patch in the array is the same when it is in stand-alone mode. Hence the original pattern of the individual patch gets multiplied by the array factor that takes into consideration the amplitudes and phases of the feed current [61].

However due to closer proximity between patches in the array there is interaction between the elements. Since each patch element induces currents to the other adjacent patches, it leads to coupling among the radiating patches [62]. Inter element location and spacing between the arrays affects the radiation pattern as well as antenna parameters. Momentum, an Advanced Design System Software (ADS), based on the Method of Moments may be used to determine the radiation pattern, current distribution and associated antenna parameters for the microstrip antenna array system. The system requirement necessitates that spacing between the patch elements in the array, in terms of wavelength in both $H$ and $E$ planes, be specified in order to obtain the desired radiation pattern.

Simple design steps of planar arrays with the schematic shown in Figure 2.4 are
1. Proper choice of spacing between patch elements primarily to minimize the grating lobes.
2. Proper selection of substrate material viz. its thickness, relative permittivity and inter element spacing within the scan volume can eliminate scan blindness.
3. In order to minimize spurious radiation, the feed network should be suitably designed or isolated from the radiating elements.

2.4.2 Microstrip Resonant Arrays

Periodically separating the patch elements fed by an open or short circuit terminated feed line results in a resonant array. As shown in the equivalent circuit of a resonant array of Figure 2.5, the inter element spacing lies between $\lambda g$ and $\lambda g/2$. The resonant feed results in the generation of broad side beam and in narrow VSWR bandwidth.

The spacing $\lambda g$ and $\lambda g/2$ between elements results in high return loss due to reflection from the patch radiating elements.
\[ y = \sum_{n=1}^{N} g_n + j0 = 1 \]

The normalized input admittance \( y \) of the array for \( N \) radiating elements may be expressed in terms of \( g_n \), the normalized conductance of the \( n^{th} \) element.[52]. In other words matching of the feed line characteristic admittance with the input admittance of the array must be ensured [63], [64].

Resonating frequencies of microstrip antenna depends on modes. Antenna characteristics such as resonant frequency, radiation pattern, polarization etc. are different for each mode. Since a square patch supports modes (1, 0) and (0, 1) with horizontal and vertical polarization, it is preferred over a rectangular patch. In view of this the design of the present work is based on the square patch array.

2.5 Single Patch Design Parameters

Patch under consideration is designed to operate in X-band. RT Duroid 5880 is the substrate chosen having thickness = 0.747 mm, permittivity \( \varepsilon_r = 2.2 \) and the substrate loss tangent \( \tan\delta = 0.0009 \). Figure 2.6 shows the ADS Momentum layout of the single microstrip patch antenna.

![Figure 2.6: ADS Momentum layout of single microstrip patch antenna.](image)

Square patch designed having length and width of 9.6mm to resonate at a frequency of 10 GHz. Details of microstrip antenna patch parameters and substrate details are
shown in Figure 2.7 and Figure 2.8 respectively.

Figure 2.7: Square Microstrip Patch Antenna Parameters

Figure 2.8: Details of Substrate considered in the design.
Calculations of feed design are given in the following subsections:

2.5.1 Impedance Calculation

The Patch Impedance is calculated from knowledge of $R_{in}$.

$$R_{in} = \left( \frac{1}{2 \times (G_1 + G_{12})} \right)$$

$$G_1 = \frac{\int_0^\pi \left( \frac{\sin \left( k_0 \frac{W}{2} \cos \theta \right)}{\cos \theta} \right)^2 \times \sin^3 \theta \, d\theta}{120 \pi^2} \quad \text{... (2.32(a))}$$

$$G_{12} = \frac{\int_0^\pi \left( \frac{\sin \left( k_0 \frac{W}{2} \cos \theta \right)}{\cos \theta} \right)^2 \times (J_0(k_0 L \sin \theta)) \sin^3 \theta \, d\theta}{120 \pi^2} \quad \text{... 2.32(b))}$$

Expressed in terms of self conductance $G_1$ and mutual conductance $G_{12}$ of the patch under consideration as given above [54], where $G_1$ and $G_{12}$ are respectively expressed by equations (2.32(a)) and (2.32(b)), where $k_0 = \frac{2 \pi}{\lambda_0}$. Using the values obtained for $G_1$ and $G_{12}$ from equations (2.32(a)) and (2.32(b)), we obtain $G_1 + G_{12} = 0.00142 \text{mho}$, hence the input impedance of the patch is found to be 350.3882 ohm.

Impedance of the matching transformer line is 161.856 ohm; impedance of the feed line is 71.8932 ohm.

Calculated Patch Impedance = \frac{(\text{Transformer Matching Impedance})^2}{\text{Feed line Impedance}}

Using the formula for patch impedance calculation is as shown above. Therefore, calculated patch impedance is \( \frac{(161.856)^2}{350.3882} = 74.76 \text{ ohm} \), which is close to 71.8932 ohm hence impedance match, is realized.
2.5.2 ADS Momentum Results

The results obtained based on ADS Momentum are as follows. Figure 2.9 shows the return loss $S_{11}$, at frequency of resonance 9.903 GHz is 24.498 dB.

![Return loss S11](image)

**Figure 2.9:** Return loss $S_{11}$ obtained for the Patch

The feed design can be considered matched to the patch input impedance. Figure 2.10 and 2.11 respectively shows the radiation plot of the microstrip patch antenna in both $\theta$ and $\phi$ plane respectively. The plot shows radiation pattern with perfect null and no sidelobes.

![Radiation plots](image)

**Figure 2.10:** Both front and back $E_{\theta}$ plot
Table 2.1 shows the antenna parameters. The antenna radiates 1.588 mW power with
directivity of 7.185 dB and gain 6.815 dB. The antenna efficiency is 94.84%.

![Antenna Parameters Table](image)

2.6 *Four Element 2x2 Antenna Array Design*

Four element planar array is designed for the study of effect of mutual coupling in
both $E$ and $H$ Plane. The square antenna element patch length and width is 9.6 mm,
height of the substrate under consideration is 0.747 mm, having permitivity $\varepsilon_r = 2.2$. In
both $E$ and $H$ plane the separation kept at $\lambda/2 = 15$ mm. Figure 2.12 shows the layout schematic of the planar four element antenna array under consideration.

### 2.6.1 Calculation of Impedance for the Planar Array

#### 2.6.1.1 Calculation of the Feed Line Impedance

$$Z = \frac{120\pi}{2 \times 2^{0.5} \times \pi \times (\varepsilon_r + 1)^{0.5}} \log \left( 1 + \left( 4 \times \frac{h}{W_c} \times (A + B) \right) \right) \times 100$$

... (2.33)

where $W_c = W + \nabla W_c$ and $\nabla W_c = \nabla W \times \left( 1 + \frac{1}{\varepsilon_{eff}} \right)$

Characteristic impedance of the strip is obtained based on the equations (2.33) [65].

---

**Figure 2.12**: Schematic of four element planar antenna array.
\[ \nabla W = \left( \frac{t}{\pi} \right) \times \log \left( 4 \times \frac{e}{ \left( \frac{t}{h} \right)^2 + \left( \frac{1}{\pi \left( \frac{W}{t} \right)} + 1.1 \right)^2} \right) \]

We obtain \( \nabla W \) from the following equation.

\[ A = \left( \frac{14 + \left( \frac{8}{\varepsilon_{\text{eff}}} \right)}{11} \times 4 \frac{h}{W_e} \right) \quad \text{and} \quad B = \left( A^2 + \left( \frac{1 + \frac{1}{\varepsilon_{\text{eff}}}}{2} \times \pi^2 \right) \right)^{0.5} \]

Also the parameters \( A \) and \( B \) are obtained using the above equations. Based on the calculation we obtain the value of \( Z=124.479 \) ohm, hence the impedance can be 125 ohm approximately. Figure 2.13 validates the calculations, as the value obtained using ADS Momentum is 125 ohm.

[Figure 2.13: Stripline Impedance calculated using ADS Momentum]

Therefore the feed impedance is taken half the impedance of the feed line i.e. 62.5 ohm [54], [66], [67].
2.6.1.2 Calculation of the Patch Impedance

Next we calculate the patch impedance considering the effect of mutual coupling due to the close proximity of the patches [54].

\[
G_{12_{Eplane}} = \left( \frac{1}{\pi} \right) \left( \frac{\varepsilon_r \times \varepsilon_0}{\mu_0} \right)^{0.5} \int_0^\pi \left[ \left( \sin \left( \frac{\pi}{\lambda_0} \frac{W}{\cos \theta} \right) \right)^2 \right] \left[ \frac{2J_0 \left( \frac{\lambda}{\lambda_0} \sin \theta \right)}{(\cos \theta)^2} \right] \left( \sin \theta \right)^2 \left( \cos \left( \frac{\pi}{\lambda_0} \frac{2\pi \sin \theta}{\lambda} \right) \right) + J_0 \left( \frac{\lambda}{\lambda_0} \sin \theta \right) \right] d\theta
\]

\ldots (2.34)

Using the equation for conductance due to effect of mutual coupling respectively in \( E \) and \( H \) plane based on equations (2.34) and (2.35). Along the \( E \) plane

\[
G_{12_{Hplane}} = \left( \frac{1}{\pi} \right) \left( \frac{\varepsilon_r \times \varepsilon_0}{\mu_0} \right)^{0.5} \int_0^\pi \left[ \left( \sin \left( \frac{\pi}{\lambda_0} \frac{W}{\cos \theta} \right) \right)^2 \right] \left[ \frac{2J_0 \left( \frac{\lambda}{\lambda_0} \sin \theta \right)}{(\cos \theta)^2} \right] \left( \sin \theta \right)^3 \left( \cos \left( \frac{\pi}{\lambda_0} \frac{2\pi \cos \theta}{\lambda} \right) \right) + J_0 \left( \frac{\lambda}{\lambda_0} \sin \theta \right) \right] d\theta
\]

\ldots (2.35)

\[
R_{in} = \left( \frac{1}{2(G_1 + G_{12_{Eplane}} + G_{12_{Hplane}} + G_{12})} \right)
\]

\ldots (2.36)

Along the \( H \) plane and then \( R_{in} \) using equation (2.36), where \( G_1 \) is the self conductance and \( G_{12} \) is the mutual conductance of the patch itself. Expression of \( G_1 \) and \( G_{12} \) are expressed by equation (2.32(a)) and (2.32(b)) respectively, the same along with equation (2.34) and (2.35) may be put in equation (2.36) to determine \( R_{in} \) of the planar array. Using \( R_{in} \), the value of the impedance of the patch is found out. The value of \( G_{12_{Eplane}} \) (due to mutual coupling in the \( E \)-Plane) is found to be 3.1173x10\(^{-4}\) mho, \( G_{12_{Hplane}} \) (due to mutual coupling in the \( H \)-Plane) works out to be 6.0377x10\(^{-4}\) mho and \( G_1 + G_{12_{Eplane}} \) of the patch itself is 0.00142 mho. Hence impedance of the patch comes out to be 213.4562 ohm. Here we have not considered the conductance due to mutual coupling between the diagonally opposite patch. Assuming the mutual
conductance of the diagonally opposite patch is in the range of the mutual conductance of the $E$-Plane ($G_{12_{E_{PLANE}}}$) along with patch $G_1 + G_{12_{E_{PLANE}}} + G_{12_{H_{PLANE}}} + G_{12}$ i.e. taking the value of the mutual conductance of the diagonally opposite patch to be $3.1173 \times 10^{-4}$ mho we get the input impedance of the patch as 188.8766 ohm. Alternatively we may also consider by taking the mutual conductance of the diagonally opposite patch to be mutual conductance of the $H$-Plane patch i.e. $6.0377 \times 10^{-4}$ mho along with patch $G_1 + G_{12_{E_{PLANE}}} + G_{12_{H_{PLANE}}} + G_{12}$, we get the input impedance of the patch as 170.1102 ohm which comes fairly close to the feed impedance. Figure 2.14 shows the ADS Momentum based value of input impedance obtained for the planar array. The input impedance obtained is 213.4484 ohm which nearly same as calculated taking into consideration the effect of mutual coupling and found to be 213.4562 ohm.

* The work reported in this chapter is based on the following research paper contributions [68]: Gupta S.D., Singh A., “Design of Microstrip Planar Antenna Array and Study of Effect on Antenna Parameters due to Mutual Coupling in both $E$ and $H$ Planes”, International Journal of Communication Engineering Applications (IJCEA), vol. 02, Issue 06, Aug. 2011.

Figure 2.14: Input Impedance obtained using ADS Momentum.
2.7 Effect of Mutual Coupling in $E$ plane and $H$ plane

**Figure 2.15**: Layout of the 2x2 microstrip patch antenna array.

Figure 2.15 shows the layout of the 2x2, four element microstrip patch antenna planar array.

**Figure 2.16**: Stripline Impedance for the Array calculated using ADS Momentum

Figure 2.16 shows the strip line impedance for the array calculated using ADS Momentum. The impedance of the feed obtained is 156.915–157 ohm, which is less
than 213.4484 ohm found based on calculations and which is expected as seen to be same that matched the result obtained using the formula.

Figure 2.17: Schematic showing Mutual Coupling due to Diagonal Patch.

Here we have considered the self conductance of the patch (same as obtained) along with conductance due to mutual coupling in the $E$ Plane and conductance due to mutual coupling in the $H$- Plane patch but not the conductance due mutual coupling due to diagonal patch, refer schematic shown in Figure 2.17. Incorporating the mutual conductance of the diagonal patch will further decrease the patch impedance value.

2.7.1 Effect of Mutual Coupling on Antenna Parameters

2.7.1.1 Analysis of Antenna Parameters Variation in $E$ Plane

Refer Table 2.2 which considers inter element spacing variation in $E$ plane starting from $0.5\,\lambda$ in steps of $0.05\,\lambda$ upto maximum range of $0.7\,\lambda$. The parameters under consideration include return loss $S_{11}$, radiated power output, gain, directivity and antenna efficiency at a deviated resonant frequency due to effect of mutual coupling. Table shows at $0.7\lambda$ the antenna directivity and gain is the maximum. It is observed that return loss of $-42.72\,\text{dB}$ is the minimum value at $0.65\lambda$. At $0.5\lambda$ radiated output power and antenna efficiency is maximum at a resonant frequency of $9.876\,\text{GHz}$ which is the closest to the design value of $10\,\text{GHz}$. 
Table 2.2: Effect of Mutual Coupling in E-Plane on Antenna Parameters

<table>
<thead>
<tr>
<th>$E$-Plane Separation</th>
<th>$S_{11}$ (dB)</th>
<th>Power Radiated (mW)</th>
<th>Gain (dB)</th>
<th>Directivity (dB)</th>
<th>Resonant Frequency ($f_r$) (GHz)</th>
<th>Efficiency (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.5\lambda$</td>
<td>-29.67</td>
<td>1.96</td>
<td>11.15</td>
<td>11.23</td>
<td>9.876</td>
<td>99.36</td>
</tr>
<tr>
<td>$0.55\lambda$</td>
<td>-25.93</td>
<td>1.84</td>
<td>10.77</td>
<td>11.03</td>
<td>9.863</td>
<td>97.64</td>
</tr>
<tr>
<td>$0.6\lambda$</td>
<td>-33.88</td>
<td>1.95</td>
<td>12.07</td>
<td>12.19</td>
<td>9.855</td>
<td>99.05</td>
</tr>
<tr>
<td>$0.65\lambda$</td>
<td>-42.72</td>
<td>1.91</td>
<td>12.25</td>
<td>12.44</td>
<td>9.847</td>
<td>98.47</td>
</tr>
<tr>
<td>$0.7\lambda$</td>
<td>-33.93</td>
<td>1.88</td>
<td>12.35</td>
<td>12.63</td>
<td>9.843</td>
<td>97.80</td>
</tr>
</tbody>
</table>

2.7.1.2 Analysis of Antenna Parameters Variation in $H$ Plane

Refering Table 2.3 considering inter element spacing variation in $H$ plane in the same range from $0.5\lambda$ in steps of $0.05\lambda$ up to maximum value of $0.7\lambda$. Here too we study variation in return loss $S_{11}$, radiated power output, gain, directivity and antenna efficiency at a changed resonant frequency due to effect of mutual coupling. We observe from the Table that at $0.5\lambda$ the antenna directivity, gain, power output and the antenna efficiency is the maximum. It is seen that at $0.55\lambda$ return loss of -52.51 dB is the minimum value. At $0.5\lambda$ $H$ plane spacing antenna resonates at frequency of 9.876 GHz same as that seen at $E$ plane. With 1.95 mW power radiated at $0.55\lambda$, a typical radio fuze application of employing such an antenna array is envisaged.

Table 2.3: Effect of Mutual Coupling in $H$-Plane on Antenna Parameters

<table>
<thead>
<tr>
<th>$H$-plane Separation</th>
<th>$S_{11}$ (dB)</th>
<th>Power Radiated (mW)</th>
<th>Gain (dB)</th>
<th>Directivity (dB)</th>
<th>Resonant Frequency ($f_r$) (GHz)</th>
<th>Efficiency (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.5\lambda$</td>
<td>-29.67</td>
<td>1.96</td>
<td>11.15</td>
<td>11.23</td>
<td>9.876</td>
<td>99.36</td>
</tr>
<tr>
<td>$0.55\lambda$</td>
<td>-52.51</td>
<td>1.95</td>
<td>11.11</td>
<td>11.22</td>
<td>9.831</td>
<td>98.96</td>
</tr>
<tr>
<td>$0.6\lambda$</td>
<td>-25.75</td>
<td>1.90</td>
<td>10.79</td>
<td>11.00</td>
<td>9.766</td>
<td>98.15</td>
</tr>
<tr>
<td>$0.65\lambda$</td>
<td>-16.73</td>
<td>1.84</td>
<td>10.77</td>
<td>11.04</td>
<td>9.728</td>
<td>97.55</td>
</tr>
<tr>
<td>$0.7\lambda$</td>
<td>-11.65</td>
<td>1.75</td>
<td>10.90</td>
<td>11.18</td>
<td>9.706</td>
<td>97.52</td>
</tr>
</tbody>
</table>

2.7.1.3 Effect of Mutual Coupling in $E$ and $H$ Plane on Radiation Pattern

Effect on radiation pattern both $E_{\theta}$ and $E_{\phi}$ due to inter element spacing respectively in $E$ and $H$ plane are shown in Figure 2.18 and Figure 2.19. In $E$ plane at antenna
element separation $0.55\lambda$ and for $H$ plane at $0.5\lambda$ we observe radiation pattern with nulls and without sidelobes in both $E_\theta$ and $E_\phi$. In both the planes as the inter element spacing is increased the effect on $E_\theta$ radiation pattern is dominant in terms of increase in sidelobe level. However $E_\phi$ radiation pattern shows marginal distortion and increase in sidelobe levels $0.7\lambda$ specifically in $E$ plane.

<table>
<thead>
<tr>
<th>$E$-Plane Separation</th>
<th>$E_\theta$</th>
<th>$E_\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.5\lambda$</td>
<td><img src="image1.png" alt="Image" /></td>
<td><img src="image2.png" alt="Image" /></td>
</tr>
<tr>
<td>$0.6\lambda$</td>
<td><img src="image3.png" alt="Image" /></td>
<td><img src="image4.png" alt="Image" /></td>
</tr>
<tr>
<td>$0.65\lambda$</td>
<td><img src="image5.png" alt="Image" /></td>
<td><img src="image6.png" alt="Image" /></td>
</tr>
</tbody>
</table>
**Figure 2.18:** Effect of Antenna Element Separation in $E$ Plane on Radiation Pattern

<table>
<thead>
<tr>
<th>$E$-Plane Separation</th>
<th>$E_\theta$</th>
<th>$E_\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7λ</td>
<td><img src="image" alt="Plot" /></td>
<td><img src="image" alt="Plot" /></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$H$-Plane Separation</th>
<th>$E_\theta$</th>
<th>$E_\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5λ</td>
<td><img src="image" alt="Plot" /></td>
<td><img src="image" alt="Plot" /></td>
</tr>
<tr>
<td>0.55λ</td>
<td><img src="image" alt="Plot" /></td>
<td><img src="image" alt="Plot" /></td>
</tr>
<tr>
<td>$H$-Plane Separation</td>
<td>$E_\theta$</td>
<td>$E_\phi$</td>
</tr>
<tr>
<td>----------------------</td>
<td>-----------</td>
<td>-----------</td>
</tr>
<tr>
<td>0.6$\lambda$</td>
<td><img src="image1.png" alt="Image" /></td>
<td><img src="image2.png" alt="Image" /></td>
</tr>
<tr>
<td>0.65$\lambda$</td>
<td><img src="image3.png" alt="Image" /></td>
<td><img src="image4.png" alt="Image" /></td>
</tr>
<tr>
<td>0.7$\lambda$</td>
<td><img src="image5.png" alt="Image" /></td>
<td><img src="image6.png" alt="Image" /></td>
</tr>
</tbody>
</table>

**Figure 2.19:** Effect of Antenna Element Separation in $H$ Plane on Radiation Pattern

### 2.7.1.4 Effect of Mutual Coupling in $E$ and $H$ Plane on Return Loss

Figure 2.20 shows effect of mutual coupling in both plane on return loss $S_{11}$. Relative comparison of return loss due to mutual coupling in $E$ plane vis-à-vis $H$ plane shows significant deterioration in $S_{11}$ in the latter after spacing is increased from 0.6$\lambda$ onwards. This attributes to drop in directivity, gain, hence antenna efficiency along
with radiated power output. It can be concluded that in $H$ plane effect of mutual coupling is more dominant as compared to mutual coupling in $E$ plane.

<table>
<thead>
<tr>
<th>Separation</th>
<th>$E$-Plane</th>
<th>$H$-Plane</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5$\lambda$</td>
<td><img src="image1.png" alt="Graph" /></td>
<td><img src="image2.png" alt="Graph" /></td>
</tr>
<tr>
<td>0.55$\lambda$</td>
<td><img src="image3.png" alt="Graph" /></td>
<td><img src="image4.png" alt="Graph" /></td>
</tr>
<tr>
<td>0.6$\lambda$</td>
<td><img src="image5.png" alt="Graph" /></td>
<td><img src="image6.png" alt="Graph" /></td>
</tr>
<tr>
<td>0.65$\lambda$</td>
<td><img src="image7.png" alt="Graph" /></td>
<td><img src="image8.png" alt="Graph" /></td>
</tr>
</tbody>
</table>
Figure 2.20: Plot Showing Effect of Mutual Coupling on Return Loss $S_{11}$

Plot shown by the Figure 2.21(a) consists of combined plot depicting variation in $S_{11}$ with inter element spacing changes in both $E$ and $H$ plane. At $0.6\lambda$, we observe that return loss in both plane are closest, an important parameter for planar antenna array design.

![Figure 2.21(a): Plot showing on variation in Return Loss $S_{11}$ due to Mutual Coupling](image)

2.7.1.5 Effect of Mutual Coupling in $E$ and $H$ Plane on Directivity, Gain, Efficiency, Resonant Frequency and Power Output

Plots shown in Figure 2.21(b) to Figure 2.21(f) considers mutual coupling in both $E$ and $H$ plane due change in inter element spacing and its effect on antenna parameters viz. directivity, gain, efficiency, resonant frequency and power output. We observe at $0.5\lambda$ identical antenna parameters in both $E$ and $H$ plane. But as the spacing in both $E$
and $H$ planes are varied in steps of $0.05\lambda$, from the initial value of $0.5\lambda$. Following observations are made, considering mutual coupling effect in $E$ and $H$ plane.

<table>
<thead>
<tr>
<th>Directivity (dB)</th>
<th>$0.5\lambda$</th>
<th>$0.55\lambda$</th>
<th>$0.6\lambda$</th>
<th>$0.65\lambda$</th>
<th>$0.7\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>E-Plane Separation</td>
<td>11.2255</td>
<td>11.0336</td>
<td>12.1873</td>
<td>12.442</td>
<td>12.628</td>
</tr>
</tbody>
</table>

**Figure 2.21(b):** Plot showing Variation in Directivity due to Mutual Coupling

<table>
<thead>
<tr>
<th>Gain (dB)</th>
<th>$0.5\lambda$</th>
<th>$0.55\lambda$</th>
<th>$0.6\lambda$</th>
<th>$0.65\lambda$</th>
<th>$0.7\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>E-Plane Separation</td>
<td>11.1533</td>
<td>10.7737</td>
<td>12.071</td>
<td>12.2514</td>
<td>12.3502</td>
</tr>
</tbody>
</table>

**Figure 2.21(c):** Plot showing Variation in Gain due to Mutual Coupling

<table>
<thead>
<tr>
<th>Efficiency (%)</th>
<th>$0.5\lambda$</th>
<th>$0.55\lambda$</th>
<th>$0.6\lambda$</th>
<th>$0.65\lambda$</th>
<th>$0.7\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>E-Plane Separation</td>
<td>99.35682152</td>
<td>97.64446781</td>
<td>99.04572793</td>
<td>98.46809195</td>
<td>97.8001267</td>
</tr>
<tr>
<td>H-Plane Separation</td>
<td>99.35682152</td>
<td>98.96202712</td>
<td>98.1434484</td>
<td>97.54986735</td>
<td>97.51757194</td>
</tr>
</tbody>
</table>

**Figure 2.21(d):** Plot showing on variation in Antenna Efficiency due to Mutual Coupling
At 0.55\( \lambda \) both the directivity and gain in \( E \) and \( H \) plane are of the order of approximately 11 dB, however antenna efficiency is of the order 97.5% at 0.7\( \lambda \). in both the plane.

- Also at 0.55\( \lambda \), the antenna resonating frequency matches.
- At 0.6\( \lambda \), the antenna radiated power output is almost of the order of 1.9 mW in both the planes.

### 2.8 Prototype 2x2 Planar Antenna Array

Figure 2.22 shows the fabricated prototype 2x2 planar antenna array. The prototype is used to validate the effect of mutual coupling on the antenna resonant frequency observed in the simulation result shown in figure 2.21 (e).
Figure 2.22: Fabricated Prototype 2x2 Planar Microstrip Antenna Array.

Figure 2.23 shows the set up to measure the return loss $S_{11}$ at the resonant frequency. The four element array is seen to resonate at $f_r = 9.731$ GHz with inter element spacing in $H$ and $E$ plane corresponding to $0.55\lambda$ and $0.65\lambda$ respectively. Measured return loss $S_{11}$ is found to be -17 dB.

Figure 2.23: Setup showing the measurement of Return Loss $S_{11}$ and Resonant Frequency $f_r$ of the Prototype 2x2 Planar Microstrip Antenna Array.
Figure 2.24(a): Polar Plot $E_\theta$.  

Figure 2.24(b): Polar Plot $E_\phi$.

Figure 2.24(a) and Figure 2.24(b) shows the polar plot corresponding to radiation pattern $E_\theta$ and $E_\phi$ respectively. The polar plot shows directive radiation pattern expected out of the planar microstrip antenna array meeting the requirement of aircraft applications. The fabricated four element antenna array gain measured corresponds to 16 dB which is 5 dB above than observed in the simulation results. Figure 2.25(a) and Figure 2.25(b) shows the $f_1=9.502$ GHz and $f_2=10.002$ GHz respectively corresponding to frequencies measured at -10 dB on either side of resonant frequency $f_r=9.731$ GHz. Hence the impedance % bandwidth obtained is

$$\frac{(f_2 - f_1)}{f_r} = \frac{(10.002 - 9.502)}{9.731} \times 100 = 5.14\%.$$
2.9 ** Six Element 2x3 Antenna Array Design

2.9.1 Modeling of the Planar Array

Figure 2.26 shows 2x3 configuration of six element microstrip antenna array designed to operate in lower S band at a frequency of resonance of 2.3 GHz. Square patches of width and length of a single patch element are taken as 30mm using commercially available FR4 ($\varepsilon_r = 4.7$, thickness =1.57 mm), taken as the substrate.

** The work reported in this chapter is based on the following research paper contributions [69]: Gupta S.D., Rahul, "Optimization of Planar Microstrip Patch Antenna Array Designed for Lower S-Band", Eleventh URSI Commission F Triennial Open Symposium on Radio Wave Propagation and Remote Sensing at Rio de Janeiro, Brazil, pp. 350-356, 30 Oct. to 02 Nov. 2007.
2.9.2 Criteria for the Substrate Selection

The Figure 2.27 shows the schematic depicting surface, space and reflected waves.

![Figure 2.27: Schematic depicting surface space and reflected waves.](image)

Microstrip line feed has been used to feed the patch. The criterion for the selection of substrate with specific dielectric constant and thickness impacts the microstrip patch antenna design necessitates the following considerations:

- Substrates with higher thickness and high dielectric constant results in smaller bandwidths and lower efficiencies due to possibility of surface-wave excitation.
- Higher dielectric constant materials with longer length of feed line results in increased losses and chances of spurious radiation [70].
- In order to eliminate multiple surface waves the substrate thickness should be made as thin as possible. A height of $0.01\lambda_o$ to $0.05\lambda_o$ is considered ideal in this regard.
- It is preferable to use low dielectric constant material and also a substrate of optimum thickness for containing the spurious feed radiation to minimum.

For simplicity of analysis a $50\Omega$ reference system is used with the microstrip feed lines of same impedance. In the analysis presented, the return loss $S_{11}$ is considered to depend on horizontal spacing $S_e(dx)$ between the elements, the width $W$, relative permittivity $\varepsilon_r$ and the resonance frequency $f_r$ and may be expressed by the following relation [71].
\[ I_m[Z_m] = 0, Z_m = Z_c \frac{1 + S_{11}}{1 - S_{11}} \]  \quad \ldots (2.37)

where \( Z_c \) is equal to 50\( \Omega \), the characteristic impedance of the system.

### 2.10 Performance Analysis of the Microstrip Planar Arrays in \( H \) plane

On the basis of the analytical discussion on the performance analysis, the return loss, resonant frequency, directivity, gain and the radiation pattern of the microstrip planar arrays are presented in the following subsections:

#### Figure 2.28(a)

\( dx = 1\lambda \)

#### Figure 2.28(b)

\( dx = 0.9\lambda \)

#### Figure 2.28(c)

\( dx = 0.8\lambda \)

#### Figure 2.28(d)

\( dx = 0.7\lambda \)

#### Figure 2.28(e)

\( dx = 0.6\lambda \)

#### Figure 2.28(f)

\( dx = 0.55\lambda \)
2.10.1 Return Loss

Limiting the distance at $dx=0.5\lambda$, the resonance frequency of the patch array is observed at around 2.2 GHz with a return loss of 21 dB. As the spacing is reduced, we observe the following results. At $0.45\lambda$ the resonance frequency shifts by 90 MHz, with a return loss of 20 dB, which is not significant. At spacing $dx=0.4\lambda$ the return loss improves considerably to 36 dB. Further reducing the antenna element spacing to $0.3\lambda$, the return loss falls to 14 dB at 2.31 GHz. At $0.25\lambda$ the antenna tunes to 2.35 GHz with a good return loss of 27 dB. This is attributed to mutual coupling and can be observed from the relation expressed by equation (2.37).

$$Z_{in} = Z_0 \frac{1 + S_{11}}{1 - S_{11}}$$

Figures 2.28(a)-2.28(j) shows Return Loss against Frequency. Closely spacing the antennas shifts the input impedance at resonance to a lower value, which in turn affects the return loss [72].
2.10.2 Resonant Frequency

Inter element separation $dx$ less than $0.5\lambda$, results in space reduction between patches. The resonance frequency reduces by 60 MHz at $0.4\lambda$ (Figure 2.29). At separation $0.3\lambda$, strong coupling is observed, which increases the resonant frequency by 20 MHz and by 60 MHz at $0.25\lambda$. The analysis has been reported in [72].

Resonance frequency rises at close inter element spacing. This shift up in $f_r$ can be attributed to the parallel line coupling which reduces $\varepsilon_r$, as a result, resonance frequency increases slightly ($f_r\alpha n_{\text{air}}\sqrt{\varepsilon_r^{-1}}$) [31].

Figure 2.29: Plot of Resonant Frequency vs. $dx(\lambda)$

Figure 2.30: Plot of Return Loss vs. $dx(\lambda)$

At inter element spacing greater than half wavelength ($dx > 0.55\lambda$), coupling is due to combined effect of surface and space waves [73], [74]. At $dx=0.25\lambda$ and $dx=0.55\lambda$, the antenna has frequency response at 2.35 GHz and 2.29 GHz with return loss of 23 dB and 27 dB respectively (Figure 2.29 and Figure 2.30). Between $0.7\lambda$ and $1\lambda$, at increments of $0.1\lambda$ there are inconsistencies in return loss. It has been observed specifically at $0.6\lambda$ and $0.8\lambda$ that return loss is poor to what has been achieved at $0.7\lambda$ and $0.9\lambda$. However, a shift of 10 MHz and 350 MHz at $0.7\lambda$ and $0.9\lambda$ respectively is observed from the tuned frequency of 2.35 GHz. Thus, no specific inference can be drawn about the return loss at $dx > 0.55\lambda$.

2.10.3 Directivity and Gain

No significant effect is observed in the directivity and gain of patch array antenna due to mutual coupling. The maximum deviation of 2 dB is observed in the directivity as shown in Figure 2.31. Similarly no significant deviation in gain is obtained except at very close spacing (about $0.25\lambda$) the gain falls drastically to 2.95 dB as shown in Figure 2.32.
2.10.4 Optimum Radiation Pattern

Figure 2.33 and Figure 2.34 respectively shows planar antenna radiation plot at $dx=0.55\lambda$ in Cartesian and polar coordinates. As seen in the Figure 2.33 and 2.34 a sharp null is observed at -40 dB for $dx =0.55\lambda$. Figures 2.35 and 2.36 shows the polar plots for two extreme cases, i.e. at $dx = 0.25\lambda$ and $dx=1\lambda$ respectively. The pattern is highly distorted in the former case due to mutual coupling. At spacing of 1\lambda the null is not sharp and the energy is distributed in the side lobes.

Figure 2.33: Cartesian Plot of Radiation at $dx=0.55\lambda$
Figure 2.34: Polar Plot of Radiation at $dx=0.55\lambda$

Figure 2.35: Polar Plot of Radiation at $dx=0.25\lambda$

Figure 2.36: Polar Plot of Radiation at $dx=1\lambda$
2.10.5 Array Elements driven through a Phase Shifter

It is always desired to excite the patch in its fundamental mode (TM$_{010}$), so that the antenna is tuned to the fundamental resonant frequency and there are no dominant higher order modes. Figure 2.37 shows the current distribution at $dx=0.55\lambda$. It has been observed that the electric field vectors in the diametrically opposite patch elements cancel each other and hence no *hot regions* are seen. Comparing this with the spacing $dx=0.25\lambda$ and $dx=1\lambda$ (Figures 2.38 and Figure 2.39 respectively), the patch elements are well excited and *hot regions* are observed. The solution is to add a 180 degree phase shifter which shall reverse the direction of the electric field vector; hence, there will be constructive interference. This serves another important purpose as well. It will ensure that the patch elements are excited only in the fundamental mode and no higher order harmonics are generated. As the absence of *hot regions* with the phase shifter is indicative of the antenna being excited in some other higher order modes.

![Figure 2.37: Antenna current distribution at $dx=0.55\lambda$](image)

![Figure 2.38: Antenna current distribution at $dx=0.25\lambda$](image)
2.11 Results

As per the results discussed above an optimum design is achieved for linear spacing of $0.55\lambda$. It is found at this spacing the antenna is working closest to the designed operating frequency with a good return loss of 23 dB. The Directivity and Gain is also found to acceptable at 7.7dB and 6.09dB respectively. Power Radiated by the patch array is 9 mW, which is a decent power level, expected of a patch antenna.

2.12 *** Optimization of Planar Microstrip Patch Antenna Array Operating at Lower S-Band based on Analysis in both $H$ and $E$ Plane

Using $2\times3$ geometry, the behavior of a six element rectangular microstrip patch antenna array may be optimized for the design parameters - gain, directivity, radiated power and radiation pattern in both $E$ and $H$ plane at Lower S-band [75]. To achieve this, the inter element spacing is varied in either horizontal or vertical plane at a time keeping the spacing at other plane constant.

*** The work reported in this chapter is based on the following research paper contributions [75]: Gupta S.D., Rahul, "Design and Optimization of Planar Microstrip Patch Antenna Array Operating at Lower S-Band Based on Analysis in both $H$ and $E$ Plane", 11th International Symposium on Microwave and Optical Technology (ISMOT 2007) Italy at Monte Porzio Catone, Roma – ITALY, pp 769-773, Dec.17-21, 2007.

This approach is based on the fact that optimized antenna characteristics can be obtained for any planar array configuration, depending upon inter element spacing. The antennas have been modeled using microstrip feed line and S-parameter data.
from individual single element. The design parameters are first calculated for square patch antenna using transmission line model equation. With configuration of a 2×3 patch array, optimum characteristics of the patch antenna array are obtained through momentum simulation using ADS software provided by Agilent Technologies. The effects of surface waves and mutual coupling have been minimized by optimizing the inter element spacing in both the plane. It is observed that for obtaining optimum antenna characteristics for any array configuration, tradeoff between various antennas parameters are to be arrived at [76].

The proposed approach is conceptually simple, practical, and efficient for designing wide-band microstrip antennas. It takes into account two major aspects in configurations of microstrip antennas for improved electrical performance and manufacturability, and in the analytical modeling of microstrip antennas and arrays.

2.12.1. Mutual Coupling Considerations in Design

The mutual coupling affects has on the resonant frequency $f_r$, resonant input impedance $R_r$ and the far field radiation pattern. It has been shown that the mutual coupling effect on $f_r$ is about 1% or a frequency shift of 10 MHz at 1 GHz, the effect on $R_r$ is about 50% and the effect on radiation pattern is about 30% Mutual coupling affects the resonance frequency of the antenna elements in the array and needs to be taken into account while tuning the elements to resonance at the desired frequency [31], [71]. A.R.Sindoris and C.M.Krowne [32] has reported that closely spacing the antenna (showing coupling) increases resonant frequency by 7.7 MHz and shifts the input impedance at resonance to a lower value. The coupling is very frequency sensitive ranging from about 3 dB at the first resonant frequency to less than 30 dB approximately half way between the first and second resonance and it also very sharply peaked about the first resonance. Further, the mutual coupling between microstrip patches may be due to both space wave and surface wave .Surface wave contributes significantly to the mutual coupling, especially in $E$-plane [32]. The mutual coupling in the $H$-plane is smaller than in $E$-plane and the increasing of $d/\lambda$ further or the distance $d$ between radiating elements reduces the mutual coupling [31]. For excitation of surface wave a distance $d_i \approx (0.5 - 0.7)\lambda$ is necessary [32]. In this zone the surface wave has a small magnitude. The distance $d_i$ is dependent on $\varepsilon_r$ and $h$ (thickness of the dielectric). However, it is difficult to determine analytically the
effect of mutual coupling between radiating elements because of the influence of a variety factors such as reflected and surface wave excited in the impedance structure and the dielectric above the metal plane [32]. This difficulty is overcome by employing ADS Momentum software. Its effect has been effectively incorporated in the array design using the software.

2.12.2 The Performance Analysis

The performance of the designed microstrip antenna in terms of the return loss, plot of $S_{11}$, resonant frequency, gain and Optimum Radiation Pattern in both $E$ and $H$ plane are next discussed.

2.12.2.1 Return Loss and plots of $S_{11}$

Figures 2.40 and 2.41 show Return Loss less than -35 dB corresponding to variation of inter element spacing in $H$ plane ($dx=0.4\lambda$, $dy=0.5\lambda$) and $E$ Plane ($dy=0.4\lambda$, $dx=0.5\lambda$) respectively.

Figure 2.40: Variation in $H$-Plane with $dx=0.4\lambda$, $dy=0.5\lambda$ & $S_{11}=-37$dB

Figure 2.41: Variation in $E$-Plane with $dy=0.4\lambda$, $dx=0.5\lambda$ & $S_{11}=-33$dB

Figure 2.42 and 2.43 show plots of $S_{11}$ variation in $E$ and $H$ plane respectively for different values inter element spacing.
2.12.2.2 Resonant Frequency

While considering mutual coupling in both $E$ & $H$ plane simultaneously for spacing $dx$ less than $0.5\lambda$, there is reduction in resonance frequency at $0.4\lambda$ in accordance with reasoning, given in section 2.12.1.
At separation of $0.3\lambda$, strong coupling is observed, which increases resonant frequency by 20 MHz and by 60 MHz at $0.25\lambda$ as shown in Figure 2.44. The analysis has been reported in [31].

At inter element spacing greater than half wavelength ($dx > 0.55\lambda$), coupling is due to combined effect of surface and space waves [72], [73]. At $dx = 0.55\lambda$, the antenna has frequency response at 2.29 GHz with return loss of 27 dB respectively.

The Resonant frequency was achieved to be 2.27 GHz at $0.4\lambda$. There is a shift in the frequency when $dy$ is varied from $0.4\lambda$ to $0.6\lambda$. Figure 2.45. This may be due to fact that the change in the feed length introduces a physical notch, which in turn introduces a junction capacitance [77]. The notch and its corresponding junction capacitance influence slightly the resonant frequency [54].

![Figure 2.45: Resonant frequency vs. dy](image)

![Figure 2.46 (a): Gain vs. dx](image)
2.12.2.3 Gain

No significant effect is observed in the gain of patch array antenna due to mutual coupling (Figure 2.46(a)) (within 1 to 2 dB). Similar results are obtained for gain, however, at very close spacing (0.25\( \lambda \)); the gain falls drastically to 2.95 dB. Figure 2.46 (b) suggests that a fair amount of gain between 7-8 dB is achieved when the spacing dy is varied between 0.4\( \lambda \) and 0.5\( \lambda \). This is remarkable achievement for microstrip patch antenna arrays.

2.12.2.4 Optimum Radiation Pattern

As observed in Figure 2.33 and 2.34, a sharp null is observed at -40 dB for dx=0.55\( \lambda \) and dy=0.5\( \lambda \) and for dx=0.5\( \lambda \) and dy=0.55\( \lambda \), in the Cartesian and polar plot of the radiation pattern a null is observed at 35 dB as shown in figure 2.47 ((a) & (b)) and 2.48 ((a) & (b)).
Figure 2.47(b): Polar Plot in $H$ Plane

Figure 2.48(a): Cartesian Plot in $E$ Plane
2.13 Conclusions

In $E$ plane at antenna element separation $0.55\lambda$ and for $H$ plane at $0.5\lambda$ in case of symmetrical 2x2 array, radiation pattern with nulls and without sidelobes in both $E_\theta$ and $E_\phi$ is observed. With the inter element spacing in both the planes increased, the effect on $E_\theta$ radiation pattern is dominant in terms of increase in sidelobe level. Similarly in 2x3 assymetrical array an optimum design for linear spacing of $dx =0.55\lambda$ and $dy =0.5\lambda$ and similarly $dx =0.5\lambda$ and $dy =0.55\lambda$, is arrived at. It is found at this spacing the antenna is working closest to the designed operating frequency with a good return loss of 23 dB. The Directivity and Gain is also found to acceptable at 7.7 dB and 6.09 dB respectively. Power Radiated by the patch array is 9 mW, which is a decent power level, expected of a patch antenna. When antenna structure is closely spaced, the return loss improves by a factor of 2.3 from $0.5\lambda$-$0.4\lambda$ in the $E$ plane. It is maximum at $0.4\lambda$ (-33.6 dB). A significant improvement in the gain of patch array antenna around 7-8 dB when the spacing is maintained around $0.4\lambda$-$0.55\lambda$. However if the array spacing is increased, it reduces by 25%. The distance $dy$ is kept constant at about $0.5\lambda$, while the inter element spacing $dx$ has been varied from $0.25\lambda$ (closest possible spacing) to $1\lambda$, to study the antenna parameters. Next the study is $E$ Plane spacing being kept constant at $0.5\lambda$, while the $H$ Plane spacing is varied for analysis.