Chapter 1

Introduction

1.1 Scope

Remote sensing involves acquiring data from the Earth’s surface without physical contact with the area being sensed. Classically, the satellite based sensors capture the data in 4 to 6 different regions in the electromagnetic spectrum covering the visible, infrared and thermal infrared wavelength bands and are well-known as the multispectral sensors. Recently the hyperspectral imaging (imaging spectroscopy) has emerged as a powerful passive remote sensing technology. The hyperspectral imagers (also known as imaging spectrometers) acquire a set of co-registered images of a scene with relatively large instantaneous field-of-view (IFOV) (about 4m × 4m to 20m × 20m) and much finer spectral resolution (10nm within more than 200 contiguous wavelength bands). This has enabled quantitative analysis of an area within an IFOV of the sensor. The unprecedented capability of the hyperspectral sensors enables the remote acquisition of images where each pixel is a vector with the high spectral resolution that enables better analysis of contents in an area [1, 2, 3].

Majority of data processing and analysis research in hyperspectral imagery can be categorized as i) detect known and/or unknown targets in a given scene, ii) classify the given image into subregions where a material is predominant, iii) detect changes in a scene over a period of time, and iv) estimate the endmembers and their proportions within a pixel location, so-called spectral unmixing, in order to better understand the scene.

The spectral unmixing of remotely sensed hyperspectral data aims at quantifying the reflectance properties of different regions especially those which are physically inac-
The quantification includes identifying number of spectrally distinct materials, estimating their spectral signatures, and respective abundances fractions [4]. Many times estimating the abundances at every location of the scene or generating the abundance maps of the scene is termed as unmixing in the remote sensing community [5]. The more general term spectral unmixing has ambitious objectives of updating the spectral library by identifying unknown materials, characterization and quantification of the materials, and to generate the abundance maps of materials found at various areas of the earth.

Apart from this, the spectral unmixing results in better analysis of remote sensing data and this is useful in various applications. To name a few, it includes better classification and segmentation of the scene [6, 7], detecting changes in the scene based on the variations in material reflectance over a period of time [8, 9, 10, 11], super-resolution of the hyperspectral images [12], and developing content-based hyperspectral image retrieval systems [13, 14]. Besides, it contributes in other areas as well which includes the agricultural product developments based on the remote sensing data [15, 16], in marine science, for example to analyze the spectral mixing in macroalgae found in the sea [17], and to analyze biochemical components in the vegetation process [18].

1.2 Hyperspectral Imaging

Hyperspectral imaging systems measure target reflectance in large number of contiguous narrow bands. This larger sampling of the electromagnetic spectrum provides a great increase in the information which is useful in different fields such as agriculture, geography, geology, mineral identification, urban planning, environmental monitoring, surveillance, target detection, and classification of the acquired data. Most hyperspectral imagers measure hundreds of spectral bands, however, the narrowness and contiguous nature of the measurements qualifies them as hyper-spectral.

The hyperspectral imaging systems are used to characterize the spectral properties of target depending on the scale of observation covering the visible, near-infrared, and shortwave infrared spectral bands. Table 1.1 briefly describes many hyperspectral imaging systems and their specifications.
1.2 Hyperspectral Imaging

Table 1.1: Hyperspectral imaging systems.

<table>
<thead>
<tr>
<th></th>
<th>AVIRIS</th>
<th>HyDICE</th>
<th>HyMAP</th>
<th>AIMS</th>
<th>Probe-1</th>
<th>Hyperion</th>
<th>CHRIS</th>
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<td>Nominal Altitude (km)</td>
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<td>5</td>
<td>3</td>
<td>2.5</td>
<td>705</td>
<td>550 - 670</td>
<td>100</td>
</tr>
<tr>
<td>Spatial Resolution (m)</td>
<td>20</td>
<td>3</td>
<td>10</td>
<td>2</td>
<td>5</td>
<td>30</td>
<td>17 &amp; 34</td>
<td>80</td>
</tr>
<tr>
<td>Spectral Resolution (nm)</td>
<td>10</td>
<td>10</td>
<td>17</td>
<td>3</td>
<td>10</td>
<td>10</td>
<td>1.3 - 11.3</td>
<td>15</td>
</tr>
<tr>
<td>Spectral Range (µm)</td>
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<td>0.4-2.5</td>
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<td>0.4-0.88</td>
<td>0.4-2.5</td>
<td>0.4-2.5</td>
<td>0.415-1.05</td>
<td>0.4-0.95</td>
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<td>Number of Spectral Channels</td>
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<td>210</td>
<td>128</td>
<td>143</td>
<td>128</td>
<td>220</td>
<td>18 &amp; 62</td>
<td>64</td>
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<td>Swath Width (km)</td>
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<td>6</td>
<td>3</td>
<td>3</td>
<td>7.7</td>
<td>0.13</td>
<td>20</td>
</tr>
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1.2.1 Principles of Imaging Spectroscopy

Figure 1.1: Passive remote sensing by satellite based sensors.

Spectroscopic analysis refers to the study of interaction between the materials and radiated energy using a laboratory spectrometer. Hyperspectral remote sensing combines imaging and the spectroscopy in a single system. The idea is to carry a spectrometer on-board the satellite or aircraft to remotely capture the data. This is done using the passive sensing (see Figure 1.1) wherein the light energy from the Sun strikes an object on the earth, and the reflected light is captured by the hyperspectral sensors. These sensors are equipped with a bank of filters designed for recording the data in various bands of the visible to infrared range. Thus the hyperspectral imaging provides a complete reflectance profile of various areas on the earth. This is depicted in Figure 1.2 for some of the pixel locations in an acquired scene data. The data acquired by these sensors is very large [28] and hence we need new methods for its processing and analysis. Hyperspectral imagery is typically collected and represented as a data cube or image cube with spatial information collected in the X-Y plane, and spectral information represented in the Z direction. Figure 1.3 illustrates an example of hyperspectral data cube which has images at different
Figure 1.2: Principles of imaging spectroscopy [19, 27]. The plots are showing the bands versus reflectance at few pixel locations.

spectral bands shown in the X-Y plane, and the spectral band range is indicated in the Z-direction.

Figure 1.4 shows some of the real hyperspectral images captured by different sensors. Figure 1.4 (a) shows an image corresponding to band 50 collected by the airborne visible/infrared imaging spectrometer (AVIRIS) [19] by the National Aeronautics and Space Administration (NASA) over the well-known Cuprite mining site located at Nevada, USA. This site is considered as a benchmark for testing the algorithms on hyperspectral data. Figure 1.4 (b) is the French Frigate Shoals image as seen by the CHRIS Proba-1 by the European Space Agency (ESA). In the Figure 1.4 (c) we display the band-31 image of the moon captured by the hyperspectral imager (HySI) on-board the Chandrayaan-1 as a part of the first mission to Moon by the Indian Space Research Organization (ISRO).
1.3 Linear Mixing Model

Researchers have attempted to solve the spectral unmixing problem by mathematical modeling of the hyperspectral image formation. However, this itself is a difficult issue since the sensor radiance due to the remote acquisition depends on many hidden parameters including material types, interaction of light intensity with the materials, the way in which they are mixed, scene topology, and the environmental effects. Both the linear and nonlinear modeling have been used for modeling the hyperspectral data [33, 34]. A general model can be considered as having a nonlinear relationship between the reflectance, i.e., pixel intensities, and the entities such as endmembers and corresponding abundances [35]. However, it is difficult to replicate the complete physical mixing phenomena in the generic nonlinear model. Besides, solving the spectral unmixing problem involving nonlinearity has many practical challenges. On the other hand, the linear mixing model presents the natural framework representing the hyperspectral data, and hence its use is prevalent in the remote sensing community. It is a balanced model having the representation accuracy and the mathematical tractability. It is also indicative of many of the real-world scenarios involving hyperspectral data analysis [33]. After analyzing many real hyperspectral datasets, majority of the researchers in the remote sensing community are using the linear mixing model for solving the problems related to hyperspectral data [36, 37]. Our approaches in this thesis are also based on the linear mixing model.

In the linear mixing model (LMM) [38], the data is considered as linear combinations of the endmembers that represent the pure spectra, i.e., spectra of the constituent mate-
1.3 Linear Mixing Model

Figure 1.4: Hyperspectral images: (a) Cuprite band-50 acquired by AVIRIS (NASA) [30], (b) French Frigate Shoals by Proba-1 CHRIS (ESA) [31], and (c) a far side of moon band-31 as seen by HySI Chandrayaan-1 (ISRO) [32].

materials, found within the scene. The LMM provides a simple and mathematically tractable framework for solving the spectral unmixing problem [5]. In general, the available radiance data from the remote sensor is first converted to equivalent reflectance values. Then the reflectance data is processed for radiometric calibration, geometric corrections, and necessary atmospheric compensations [39]. Now, considering the LMM at each pixel location in the remotely acquired scene, one may write

\[ \mathbf{r} = \mathbf{M} \mathbf{\alpha} + \mathbf{n}, \]  

(1.1)

where \( \mathbf{r} \) is the data (reflectance) vector at a location considering \( W \) number of bands, the size of which is \( W \times 1 \). Given \( e \) number of endmembers in the scene, \( \mathbf{M} \) represents the endmember matrix of size \( W \times e \). The corresponding abundances are represented as a vector \( \mathbf{\alpha} \) of size \( e \times 1 \). Here, the \( \mathbf{n} \) denotes independent and identically distributed Gaussian noise having same size as the \( W \times 1 \). There are various types of noise sources in the remotely sensed data. It includes the sensor noise due to limiting quality/aging of components, environmental noise, channel noise such as stripping noise and impulse noise, thermal noise present in the circuits, drifts & imperfections due to control mechanisms, etc. However, in general it is difficult to predict the amount and types of noises in the real data acquired by the hyperspectral sensors. By using the theory of Central Limit
1.3 Linear Mixing Model

Theorem, researchers often model it as additive white Gaussian noise (AWGN). Mostly the noise $n$ in equation (1.1) includes background noise, instrument noise and can also include errors due to modeling inaccuracies. In order to handle the random nature of noise, it is a common practice in the spectral unmixing research community to carry out the noise sensitivity analysis on the synthetic hyperspectral data.

It can be seen that use of the equation (1.1) at every pixel of the hyperspectral data forms a vector space. Since the images are acquired using the principles of reflectance spectroscopy by the hyperspectral (passive) sensors, the resultant data lies in the $W$ dimensional nonnegative real vector space. The endmember vectors representing the columns of $M$ are the basis vectors that represent the pure spectra of the constituent materials. These vectors span the $W$ dimensional vector space formed by the hyperspectral data in which the abundances correspond to the weights of the endmember vectors. The hyperspectral data inherently imposes the nonnegativity constraint on the endmembers while the abundance fractions are constrained by the nonnegativity as well as sum-to-one at each location due to the LMM on data.

The LMM can be easily illustrated using a noiseless scenario in the data. Let us consider a data vector $r \in \mathbb{R}^{+W}$, where $W$ is the available number of wavelength bands, is a linear combination of endmembers weighted by their abundances. Thus,

$$ r = M\alpha, $$

where endmember matrix $M$ represents the spectral signatures and it has the size of $W \times e$, with $e$ being the number of endmembers. The corresponding abundances are denoted by $e$-dimensional vector $\alpha = [\alpha_1, \alpha_2, ..., \alpha_e]^T$ where, $\alpha_i$ represents the fractional area covered by the $i^{th}$ endmember.

Figure 1.5 represents the hyperspectral data vectors of a scene as points in the $W$ dimensional Euclidean space. A convex hull is formed due to the data points representing the endmembers of the scene since the corresponding abundances are nonnegative and sum-to-one for each data point. As shown in the figure, all the data points lie within the polygon formed by the endmembers of the scene.
1.3 Linear Mixing Model

Data point
Endmember

Figure 1.5: Linear mixing model representing data vectors as points in W-dimensional Euclidean space. Endmembers represent the vertices of the polygon comprising the data points. The physical constraints on the abundances lead to the convex hull representation.

1.3.1 Spectral Mixing in Hyperspectral Data

In general, the processing of remotely sensed imagery is a challenging task mainly due to limited resolution offered by high altitude satellite sensors [40]. The hyperspectral data is envisioned for recording material reflectance across the contiguous wavelength bands. A hyperspectral-pixel is a vector which constitutes reflectance across the bands within an IFOV, i.e., it represents spectral response at the IFOV. Pixel values within a band are dependent on many factors that include spatial resolution of camera, scene content, day/time of acquisition, ground area, etc. In order to avail the higher spectral resolution, the sensors offer relatively poor spatial resolution due to the technology trade-off and the hardware limitations. Thus one can consider the reflectance or pixel intensity values as mix of more than one material reflectance within a pixel location, in addition to the various parameters that contribute to its formation. This physical mixing among the spectral signatures is a function of wavelength ($\lambda$) and the radiance $\rho$ sensed by the sensor can be simplified at each location of the scene as,

$$\rho = \frac{\int_0^\infty B(\lambda)\phi(\lambda)d\lambda}{\int_0^\infty \phi(\lambda)d\lambda},$$

where, $B(\lambda)$ is the Planck distribution function and $\phi(\lambda)$ is the spectral response of the detector within an IFOV. Hyperspectral imaging constitute data captured at large num-
1.3 Linear Mixing Model

ber of bands representing the radiance given in equation (1.2). Though (1.2) represents mixing using integration of product of two factors, in practice, this mixing phenomenon has been modeled as either linear or nonlinear mixture of finite number of spectral signatures of constituent materials called *endmembers* present in a scene. In the linear model, reflectance $r$ is approximated as a linear combination of the endmember signatures constituted by the endmember matrix $M$. It is conveniently expressed as the linear mixing model (LMM) (1.1) that includes the uncertainty inherent in the data acquisition.

### 1.3.2 Spectral Unmixing: An *Ill-posed* Inverse Problem

An inverse problem [41, 42, 43] can be considered as given the effects, find out the causes. This is *inverse*, because the effects are normally observed given the causes, which constitutes a forward or direct problem. The solution to an inverse problem allows us to know about the physical parameters that cannot be directly measured or observed. The inverse problems are challenging to solve because of the difficulties involved in inverting in order to get a unique solution. Inverse problems arise in many of the practical situations in the field of science and engineering that include imaging systems. An example of inverse problem is in the area of computer vision where the estimation of depths, i.e., distances of object points, is often of interest. Here, given a 2-D image, one has to estimate the unknown depths which is the third dimensional parameter. In order to solve such problems, the physical phenomena can be suitably represented by mathematical model. The linear model is generally preferred in many applications due to its mathematical tractability. Inverse problems using the linear model aim to estimate the model parameters, given the data. If we have $y = Ax$, where $y$ is the observation vector, the issue in the linear inverse problem is to estimate matrix $A$ and vector $x$, given the $y$. Estimating $x$, given $y$ and $A$ is also an inverse problem. Several problems such as image restoration [44], image deconvolution [45] in the field of image processing, and the signal estimation [46] in the area of communications often use the linear model, and they solve the inverse problem in order to estimate the image or the signal.

The term *ill-posed* problem in mathematics is due to the definition given by Jacques Hadamard for a well-posed problem [47, 48]. Well-posed problems guarantee the following:

2. Uniqueness: the solution is unique.

3. Stability: the solution is consistent with changes in the input conditions.

As an example consider the \( y = Ax \), where \( y \) is observed and the \( x \) has to estimated. If \( A \) is a square matrix and if its inverse exist, i.e., \( A \) is well-conditioned matrix, then one can obtain the \( x = A^{-1}y \) which is the unique solution. If any one of the above mentioned properties is not satisfied, then it becomes difficult to invert in order to get the solution. A situation may arise when one does not have a solution at all. In this case, the equations are inconsistent. For example, consider a linear system with two variables \( x \) and \( y \), i.e., \( x + y = 2 \) and \( x + y = 3 \), which are inconsistent and hence do not have a solution. Many times one may have multiple solutions and it is difficult to decide which is the correct answer. Let’s say, an equation \( x + y = 2 \) which has infinite solutions. Such problems are not well-posed and are termed as \textit{ill-posed} problems in the context of the Hadamard definition.

A problem can also be ill-posed when the transformation matrix \( A \) is ill-conditioned. In this case, a small error in the data can result in much larger errors in the solution. Returning to the linear system \( y = Ax \), with the matrix \( A \) given in [49]:

\[
A = \begin{bmatrix}
1 & 1 \\
1 & 1.0001
\end{bmatrix} \quad \therefore \quad \det(A) = 1.0000e^{-004}, \quad \text{and} \quad \text{cond}(A) = 4.0002e^{004}. \quad (1.3)
\]

Here, \( \det(A) \) and \( \text{cond}(A) \) represent the determinant and condition number of the \( A \), respectively. In this case one can see that although the determinant exists, the eigenvalues of \( A \), i.e., \( \lambda_1 = 10^{-4}/2 \) and \( \lambda_2 = 2 \), result in a high condition number. Thus, \( A \) becomes nearly singular matrix. Hence, a slight change in the observation \( y \) can drastically change the solution \( x \). This leads to a severe ill-posedness.

It is observed that the inverse problems are generally ill-posed in nature. Hence, it is very difficult to obtain a unique solution. One has to impose certain constraints on the solution in order to obtain a better solution. In practice, the observations are often noisy, i.e., \( y = Ax + n \), where \( n \) represents the noise. This effectively avoids the constrained minimization and one may go for unconstraint minimization where prior information
about the solution is used in order to arrive at a better solution. The minimization is carried out without imposing the strict constraint. The method of solving an ill-posed problem using the prior is known as regularization. This can also be done by using the Bayesian perspective [50, 51, 52]. One can include number of priors as constraints on the solution in order to better restrict the solution space that helps in making the problem well-posed.

Solving for the spectral unmixing using the mathematical model as given in equation (1.1) is an ill-posed inverse problem. First of all the noise and outliers are inherent in the remote sensing data acquisition. Apart from them, there are three different inverse problems involved while performing the complete spectral unmixing. In the hyperspectral data, the number of endmembers $e$ is found to be significantly less than the available bands $W$. This is similar to pigeon-hole problems wherein the pigeons, i.e., number of endmembers, are far less than the holes, i.e., spectral bands. This leads to multiple solutions and hence it is hard to determine the exact value of $e$ from the available data. Looking at (1.1) we see that there are less number of knowns, i.e., reflectance. The endmembers extracted from the data need to be linearly independent since they correspond to distinct constituent spectra with their values in each band satisfying the nonnegativity constraint due to the passive remote sensing. Finally, the abundances must be constrained by both nonnegativity and sum-to-one. This leads to solving an overdetermined system of equations satisfying the number of physical constraints. Hence, solving for the complete spectral unmixing or solving for any of the single entity is difficult since it results in ill-posed inverse problem due to reasons mentioned above.

In this thesis, we propose different approaches for solving the ill-posed inverse problem of spectral unmixing. Our approaches are novel and inspired from the Bayesian inference and the regularization frameworks.

### 1.3.3 Mixed Pixels in Hyperspectral Imagery

The hyperspectral sensors have a high spectral resolution, but they have limited spatial resolution. This results in mixed-pixels in the hyperspectral imagery, i.e., presence of multiple spectral sources within a pixel location. Figure 1.6 illustrates the concept of the mixed pixels considering the data as linear combinations of the endmembers or spectral
signatures that represent the pure spectra found within the scene. Pixels corresponding to a single spectral signature are called pure pixels. Note that the reflectance corresponding to such a pixel location indicate the presence of one of the endmembers at that pixel. Mixed pixels are inherently found at almost every location of the scene due to the relatively poor spatial resolution of hyperspectral image. However, one may also find mixed pixels in the data even when the spatial resolution is high [6]. A part of an exemplary scene shown in Figure 1.6 having a spatial resolution of $3m \times 3m$ has such mixed pixels with three endmembers, viz, grass, tree and soil. A pixel marked with the thick borders has the mixing proportions (abundances) as given in the box.

Our work in this thesis involves estimating the abundances given the other two unknowns, i.e., number of endmembers and their signatures, and to extract the endmembers, given the abundances and number of endmembers. Finally, we solve the complete spectral unmixing problem in which we estimate all the three entities, given the data. This is similar to the blind source separation problem for which different methods and frameworks including the independent component analysis (ICA) have been proposed [54, 55, 56, 57, 58, 59]. However, most of these methods/frameworks can not be applied for solving spectral unmixing because of the severe ill-posedness in the problem. Since the sources are actually dependent in the hyperspectral data, the ICA based techniques are not suitable to perform the spectral unmixing [60, 61].
1.4 Thesis Contributions

This section summarizes the topics addressed in the thesis and our main contributions. We begin by estimating the abundances considering the cluttered endmembers. Next problem considers the abundance estimation given the true endmembers of the scene. Then the problem of endmember extraction is addressed. Finally, number of endmembers, their signatures and corresponding abundances are simultaneously estimated in a single algorithm that is we perform blind decomposition of the given data. We call this as the complete spectral unmixing of the hyperspectral data.

Towards this end, we first discuss the linear mixing model (LMM) for the remotely acquired hyperspectral data. The LMM accounts for modeling the general degradations found in hyperspectral data including the instrument noise. We then review the literature on our works estimating each of the three entities in the spectral unmixing, i.e., estimating the number of endmembers, their signatures and corresponding abundances, as well as works involving joint estimation of more than one entity. Finally, approaches incorporating the usage of multitemporal data for change detection and unmixing are also reviewed. Our thesis contribution include following:

- To begin with, we consider an unmixing problem wherein given the noisy data and the corrupted endmembers, we estimate the corresponding abundances. We propose a regularization based approach within a total least-squares (TLS) framework for recovering the underlying abundance maps. The ill-posedness is handled by considering a Tikhonov prior on the abundances. The formulated objective function is minimized using gradient based method. A theoretical analysis is carried out to show the effectiveness of the regularization for restricting the solution space in order to arrive at a better solution. We also analyze the role of regularization in solving the unmixing problem. The experiments are conducted on synthetically generated data constructed using the USGS library signatures with increasing levels of noise in the data. After verifying consistency in the spatial abundance patterns within the estimated abundance maps, the results are quantitatively compared using different measures with the existing TLS-based approaches. The performance of the proposed approach is also tested on the real AVIRIS Indian Pines data. The results are validated for the consistency in the spatial patterns in the maps and then the
quantification of the unmixing errors is done in terms of the data reconstruction error. Our algorithm outperforms the existing TLS-based approaches.

• The hyperspectral signatures representing the endmembers are strongly correlated due to nature of the data. This leads to an ill-conditioned endmember matrix. The underlying abundances of the scene are either smoothly varying or have sudden variations depending upon the mixture of the signatures at a location. The inevitable presence of noise and outliers in the data adds the uncertainty while solving the unmixing problem. Hence, estimating the abundances is a challenging problem even with the ground truth endmembers. To this end, we next propose a new data-driven prior and the same is used to take care of the ill-posedness in the unmixing. For this, we develop a two-step Bayesian method in which the dependencies in the abundances are estimated in the first step to yield a data-dependent Huber-Markov random field (dHMRF) model for the abundances. The posterior probability is then maximized in the second step using particle swarm optimization (PSO) which is guaranteed to converge. The MAP optimization is initialized by particles drawn from Dirichlet distribution enforcing the required physical constraints on the solution. The proposed method uses only the available data to estimate the necessary parameters and hence effectively avoids the learning of the parameters from the large datasets. A theoretical analysis along with the geometric illustration is carried out to show the competency of the method. The experiments are conducted on synthetically generated scene having different mixing proportions. The noise sensitivity analysis is then carried out and the results are compared with the existing state-of-art methods using different quantitative measures. The processing time of the proposed algorithm is also compared with the other approaches. Finally the abundance maps of the real AVIRIS Cuprite data are estimated using the proposed approach. The results are first validated for the visual consistency of the maps with the existing state-of-art algorithms and then the quantification of the unmixing errors is done in terms of the data reconstruction error. Our algorithm outperforms the existing state-of-art approaches.

• The hyperspectral data is composed of the endmembers and their distribution representing the abundances over the scene. Hence, given the endmembers, the corre-
1.4 Thesis Contributions

Corresponding abundances can be estimated; and given the abundances, the endmembers can also be estimated since these two together form the reflectance values. Hence, after the work on abundance estimation, a novel approach for estimating endmembers of the scene is developed in this thesis. This method integrates the spatial, spectral as well as temporal characteristics of a scene to better constrain the endmember values in each band. An overdetermined set of equations are formulated using the multi-temporal data and the knowledge of estimated abundances. The formulated objective function is minimized using the nonnegative constrained least-squares framework. Our approach is validated by conducting experiments on the simulated data and comparing our results with the state-of-art approaches. The proposed algorithm competes with the current state-of-art approaches, yet it has much lower processing time as well as computational complexity. This algorithm requires the availability of the abundances, and the same is overcome in our next contribution.

- Finally, given the multitemporal data, we solve the complete spectral unmixing problem which is a blind decomposition of the data. We develop a completely new framework for simultaneous estimation of number of endmembers, their signatures and abundances. This method provides the complete solution in a single algorithm. The framework is inspired from the principles of bootstrapping used in the theory of linear electronics. To carry out this work, we use our endmember extraction algorithm. Data reconstruction error (DRE) is used as a feedback to obtain the complete spectral unmixing iteratively. This is a self-regulatory mechanism which converges in the least squares sense. The experimentation is carried out using the multitemporal data synthesized using the real hyperspectral signatures of the USGS spectral library. First the algorithm is tested for the correctness of number of endmembers and the final results are compared with the existing approaches. Error analysis is carried out and the time complexity is calculated. To the best of our knowledge, this is the first method which solves for all the three unknowns in a single algorithm. Hence, in order to compare our results, we combine the state-of-art approaches for each of the unknowns in a chain. The proposed framework outperforms the standard chains of spectral unmixing comprising of the current
state-of-art approaches.

All our algorithms are run on a Desktop PC with Intel® Core™ i5-3210M CPU at 2.5 GHz with 4 GB of RAM. These algorithms are implemented in MATLAB® running on a 64-bit operating system.

1.5 Thesis Organization

The organization of the thesis mentioning the contribution is shown in Figure 1.7 showing flow of the work. It begins with an introductory chapter on the orientation of the problem with the linear mixing model. Subsequently, each of the work is arranged as an individual chapter (see Figure 1.7). Finally, the thesis provides the concluding remarks and future research lines.

The thesis is organized as follows. Chapter 1 discusses the principles of imaging spectroscopy, spectral unmixing problem and linear mixing model. Chapter 2 reviews the research work carried out in the area of spectral unmixing. The TLS-Tikhonov regularization method for estimating the abundances, given the data and perturbed endmembers is presented in Chapter 3. In Chapter 4 we discuss the MAP-dfHMRF based data-driven stochastic approach to unmix the hyperspectral data. A new approach for endmember extraction is discussed in Chapter 5 in which we show how the accuracy of endmembers is improved by using the multi-temporal data. Chapter 6 presents a novel framework for a blind estimation of number of endmembers, their signatures and abundances, called Iterative Bootstrapping (IB). Finally in Chapter 7 we conclude the thesis by summarizing the main contributions and by listing out future research directions.
1.5 Thesis Organization

Figure 1.7: Thesis Organization.